

Color-flavor locked strange matter

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We analyze and quantify how the color-flavor locked (CFL) phase in dense matter enhances the parameter space for absolute stability (strange matter). We find that the “CFL strange matter” phase can be the true ground state of hadronic matter for a much wider range of the parameters of the model (the gap of the QCD Cooper pairs Δ , the strange quark mass m_s , and the bag constant B) than the state without any pairing, and derive a full equation of state and an accurate analytic approximation to the lowest order in Δ and m_s which may be directly used for applications. The effects of pairing on the equation of state are found to be small (as previously expected), but not negligible, and may be relevant for astrophysics.

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I. INTRODUCTION

A great deal of activity lasting more than two decades was generated by the hypothetical stability of strange quark matter (SQM) put forward in Witten’s seminal paper [1] and a few important precursors [2]. This work actually questioned the nature of the true ground state of hadronic matter and showed within simple models that the hypothesis of a stable form of cold catalyzed plasma was tenable. Following this work a comprehensive discussion of strange matter by Farhi and Jaffe [3] in the framework of the MIT bag model of confinement [4] presented the so-called “windows of stability,” or regions in the plane m_s - B inside which the stability of SQM can be realized. Other models of confinement have also shown a fairly large range of conditions for SQM to be absolutely bound [5,6]; although it has always been clear that the availability of a $\sim 1\%$ binding energy difference for SQM to be bound is ultimately an experimental matter.

Nevertheless, and while sophisticated experiments push the search for SQM in laboratory and astrophysical environments beyond their present limits, important theoretical developments have taken place. The main one is probably the revival of interest in pairing interactions in dense matter, a subject already addressed in the early 1980s [7] which came back a few years ago and prompted new calculations of the pairing energy and related physics. It is now generally agreed [8–10] that (at least for asymptotic densities) the color-flavor locked (CFL) state is likely to be the ground state, even if the quark masses are unequal [11]. Moreover, the equal number of flavors is enforced by the symmetry, and electrons are absent since the mixture is automatically neutral [12].

Given these important modifications in the character of the ground state indicated by theoretical improvements, we consider in Sec. III the problem of SQM in the light of the CFL state to address whether there is still room for the Bodmer-Witten-Terazawa conjecture. Independent of stability considerations, the equation of state for CFL matter is

studied in the next section and differences with respect to unpaired matter quantified.

II. THERMODYNAMICS OF THE CFL PHASE

To order Δ^2 , the thermodynamical potential Ω_{CFL} can be found quite simply [13]. One begins with Ω_{free} of a fictional state of unpaired quark matter in which all quarks which are “going to pair” have a common Fermi momentum ν , with ν chosen to minimize Ω_{free} of this fictional unpaired state. The binding energy of the diquark condensate is included by subtracting the condensation term $3\Delta^2\mu^2/\pi^2$. Given that the mixture does not show automatic confinement, it may be introduced at this point by means of the phenomenological vacuum energy density or bag constant B . The advantages and inconveniences of this particular implementation of confinement forces have been discussed many times and will not be repeated here. The expression for Ω_{CFL} in this model is then [13]

$$\begin{aligned} \Omega_{CFL} &= \Omega_{free} - \frac{3}{\pi^2} \Delta^2 \mu^2 + B \\ &= \frac{6}{\pi^2} \int_0^\nu [p - \mu] p^2 dp + \frac{3}{\pi^2} \int_0^\nu [(p^2 + m_s^2)^{1/2} - \mu] p^2 dp \\ &\quad - \frac{3}{\pi^2} \Delta^2 \mu^2 + B, \end{aligned} \quad (1)$$

$$\begin{aligned} &= \sum_{i=u,d,s} \frac{1}{4\pi^2} \left[\mu_i \nu \left(\mu_i^2 - \frac{5}{2} m_i^2 \right) \right. \\ &\quad \left. + \frac{3}{2} m_i^4 \log \left(\frac{\mu_i + \nu}{m_i} \right) \right] - \frac{3}{\pi^2} \Delta^2 \mu^2 + B, \end{aligned} \quad (2)$$

where $3\mu = \mu_u + \mu_d + \mu_s$, and the common Fermi momentum $\nu = (\mu_i^2 - m_i^2)^{1/2}$ is given by

