Radiative decay of Υ into a scalar glueball

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We study the radiative decay of Υ into a scalar glueball $\Upsilon \to \gamma G_s$ using QCD factorization. We find that for this process the nonperturbative effects can be factorized into a matrix element well defined in nonrelativistic QCD (NRQCD) and the gluon distribution amplitude. The same NRQCD matrix element appears also in the leptonic decay of Υ and therefore can be determined from data. In the asymptotic limit the gluon distribution amplitude is known up to a normalization constant. Using a QCD sum-rule calculation for the normalization constant, we obtain $\mathrm{Br}(\Upsilon \to \gamma G_s)$ to be in the range $(1–2)\times 10^{-3}$. We also discuss some of the implications for $\Upsilon \to \gamma f_i$ decays. Near future data from CLEO-III can provide crucial information about scalar glueball properties.

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I. INTRODUCTION

The existence of glueballs is a natural prediction of QCD. Some of the low lying states are 0^{++}, 0^{-+}, 1^{+-}, and 2^{++} with the lowest mass eigenstate 0^{++} in the range of 1.5–1.7 GeV from theoretical calculations [1]. There are indications that $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ contain substantial scalar glueball content. In searching for glueballs, decays of quarkonia are well suited processes because the decays are mediated by gluons. Among these decays, two-body radiative decays are ideal places to study this subject, because there is no complication from interactions between light hadrons. Radiative decays of Υ have been studied before, in particularly by the CLEO Collaboration [2,3] recently. With the current data sample, there are already several observations of the radiative decay of Υ into mesons. Among them only a few have good precision, such as the decays $\Upsilon \to \gamma f_2(1270)$, $\Upsilon \to \gamma f_0(1710) \to \gamma K\bar{K}$, while the others have large errors [3]. About 4 fb^{-1} $b\bar{b}$ resonance data are planned to be taken at CLEO-III in the year prior to conversion to low energy operation (CLEO-C) [4]. This will produce the largest data sample of Υ in the world. More radiative decay modes of Υ may be observed. By combining experimental data in the near future and theoretical results, glueball properties can be studied in detail.

In this paper we carry out a theoretical study of the radiative decay of Υ into a scalar glueball by using QCD factorization. We find that the nonperturbative effects can be factorized into a matrix element well defined in nonrelativistic QCD NRQCD, and the gluon distribution amplitude. The NRQCD matrix element can be determined from leptonic Υ decays. The asymptotic form of the gluon distribution amplitude is known in QCD up to a normalization constant. Using a QCD sum-rule calculation for this constant, the branching ratio $\mathrm{Br}(\Upsilon \to \gamma G_s)$ is predicted to be in the range of $(1–2)\times 10^{-3}$. Combining this result with experimental data, we find that $f_0(1710)$ is unlikely to be a pure glueball. Existing information on glueball mixing allows us to predict the branching ratios for several radiative decays, such as $\Upsilon \to \gamma f_0(1370,1500,1710) \to \gamma K\bar{K}(\pi\pi)$. Near future experimental data from CLEO-III will provide crucial information about scalar glueball properties.

II. QCD FACTORIZATION OF $\Upsilon \to \gamma G_s$

It is known that properties of Υ can be well described with nonrelativistic QCD [5]. The decay of $\Upsilon \to \gamma G_s$ can be thought of as follows: a free $b\bar{b}$ quark pair is first freed from the Υ with a probability that is characterized by matrix elements defined in NRQCD; this pair of quarks decays into a photon and gluons; and then the gluons are subsequently converted into a scalar glueball. In the heavy quark limit $m_b \to \infty$, the glueball has a large momentum; this allows a twist expansion to describe the conversion. Also, the gluons are hard, and perturbative QCD can be applied for the decay of the $b\bar{b}$ pair into a photon and gluons. This implies that the decay width can be factorized. In the real world, the b quark mass is 5 GeV and a scalar glueball has a mass around 1.5 GeV as suggested by lattice QCD simulations [2]. This may lead to a question of whether the twist expansion is applicable. For radiative decay of the Υ, the glueball has a momentum of order of m_b. The twist expansion means a collinear expansion of the momenta of gluons in the glueball; components of these momenta have the order of $(\mathcal{O}(k^+),\mathcal{O}((k^-),\mathcal{O}(\Lambda_{QCD}),\mathcal{O}(\Lambda_{QCD}))$, where k is the mo-

