Monopole clusters, center vortices, and confinement in a $\mathbb{Z}_2$ gauge-Higgs system

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(Received 2 July 2002; published 30 October 2002)

We propose to use the different kinds of vacua of gauge theories coupled to matter as a laboratory to test confinement ideas of pure Yang-Mills theories. In particular, the very poor overlap of the Wilson loop with the broken string states supports the 't Hooft and Mandelstam confinement criteria. However, in the $\mathbb{Z}_2$ gauge-Higgs model we use as a guide we find that the condensation of monopoles and center vortices is a necessary but not sufficient condition for confinement.

DOI: 10.1103/PhysRevD.66.074511 PACS number(s): 11.15.Ha, 14.80.Cp

I. INTRODUCTION

Center vortices [1] and magnetic monopoles [2] are widely believed to be the collective degrees of freedom responsible for the nonperturbative features of SU(N) Yang-Mills theories, in particular for confinement.

Condensation of magnetic monopoles implies a dual Meissner effect: the chromolelectric field is expelled from the vacuum and gives the well-known physical picture of confinement in terms of dual Abrikosov vortices which describe the flux tubes joining static sources.

Center vortices are stringlike excitations formed out of the center of the gauge group. They produce a very efficient disordering mechanism of the gauge configurations which could lead to area law decay of large Wilson loops.

Although the above description for almost all known models remains at a conjectural stage,[1] in the last few years many numerical lattice studies have given strong support to the relevant role played by center vortices and monopole condensation in confinement. For instance, in the SU(2) gauge model in $3+1$ dimensions both magnetic monopole condensation [4–7] and the phenomenon of center dominance [8], namely, the fact that the string tension obtained from center projected configurations in the maximal center gauge agrees with the same quantity calculated in the full theory, have been observed.

Center dominance is verified in a trivial way in Abelian theories, where the whole dynamics is described by center vortices. Here the distinction between a confined and an unconfined phase is related to the maximal size of the clusters of vortices: it can be shown that confinement requires the presence of an infinite cluster [10]. This agrees with an earlier observation of percolating center vortices in the cold phase of the SU(2) gauge model at finite temperature [11]. For the sake of brevity we call the appearance of such an infinite cluster "vortex condensation."

There are many open, intertwined questions about the actual validity of the 't Hooft criteria of confinement, namely, that the monopole and/or vortex condensations imply the decay with an area law of large Wilson loops. In particular, is it possible to derive monopole condensation from vortex condensate or vice versa? If not, are both condensations necessary for confinement? Are they also sufficient?

In this work we propose to answer these questions by studying gauge systems coupled to matter. Indeed it is worth noting that, while center vortices and monopoles are generally studied in pure Yang-Mills models, they are well defined also in gauge theories coupled to matter. In these models there are different vacua, distinguished by different entities which condense. If, for instance, the region where the vortices condense does not coincide with that with monopole condensation, one can infer that these two properties are logically independent. This is precisely what happens in a $2+1$ $\mathbb{Z}_2$ gauge-Higgs model [9]. A simple argument shows [12] that the only vacua compatible with an area law decay of the large Wilson loops are those in which both magnetic monopoles *and* center vortices condense. In other terms, the 't Hooft criteria are both necessary for confinement.

Assuming that they are also sufficient can explain a surprising phenomenon observed in almost all coupled gauge systems studied up to now: although the potential between static sources flattens at large distances because of the screening produced by pair creation, this flattening (called string breaking) is invisible in the Wilson loop: it continues to obey an area law in full QCD [13] even at distances where the static charges are completely screened. The point is that in gauge theories coupled to matter the basis of the operators has to be enlarged [14] in order to get a reliable estimate of the potential. In this way the breaking of the confining string in Higgs models [15] and in QCD [16] has been observed. So the fact that large Wilson loops obey an area law even in coupled systems, as first suggested in [17], may be considered as further support to the usual plausibility arguments for the confinement mechanisms. Of course numerical experiments cannot give a true proof of the sufficiency of these criteria: it could well happen that Wilson loops of much larger size exhibit string breaking explicitly.

In this work we study the $2+1$ $\mathbb{Z}_2$ gauge-Higgs model to investigate a subtler issue: by probing the different vacua characterized by infinite vortex and monopole clusters we study the effect of other condensates on confinement. In this model there are two vacua having center vortex and magnetic monopole condensates; one of them has also an electric

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[1]The only explicit, analytic example of these mechanisms can be found in the Polyakov [3] proof of the confinement of compact U(1) gauge model in $2+1$ dimensions.

