QED radiative corrections in processes of exclusive pion electroproduction

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A formalism for radiative correction (RC) calculation in exclusive pion electroproduction on the proton is presented. A FORTRAN code EXCLURAD is developed for the RC procedure. The numerical analysis is done in the kinematics of current Jefferson Lab experiments.

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I. INTRODUCTION

Understanding the electromagnetic transition amplitudes from the nucleon ground state to excited states provides valuable insight into the electromagnetic structure of the nucleon. Exclusive pion electroproduction is one of the major sources to provide the most direct information about the spatial and spin structure of the excited states. With the development of a high intensity and high duty-factor electron beam with a high degree of polarization, this field reaches a new level of quality. For the past several years, exclusive pion electroproduction has been the main subject of extensive studies at various accelerator laboratories, such as MIT-Bates, ELSA, MAMI, and Jefferson Lab.

New measurements with the CLAS detector at Jefferson Lab/Hall B [1] are expected to greatly improve the systematic and statistical precision and cover a wide kinematic range in four-momentum transfer Q^2 and invariant mass W, as well as the full angular range of the resonance decay into the nucleon-pion final state:

$$\begin{aligned} &e(k_1)+p(p)\rightarrow e'(k_2)+\pi^+(p_h)+n(p_u),\\ &e(k_1)+p(p)\rightarrow e'(k_2)+p(p_h)+\pi^0(p_u), \end{aligned} \tag{1}$$

where p_h (p_u) denotes the momentum of the detected (undetected) hadron. The adequate calculation of radiative corrections (RC) becomes important in interpreting the measured observables such as unpolarized coincidence cross sections and polarization asymmetries.

While solving the RC problem, one is commonly referred to the classical approach developed by Mo and Tsai [2] and used for inclusive and elastic electron scattering for decades. However, this approach cannot be directly applied to exclusive pion electroproduction for the following reasons.

First, we now deal with *exclusive* electroproduction, where the hadron is detected in addition to the final electron. It reduces the room for the phase space allowed for the final radiated photon. The formulas of Mo and Tsai as well as any other inclusive formulas cannot be applied do this case without additional strong assumptions.

Second, there are only contributions of two structure functions in the inclusive case. The exact formalism for the exclusive process requires consideration of four structure functions for the unpolarized case with additional angular dependence associated with them. Note that the transition to the case of two structure functions would not be possible even within realistic approximations. The Mo-Tsai approach predicts neither RC to polarization asymmetries, nor dependencies on the outgoing hadron angles.

The third reason is a known shortcoming of the Mo-Tsai approach, namely, the dependence on an unphysical parameter splitting soft and hard regions of the phase space of radiated photon in order to cancel the infrared divergence.

In our approach, which is based on a covariant procedure of infrared divergence cancellation proposed by Bardin and Shumeiko in Ref. [3], such an unphysical parameter is not required. Previously, this approach was applied for the calculation of RC for inclusive [4–6], semi-inclusive [7,8] and exclusive diffractive [9] reactions. Recent reviews of the approach, higher order effects and calculation for specific experiments can be found in papers [10–12]. Based on these results, a FORTRAN code POLRAD [13] for RC calculation in polarized inclusive and semi-inclusive processes was developed. Besides, some specific tasks such as the Monte Carlo (MC) generator RADGEN [14], the MC approach to diffractive vector meson electroproduction [15], RC to spin-density matrix elements in exclusive vector meson production [16], and the quasielastic tail for a polarized He-3 target [17] were solved. Recently the Bardin-Shumeiko approach was applied to the measurements of elastic polarized electron-proton scattering at Jefferson Lab [18,19]. A comprehensive analysis of results obtained in Refs. [2] and [3] was made in Refs. [14,20].

Note that there are also other approaches for the calculation of RC in electroproduction processes. For example, the results of Ref. [21] are actively used for DESY *ep* collider HERA experiments, while the approach developed in papers [22] is applied to specific measurements at Jefferson Lab [23].

The Feynman diagrams needed to calculate RC are presented in Fig. 1. They include QED processes of radiation of an unobserved real photon, vacuum polarization, and lepton-photon vertex corrections. These processes give the largest contribution due to a large logarithmic term $\ln(Q^2/m_e^2)$. They can be calculated exactly from QED, and uncertainties of such a calculation are only due to the fits and data used for the hadronic structure functions. These uncertainties are demonstrated in the present article. Additional mechanisms (box-type diagrams, emission by hadrons) are smaller by

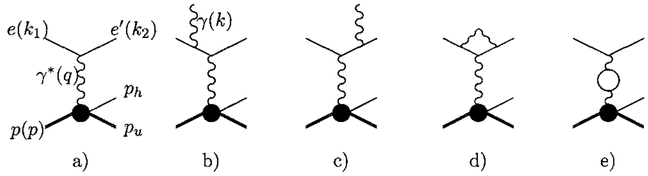

FIG. 1. Feynman diagrams contributing to the Born and the next-order electroproduction cross sections. (a) Born process, (b) and (c) bremsstrahlung, (d) vertex correction, and (e) vacuum polarization. The momentum $p_h(p_u)$ is assigned to the detected (undetected) hadron.

about an order of magnitude and they contain considerable theoretical uncertainties. Most recent studies of two-photon exchange effects for elastic ep scattering were reported in Refs. [24,25]. Previous RC calculations [26] for inclusive polarized deep-inelastic scattering included the above-mentioned additional mechanisms. Generalization to the case of meson electroproduction will be subject to a separate study.

The paper is organized as follows. We introduce kinematics and definitions (Sec. II), derive a cross section of the radiative process (Sec. III), solve the problem of infrared divergence (Sec. IV), obtain RC in the leading log arithmic approximation (Sec. V), verify the relation between exclusive and inclusive RC (Sec. VI), perform numerical analysis (Sec. VII), and summarize the results in Sec. VIII.

