

Double parton scatterings in b -quark pair production at the CERN LHC

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A sizable rate of events where two pairs of b quarks are produced contemporarily is foreseen at the CERN LHC, as a consequence of the large parton luminosity. At very high energies both single and double parton scatterings contribute to the process, the latter mechanisms, although power suppressed, giving the dominant contribution to the integrated cross section.

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I. INTRODUCTION

One of the main topics at the CERN Large Hadron Collaboration (LHC) is the production of b quarks, both to search for CP violation, looking at b decays, and to test QCD, by studying the production mechanism [1]. Bottom quarks are also a large source of background to several processes of interest, as in various promising channels for Higgs detection [2]. The production mechanism of heavy quarks in hadronic collisions pose, on the other hand, nontrivial problems already at smaller energies, where also the simplest observable quantity, the integrated inclusive cross section, is not reproduced trivially.

The inclusive cross section of b -quark production has been evaluated in perturbative QCD (PQCD) at the next to leading order in α_S [4]. Unfortunately comparisons with the recent experimental data of the D0 Collaboration [5] at the fermilab Tevatron have shown that the next leading order (NLO) PQCD calculations underestimate the cross section by a factor $\sim 2,3$, showing that NNLO corrections, whose explicit evaluation is still an open question, give a large contribution to the cross section. A complementary approach to heavy quarks production, which takes explicitly into account that transverse momenta and virtualities of the interacting partons become increasingly important in the kinematical regime of $s \gg m_b^2 \sim \hat{s} \gg \Lambda^2$, and includes terms at every order in α_S in the calculation of the cross section, is k_t factorization, where the interaction is factorized into unintegrated structure functions and off shell matrix elements [6–8]. From the phenomenological point of view k_t factorization is not inconsistent with the DESY ep collider HERA and Tevatron data, allowing one to reproduce both the value of the integrated inclusive cross section and various differential distributions, including the correlation in the azimuthal angle between the produced b quarks, where different approaches are less successfully compared with experiment (see [3] and references therein).

Interestingly, although the value of the integrated inclusive cross section cannot be obtained trivially, one may find several cases where the overall effect of higher order correc-

tions amounts to a simple rescaling of the lowest order parton model result. There are in fact several distributions, derived either using the k_t -factorization approach or by working out the cross section at the NLO PQCD, which are rather similar (apart from the normalization factor) to those obtained with a simple lowest order calculation [9]. Hence, in a few cases, the whole effect of higher order corrections is (approximately) reduced to a single numerical value, the so-called K factor:

$$K = \frac{\sigma(b\bar{b})}{\sigma_{LO}(b\bar{b})}, \quad (1)$$

where $\sigma(b\bar{b})$ is the inclusive cross section for $b\bar{b}$ production and $\sigma_{LO}(b\bar{b})$ the result of the lowest order calculation in PQCD.

When looking at extrapolations of the cross section at high energies, one finds that the result is affected by several uncertainties, such as the knowledge of the parton structure functions at very small x and the values of the heavy quark mass and of the running coupling constant. Although the expected inclusive cross section of b production is hence still pretty uncertain at LHC energy, all estimates point in the direction of rather large values, as a consequence of the high parton luminosity [1]. The fairly large flux of partons make it also plausible to expect a sizable rate of events, where two or more $b\bar{b}$ pairs are produced contemporarily by different partonic collisions in a given pp interaction [10]. Although at the present stage all quantitative predictions for this much more structured interaction process are unavoidably pretty uncertain, the large cross sections foreseen at the LHC are, in our opinion, a strong motivation to make an attempt of giving a few quantitative indications on the production rate of multiple $b\bar{b}$ pairs through multiparton interactions at the LHC, comparing with the rates to be expected by the more conventional single parton scattering mechanism.