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mentum of the glueball. Here we used the light-cone coordinate system. Hence the expansion parameters are

$$\frac{k^-}{k^+}=\frac{m_G^2}{M_\Upsilon^2}\sim 0.02,\quad \frac{\Lambda_{QCD}}{k^+}\sim 0.1, \tag{1}$$

where m_G is the mass of the glueball and we have taken $\Lambda_{QCD}\approx 500$ MeV. In the above estimation we have used the fact that the probability for the conversion of gluons into a glueball is suppressed if the $+$ component of the momentum of a gluon is very small. We see that the relevant expansion parameters are small, and therefore the twist expansion is expected to be a good one. We note that the same approximation may not be applied to the J/ψ system because in this case the relevant expansion parameters are not small. We now provide some details of the calculations.

The leading Feynmann diagrams for $\Upsilon\to\gamma G_s$ are from $b\bar{b}$ annihilation into two gluons and a photon. The basic formalism for such calculations has been developed in Ref. [6] and has been used in the case of $\Upsilon\to\gamma\eta(\eta')$ to obtain a result consistent with experimental data [7]. With appropriate modifications we can obtain the S matrix for $\Upsilon\to\gamma G_s$ decay. It is given by

$$\begin{aligned}\langle\gamma G_s|S|\Upsilon\rangle=&-i\frac{1}{2}eQ_b g_s^2\epsilon_\rho^*\int d^4x d^4y d^4z d^4x_1 d^4y_1 e^{iq\cdot z}\\&\times\langle G_s|G_\mu^a(x)G_\nu^a(y)|0\rangle\langle 0|\bar{b}_j(x_1)\\&\times b_i(y_1)|\Upsilon\rangle\cdot M_{ij}^{\mu\nu\rho,ab}(x,y,x_1,y_1,z),\end{aligned} \tag{2}$$

where $M_{ij}^{\mu\nu\rho,ab}$ is a known function from evaluation of the Feynman diagrams, i and j stand for Dirac and color indices, and a and b are the color indices of the gluon. ϵ^* is the polarization vector of the photon and $Q_b=-1/3$ is the b quark electric charge. Since the b quark is heavy and moves with small velocity v, one can expand the Dirac fields in NRQCD fields:

$$\begin{aligned}\langle 0|\bar{b}_j(x)b_i(y)|\Upsilon\rangle=&-\frac{1}{6}(P_+\gamma^l P_-)_{ij}\langle 0|\chi^\dagger\sigma^l\psi|\Upsilon\rangle\\&\times e^{-ip\cdot(x+y)}+O(v^2),\end{aligned} \tag{3}$$

where $\chi^\dagger$ (ψ) is the NRQCD field for the $\bar{b}$ (b) quark and $P_\pm=(1\pm\gamma^0)/2$. The b is almost at rest; then $p_\mu=(m_b,0,0,0)$ with m_b being the b quark pole mass.