II. KINEMATICS AND BORN PROCESS

At the Born level [Fig. 1(a)], the cross section of the processes (1) is described by four kinematic variables. Following tradition, we choose them as the squared virtual photon momentum Q^2, invariant mass of initial proton and the virtual photon W, and detected pion (or proton) angles θ_h and ϕ_h in the center-of-mass of the final hadrons. The obtained formulas are equally valid for both electron and muon scattering.

We use the following Lorentz invariants defined from leptonic 4-momenta:

$$\begin{gathered}
S=2k_1p, \quad X=2k_2p, \quad Q^2=-(k_1-k_2)^2,\\
u_1=S-Q^2, \quad u_2=X+Q^2, \quad \lambda_{1,2}=u_{1,2}^2-4m^2W^2,\\
S_{p,x}=S\pm X, \quad \lambda_S=S^2-4m^2M^2 \qquad (2)\\
W^2=S_x-Q^2+M^2, \quad \lambda_q=S_x^2+4M^2Q^2,\\
\lambda=Q^2u_1u_2-Q^4W^2-m^2\lambda_q,
\end{gathered}$$

where m (M) is the lepton (proton) mass.

We use the c.m. system of virtual photon ($q=k_1-k_2$) and initial nucleon. The axis OZ is chosen along $\vec{q}$ (see Fig. 2). The energies and angles (Fig. 2) in the selected frame can be expressed in terms of invariants (2) as follows:

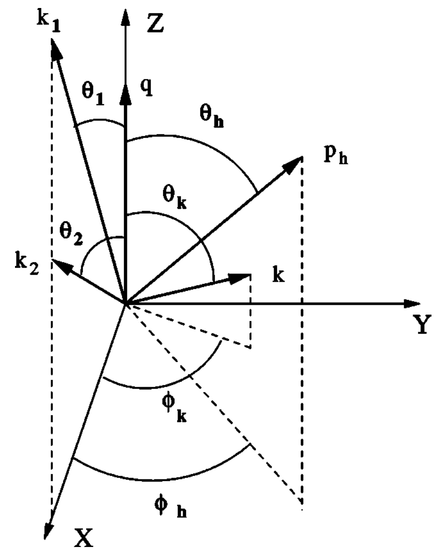

FIG. 2. Definition of momenta and angles in the center-of-mass frame.

$$\begin{gathered}
E_{1,2}=\frac{u_{1,2}}{2W}, \quad E_q=\frac{S_x-2Q^2}{2W}, \quad E_p=\frac{S_x+2M^2}{2W},\\
p_{1,2}=\frac{\sqrt{\lambda_{1,2}}}{2W}, \quad p_p=p_q=\frac{\sqrt{\lambda_q}}{2W},\\
\cos\theta_{1,2}=\frac{u_{1,2}(S_x-2Q^2)\pm 2Q^2W^2}{\sqrt{\lambda_{1,2}}\sqrt{\lambda_q}}, \qquad (3)\\
\sin\theta_{1,2}=\frac{2W\sqrt{\lambda}}{\sqrt{\lambda_{1,2}}\sqrt{\lambda_q}}.
\end{gathered}$$

Here subscripts $1,2,q,p$ denote the initial lepton, final lepton, virtual photon and initial proton, respectively.

All the introduced kinematic variables have the same definition for both Born and radiative kinematics. But the situation is different for the final hadrons. The reason is that the energy of the observed hadron is not fixed by measurements of the chosen kinematic variables: Q^2, W^2, θ_h ϕ_h. As a result, it is different for these cases. In the Born case the hadron energy can be defined via conservation laws, but for the radiative process it depends on the unknown energy of the radiated photon. In order to distinguish between the variables, we use the superscript ''0'' for the quantities calculated for Born kinematics:

$$\begin{gathered}
E_h^0=\frac{W^2+m_h^2-m_u^2}{2W}, \quad p_h^0=\frac{\sqrt{\lambda_W^0}}{2W}, \qquad (4)\\
\lambda_W^0=(W^2-m_h^2-m_u^2)^2-4m_h^2W^2.
\end{gathered}$$

All invariants defined via the measured 4-momentum of hadron p_h are also different for Born and radiative cases:

$$V^0_{1,2}=2k_{1,2}p_h=2[E_{1,2}E^0_h-p_{1,2}p^0_h(\cos\theta_h\cos\theta_{1,2}+\sin\theta_h\sin\theta_{1,2}\cos\phi_h)],\tag{5}$$

$$S^0_t=2pp_h=W^2+m_h^2-m_u^2+V^0_2-V^0_1.$$

The last expression follows from 4-momentum conservation.

The Born cross section of exclusive pion electroproduction in terms of the introduced variables reads:

$$d\sigma_0=\frac{M_0^2}{2S(2\pi)^5}\frac{d\vec{k}_2}{2E_2}\frac{d\vec{p}_u}{2E_u}\frac{d\vec{p}_h}{2E_h}\delta(\Lambda-p_u)$$
$$=\frac{(4\pi\alpha)^2}{2(4\pi)^4S^2}\frac{L^0_{\mu\nu}W_{\mu\nu}}{Q^4}\frac{\sqrt{\lambda^0_W}}{W^2}dQ^2dW^2d\Omega_h,\tag{6}$$

where Λ is the total 4-momentum of the undetected particles, $\Lambda=p+k_1-k_2-p_h$. For the Born process [Fig. 1(a)], Λ is equal to the momentum of the undetected hadron, p_u, while for the radiative process [Figs. 1(b) and 1(c)], it is a sum of the undetected hadron and bremsstrahlung photon momenta, $\Lambda=p_u+k$.