Since the details of the elementary production of heavy quarks are still a matter of debate, we limit our considerations, for the production of multiple $b\bar{b}$ pairs, to the simplest cases, where the whole effect of higher order corrections is taken into account by the overall normalization factor. Given the lack of information on higher order corrections in the $2 \rightarrow 4$ processes, we make moreover the assump-

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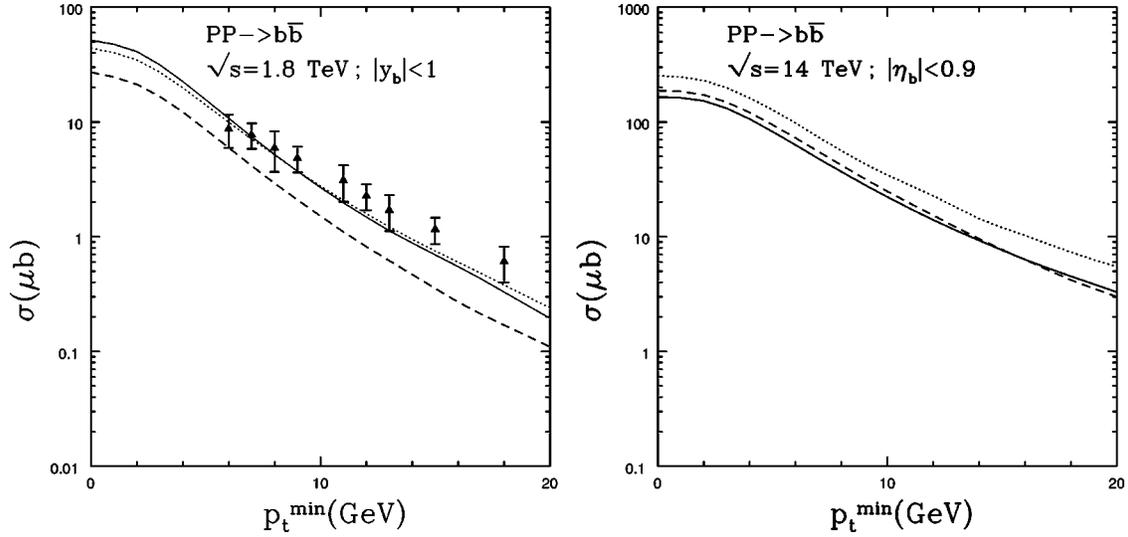


FIG. 1. $p\bar{p} \rightarrow b\bar{b}$ production cross section as a function of p_t^{\min} at $\sqrt{s}=1.8$ TeV, with the b quark within the rapidity range $|y_b|<1$, experimental data from Ref. [5], and at $\sqrt{s}=14$ TeV with the b quark within the pseudorapidity range $|\eta|<0.9$.

tion that the K factors of the $gg \rightarrow b\bar{b}b\bar{b}$ and of the $gg \rightarrow b\bar{b}$ processes are equal. Hence we work out the $gg \rightarrow b\bar{b}$ process in the k_t -factorization approach, fixing the input parameters by comparing with the Tevatron data, and extrapolate the cross section at LHC energies, identifying a few distributions where the effect of higher order corrections reduces to a simple rescaling of the lowest order result. The value of the K factor derived in this way is then used to renormalize the double $(gg \rightarrow b\bar{b})^2$ and the single $gg \rightarrow b\bar{b}b\bar{b}$ parton scattering cross sections, which we evaluate by working out all Feynman diagrams at order α_s^4 .

II. $b\bar{b}$ CROSS SECTION AT TEVATRON AND LHC AND K FACTOR

In the k_t -factorization approach the $b\bar{b}$ production cross section is expressed as [6,7]

$$\sigma(pp \rightarrow b\bar{b}) = \int \frac{d^2q_{t1}}{\pi} \frac{d^2q_{t2}}{\pi} dx_1 dx_2 f(x_1, q_{t1}, \mu) \times f(x_2, q_{t2}, \mu) \hat{\sigma}(x_1, q_{t1}; x_2, q_{t2}; \mu) \quad (2)$$

where $f(x, q_t, \mu)$ is the unintegrated structure function, representing the probability to find a parton with momentum fraction x , transverse momentum q_t at the factorization scale μ , while $\hat{\sigma}$ is the off-shell partonic cross section of the process $g^*g^* \rightarrow Q\bar{Q}$.