From the above we obtain the decay amplitude for $\Upsilon\to\gamma G_s$ as

$$\mathcal{T}=\frac{eQ_b g_s^2}{6}\langle 0|\chi^\dagger\epsilon\cdot\sigma\psi|\Upsilon\rangle\int_0^1 dz\frac{1}{z(1-z)}\mathcal{F}_s(z); \tag{4}$$

the decay width then reads

$$\begin{aligned}\Gamma=&\frac{2}{9m_b^4}\pi^2 Q_b^2\alpha\alpha_s^2\langle\Upsilon|O_1(^3S_1)|\Upsilon\rangle\\&\times\left|\int_0^1 dz\frac{1}{z(1-z)}\mathcal{F}_s(z)\right|^2.\end{aligned} \tag{5}$$

In the above, $\mathcal{F}_s$ is the gluon distribution amplitude of G_s and is given by

$$\begin{aligned}\mathcal{F}_s(z)=&\frac{1}{2\pi k^+}\int dx^- e^{-izk^+x^-}\\&\times\langle G_s(k)|G^{a,+\mu}(x^-)G^{a,+}{}_\mu(0)|0\rangle.\end{aligned} \tag{6}$$

Here we have used a gauge with $G^+=0$ such that the gauge link between the field strength operators vanishes. This distribution essentially characterizes how two gluons are converted into G_s, where one of the two gluons has the momentum $(zk^+,0,\mathbf{O_T})$.

In the above equations, the matrix element $\langle\Upsilon|O_1(^3S_1)|\Upsilon\rangle$ defined in NRQCD contains the bound state effect of b quarks in the Υ [5] and can be extracted from leptonic $\Upsilon\to l^+l^-$ decay. A prediction can be made for $\Upsilon\to\gamma G_s$ if the distribution amplitude is known.

The distribution amplitude can be written as

$$\mathcal{F}_s(z)=f_s f(z)\quad\text{with}\quad\int_0^1 dz f(z)=1, \tag{7}$$

where $f(z)$ is a dimensionless function and its asymptotic form is

$$f(z)=30z^2(1-z)^2. \tag{8}$$

With the asymptotic form in Eq. (6) we have

$$R_s=\frac{\Gamma(\Upsilon\to\gamma+G_s)}{\Gamma(\Upsilon\to\ell^+\ell^-)}=\frac{25\pi\alpha_s^2}{3\alpha}\cdot\frac{|f_s|^2}{m_b^2}. \tag{9}$$

In the above we have used the fact that both $\Upsilon\to\gamma G_s$ and $\Upsilon\to l^+l^-$ are proportional to $\langle\Upsilon|O_1(^3S_1)|\Upsilon\rangle$.

The use of the asymptotic form for $f(z)$ may introduce some errors, because the scale μ here is actually m_b, not $\mu\to\infty$. However, with large m_b one may expect that it can provide a good order of magnitude estimate with the asymptotic form. Also, it has been shown that for a pseudoscalar glueball the distribution amplitude is rather close to its asymptotic form from a QCD sum-rule calculation [8]. We expect that the same is true for the scalar glueball. We will use Eq. (9) later for our numerical discussion.

We note that at this stage the state G_s can be any particle with the same quantum number as $G^{a,+\mu}G_\mu^{a,+}$, i.e., $J^{PC}=0^{++}$. The normalization constant f_s depends on the properties of the specific particle. In order to obtain the branching ratio of G_s as a scalar glueball, we have to evaluate f_s with G_s specified to be the scalar glueball. The evaluation of f_s is a difficult task because it is dominated by nonperturbative effects. One of the best ways of handling such effects is the

QCD sum-rule method [9]. In the following we provide an estimate for f_s based on a QCD sum-rule calculation.

III. QCD SUM-RULE CALCULATION OF THE NORMALIZED CONSTANT

The constant f_s has dimension 1 in mass and is related to the the product of local operators

$$\langle G_s(k)|G^{\mu\rho}G^{\nu}{}_{\rho}|0\rangle = f_0 m_G^2 g^{\mu\nu} + f_s k^\mu k^\nu. \tag{10}$$

The fact that the same f_s appears in Eq. (9) and Eq. (12) can be checked by integrating over z on both sides of Eq. (6).