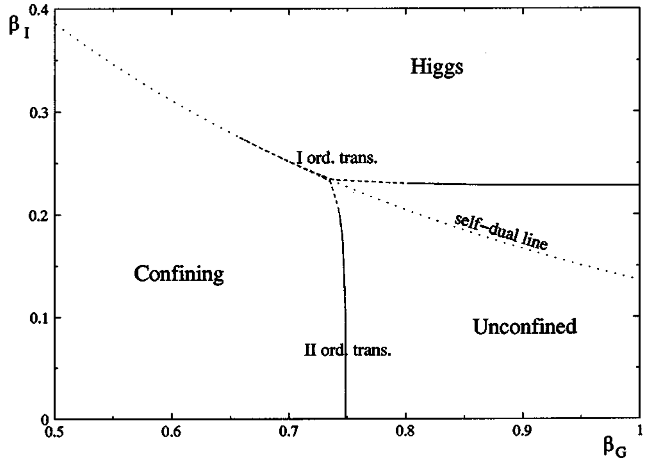


FIG. 1. Phase diagram for the $2+1$ $\mathbb{Z}_2$ gauge-Higgs system. The dashed (solid) lines denote the first-order (second-order) transitions. The dotted line is the self-dual line.

condensate, i.e., the Higgs field has a vacuum expectation value different from zero.

The former vacuum satisfies all the requirements of the confinement criteria, so an area law is expected; here we find that the Wilson loop obeys a perfect area law even at distances larger than five times the string breaking scale.

The latter can be identified with the torn phase predicted in Ref. [17]: the Wilson loop decays with an area law below a given scale which varies very rapidly as a function of the couplings of the model and is unrelated to the string breaking scale. Above this threshold we observe a perimeter law: the infrared properties of this vacuum are indistinguishable from those of the perturbative unconfined vacuum.[2]

II. THE MODEL

The action of a 3D $\mathbb{Z}_2$ gauge theory coupled to matter in a cubic lattice Λ can be written as

$$S(\beta_G,\beta_I)=-\beta_I\sum_{\langle ij\rangle}\varphi_i U_{ij}\varphi_j-\beta_G\sum_{plaq.}U_\Box, \qquad (1)$$

where both the link variable $U_{ij}\equiv U_\ell$ and the matter field φ_i take values ± 1 and $U_\Box=\Pi_{\ell\in\Box}U_\ell$.

This model is self-dual: the Kramers-Wannier transformation maps the model into itself. Its partition function

$$Z(\beta_G,\beta_I)=\sum_{\{\varphi_i=\pm 1,\ U_\ell=\pm 1\}}e^{-S(\beta_G,\beta_I)} \qquad (2)$$

satisfies the functional equation

$$Z(\beta_G,\beta_I)=(\sinh 2\beta_G \sinh 2\beta_I)^{3N/2}Z(\tilde{\beta}_I,\tilde{\beta}_G) \qquad (3)$$

with $\tilde{\beta}=-\frac{1}{2}\log(\tanh\beta)$.

The phase diagram of this model (see Fig. 1) was studied long ago [22]. There is an unconfined region surrounded by lines of phase transitions toward the Higgs phase and its dual. These lines are second order until they are near each other and the self-dual line, where a first-order transition occurs.[3]

A. $\mathbb{Z}_2$ vortices and monopoles

In this model the construction of center vortex configurations is straightforward: to each frustrated plaquette (i.e., $U_\Box=-1$) assign a vortex in the dual link. Since the product of the plaquettes belonging to any elementary cube is 1, center vortices form closed subgraphs of even coordination number. Thus a connected vortex subgraph contributes to a given Wilson loop $W(C)$ only if an *odd* number of lines are linked to it: the Wilson loop is a vortex counter modulo 2.