$$\nu = 2\mu - \left(\mu^2 + \frac{m_s^2}{3} \right)^{1/2}. \quad (3)$$

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The pressure, baryon number density n_B , and particle number densities are easily derived and read

$$P = -\Omega_{CFL}, \quad (4)$$

$$n_B = n_u = n_d = n_s = \frac{(\nu^3 + 2\Delta^2\mu)}{\pi^2}. \quad (5)$$

Since we work at zero temperature, the energy density is given by

$$\varepsilon = \sum_i \mu_i n_i + \Omega_{CFL} = 3\mu n_B - P. \quad (6)$$

We emphasize that, to this order, the exact nature of the interaction that generates Δ does not matter. Ω_{CFL} is given by this prescription regardless of whether the pairing is due to a pointlike four-Fermi interaction, as in Nambu-Jona-Lasinio (NJL) models, or due to the exchange of a gluon, as in QCD at asymptotically high energies [10]. Of course, the strength and form of the interaction determine the value of Δ , and also its dependence with the density. Lacking an accurate calculation for Δ , which may be as high as ~ 100 MeV, we shall keep it as a free constant parameter.

In the general case for unpaired uds matter the equation of state can be derived from the chemical potentials of Ref. [3]. As is well known, in the limit $m_s \rightarrow 0$ not only do the particle densities become equal but also the equation of state takes the simple form $\varepsilon = 3P + 4B$. Pairing introduces the Δ^2 term in Eq. (1); thus the equation of state picks up an additional term $\varepsilon = 3P + 4B - (6\Delta^2\mu^2)/\pi^2$. The situation is much more complicated when $m_s \neq 0$ because the equation of state must be calculated numerically. However, since the mass is not large when compared to the natural scale introduced by the chemical potential, it is generally sufficient to keep Ω_{free} to order m_s^4 [13]:

$$\begin{aligned} \Omega_{CFL} = & \frac{-3\mu^4}{4\pi^2} + \frac{3m_s^2\mu^2}{4\pi^2} - \frac{1 - 12\log(m_s/2\mu)}{32\pi^2} m_s^4 \\ & - \frac{3}{\pi^2} \Delta^2 \mu^2 + B. \end{aligned} \quad (7)$$

We have checked that the errors are small enough even to work to the order m_s^2 . The main advantage of the lowest approximation is to keep the equation of state very simple, yet useful for most calculations, and also to make clear the effect of each parameter of the model. To this order we have

$$P = \frac{3\mu^4}{4\pi^2} - \frac{3m_s^2\mu^2}{4\pi^2} + \frac{3}{\pi^2} \Delta^2 \mu^2 - B, \quad (8)$$

$$\varepsilon = \frac{9\mu^4}{4\pi^2} - \frac{3m_s^2\mu^2}{4\pi^2} + \frac{3}{\pi^2} \Delta^2 \mu^2 + B, \quad (9)$$

$$n_B = \frac{\mu^3}{\pi^2} - \frac{m_s^2\mu}{2\pi^2} + \frac{2}{\pi^2} \Delta^2 \mu, \quad (10)$$

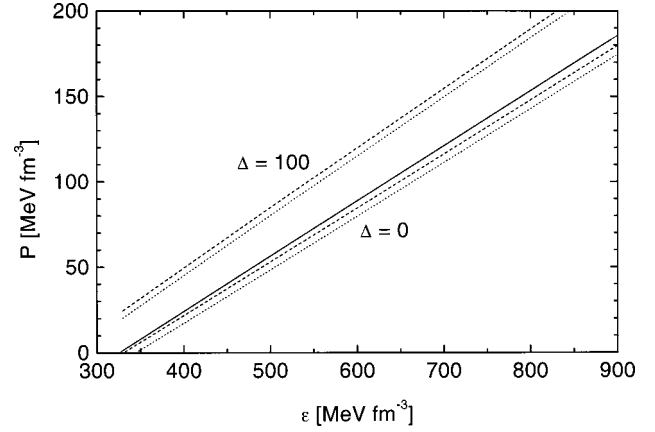


FIG. 1. The EOS for CFL SQM and for SQM without color-flavor locking. We have chosen $B = 75 \text{ MeV fm}^{-3}$ and $m_s = 150 \text{ MeV}$ for all the curves, which are shown for two different values of the gap Δ as indicated in the figure. The solid line corresponds to SQM (no CFL); the dashed lines are the CFL calculated to all orders in m_s ; and the dotted lines are the approximate EOS to the order m_s^2 , which gives quite accurate results. Note the change of stiffness according to the value of Δ , as discussed in the text.

$$\nu = \mu - \frac{m_s^2}{6\mu}. \quad (11)$$

Since $m_s \sim 150 \text{ MeV}$ and μ is greater than $\sim 300 \text{ MeV}$ this approximation is quite accurate, especially at high densities, as is apparent from Fig. 1.