The phase space is calculated as

$$\frac{d\vec{k}_2}{2E_2}=\frac{\pi}{2S}dQ^2dW^2\tag{7}$$

and

$$\frac{d\vec{p}_u}{2E_u}\frac{d\vec{p}_h}{2E_h}\delta(\Lambda-p_u)=\frac{p^0_h}{4W}d\Omega_h=\frac{\sqrt{\lambda^0_W}}{8W^2}d\Omega_h.\tag{8}$$

To calculate the matrix element squared, one needs to contract the leptonic and hadronic tensors,

$$M_0^2=\frac{e^4}{Q^4}L^0_{\mu\nu}W_{\mu\nu}=\frac{2e^4}{Q^4}\sum_{i=1}^{5}\theta^0_i\mathcal{H}^0_i.\tag{9}$$

We consider the longitudinally polarized lepton beam. In this case the leptonic tensor reads

$$L^0_{\mu\nu}=\frac{1}{2}\mathrm{Tr}(\hat{k}_2+m)\gamma_\mu(\hat{k}_1+m)(1+i\gamma_5\hat{\xi})\gamma_\nu.\tag{10}$$

Here the lepton polarization vector is kept in a general form. If the lepton is longitudinally polarized and its helicity is positive, then the vector ξ can be expressed as [6]

$$\xi=\frac{1}{\sqrt{\lambda_s}}\left(\frac{S}{m}k_1-2mp\right).\tag{11}$$

In the Born approximation, the second term can be dropped. However, it gives a nonzero contribution to RC.

For the hadronic tensor, we use a general covariant form

$$W^{\mu\nu}=-\tilde{g}^{\mu\nu}\mathcal{H}_1+\tilde{p}^\mu\tilde{p}^\nu\mathcal{H}_2+\tilde{p}^\mu_h\tilde{p}^\nu_h\mathcal{H}_3+(\tilde{p}^\mu\tilde{p}^\nu_h+\tilde{p}^\mu_h\tilde{p}^\nu)\mathcal{H}_4+(\tilde{p}^\mu_h\tilde{p}^\nu-\tilde{p}^\mu\tilde{p}^\nu_h)\mathcal{H}_5,\tag{12}$$

where the tilde for an arbitrary 4-vector a^μ denotes the substitution $\tilde{a}^\mu=a^\mu-aq/q^2q^\mu$ (to ensure electromagnetic gauge invariance). The first four structure functions have tensor coefficients symmetric over Lorentz indices, but the last one is antisymmetric, it contributes to the polarization-dependent part of the cross section. The quantities θ^0_i have the following form:

$$\theta^0_1=Q^2,$$
$$\theta^0_2=\frac{1}{2}(SX-M^2Q^2),$$
$$\theta^0_3=\frac{1}{2}(V^0_1V^0_2-m_h^2Q^2),\tag{13}$$
$$\theta^0_4=\frac{1}{2}(SV^0_2+XV^0_1-S^0_tQ^2),$$
$$\theta^0_5=-2\epsilon(k_1,k_2,p,p_h),$$

where the operator in the last line is defined as

$$\epsilon(p_1,p_2,p_3,p_4)=p_1^\alpha p_2^\beta p_3^\gamma p_4^\sigma\epsilon_{\alpha\beta\gamma\sigma}.\tag{14}$$

In the chosen c.m. frame it is equal to

$$\epsilon(k_1,k_2,p,p_h)=WP_qp_hp_1\sin\theta_1\sin\theta_h\sin\phi_h=\frac{\sqrt{\lambda^0_W\lambda}}{4W}\sin\theta_h\sin\phi_h.\tag{15}$$

As a result, we obtain for the Born cross section ($\sigma_0=d\sigma/dW^2dQ^2d\Omega_h$)

$$\sigma_0=\frac{\alpha^2\sqrt{\lambda^0_W}}{32\pi^2S^2W^2Q^4}\sum_{i=1}^{5}\theta^0_i\mathcal{H}^0_i.\tag{16}$$

The expression for the Born cross section in the form (16) is convenient for our further calculation. It is equivalent to the well known formula in terms of photoabsorption cross sections (or response functions), where each term corresponds to certain polarization states of the virtual photon [27],

$$\frac{1}{N_0}\sigma_0=\sigma_T+\epsilon\sigma_L+\epsilon\cos 2\phi_h\sigma_{TT}+\sqrt{\epsilon(1+\epsilon)/2}\cos\phi_h\sigma_{LT}+h_e\sqrt{\epsilon(1-\epsilon)/2}\sin\phi_h\sigma'_{LT},\tag{17}$$

where

$$N_0=\alpha\frac{W^2-M^2}{2S^2Q^2(1-\epsilon)},\tag{18}$$

h_e is the longitudinal polarization degree of the incoming leptons, and ϵ [27] describes the virtual photon polarization.