Magnetic monopoles can be defined exactly as in the pure gauge model. They live on the dual lattice $\tilde{\Lambda}$. To create a monopole in a site $\tilde{x}\in\tilde{\Lambda}$, corresponding to the center of an elementary cube of Λ, it is sufficient to draw an arbitrary, continuous line $\gamma(\tilde{x},\tilde{y})$ joining $\tilde{x}$ to another monopole located at $\tilde{y}$ (or to ∞) and flipping the sign of the coupling of the plaquettes crossed by γ. This flipping is generated by the nonlocal operator

$$\Psi_\gamma(\tilde{x},\tilde{y})=\exp\left(-2\beta_G\sum_{\Box\in\gamma}U_\Box\right). \qquad (4)$$

As a consequence, the flux across any closed surface with $\tilde{x}$ inside and $\tilde{y}$ outside is equal to -1: this monopole field $\Psi_\gamma(\tilde{x},\tilde{y})$ creates one unity of $\mathbb{Z}_2$ flux joining $\tilde{x}$ to $\tilde{y}$. Monopole condensation occurs when

$$\lim_{|\tilde{x}-\tilde{y}|\to\infty}\langle\Psi_\gamma(\tilde{x},\tilde{y})\rangle\neq 0. \qquad (5)$$

Why should this condensation imply confinement? A useful piece of information comes from the Kramers-Wannier duality. One can easily show that under this duality map the monopole correlator transforms as follows:

$$\langle\Psi_\gamma(\tilde{x},\tilde{y})\rangle_{\beta_G,\beta_I}=\left\langle\varphi_{\tilde{x}}\prod_{\ell\in\gamma}U_\ell\varphi_{\tilde{y}}\right\rangle_{\tilde{\beta}_I,\tilde{\beta}_G}. \qquad (6)$$

Thus the monopole condensation is associated with the dual Higgs phase, where the $\mathbb{Z}_2$ symmetry is spontaneously broken.

The same transformation maps the Wilson loop $W(C)$ associated with any closed curve $C\in\Lambda$ into the corresponding 't Hooft loop $\tilde{W}(C)$ of the dual phase:

$$\langle W(C)\rangle_{\beta_G,\beta_I}=\langle\tilde{W}(C)\rangle_{\tilde{\beta}_I,\tilde{\beta}_G} \qquad (7)$$

with

[2] A similar phase has been reported in the 4D SU(2)-Higgs model [18].

[3] For a more detailed description of the phase diagram see the poster presented by A. Rago at "Lattice 2002."

$$\widetilde{W}(C)=\exp\left(-2\widetilde{\beta}_G\sum_{\langle ij\rangle\in\Sigma}\varphi_i U_{ij}\varphi_j\right),\quad \partial\Sigma=C, \tag{8}$$

where Σ is an arbitrary surface bounded by C. $\widetilde{W}(C)$ creates an elementary $\mathbb{Z}_2$ flux along C which manifests itself as a topological defect: the action of $\widetilde{W}(C)$ on an arbitrary configuration maps the product $\eta_{\widetilde{C}}=\Pi_{\ell\in\widetilde{C}}U_\ell$ along any loop $\widetilde{C}\in\widetilde{\Lambda}$ having an odd linking number with C into $-\eta_{\widetilde{C}}$.

B. A microscopic picture of confinement

Confinement at β_G,β_I requires area law decay of $\langle\widetilde{W}(C)\rangle_{\widetilde{\beta_I\beta_G}}$. To understand this property at a microscopic level it is convenient to resort to the Fortuin-Kasteleyn (FK) random cluster representation [19] of the model. Starting from the obvious identity

$$e^{\beta_I\varphi_x U_{xy}\varphi_y}=e^{\beta_I}(1-p+p\,\delta_{1,\varphi_x U_{xy}\varphi_y}),\quad p=1-e^{-2\beta_I}. \tag{9}$$

It is easy to perform explicitly the sum on the matter fields φ_x, which yields

$$Z(\beta_G,\beta_I)=\sum_{U_\ell}e^{\Sigma_\Box\beta_G U_\Box}\sum_{G\subseteq\Lambda}\varpi_U(G)v^{b_G}2^{c_G},v=\frac{p}{1-p}. \tag{10}$$