To work out the inclusive cross section we use two different prescriptions for constructing the k_t distributions from the usual integrated parton densities. The first prescription is based on the conventional Dokshitzer-Gribov-Altarelli-Parisi (DGLAP) evolution equations [11], with virtual corrections

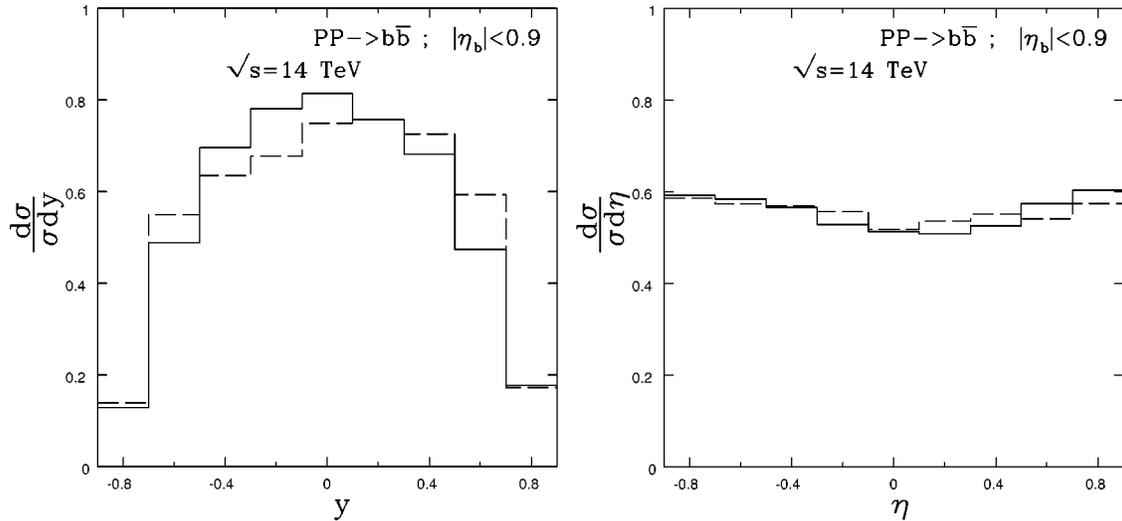


FIG. 2. Normalized rapidity (y) and pseudorapidity (η) distributions for $b\bar{b}$ production at ALICE, with the k_t factorization approach (dashed histograms) and at the lowest order in PQCD multiplied by the K factor (continuous histograms).

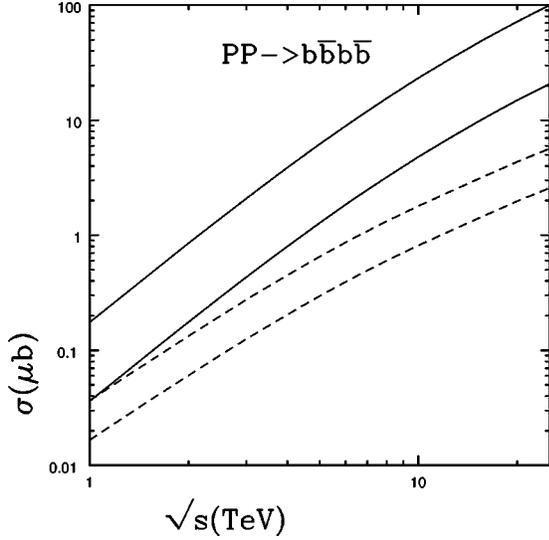


FIG. 3. $b\bar{b}b\bar{b}$ total cross section as a function of the center of mass energy. Lower curves $K=2.5$; higher curves $K=5.5$.

resummed in the survival probability factor $T_a(k_t^2, \mu^2)$ [12]. Hence the unintegrated structure function for the parton a reads

$$f_a(x, k_t^2, \mu^2) = T_a(k_t^2, \mu^2) \left[\frac{\alpha_S(k_t^2)}{2\pi} \int_x^{1-\delta} P_{aa'}(z) \times a' \left(\frac{x}{z}, k_t^2 \right) dz \right] \quad (3)$$

where $P_{aa'}(z)$ is the splitting function, $a'(x, k_t^2)$ the integrated structure function and δ a cutoff parameter introduced to give sense to the integral. Although not written explicitly $T_a(k_t^2, \mu^2)$ also depends on δ , in such a way that $f_a(x, k_t^2, \mu^2)$ is a smooth function when δ becomes small.