The basic idea of the QCD sum-rule calculation for our estimate is to consider the two-point correlator

$$\begin{aligned}\Pi_{\mu\nu,\mu'\nu'}(Q^2) &= \int d^4x e^{iq\cdot x} i\langle 0|T G_{\mu\alpha}G^{\alpha}_{\nu}(x), G_{\mu'\beta}G^{\beta}_{\nu'}(0)|0\rangle \\ &= T_{\mu\nu\mu'\nu'}\Pi_T(Q^2) + + V_{\mu\nu\mu'\nu'}\Pi_V(Q^2) \\ &\quad + S^1_{\mu\nu\mu'\nu'}\Pi_{S1}(Q^2) + S^2_{\mu\nu\mu'\nu'}\Pi_{S2}(Q^2) \\ &\quad + S^3_{\mu\nu\mu'\nu'}\Pi_{S3}(Q^2),\end{aligned} \tag{11}$$

for a region of Q in which one can incorporate the asymptotic freedom property of QCD via the operator product expansion (OPE), and then relate it to the hadronic matrix elements via the dispersion relation. The tensors in Eq. (11) are defined as

$$\begin{aligned}&T_{\mu\nu\mu'\nu'} = g^t_{\mu\mu'}g^t_{\nu\nu'} + g^t_{\mu\nu'}g^t_{\nu\mu'} - \frac{2}{3}g^t_{\mu\nu}g^t_{\mu'\nu'}\\ &V_{\mu\nu\mu'\nu'} = g^t_{\mu\mu'}q_\nu q_{\nu'} + g^t_{\nu\nu'}q_\mu q_{\mu'} + g^t_{\mu\nu'}q_\nu q_{\mu'} + g^t_{\nu\mu'}q_\mu q_{\nu'}\\ &S^1_{\mu\nu\mu'\nu'} = g^t_{\mu\nu}g^t_{\mu'\nu'}, \qquad S^2_{\mu\nu\mu'\nu'} = g^t_{\mu\nu}q_{\mu'}q_{\nu'} + g^t_{\mu'\nu'}q_\mu q_\nu,\\ &S^3_{\mu\nu\mu'\nu'} = q_\mu q_\nu q_{\mu'}q_{\nu'},\end{aligned} \tag{12}$$

where $g^t_{\mu\nu} = g_{\mu\nu} - q_\mu q_\nu/q^2$. The corresponding terms $\Pi_T(Q^2)$, $\Pi_V(Q^2)$, $\Pi_{S1}(Q^2)$, $\Pi_{S2}(Q^2)$, and $\Pi_{S3}(Q^2)$ are from the contributions of 2^{++}, 1^{-+}, and 0^{++} states, respectively.

In the deep Euclidean region $Q^2 = -q^2 \gg \Lambda_{QCD}$, they can be expanded as

$$\begin{aligned}\Pi_i(Q^2) &= C_i^0(Q^2)I + C_i^1(Q^2)\alpha_s\langle G_{\mu\nu}G^{\mu\nu}\rangle \\ &\quad + C_i^2(Q^2)\langle g_s f^{abc}G^{a\ \mu}{}_{\alpha}G^{b\ \alpha}{}_{\beta}G^{c\ \beta}{}_{\mu}\rangle + \cdots,\end{aligned} \tag{13}$$

where C_i^j are Wilson coefficients which need to be determined later.

On the other hand, the correlator in Eq. (11) can be saturated by all possible resonances and the continuum. We have

$$\begin{aligned}\mathrm{Im}\,\Pi_{\mu\nu,\mu'\nu'}(Q^2) &= \sum_R \langle 0|G_{\mu\alpha}G^{\alpha}_{\nu}|R\rangle\langle R|G_{\mu'\beta}G^{\beta}_{\nu'}|0\rangle \\ &\quad \times \pi\delta(Q^2 + m_R^2) + \text{continuum},\end{aligned} \tag{14}$$