The summation is over all spanning subgraphs $G\subseteq\Lambda$. b_G is the number of links of G, called active bonds (which are the matter degrees of freedom replacing the φ's in this representation), and c_G is the number of connected components, called FK clusters. $\varpi_U(G)$ is a projector on the subgraphs which are compatible with a given gauge configuration $U=\{U_\ell\}$: only those subgraphs are allowed for which no closed path within each FK cluster is linked to an elementary $\mathbb{Z}_2$ flux [20,21]. Put differently, no frustration is permitted in G. In a sense, this is a microscopic realization of a sort of dual Meissner effect: the FK clusters behave like pieces of dual superconducting matter of type I and no $\mathbb{Z}_2$ flux can go through the circuits of G. This is the only constraint generated by the interaction between matter and gauge fields. In the limit $\beta_G\to\infty$ all the plaquettes have $U_\Box=1$ (trivial gauge vacuum); hence the sum over the subgraphs is unconstrained and one gets the standard Ising model. Introducing a further projector $\varpi_C(G)$ on the space of configurations $\{G\subseteq\Lambda\}$, which takes the value 0 whenever there is a FK cluster (at least) linked to C, and the value 1 in all other cases, yields the very useful identity

$$\langle\widetilde{W}(C)\rangle=\langle\varpi_C\rangle=\frac{\text{number of compatible config.}}{\text{total number of config.}}. \tag{11}$$

Assume that there are only FK clusters of finite size (this is the case of region V in Fig. 2). If C is much larger than the mean size of the clusters, the configurations contributing to $W(C)$, i.e., those with $\varpi_C(G)=1$, are characterized by the fact that there is no cluster linked to C. The weight of this class of configurations, when compared with the total ensemble $G\subseteq\Lambda$, is clearly suppressed by a factor $e^{-\alpha\,p(C)}$, where $p(C)$ is the length of C. Thus in this phase the Wilson loop obeys a perimeter law.

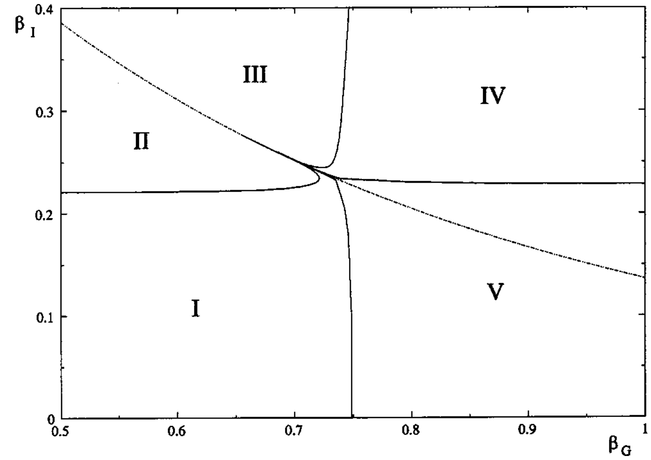


FIG. 2. The different vacua of the $\mathbb{Z}_2$ gauge-Higgs model, characterized by different kinds of infinite FK or center vortex clusters (see Table I).

Conversely, a large Wilson loop decaying with an area law implies by necessity the presence of an infinite FK cluster. In order to see whether this condition is also sufficient for confinement in the dual phase, note that according to Eq. (11) an infinite FK cluster fk_∞ gives a nonvanishing contribution to $\widetilde{W}(C)$ only if there is at least a simply connected surface Σ bounded by C which is not pierced by the loops of the cluster. If the links of fk_∞ are weakly correlated, one is led to argue that the weight of the configurations compatible with $\widetilde{W}(C)$ is suppressed by a factor $e^{-\sigma\,a(\Sigma)}$, where $a(\Sigma)$ is the area of the minimal surface with $\partial\Sigma=C$. This leads to the area law decay of $\langle\widetilde{W}(C)\rangle$.

A crucial assumption in the above argument is the weak correlation of the links of fk_∞ which describe the monopole condensation at the microscopic level. We shall see that there is a phase in which, although such a condensation occurs, this assumption is no longer true. Correspondingly we observe a violation of the area law.

We can now apply the same line of reasoning to the center vortices (CVs). Finite CV clusters can link with the 't Hooft loop only along its perimeter. Therefore they contribute only to the perimeter term. An infinite CV cluster is necessary for the area law: confinement implies both an infinite center vortex subgraph *and* magnetic monopole condensation. In pure gauge theory these two requirements coincide, while in the coupled system the vacuum structure is more intriguing.

C. The vacua

In the coupled theory we have two kinds of dynamical subgraphs: CV *or* FK clusters in the *dual* lattice describe the gauge field degrees of freedom; FK clusters in the *direct* lattice describe the charged Higgs matter.