As for the second prescription we follow Ref. [13], where the unintegrated structure functions are obtained from the leading order Balitskii-Fadin-Kuraev-Lipatov (BFKL) equation and are expressed as the convolution of the usual collinear gluon densities $G(x, \mu^2)$ with the universal function $\mathcal{G}(x, k_t^2, \mu^2)$

$$\mathcal{F}(x, k_t^2, \mu^2) = \int_x^1 d\xi \mathcal{G}(\xi, k_t^2, \mu^2) G\left(\frac{\xi}{x}, \mu^2\right). \quad (4)$$

The weight factors $\mathcal{G}(\xi, k_t^2, \mu^2)$ have a known analytic expression, in double-logarithmic approximation, in terms of Bessel functions and depend on the quantity $\bar{\alpha}_s = 3\alpha_s/\pi$, which in the BFKL formalism is a fixed parameter, related to the Pomeron intercept $\alpha(0) = 1 + \Delta$, where Δ in leading log approximation is $\Delta = 4\bar{\alpha}_s \log 2$. Following [14] we take the value $\Delta = 0.35$.

To generate the unintegrated structure functions we have used the parton distributions set GRV94 [15] with factorization scale $\mu_F^2 = \hat{s}$. Hence in evaluating the cross sections, in the k_t -factorization approach, we have set the renormalization scale equal to the gluon virtuality. To obtain the cross section at the lowest order in PQCD we have used the Martin-Roberts-Stirling 1999 (MRS99) parton distributions [16], with factorization and renormalization scale equal to the transverse mass of the b quark. Comparing the total cross section values at the LHC in the parton model and in the k_t factorization approach, using the first prescription for the unintegrated structure functions, we have obtained for the K factor the value $K \sim 5.5$.

In Fig. 1 we plot the integrated cross section of $b\bar{b}$ production at Tevatron ($\sqrt{s} = 1.8$ TeV) and at LHC ($\sqrt{s} = 14$ TeV), as a function of the minimum value of the transverse momentum p_t^{\min} of the b quark. The dotted curves

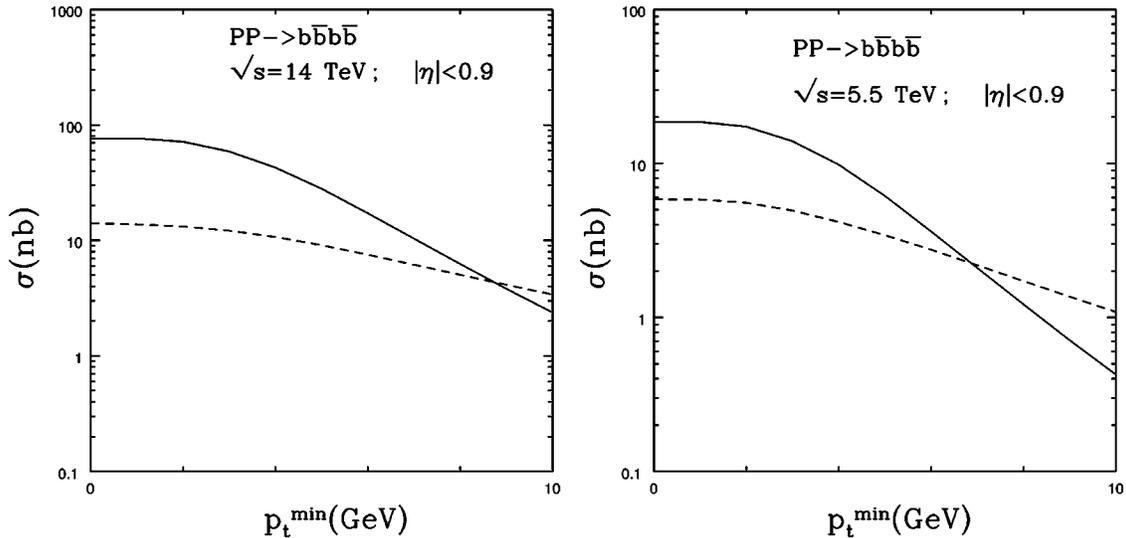


FIG. 4. $b\bar{b}b\bar{b}$ production cross section at $\sqrt{s} = 14$ TeV and at $\sqrt{s} = 5.5$ TeV as a function of p_t^{\min} with all four b quarks in the pseudorapidity interval $|\eta| < 0.9$.