where the sum on R is for all possible resonances. The term $\langle|G_{\mu\alpha}G^{\alpha}_{\nu}|R\rangle\langle R|G_{\mu'\beta}G^{\beta}_{\nu'}|0\rangle$ in the above equation contains the information on f_0 and f_s when R is the scalar glueball. The T and V tensors are not related to f_s. They are irrelevant to our calculations. The functions $\Pi_{S1,S2,S3}$ contain linear combinations of f_0 and f_s. QCD sum-rule calculations for $\langle G_s|G^{\mu\nu}G_{\mu\nu}|0\rangle = (4f_0 + f_s)m_G^2$ have been carried out before [10]. Therefore if one of the $\Pi_{S1,S2,S3}$ is known, one can obtain f_s. From Eq. (10) and the tensor structure of Eq. (11), we find that Π_{S3} is proportional to $(f_0 + f_s)^2$. Therefore the study of Π_{S3} is sufficient for our purpose of determining f_s. $\Pi_{S1,2}$ also contain information about f_0 and f_s. However, the nonperturbative contributions for them begin at the level of dimension-8 operators. The results obtained are not as reliable as the ones from Π_{S3} which have a lower dimension. We now concentrate on Π_{S3}.

There may be several bound states with the same quantum numbers to include in the QCD sum-rule calculation, such as a pure scalar glueball, quark bound states, and higher excited states. The contributions from higher excited states are suppressed upon the use of the Borel transformation, which is discussed below. For the quark bound states, the OZI rule implies that the conversion of a bound quark state into a scalar glueball is suppressed compared with the conversion of two gluons into a scalar glueball [13], perturbatively suppressed by powers in α_s. If this is indeed true, the corresponding f_s parameters for quark bound states will be smaller than for the pure glueball state. We will work with this approximation in the following discussion. To be consistent with our previous expansion, we again work to order α_s. To this order, using the method in Ref. [12], we find

$$\begin{aligned}\Pi_{S3}(Q^2) &= \frac{1}{8\pi^2}\ln\frac{\mu^2}{Q^2} + \frac{1}{2Q^4} \\ &\quad \times \left(\langle G_{\mu\nu}G^{\mu\nu}\rangle + \frac{2g_s}{Q^6}\langle f^{abc}G^{a\ \mu}{}_{\alpha}G^{b\ \alpha}{}_{\beta}G^{c\ \beta}{}_{\mu}\rangle\right).\end{aligned} \tag{15}$$

The correlator in Eq. (15) obtained by using OPE is related to Eq. (14) via the standard dispersion relation

$$\Pi_{\mu\nu,\mu'\nu'}(Q^2) = \frac{1}{\pi}\int_0^\infty ds\,\frac{\mathrm{Im}\,\Pi_{\mu\nu,\mu'\nu'}(-s)}{s+Q^2}. \tag{16}$$

In practice one may only include ground states in the calculation. In order to reduce the uncertainty due to higher excited states and also continuum states, we apply the Borel transformation and obtain

$$\hat{B}\Pi_{S3}(Q^2) = \frac{1}{M^2}\int_0^{s_0} ds e^{-s/M^2}\rho_{S3}(s), \tag{17}$$

where $\rho_{S3}(s)=(1/\pi)\mathrm{Im}\,\Pi_{S3}(-s)$, and

$$\hat{B}\Pi(Q^2)=\lim_{Q^2,n\to\infty}\frac{1}{(n-1)!}(Q^2)^n\left(-\frac{d}{dQ^2}\right)^n\Pi(Q^2). \tag{18}$$

Here one also needs to have the limit $Q^2/n=M^2=\text{const}$.

In our numerical calculation we have varied s_0 in the range of 3–6 GeV^2, and found that the uncertainty is around 10%. The parameters determined are reasonably stable.