In order to recognize an infinite cluster in practical simulations one has to look at the cluster size s as a function of the lattice volume V. We say that there is an infinite cluster

TABLE I. Distribution of the infinite clusters in the phase diagram.

Phase	Magnetic condensate	Electric condensate	Center vortices	Dual center vortices
I	yes	no	yes	no
II	yes	yes	yes	no
III	yes	yes	no	yes
IV	no	yes	no	yes
V	no	no	no	no

whenever $s \propto V$ for large enough V. Straightforward numerical experiments show that they are distributed in the phase diagram according to Fig. 2 and Table I. There are four kinds of infinite clusters: (i) FK cluster in the dual lattice (magnetic condensate), (ii) FK cluster in the direct lattice (electric condensate), (iii) CV cluster in the dual lattice, and (iv) CV cluster in the direct lattice (dual CV).

In region V there is no infinite cluster of any type. This is the perturbative, weak coupling vacuum, where large Wilson loops *and* 't Hooft loops obey a perimeter law.

Region IV is characterized by an electric condensate: it is the normal Higgs phase, where the Wilson loops decay with a perimeter law, while the 't Hooft loops obey an area law.

Region III has a magnetic condensate but no large center vortices; thus there is no confinement, as confirmed by numerical tests. This region is dual to the region II, where we shall see that the Wilson loops follow a perimeter-law decay. Because of the duality relation (7) we can infer also $\widetilde{W}(C)$ that decays in the same way.

Regions I and II have both CV and FK infinite clusters in the dual lattice.

The former is a normal confining phase: the potential extracted from an enlarged basis shows the expected string breaking (see Fig. 3b), while the Wilson loop obeys a perfect area law even at a large scale (see Fig. 4). This has been checked up to distances of the order of five times the string breaking scale (compare Fig. 3b and Fig. 4).

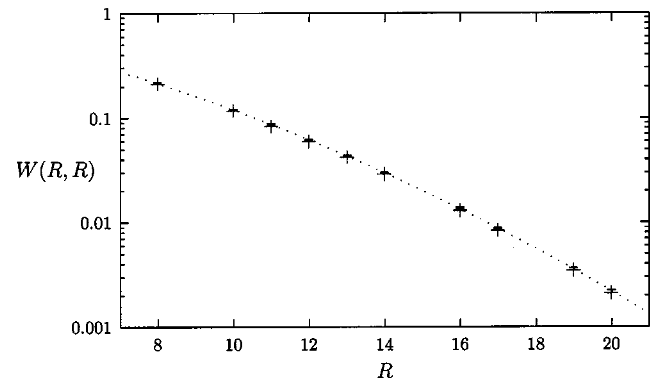
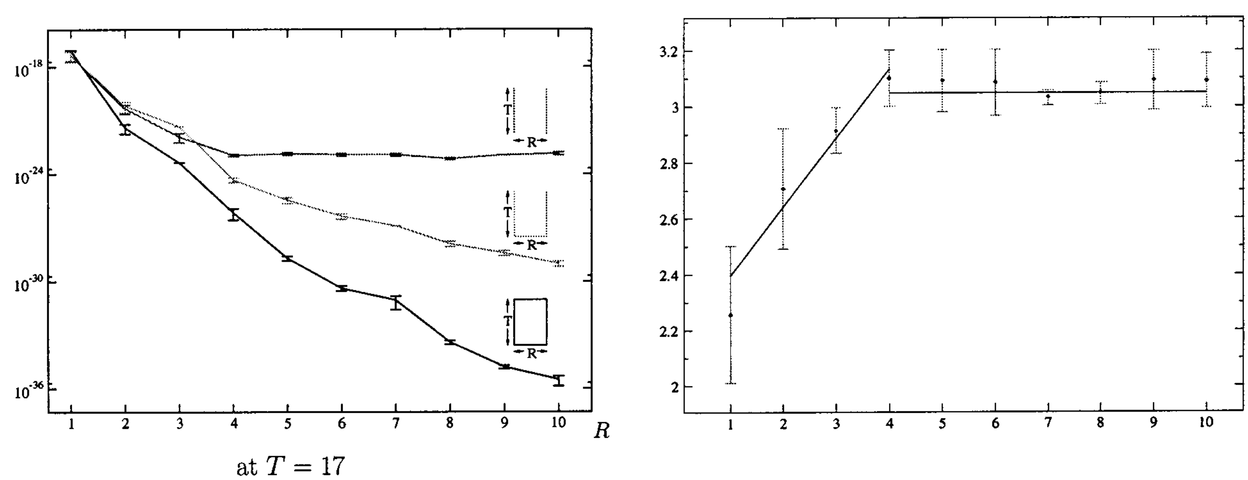