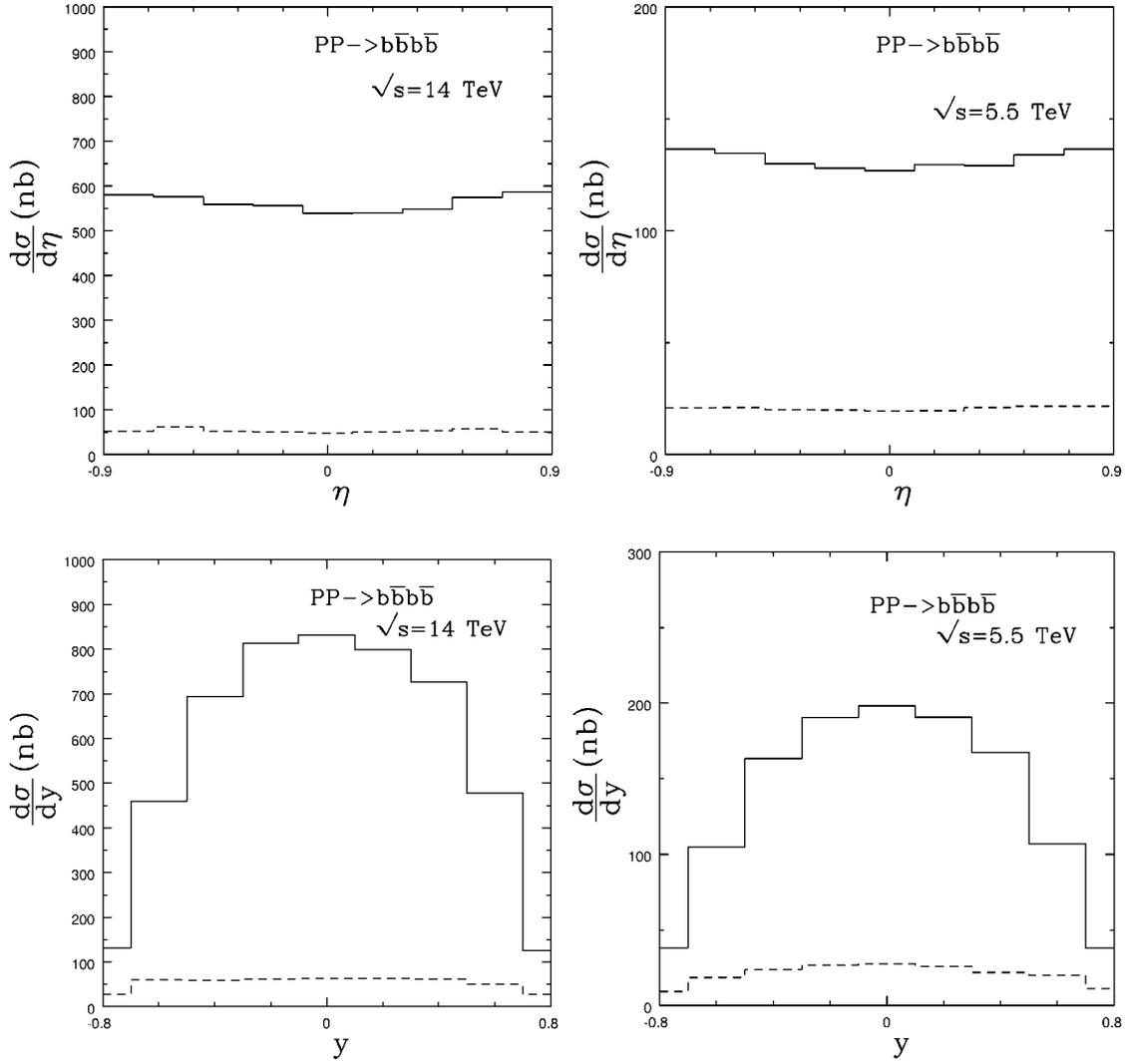


FIG. 5. $b\bar{b}b\bar{b}$ production with the two equal sign b quarks in the pseudorapidity interval $|\eta_b| < 0.9$. η distributions and y_b distributions at $\sqrt{s} = 14$ TeV and at $\sqrt{s} = 5.5$ TeV. The continuous histograms refer to the contribution of double parton scatterings while the dashed histograms refer to the single parton scatterings.

represent the cross section derived using the unintegrated gluon structure function, according to the BFKL prescription of Eq. (4), whereas the dashed lines have been obtained by using the prescription in Eq. (3). The continuous lines represent the result of the lowest order calculation multiplied by the K factor. At Tevatron energy the b -quark distributions are within the rapidity interval $|y| < 1$ and are compared with the D0 experimental data [5]. The same distributions, extrapolated at LHC energy, are then plotted as a function of p_t^{min} within the pseudorapidity interval $|\eta| < 0.9$, corresponding to the acceptance of the ALICE detector.