It is also desirable to have an expression of P as an explicit function of ε . From Eqs. (9) and (4) we obtain

$$\varepsilon = 3P + 4B - \frac{6\Delta^2\mu^2}{\pi^2} + \frac{3m_s^2\mu^2}{2\pi^2} \quad (12)$$

where μ^2 is given by

$$\mu^2 = -\alpha + \left(\alpha^2 + \frac{4}{9} \pi^2 (\varepsilon - B) \right)^{1/2} \quad (13)$$

and

$$\alpha = -\frac{m_s^2}{6} + \frac{2\Delta^2}{3}. \quad (14)$$

Equation (12) resembles the equation of state (EOS) for strange quark matter with massless quarks with the addition of the last two terms. The term proportional to Δ^2 tends to stiffen the EOS compared to the SQM case since it induces a higher pressure for a given energy density. The term with m_s^2 has the opposite effect, although it is not as large. The CFL state may be preferred to SQM in spite of the finite m_s value because of the importance of the Δ term. The effect of color-flavor locking in the equation of state is not negligible although it is not extreme either. Given that $\Delta \sim 100 \text{ MeV}$ and that a typical μ is $\geq 300 \text{ MeV}$ the effect of CFL in the EOS may be important, specially at low densities. We show in Fig. 1 the EOSs in the different approximations. From the expres-

sions above, it is readily noticed that, provided Δ is higher than $m_s/2$, the EOS is stiffer than the SQM, that is, it produces more pressure for a given energy density. Since the actual value of Δ is not well known, we expect either a stiffer or a softer EOS (for a given B). It should be kept in mind that there are other caveats, for example, the likely dependence of Δ on the density, which may cause a cross-over from stiffer to softer EOS depending on the parameters.

III. STABILITY OF THE CFL PHASE

For a given EOS the energy per baryon of the deconfined phase (at $P=0$ and $T=0$) must be lower than 939 MeV (the neutron mass) if matter is to be absolutely stable. The other condition that must be considered comes from the empirically known stability of normal nuclear matter against deconfinement at zero pressure [3]. In other words, the energy per baryon of deconfined matter (a pure gas of quarks u and d) at zero pressure and temperature must be higher than the neutron mass value. In the framework of a MIT-based EOS it has been shown that the latter condition imposes that the MIT bag constant must be greater than 57 MeV fm^{-3} [3].

From Eq. (6) we can write the absolute stability condition as

$$\left. \frac{\varepsilon}{n_B} \right|_{P=0} = 3\mu \leq m_n = 939 \text{ MeV}. \quad (15)$$

This simple result is a direct consequence of the existence of a common Fermi momentum for the three flavors and is valid at $T=0$ without any approximation. Since this must hold at the zero pressure point, then from Eq. (4) we have

$$B = -\Omega_{free}(m_s, \mu_0) + \frac{3}{\pi^2} \Delta^2 \mu_0^2 \quad (16)$$

with $\mu_0 = 313 \text{ MeV}$.

The last equation defines a curve in the m_s - B plane on which the energy per baryon is exactly $\varepsilon/n_B = m_n$ for a given Δ . To order m_s^2 we can obtain a very simple parabolic expression for Eq. (16):

$$B = -\frac{m_s^2 m_n^2}{12\pi^2} + \frac{\Delta^2 m_n^2}{3\pi^2} + \frac{m_n^4}{108\pi^2}. \quad (17)$$

Since this analytic expression is calculated to order m_s^2 , it deviates from Eq. (16) when $m_s \sim \mu$; in practice, the approximation holds for masses up to about 150 MeV, expected to be quite realistic.