The following expressions relate the structure functions (12) and Born photoabsorption cross sections (17):

$$\mathcal{H}_1(W^2,Q^2,t)=C(\sigma_T-\sigma_{TT}),$$

$$\mathcal{H}_2(W^2,Q^2,t)=\frac{2C}{\lambda_q}[2Q^2(\sigma_T-\sigma_{TT}+\sigma_L), -TQ\sigma_{LT}+T^2\sigma_{TT}],$$

$$\mathcal{H}_3(W^2,Q^2,t)=\frac{2C\lambda_q}{\lambda_l}\sigma_{TT}, \tag{19}$$

$$\mathcal{H}_4(W^2,Q^2,t)=\frac{C}{\sqrt{\lambda_l}}(2T\sigma_{TT}-Q\sigma_{LT}),$$

$$\mathcal{H}_5(W^2,Q^2,t)=\frac{C}{\sqrt{\lambda_l}}Q\sigma'_{LT},$$

where

$$C=\frac{16\pi^2(W^2-M^2)W^2}{\alpha\sqrt{\lambda_W^0}},$$

$$T=\frac{S_x t_q-2Q^2S_t}{\sqrt{\lambda_l}}, \tag{20}$$

$$\lambda_l=Q^2S_t^2-S_tS_xt_q-M^2t_q^2-m_h^2\lambda_q,$$

and $Q=\sqrt{Q^2}$. Note that both structure functions $\mathcal{H}_i$, as well as cross sections σ's, are functions of three independent invariant variables, which usually are chosen as Q^2, W^2 and t. Therefore their transformation coefficients depend only on these variables but not on ϕ_h. For the radiative case the variables will be defined in the next section [see Eq. (30)]. For the Born case, they are taken as W^2, Q^2 and $t=t_0=V_2^0-V_1^0-Q^2+m_h^2$.

III. EXACT FORMULAS FOR RADIATIVE CORRECTION

The cross section of the radiative process is given by

$$d\sigma_r=\frac{M_r^2}{2S(2\pi)^8}\frac{d\vec{k}_2}{2E_2}\frac{d\vec{k}}{2\omega}\frac{d\vec{p}_u}{2E_u}\frac{d\vec{p}_h}{2E_h}\delta(\Lambda-k-p_u)$$
$$=\frac{(4\pi\alpha)^3dQ^2dW^2d\Omega_h}{2(4\pi)^7S^2W^2}\int d\Omega_k dv\,\frac{v\sqrt{\lambda_W}}{f_W^2\tilde{Q}^4}L^R_{\mu\nu}W_{\mu\nu}, \tag{21}$$

where $\Omega_{k(h)}$ stands for the solid angle of the bremsstrahlung photon (detected hadron).

Let us first consider the phase space for the radiative process. Integrating over the 3-momentum of unobserved hadron and using the Dirac δ function to eliminate integration over the hadron energy, we have

$$\int\frac{d\vec{k}}{2\omega}\frac{d\vec{p}_u}{2E_u}\frac{d\vec{p}_h}{2E_h}\delta(\Lambda-k-p_u)$$
$$=\frac{1}{64}\int d\Omega_h d\Omega_k\int_0^{v_m}\frac{v\,dv}{f_W^2}\sqrt{\lambda_W}, \tag{22}$$

where

$$f_W=W-E_h+p_h[\cos\theta_h\cos\theta_k+\sin\theta_h\sin\theta_k\cos(\phi_h-\phi_k)]. \tag{23}$$

Here we introduce a quantity v that describes the missing mass (or inelasticity) due to the emission of a bremsstrahlung photon, $v=\Lambda^2-m_u^2$. Note that $v=0$ for the Born process [Fig. 1(a)], as well as in the soft-photon limit $(k=0)$. As can be seen from Eq. (22), both the photon and hadron energies are now functions of v. It is related to another quantity, f_W, as follows:

$$v=W^2+m_h^2-m_u^2-2WE_h=2\omega f_W. \tag{24}$$

The largest value of inelasticity allowed by kinematics (v_m) corresponds to the threshold of electroproduction. It is therefore defined from the relation $E_h=m_h$, yielding

$$v_m=(W-m_h)^2-m_u^2. \tag{25}$$

Note that v_m is always smaller for the heavier hadron, namely, the nucleon, detected in the final state. It does not depend on photon angles, therefore the integration region is a rectangle.

The maximum inelasticity v_m is an important quantity for the RC calculation. All kinematic cuts made by experimentalists in data analysis influence RC. It is often possible to reduce all these cuts to one effective cut on the inelasticity (or missing mass), v_{cut}, in which case v_{cut} should replace v_m as the upper limit of integration in Eq. (22), and thus RC can be calculated within the cuts using the obtained formulas. If no cuts are applied, the maximum value of v equals v_m, as given by energy-momentum conservation (25).

Now we can fix the kinematics of the radiative process. We have to express all scalar products and kinematic variables in terms of seven variables: Four variables that define the differential cross section and three integration variables. First, note that all definitions of leptonic variables given in Eq. (2) hold in this case. Hadron and real photon energies are defined by Eq. (24). Hadron momentum and scalar products are given as

$$p_h=\frac{\sqrt{\lambda_W}}{2W},\quad \lambda_W=(W^2-m_h^2-m_u^2-v)^2-4m_h^2W^2,$$

$$V_{1,2}=2k_{1,2}p_h=2[E_{1,2}E_h-p_{1,2}p_h(\cos\theta_h\cos\theta_{1,2}+\sin\theta_h\sin\theta_{1,2}\cos\phi_h)],\tag{26}$$

$$S_t=2pp_h=W^2+m_h^2-m_u^2+V_2-V_1-v$$
$$=S_x+t+M^2-m_u^2-v,$$
$$t=V_2-V_1-Q^2+m_h^2.$$

As in Eq. (5), the last expression is obtained from conservation laws. All hadron kinematic variables depend on inelasticity v but not on photon angles Ω_k. Scalar products containing the photon 4-momentum read

$$2pk=R_w(1-\tau)=2\omega(E_p-p_p\cos\theta_k),$$
$$2k_{1,2}k=\kappa_{1,2}=2\omega[E_{1,2}-p_{1,2}(\cos\theta_k\cos\theta_{1,2}+\sin\theta_k\sin\theta_{1,2}\cos\phi_k)],\tag{27}$$
$$2p_hk=\mu=2\omega[E_h-p_h(\cos\theta_k\cos\theta_h+\sin\theta_k\sin\theta_h\cos(\phi_k-\phi_h)].$$

Here we use also invariant variables $R_w=2k(p+q)$ and $\tau=2kq/R_w$; $R_w=f_Wv/W$.