In Fig. 2 we show the rapidity (y) and pseudorapidity (η) distributions normalized to one and within $|\eta| < 0.9$. Here the continuous histograms refer to the result of the lowest order calculation, rescaled by the K factor, whereas the dashed histograms represent the distributions evaluated with the k_t -factorization approach Eq. (3). As one may see also in this case the whole effect of higher orders reduces to a simple rescaling.

III. $b\bar{b}b\bar{b}$ CROSS SECTION

The leading order QCD process to produce two pairs of heavy quarks is given by the single parton scattering term at the fourth order in the coupling constant [17]. A competing mechanism at the LHC energy is the double parton scattering [18]. We compare the two mechanisms in proton-proton collisions in the kinematical range of the ALICE and of the LHCb detectors, namely at center-of-mass energies of 5.5 and 14 TeV, within the pseudorapidity regions $|\eta| < 0.9$ and $1.8 < \eta < 4.9$, down to very low transverse momenta.

The single scattering PQCD subprocesses at the lowest order in α_s , in $pp \rightarrow b\bar{b}b\bar{b}$, are the quarks initiated process, $q\bar{q} \rightarrow b\bar{b}b\bar{b}$, whose amplitude is given by the sum of 14 Feynman diagrams for each flavor in the initial state, and gluon fusion, $gg \rightarrow b\bar{b}b\bar{b}$, represented by 76 diagrams altogether, the latter amplitude giving the dominant contribution to the cross section at small x . To evaluate the cross section we have generated the matrix elements of the partonic am-