We obtain the range for f_s as

$$f_s=(100\text{–}130)\ \ \mathrm{MeV}, \tag{19}$$

with $f_0=190$ MeV and $f_s=100$ MeV for $m_{0^{++}}=1.5$ GeV, and $f_0=130$ MeV and $f_s=130$ for $m_{0^{++}}=1.7$ GeV. In obtaining the above result, we have reevaluated f_0 also using the same parameters. The input parameters used are [11] $\alpha_s(\mu)=4\pi/9\ln(\mu^2/\Lambda_{QCD}^2)$, $\Lambda_{QCD}=0.25$ GeV, $\mu=M$, $\langle\alpha_s G_{\mu\nu}G^{\mu\nu}\rangle=0.06\pm0.02\ \mathrm{GeV}^4$, and $g_s\langle f^{abc}G^{a\ \mu}{}_{\alpha}G^{b\ \alpha}{}_{\beta}G^{c\ \beta}{}_{\mu}\rangle=(0.27\ \mathrm{GeV}^2)\langle\alpha_s G^{\mu\nu}G_{\mu\nu}\rangle$.

For consistency, we also calculated the glueball masses. We find that for the 0^{++} state the mass is 1.5–1.7 GeV, and for 2^{++} the mass is 2.0–2.2 GeV. These values are in agreement with other calculations [10].

If the scalar glueball is a pure one, using the above results we obtain the branching ratio for $\Upsilon\to\gamma G_s$ in the range

$$\mathrm{Br}(\Upsilon\to\gamma G_s)=(1\text{–}2)\times10^{-3}, \tag{20}$$

with a larger branching ratio for a larger glueball mass up to 1.7 GeV. Here we have used $\alpha_s=0.18$ which is the typical value for α_s in the energy range of the decay. We obtain a large branching ratio for $\Upsilon\to\gamma G_s$. We would like to point out that, considering the several uncertainties, the assumptions of factorization, and of a single pure glueball state in the QCD sum-rule calculation, the above numbers should be used as an order of magnitude estimate.

IV. DISCUSSIONS OF PHENOMENOLOGICAL IMPLICATIONS

Experimental measurement of $\Upsilon\to\gamma G_s$ may be nontrivial. One has to rely on the decay products of glueballs. There are several ways the glueball can decay with reasonably large branching ratios: $G_s\to K\bar{K}$ or G_s to multipions. As mentioned earlier there are several candidates for scalar glueball, the $f(1370)$, $f_0(1500)$, and $f_0(1710)$. Decays of $\Upsilon\to\gamma f_0(i)\to\gamma(K\bar{K}$ or multipions) can provide important information.

Experimentally there is only an upper bound [3] of $\mathrm{Br}(\Upsilon\to\gamma f_0(1710)\to\gamma K\bar{K})<2.6\times10^{-4}$ at 90% C.L. If $f_0(1710)$ is a pure glueball, experimental measurement [3] of $\mathrm{Br}(f_0(1710)\to K\bar{K})=0.38^{+0.09}_{-0.19}$ [3] would imply $\mathrm{Br}(\Upsilon\to\gamma f_0(1710)\to\gamma K\bar{K})$ to be in the range of $(0.4\text{–}1.0)\times10^{-3}$ which seems to indicate that $f_0(1710)$ may not be a pure glueball. At present we cannot rule out the possibility that one of the $f_0(1370)$ or $f_0(1500)$ states is a pure glueball state. The data also allow some mixing between glueball states and other quark bound states.

Theoretical calculation of the mixings among glueball and quark bound states is a very difficult task. There is no reliable theoretical calculation. Lattice calculations may eventually give accurate predictions for the mixing parameters. At present there are some phenomenological studies of glueball mixing. We now study some implications of the branching ratio for the radiative decay of a Υ into a pure scalar glueball obtained in the previous section for a mixing pattern suggested in Ref. [14].