FIG. 4. Improved square Wilson loops evaluated using the duality relation (7) and the projection on the largest FK cluster. The simulation was performed on a 40^3 cubic lattice using 3.0×10^5 Monte Carlo configurations.

The latter is a phase with the simultaneous presence of infinite clusters in direct and dual lattices. The infinite cluster of Λ is associated with the condensation of the Higgs field φ_x, while the infinite CV cluster of $\widetilde{\Lambda}$ is typical of a confining phase. However, there is no confinement in the IR limit: the no-frustration constraint induces strong correlations among $\mathbb{Z}_2$ flux lines, because only an even number of them can pass through the closet paths within the FK clusters.

As a matter of fact, large Wilson loops obey a perimeter law decay, even if at intermediate distances an area law seems recovered. A first example of this behavior is reported in [9]. The cross over scale varies rapidly as a function of the coupling constant and is unrelated to the string breaking scale.



(a) (b)

FIG. 3. Results of Monte Carlo simulations of 6.0×10^5 sweeps of update on cubic lattice of size 40^3. (a) Plot of the rectangular Wilson $W(R,T)$ loop and the other two operators defined in Eqs. (12) and (13) at $T=17$ as a function of R. (b) The static potential extracted by the lowest eigenvalue of the correlation matrix obtained for six different values of T ($T=15,\ldots,20$).

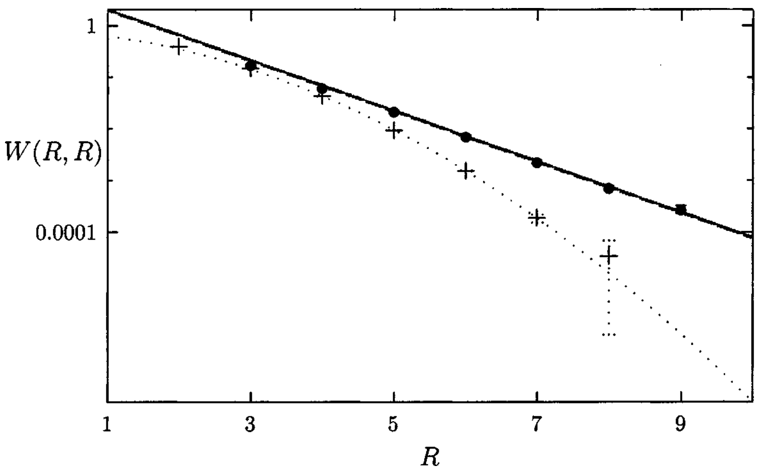
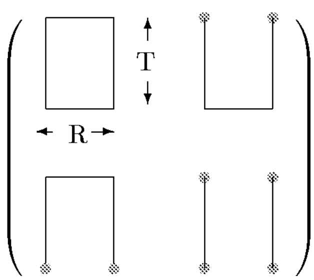


FIG. 5. Square Wilson loops in the torn phase (circles) and in the normal, confining phase (crosses) resulting from two simulations in a cubic lattice of size 40^3 with 2.0×10^5 Monte Carlo configurations. The couplings are chosen in such a way that the density of center vortices is the same in the two cases. The straight line is a fit to a perimeter-law decay, while the dotted line fits an area law decay with $\sigma=0.1541(2)$.