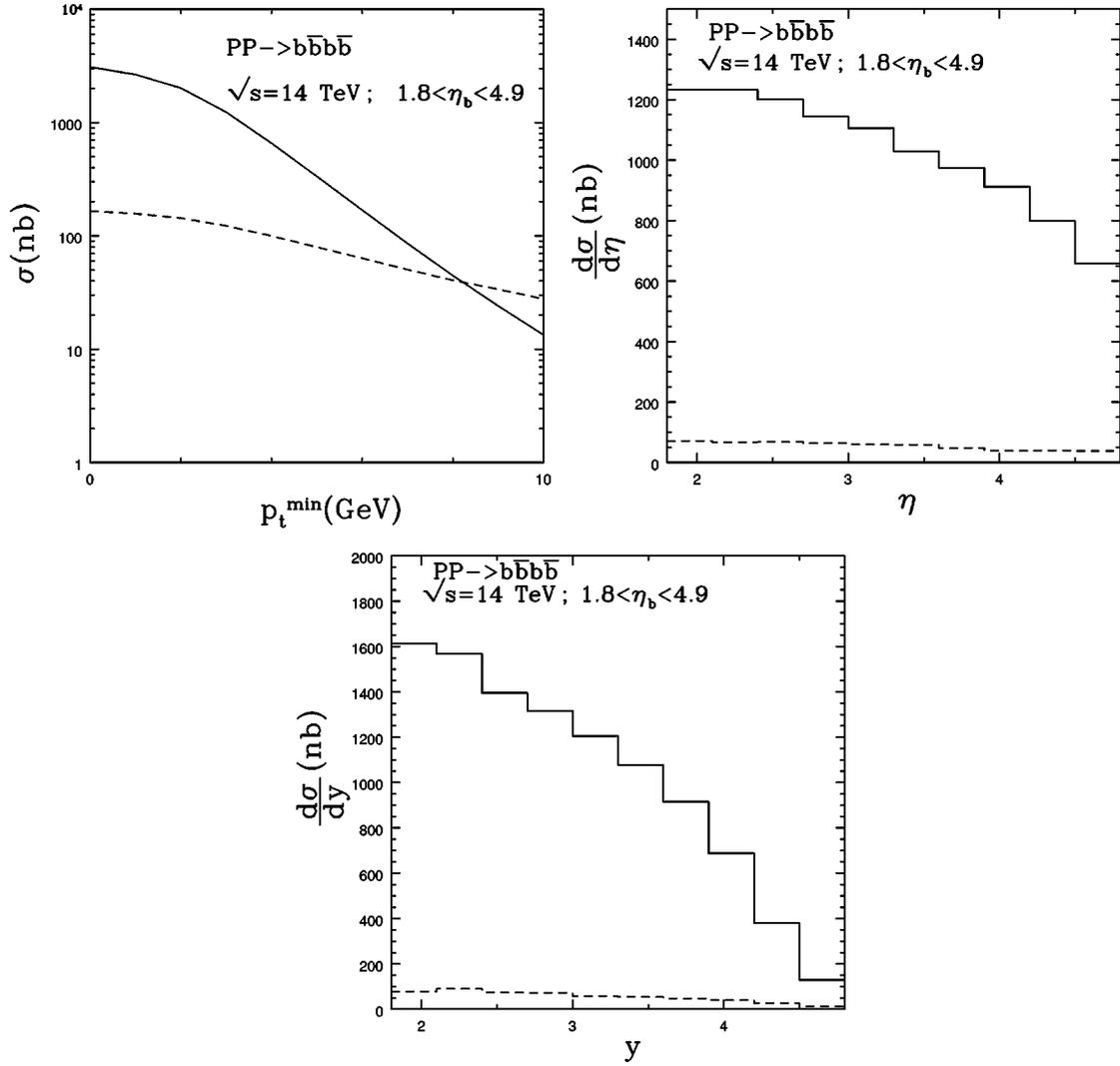


FIG. 6. $b\bar{b}b\bar{b}$ production with the two equal sign b quarks in the pseudorapidity interval $1.8 < \eta < 4.9$ at $\sqrt{s} = 14$ TeV. Production cross section as a function of p_t^{\min} , η and y . The continuous lines and histograms refer to the contribution of double parton scatterings while the dashed lines and histograms refer to the single parton scatterings.

plitudes with MADGRAPH [19] and HELAS [20] and we have used the MRS99 parton distributions [16], with the factorization scale equal to the renormalization scale $\mu_F = \mu_R$, which we have kept fixed at the value of the transverse mass of the produced b quark. For the mass of the bottom quark we have used the value $m_b = 4.6$ GeV. The multidimensional integrations have been performed by VEGAS [21] and the resulting cross section has been finally multiplied by the K factor obtained as described in the previous section.

The evaluation of the double parton scattering contribution to the cross section is considerably more uncertain because of the unknown nonperturbative input to the process, given by the two-body parton distribution functions $\Gamma(x_1, x_2, \beta)$ [10], where $x_{1,2}$ are the fractional momenta of the two partons belonging to the same hadron and β their distance in transverse space. Although not explicitly written, the distributions depend also on the scale factors characterizing each elementary interaction and on the different kinds of partons involved. Given the large parton population at low

x , to proceed further we make the usual simplifying assumption of neglecting correlations in fractional momenta and we factorize the two-body parton distribution as $\Gamma(x_1, x_2, \beta) = G(x_1)G(x_2)F(\beta)$, where $G(x)$ are the usual one-body parton distributions and $F(\beta)$ is a function normalized to 1 and representing the parton pair density in transverse space. With these assumptions the cross section acquires the simplified form [22]

$$\sigma_D(b\bar{b}; b\bar{b}) = \frac{1}{2} \sum_{ij} \Theta^{ij} \sigma_i(b\bar{b}) \sigma_j(b\bar{b}) \quad (5)$$

where the indices i, j label the different cases where each $b\bar{b}$ pair is originated either by a $q\bar{q}$ annihilation, discriminating the cases of sea and valence, or by two gluons and $\sigma_i(b\bar{b})$ represents the inclusive cross sections for $b\bar{b}$ production in a hadronic collision, with the index i labeling a definite parton process. The weight factors Θ^{ij} have dimension as an in-

verse cross section and result from integrating the product of the two-body parton distributions in transverse space, while the factor 1/2 is a consequence of the symmetry of the expression for exchanging i and j . The dependence of Θ^{ij} on the indices i, j accounts for the possibility, for different pairs of partons in the hadron structure, to be characterized by different values of their relative average transverse distance [22,18]. Notice that by measuring the double parton collisions one has access to new information on the hadron structure, summarized in these weight factors, which cannot be obtained in hard processes with a single parton interaction only.