The leptonic tensor of the radiative process has a more complicated form. It can be written as

$$L^R_{\mu\nu}=\frac{1}{2}\mathrm{Tr}(\hat{k}_2+m)\Gamma_{\mu\alpha}(\hat{k}_1+m)(1+i\gamma_5\hat{\xi})\hat{\Gamma}_{\alpha\nu},$$

$$\Gamma_{\mu\alpha}=\left[\left(\frac{k_{1\alpha}}{kk_1}-\frac{k_{1\alpha}}{kk_1}\right)\gamma_\mu-\frac{\gamma_\mu\hat{k}\gamma\alpha}{2kk_1}-\frac{\gamma_\alpha\hat{k}\gamma\mu}{2kk_2}\right],\tag{28}$$

$$\hat{\Gamma}_{\alpha\nu}=\left[\left(\frac{k_{1\alpha}}{kk_1}-\frac{k_{1\alpha}}{kk_1}\right)\gamma_\mu-\frac{\gamma_\alpha\hat{k}\gamma\nu}{2kk_1}-\frac{\gamma_\nu\hat{k}\gamma\alpha}{2kk_2}\right].$$

After contraction of the radiative leptonic tensor, we obtain

$$M_R^2=-\frac{2e^6}{\tilde{Q}^4}L^R_{\mu\nu}W_{\mu\nu}=-\frac{2e^6}{\tilde{Q}^4R_w}\sum_{i=1}^{5}\theta_i\mathcal{H}_i.\tag{29}$$

R_w was extracted explicitly in order to cancel v coming from Jacobian (22). Arguments of $\mathcal{H}_i$ can be expressed as

$$\tilde{Q}^2=-(q-k)^2=Q^2+R_w\tau,\tag{30}$$

$$\tilde{W}^2=(p+q-k)^2=W^2-R_w,$$
$$\tilde{t}=(q-k-p_h)^2=t-R_w(\tau-\mu).\tag{31}$$

It is well known that the cross section of the radiative process is infrared divergent, which requires careful consideration in order to cancel in the difference. The procedure will be discussed in the next section. Here we can extract the infrared convergent terms in θ_i in separate pieces:

$$\theta_i=\frac{4}{R_w}F_{IR}\theta_i^B+\theta_i^F.\tag{32}$$

The quantities θ_i^B are defined by expressions for θ_i^0 with a reservation that the hadronic quantities V_1,V_2,S_t and vector p_h itself have to be calculated for the radiative kinematics (26). This term originates from the first terms of $\Gamma_{\mu\alpha}$ and $\overline{\Gamma}_{\alpha\nu}$ in definition of Eq. (28). The explicit form of finite parts of these functions θ_i^F are given in the Appendix.

Finally, the cross section of the radiative process ($\sigma_R=d\sigma_R/dQ^2dW^2d\Omega_h$) is given as

$$\sigma_R=-\frac{\alpha^3}{2^9\pi^4S^2W^4}\int d\Omega_k dv\frac{\sqrt{\lambda_W}}{f}\sum_i\frac{\theta_i\mathcal{H}_i}{\tilde{Q}^4},\tag{33}$$

where $f=f_W/W$.

As a cross-check, we consider the soft-photon limit $\omega_{min}<R/2M<\omega_{max}\ll$ all energies and masses. In this case only the first term in Eq. (32) survives,

$$\sigma_R=\frac{2\alpha}{\pi}\left(\log\frac{Q^2}{m^2}-1\right)\log\frac{w_{max}}{w_{min}}\sigma_0.\tag{34}$$

Integration over photonic angles is performed analytically,

$$\int d\Omega_kF_{IR}=-2(l_m-1),\quad l_m=\log\frac{Q^2}{m^2}.\tag{35}$$

IV. INFRARED DIVERGENCE

As was already mentioned, this cross section contains an infrared divergence. Therefore, in order to compute RC, we have to use some regularization method first. We use the method of Bardin and Shumeiko for a covariant treatment of an infrared divergence problem. Basically, we follow the original papers devoted to this topic, namely, Ref. [3] for $e\mu$ elastic scattering and Ref. [9] where exclusive electroproduction was considered. One can find a good and detailed review in Ref. [10].

Following the rules of dimensional regularization, we apply an identity transformation to the radiative cross section assuming that we deal with n-dimensional space:

$$\sigma_R=\sigma_R-\sigma_{IR}+\sigma_{IR}=\sigma_F+\sigma_{IR}.\tag{36}$$

The purpose of the procedure is to separate the cross section into two pieces. The first term $d\sigma_F$ is complicated but infrared free. Infrared divergence is contained in the separate term $d\sigma_{IR}$, which has a quite simple structure and can be analytically calculated within n-dimensional space. In principle, there is some arbitrariness in choosing the form of the subtracted term. Actually only the asymptotical form for R_w (or w) $\rightarrow 0$ is fixed. Another limitation comes from the theorem about a possibility to switch the order of integration and the limit $n \rightarrow 4$. It means that we have to provide uniform convergence of $d\sigma_F$ in the limit. Practically, it is ensured if the subtracted term has a structure F/w, and the difference is $[F(w)-F(0)]/w$.