III. NUMERICAL RESULTS

As we anticipated, even in this model, as in the other gauge systems coupled to matter analyzed up to now, the Wilson loop fails to exhibit string breaking in the normal, confining phase, due to its poor (or vanishing) projection onto the screened potential. One has to use additional operators with good projection on the ground state, as first observed in Ref. [14]. The static potential $V(R)$ and its excitations can be extracted from measurements of the matrix correlator, represented pictorially as

$C(R,T) =$ 

where $C_{11}(R,T)$ is the rectangular Wilson loop $\langle W(R,T)\rangle$, the U-shaped operator is

$$C_{12}(R,T)=\langle\varphi(0)U(0,-\vec{j}T)U(-\vec{j}T,-\vec{j}T+\vec{k}R)\times U(\vec{k}R-\vec{j}T,\vec{k}R)\varphi(\vec{k}R)\rangle, \qquad (12)$$

where $\vec{j}$ and $\vec{k}$ are two orthogonal unit vectors, and $U(x,y)$ is a shorthand notation for a straight line of U_ℓ connecting the sites x and y. The correlator is symmetric, $C_{12}(R,T)=C_{21}(R,T)$, and

$$C_{22}(R,T)=\langle\varphi(0)U(0,\vec{j}T)\varphi(\vec{j}T)\varphi(\vec{k}R)U(\vec{k}R,\vec{j}T+\vec{k}R)\times\varphi(\vec{j}T+\vec{k}R)\rangle. \qquad (13)$$

Denoting by $\lambda_0\leqslant\lambda_1$ the two eigenvalues, the ground state potential is defined as

$$V(R)=-\lim_{T\to\infty}\frac{1}{T}\log\lambda_0. \qquad (14)$$

In practical simulations, the limit $T\to\infty$ is not realized. In our case T was typically less than 20 lattice spacings when the the signal was lost in noise.

The first point chosen in the present investigation is given by the couplings $\beta_G=0.75245$ and $\beta_I=0.16683$. This point in the phase diagram of Fig. 2 is in phase I, fairly close to the deconfinement transition, where the model has properties very similar to the confining phase of pure gauge theory [23]. The estimates of the above three operators in a lattice of size 40^3 as functions of R are reported in Fig. 3a and the static potential in Fig. 3b. We observe the typical string breaking phenomenon with a Wilson loop which seems to follow the area law decay even in the region where the string is broken. In order to see this property better, we used a powerful algorithm based on Eq. (11) and already used in pure gauge theory [24]. We also modified each configuration in the Monte Carlo ensemble by eliminating all the FK clusters not belonging to the largest cluster. It has been shown that this transformation does not change the value of the string tension but greatly reduces the noise of the measurements. The results are reported in Fig. 4: the square Wilson loops obey a perfect area law even at distances five times the string breaking scale at this point.

In order to investigate the effect of the electric condensate on the Wilson loop, we considered two different points in the phase diagram, one in region I, at $\beta_G=0.6867$ and $\beta_I=0.20$, which is a normal confining phase; and the other in region II, at $\beta_G=0.66$ and $\beta_I=0.27$, where in addition to the CV (and the FK) infinite cluster in the dual lattice there is also a percolating FK cluster in the direct lattice, associated with the "electric" condensation. These two points are chosen in such a way that the total sizes of the CV vortices are the same (within statistical errors) in the two cases. It turned out that the size of the maximal CV cluster is also approximately the same. We also verified that this quantity scales with the volume for large enough lattices.

The dramatic effect produced by the electric condensate is demonstrated by the completely different behaviors of the square Wilson loop in the two cases, as is evident in Fig. 5. In region I we see again the characteristic area law of the normal confining phase, while the data of region II are compatible, for Wilson squares of larger size, with a decay with the perimeter. This is the behavior expected for the torn phase [17].

IV. CONCLUSIONS

In this study we proposed to use gauge theories coupled to charged matter to test the confinement ideas of 't Hooft and Mandelstam. In particular, in the case of the $\mathbb{Z}_2$ gauge-Higgs model, we gave a detailed microscopic description of the center vortex and monopole condensates. The analysis of the different vacua of this theory led us to conclude that confinement requires that both magnetic monopoles *and* center vortices condense, the condensation of the former does not imply necessarily the condensation of the latter: there are vacua with a percolating FK cluster (i.e., the microscopic description of the magnetic condensate) which is not associated with an infinite CV cluster, plausibility arguments suggest that these two kinds of infinite clusters are responsible for the area law decay of Wilson loop only if the links of these clusters are weakly correlated, and there are vacua where the condensation of the gauge degrees of freedom (monopoles or center vortices) is associated with a condensate of the matter field (the Higgs field in our example). This yields strong correlations among vortices and monopoles so that the plausibility arguments for confinement we mentioned above are no longer justified. In fact we observed a perimeter law decay of large Wilson loops. This could be identified with the torn phase described in [17].

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