The experimental information on the double parton scatterings is due to the four-jet production measurement in pp collisions at $\sqrt{s}=63$ GeV, performed by the AFS Collaboration [23], and to the study of final states with three minijets and one photon in $p\bar{p}$ collisions at $\sqrt{s}=1800$ GeV, because of the collider Detector at fermilab (CDF) [24,25]. In both cases the cross section was expressed as

$$\sigma_D = \frac{m}{2} \frac{\sigma_S(A)\sigma_S(B)}{\sigma_{eff}} \quad (6)$$

where $m=1$ if the two parton processes A and B are identical, while $m=2$ if they are different and σ_S is the single scattering inclusive cross section. The overall output of the experiment hence reduces to the value of a single parameter, the scale factor σ_{eff} , whose value is $\sigma_{eff}=5$ mb for AFS, while $\sigma_{eff}=14.5$ mb for CDF. The two experimental results are not inconsistent, given the different content of partons in the two cases, mainly valence quarks in the former case and mostly gluons and sea quarks in the latter, the experimental indication hence pointing in the direction of a sizable dependence of the factors Θ^{ij} in Eq. (5) on the different elementary processes. Interestingly, the measurement of σ_{eff} for different final states, as a function of the c.m. energy and cuts applied, allows one to obtain the values of the scale factors Θ^{ij} , allowing in this way access to the three dimensional structure of the proton [18].

The dominant contribution to $b\bar{b}b\bar{b}$ production at the LHC is gluon fusion, so, for the present purposes, the sum in Eq. (5) may be approximated well by a single term, where the scale factor could not be strongly different with respect to the case of the CDF experiment. Hence, to evaluate the double scattering cross section, we have used the simplest expression

$$\sigma_D(b\bar{b}b\bar{b}) = \frac{\sigma(b\bar{b})^2}{2\sigma_{eff}}, \quad (7)$$

where for σ_{eff} we have taken the value reported by CDF. Notice that since σ_D is proportional to σ_S^2 , the effect of higher order corrections is enhanced on σ_D :

$$\begin{aligned} \sigma_S &= K\sigma_S^{LO} \\ \sigma_D &= K^2\sigma_D^{LO} \end{aligned} \quad (8)$$

where $\sigma_{S,D}^{LO}$ refers to the lowest order expressions of the cross section.

IV. RESULTS

In Fig. 3 we plot the expected rise of the total $b\bar{b}b\bar{b}$ production cross section as a function of the c.m. energy. The continuous curves refer to the double parton scattering contribution, while the dashed curves to single scattering. In each case the lower curve refers to the value $K=2.5$, while the higher curve to $K=5.5$, which are the typical estimate of the NLO-QCD and the result of our calculation within the k_t -factorization approach. Notice that at the LHC the double parton scattering gives a contribution to the integrated cross section about ten times larger than the single scattering.

In the following results we have used the value $K=5.5$. To see how the cross section depends on the transverse momenta we have plotted in Fig. 4 the two contributions to the integrated cross section, as a function of p_t^{min} , the minimum value of the transverse momenta of the b quarks (which we require to be all inside the pseudorapidity interval $|\eta| < 0.9$), for the center-of-mass energy values of 14 and 5.5 TeV. The continuous histograms refer to the double parton scattering contribution, while the dashed histograms refer to single scattering. The double parton cross section decreases faster with p_t^{min} than the single parton cross section, the two contributions being of the same order at $p_t^{min}=8-10$ GeV. Pseudorapidity, and rapidity distributions at 14 and 5.5 TeV, are plotted in Fig. 5 (always requiring for the two b quarks $|\eta| < 0.9$), where continuous and dashed histograms have the same meaning as in the previous cases.

To see how the results depend on rapidity, we have plotted in Fig. 6 the same rapidity (y) and pseudorapidity (η) distributions at $\sqrt{s}=14$ TeV, requiring both b -quarks to be in the pseudorapidity interval $1.8 < \eta < 4.9$, which corresponds to the acceptance of the LHCb experiment. In the same figure we also compare the two contributions of single and double parton scattering, integrated within the rapidity acceptance of the LHCb, as a function of p_t^{min} .

The overall indication which one obtains from the present study is that double parton scatterings dominate the $b\bar{b}b\bar{b}$ integrated cross section by a large factor, both in the central rapidity region and at the larger rapidity values of the LHCb experiment. In both cases the contribution of the single parton scattering term becomes important only after applying cuts to the transverse momenta of the order of 8–10 GeV. The large values expected for the $b\bar{b}b\bar{b}$ cross section, which at 14 TeV are of the order of one μb , inside the ALICE and the LHCb detectors, and the localization of the double scattering contribution at relatively low p_t values, suggest that heavy quark pair production at the LHC might represent an efficient tool for studying the gluon initiated double parton scattering process.

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