We display in Fig. 2 the stability window for the CFL phase (i.e., the region in the m_s versus B plane where E/n_B is lower than 939 MeV at zero pressure). Equation (16) gives the right side boundary of the window while the left side boundary is given by the minimum value $B = 57 \text{ MeV}$. As it stands, the window is greatly enlarged for increasing values of Δ . This is to be compared, for example, with Fig. 1 of

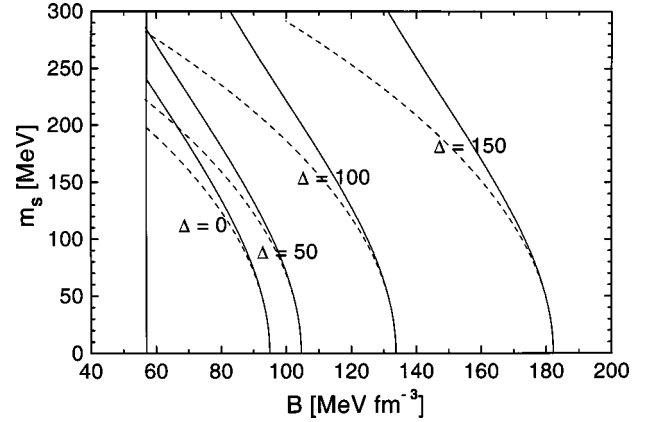


FIG. 2. The windows of stability for CFL strange matter. The symmetric CFL state is absolutely bound if the strange quark mass m_s and the vacuum energy density B lie inside the bounded region. Each window has been calculated for a given value of the gap Δ as indicated by the label, to be compared with the SQM results of [3]. The solid lines are calculated to all orders in m_s , while the dashed lines are the approximate regions to order m_s^2 as given by Eq. (17). As expected, the approximation is worse for increasing values of m_s . The vertical solid line is the limit imposed by requiring instability of two-flavor quark matter. The large increase of the stable region is the main feature of interest.

Ref. [3] in which no pairing was included. The Δ term actually produces this effect of enlargement of the parameter space.

IV. DISCUSSION

The CFL phase at zero temperature has been modeled as an electrically neutral and colorless fluid, in which quarks are paired in such a way that all the flavors have the same Fermi momentum and hence the same number density, as long as m_s is not too large [12]. The CFL phase is strongly favored over a pure mixture of quarks u and d and pure neutron matter for a wide range of parameters of the theory (namely, B , m_s , and Δ). Although some energy must be paid in order to maintain the same Fermi momentum for all three flavors, more energy is gained by opening the strange quark channel and from the energy gap of the pairing.

The CFL phase treated here as a gas of Cooper pairs following Ref. [13] shows a (qualitatively and quantitatively) different behavior from that developed for quark-diquark matter in [14], where diquarks are treated as bosons much in the same way as in Refs. [15–17]. In the latter case, the effect of Bose condensation is much more important than the energy gap of the pairing itself, while in the present case the gap energy is essential to widen the stability window. The gap effect does not dominate the energetics, being of the order $(\Delta/\mu)^2$ (a few percent), but it may be large enough to allow a CFL strange matter for the same parameters that would otherwise produce unbound strange matter without pairing. Similar conclusions have been recently presented by Madsen [18] in a study focused on CFL strangelets (not addressed here). We believe that the explicit analytic expressions derived in Sec. II may be useful in studying strange

stars and related problems, while Fig. 2 quantifies the expected enlargement of the stability windows in a convenient manner for comparison with “ordinary” SQM [3].

Even if the EOS is very simple and the confinement has been introduced by brute force, it is remarkable that the strange matter hypothesis may be boosted by pairing interactions, and clearly more detailed studies are desirable. The dynamics of the transition itself is also a matter of interest. While it is likely that a two-flavor color superconducting phase (2SC) may be bypassed in favor of a CFL state [19], the original flavor content of the hadronic phase is generally not the one needed by the CFL flavor symmetry. Therefore, it is still reasonable to assume that the transition dynamics is dominated by the rate of strangeness production needed to achieve the CFL flavor symmetry. The energy liberated in the

transition from the hadronic to the CFL state could be much higher than that liberated in the process of unpaired SQM formation and could lead to very energetic explosive phenomena [20–22]. It is also worth remarking that stable diquark states were suggested some time ago although within a different (naiver) model [23].

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