We define the subtracted part of the radiative cross section as follows: Only the first term from the right-hand side (rhs) of Eq. (32) is kept. It gives the required $1/R_w$ behavior of the radiative cross section. Everywhere else, except for the δ function, we assume $v=0$. This allows us to factorize the Born cross section when calculating the correction:

$$\sigma_{IR} = -\sigma_0 \frac{\alpha}{\pi^2} \int dv \int \frac{d\vec{k}}{\omega} F_{IR} \delta((\Lambda - k)^2 - m_u^2). \quad (37)$$

In the cross section σ_F we can now remove the regularization. The result is

$$\sigma_F = -\frac{\alpha^3}{2^9 \pi^4 S^2 W^2} \int d\Omega_k \frac{dv}{f} \times \sum_i \left[\frac{\sqrt{\lambda_W}}{\tilde{Q}^4} \theta_i \mathcal{H}_i - \frac{4F_{IR}\sqrt{\lambda_W^0}}{Q^4} \theta_i^0 \mathcal{H}_i^0 \right]. \quad (38)$$

For calculation purposes, we split the integration region into two parts separated by the infinitesimal value of inelasticity $\bar{v}$:

$$\sigma_{IR} = \frac{\alpha}{\pi} \delta_R^{IR} \sigma_0 = \frac{\alpha}{\pi} (\delta_S + \delta_H) \sigma_0 \quad (39)$$

with

$$\delta_S = \frac{-1}{\pi} \int_0^{\bar{v}} dv \int \frac{d^{n-1}k}{(2\pi\mu)^{n-4} k_0} F_{IR} \delta((\Lambda - k)^2 - m_u^2),$$
$$\delta_H = \frac{-1}{\pi} \int_{\bar{v}}^{v_m} dv \int \frac{d^3k}{k_0} F_{IR} \delta((\Lambda - k)^2 - m_u^2). \quad (40)$$

The term δ_S^{IR} corresponds to the soft photon contribution, while the term δ_H^{IR} is caused only by hard photons and therefore it does not contain the infrared singularity. Again, in the second contribution the regularization can be removed.

We keep integration over the 3-momentum of the radiated photon in a covariant form. It makes it possible to calculate the integral in any frame. We choose the frame where $\vec{\Lambda} = 0$ (so-called R frame). Integration over inelasticity is external, therefore v is fixed for the integral.

Calculation of the hard-photon contribution is straightforward,

$$\delta_H = (l_m - 1) \log \frac{v_m}{\bar{v}}. \quad (41)$$

For δ_S, we follow Ref. [3] generalizing these calculations to the exclusive electroproduction case.

Using the spherical n-dimensional frame, we have [3,28,29]

$$\delta_v = \frac{1}{\pi} \int_0^{v_m} dv \frac{2\pi^{n/2-1}}{(2\pi\mu_0)^{n-4} \Gamma\left(\frac{n}{2}-1\right)} \int dk_0 k_0^{n-3} \times \int \sin^{n-3}\theta d\theta \left(\frac{k_1}{2k_1k} - \frac{k_2}{2k_2k} \right)^2 \delta(v - 2k_0 m_u). \quad (42)$$

$$F(\alpha, \cos(\theta)) = \left(\frac{k_1}{2k_1k} - \frac{k_2}{2k_2k} \right)^2 = \frac{1}{4k_0^2} \left[\frac{m^2}{E_{1R}^2 (1 - \beta_1 \cos\theta)^2} + \frac{m^2}{E_{2R}^2 (1 - \beta_2 \cos\theta)^2} - \int_0^1 \frac{d\alpha Q^2}{E_\alpha^2 (1 - \beta_\alpha \cos\theta)^2} \right]. \quad (43)$$

Here Feynman parameters are introduced in order to join two denominators. Thus we define the new vector

$$k_\alpha = \alpha k_1 + (1-\alpha) k_2 \quad (44)$$

and quantities $\beta_{1,2,\alpha}$ as a ratio of energy to momentum in the R frame. In terms of invariants, we have

$$\beta_1 = \sqrt{1 + \frac{4m^2 m_u^2}{S_0'^2}}, \quad \beta_2 = \sqrt{1 + \frac{4m^2 m_u^2}{X_0'^2}},$$
$$\beta_\alpha = \sqrt{1 + \frac{4m_u^2[m^2 + \alpha(1-\alpha)Q^2]}{[\alpha S_0' + (1-\alpha) X_0']^2}}. \quad (45)$$

It should be noted that the same polar angles are used in Eq. (43) for all three terms. That is why we can rotate the coordinate system for all the terms and choose the OZ axis along $\vec{k}_{1,2,\alpha}$, respectively. After straightforward integration over the polar angle, we obtain Eq. (43).

The next two steps involve integration over k_0 using the δ function and over v:

$$\int_0^{\bar{v}} dv \int dk_0 k_0^{n-5} \delta(v - 2k_0 m_u) = \frac{1}{n-4} \left(\frac{\bar{v}}{2m_u} \right)^{n-4}. \quad (46)$$

Since the pole $(n-4)^{-1}$ is extracted, we can use expansion in series of $n-4$,

$$\frac{1}{n-4}\left(\frac{\overline{v}}{4\pi\mu_0 m_u}\right)^{n-4}\frac{[\pi(1-xi^2)]^{n/2-2}}{\Gamma\left(\frac{n}{2}-1\right)}$$

$$\to P_{IR}+\log\frac{\overline{v}}{2\mu_0 m_u}+\log\sqrt{1-\xi^2} \qquad (47)$$

with $\xi = \cos\theta$ and standard P_{IR} defined as

$$P_{IR}=\frac{1}{n-4}+\frac{1}{2}\gamma_E+\ln\frac{1}{2\sqrt{\pi}}, \qquad (48)$$