An analysis combining other experimental data in Ref. [14] showed that the three 0^{++} states $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$ all contain substantial glueball content. Reference [14] obtained the mixing matrix of physical states in terms of pure glueball and other quark bound states as [14]

	$f_{i1}^{G_s}$	f_{i2}^{S}	$f_{3i}^{(N)}$
$f_0(1710)$	0.39 ± 0.03	0.91 ± 0.02	0.15 ± 0.02
$f_0(1500)$	-0.65 ± 0.04	0.33 ± 0.04	-0.70 ± 0.07
$f_0(1370)$	-0.69 ± 0.07	0.15 ± 0.01	0.70 ± 0.07

(21)

where the states G_s, $S=|s\bar{s}\rangle$, and $N=|u\bar{u}+d\bar{d}\rangle/\sqrt{2}$ are the pure glueball and quark bound states. $f_{i1}^{G_s}$ indicate the amplitude of glueball G_s in the three physical $f_0(i)$ states.

Because of the mixing, when applying our calculations to radiative decay of Υ into a physical state which is not a purely gluonic state the parameters will be modified. If the mixing parameter is known one can obtain the R_s ratios for $\Upsilon\to\gamma f_0(1370{,}1500{,}1710)$ as

$$R_s(\Upsilon\to\gamma f(i))=\frac{25\pi\alpha_s^2}{3\alpha}\cdot\frac{|f_s|^2}{m_b^2}|f_{i1}^{(G_s)}|^2. \tag{22}$$

$\Upsilon\to\gamma f(i)$ may also result from Υ decays into a γ and S,N quark bound states. However, these processes are suppressed by α_s^2.

Using the mixing amplitudes in Eq. (21), one obtains the branching ratios of, $\Upsilon\to\gamma f_0(1370{,}1500{,}1710)$ in the ranges $(4.8\text{–}9.6{,}4.2\text{–}8.4{,}1.5\text{–}3.0)\times10^{-4}$. Combining the branching ratios of $f_0(1370{,}1500{,}1710)\to K\bar{K}(\pi\pi)=(0.38^{+0.09}_{-0.19}(0.039^{+0.002}_{-0.024}),0.044^{+0.021}_{-0.021}(0.454^{+0.104}_{-0.104}),0.35^{+0.13}_{-0.13}(0.26^{+0.09}_{-0.09}))$ [3] we obtain

$$\mathrm{Br}(\Upsilon\to\gamma f_0(1710)\to\gamma K\bar{K})\approx0.6\text{–}1.2,$$

$$\mathrm{Br}(\Upsilon\to\gamma f_0(1710)\to\gamma\pi\pi)\approx0.06\text{–}0.12,$$

$$\mathrm{Br}(\Upsilon\to\gamma f_0(1500)\to\gamma K\bar{K})\approx0.2\text{–}0.4,$$

$$\mathrm{Br}(\Upsilon\to\gamma f_0(1500)\to\gamma\pi\pi)\approx1.9\text{–}3.8,$$

$$\mathrm{Br}(\Upsilon\to\gamma f_0(1370)\to\gamma K\bar{K})\approx1.7\text{–}3.4,$$

$$\mathrm{Br}(\Upsilon\to\gamma f_0(1370)\to\gamma\pi\pi)\approx1.2\text{–}2.4.$$

In the above the branching ratios are in units of 10^{-4}. The branching ratios predicted above can provide a further test for QCD factorization. Future experimental data from CLEO III will provide us with important information.

To summarize, we have estimated the branching ratio of $\Upsilon \to \gamma + G_s$ with G_s as a glueball. Our results show that $f_0(1710)$ may not be consistent with the assumption that it is a pure glueball, but cannot rule out the possibility that one of the $f(1370)$ and $f_0(1500)$ sates is pure glueball state. We also predicted several $\Upsilon \to \gamma KK(\pi\pi)$ branching ratios using a phenomenological glueball mixing pattern, which can provide further tests for QCD factorization calculations and glueball mixing. To have a better understanding of the situation, we have to rely on future improved experimental data. Fortunately, CLEO-III will provide us with more data in the near future. We have a good chance of understanding the properties of scalar glueballs. We strongly encourage our experimental colleagues to carry out a study of the radiative decay of Υ into a scalar glueball.

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