γ_E being the Euler constant. After then we reduce the expression for δ_S to the form

$$\delta_S=\int d\alpha d\xi\left[P_{IR}+\log\frac{\overline{v}}{2\mu_0 m_u}+\log\sqrt{1-\xi^2}\right]F(\alpha,\xi), \qquad (49)$$

which includes only standard integration that can be done using, for instance, tables from Appendix D of Ref. [10]. Integration is straightforward, but it contains an integral usually associated with a so-called S_ϕ function in the Bardin and Shumeiko approach. Here we skip the discussion about properties of the function and give the result in the ultrarelativistic approximation which we use for the entire calculation:

$$S_\phi=\frac{Q^2}{2}\int_0^1\frac{d\alpha}{\beta_\alpha[m^2+\alpha(1-\alpha)Q^2]}\log\frac{1-\beta_\alpha}{1+\beta_\alpha}$$
$$=\frac{1}{2}l_m^2-l_m\log\frac{S_0'X_0'}{m^2m_u^2}-\frac{1}{2}\log^2\frac{S_0'}{X_0'}+\mathrm{Li}_2\left(1-\frac{Q^2m_u^2}{S_0'X_0'}\right)-\frac{\pi^2}{3}. \qquad (50)$$

Finally, we have for δ_S

$$\delta_S=2\left(P_{IR}+\log\frac{\overline{v}}{\mu M}\right)(l_m-1)+\log\frac{S'X'}{m^2M^2}+S_\phi. \qquad (51)$$

The infrared-divergent terms P_{IR}, as well as the parameters μ and $\overline{v}$ are completely canceled in the sum $\delta_S+\delta_H$ with δ_V which is a contribution of the vertex function [Fig. 1(d)]:

$$\delta_V=-2\left(P_{IR}+\log\frac{m}{\mu}\right)(l_m-1)-\frac{1}{2}l_m^2+\frac{3}{2}l_m-2+\frac{\pi^2}{6}. \qquad (52)$$

For this sum, we have

$$\frac{\alpha}{\pi}(\delta_S+\delta_H+\delta_V)=\delta_{inf}+\delta_{VR}, \qquad (53)$$

where

$$\delta_{VR}=\frac{\alpha}{\pi}\left(\frac{3}{2}l_m-2-\frac{1}{2}\ln^2\frac{X_0'}{S_0'}+\mathrm{Li}_2\left[1-\frac{Q^2M^2}{S_0'X_0'}\right]-\frac{\pi^2}{6}\right),$$
$$\delta_{inf}=\frac{\alpha}{\pi}(l_m-1)\ln\frac{v_m^2}{S_0'X_0'}. \qquad (54)$$

Higher-order corrections can be partially taken into account using a special procedure of exponentiation of multiple soft photon radiation. There is an uncertainty about which part of δ_{VR} has to be exponentiated. Within the approach [30], we have to change $1+\delta_{inf}$ to $\exp\delta_{inf}$.

Collecting all the terms, we obtain for the case of meson electroproduction:

$$\sigma_{obs}=\sigma_0 e^{\delta_{inf}}(1+\delta_{VR}+\delta_{vac})+\sigma_F. \qquad (55)$$

Here the corrections δ_{inf} and δ_{vac} come from the radiation of soft photons and the effects of vacuum polarization, the correction δ_{VR} is an infrared-free sum of factorized parts of real and virtual photon radiation, and σ_F is an infrared-free contribution from the bremsstrahlung process.

V. LEADING LOGARITHM APPROXIMATION

In this section we extract the leading logarithm contribution from formulas obtained in the previous section. After then we show that the result coincides with what we obtain from a generalized leading logarithm approximation.

The method of extraction of leading logarithm (or peaking) contributions was first suggested in Ref. [2] (see also papers [30,31]). Formally, we have to calculate residues of the terms $1/z_1$ and $1/z_2$. The corresponding poles appear in τ for $\tau=\tau_s=-Q^2/S$ and $\tau=\tau_x=Q^2/X$. As a result, we have (apart from the factorizable correction) two contributions of hard radiation,

$$\sigma_{LL}=\frac{\alpha}{2\pi}(l_m-1)\left[\left(3+2\log\frac{v_m^2}{S'X'}\right)\sigma_0+\sigma_S+\sigma_X\right]. \qquad (56)$$

Technically, the contributions can be obtained as

$$\int d\Omega_k\theta_i=-8\pi l_m\frac{W^2}{S-Q^2}\frac{1+z_1^2}{z_1(1-z_1)}\theta_i^B,$$
$$\int d\Omega_k\theta_i=-8\pi l_m\frac{W^2}{X+Q^2}\frac{1+z_2^2}{1-z_2}\theta_i^B. \qquad (57)$$

For the two peaks the contributions are, respectively,

$$\sigma_s=\int_0^{v_m}\frac{dv}{S'}\left(\frac{1+z_1^2}{1-z_1}C_s\sigma_s-\frac{2\sigma_0}{1-z_1}\right),$$
$$\sigma_x=\int_0^{v_m}\frac{z_2dv}{X'}\left(\frac{1+z_2^2}{1-z_2}C_x\sigma_x-\frac{2\sigma_0}{1-z_2}\right), \tag{58}$$

where $z_1=1-v/S'$, $z_2=X'/(X'+v)$ and

$$C_{s,x}=\frac{\sqrt{\lambda_W}}{\sqrt{\lambda^0_{Ws,x}}}\frac{W^2_{s,x}}{W^2},$$
$$W_s^2=z_1S-X-z_1Q^2-M^2, \tag{59}$$
$$W_s^2=S-\frac{X}{z_2}-\frac{Q^2}{z_2}-M^2,$$

and $\lambda^0_{Ws,x}$ are calculated in accordance with Eq. (4) using $W^2=W^2_{s,x}$.

These leading logarithm formulas were extracted from our exact expressions given in the previous section. The result can also be obtained using standard leading logarithm techniques [32] from our expression (21). The leading term comes from angular integration of denominators like kk_1 and kk_2. They can be extracted at the level of leptonic tensor

$$L^R_{\mu\nu}=L_{\mu\nu}\left[\frac{1+z_1^2}{1-z_1}\frac{1}{kk_1}+\frac{1+z_2^2}{z_2(1-z_2)}\frac{1}{kk_2}\right]. \tag{60}$$

Only these scalar products in the denominators are subject to angular integration. For our exclusive process, integration yields

$$\int\frac{d\vec{p}_u}{2\epsilon_u}\frac{d\vec{k}}{2\omega}\delta(\Lambda-k-p_u)\frac{1}{kk_{1,2}}=\left[\frac{\pi l_m}{S'},\frac{\pi l_m}{X'}\right]. \tag{61}$$

Now, using Eqs. (21), (60), and (61), we obtain the following leading-logarithm cross section for the radiative process:

$$\frac{d^6\sigma}{d\Gamma^6}=-\frac{\alpha l_m}{2\pi}\left[\frac{1}{S'}\frac{1+z_1^2}{1-z_1}\frac{d^6\sigma_0(z_1k_1,k_2)}{d\tilde{\Gamma}^6}+\frac{1}{X'}\frac{1+z_2^2}{1-z_2}\frac{d^6\sigma_0(k_1,k_2/z_2)}{d\tilde{\tilde{\Gamma}}^6}\right], \tag{62}$$

$$d\Gamma^6=\frac{d^3\vec{k}_2}{2\epsilon_2}\frac{d^3\vec{p}_h}{2\epsilon_h}.$$

While deriving the above results, Eqs. (57), (58), we take into account relations between $z_{1,2}$, v and S' (X'), which follow from the constraint $[\Lambda^2-(1-z_1)k_1]^2-m_u^2=0$ or $[\Lambda^2-(1-1/z_2)k_2]^2-m_u^2=0$. They are

$$v=(1-z_1)S',\quad v=\frac{1-z_2}{z_2}X'. \tag{63}$$

VI. EXCLUSIVE AND INCLUSIVE RADIATIVE CORRECTION

An important consistency test is to show that inclusive RC can be obtained by integration over hadronic angles. It should be noted that this is not trivial because the hadronic angles of radiative and Born cross sections are defined even in different frames. For a definition of the Born angles, we use the center-of-mass frame, while for angles of the radiative process we have to use another frame defined by vectors p and $q-k$. As a result, it leads to quite complicated kinematic relations between these angles.

We start with Eq. (56). After integration over hadronic angles, we have to obtain the leading logarithm cross section ($\sigma^{inc}_{LL}=d\sigma/dW^2dQ^2$) for the inclusive case. The inclusive formula can be found in Refs. [31,13], for example. For double differential cross sections in Q^2 and W^2, it reads

$$\sigma^{inc}_{LL}=\frac{\alpha}{2\pi}(l_m-1)\left[\left(3+2\log\frac{v_m^2}{u_1u_2}\right)\sigma_0^{inc}+\sigma_S^{inc}+\sigma_X^{inc}\right], \tag{64}$$

with

$$\sigma_S^{inc}=\frac{\alpha}{2\pi}l_m\int_{z_1^m}^1 dz_1z_1\frac{1+z_1^2}{1-z_1}\sigma^{inc}_{0S}-\frac{2\sigma_0^{inc}}{1-z_1}, \tag{65}$$

$$\sigma_X^{inc}=\frac{\alpha}{2\pi}l_m\int_{z_2^m}^1\frac{dz_2}{z_2}\frac{1+z_2^2}{1-z_2}\sigma^{inc}_{0X}-\frac{2\sigma_0^{inc}}{1-z_2}, \tag{66}$$

where $z_1^m=X/u_2$ and $z_2^m=u_1/S$.

Let us consider integration over Ω_h of radiative cross sections (i.e., the first term on the left-hand side in Eq. (65)]. The simplest way to relate angles in different frames is to express them in terms of kinematic invariants. We use the following relation:

$$d\Omega_h=\frac{W^2}{\sqrt{\lambda_W^0}}\frac{dV_1^0dV_2^0}{\sqrt{-D(V_1^0,V_2^0)}}, \tag{67}$$

where $D(V_1^0,V_2^0)=[\epsilon(k_1,k_2,p,p_h)]^2$ is the Gramm determinant [see Eq. (14) and Ref. [33]]. For the cross sections σ_S^{inc} and σ_X^{inc} the corresponding expressions are

$$d\Omega_h=\frac{W_s^2}{\sqrt{\lambda^0_{Ws}}}\frac{z_1dV_1dV_2}{\sqrt{-D(z_1V_1,V_2)}} \tag{68}$$

and

$$d\Omega_h=\frac{W_x^2}{\sqrt{\lambda^0_{Wx}}}\frac{dV_1dV_2}{z_2\sqrt{-D(V_1,V_2/z_2)}}. \tag{69}$$

The expression for radiative kinematics is

$$16D(V_1,V_2)=[u_1V_2+u_2V_1-(W^2+m_h^2-m_u^2-v)Q^2]^2 -4(SX-M^2Q^2)(V_1V_2-m_h^2Q^2). \quad (70)$$

The Born case is reached in the limit $v\to 0$. Two comments are in order before explicit integration. The integration area in variables V_1 and V_2 is defined by equation $D(V_1,V_2)=0$ that produces an ellipse. For radiative cross sections σ_S^{inc} and σ_X^{inc} this area is z_1 (or z_2) dependent.

A simple integration procedure follows:

$$\int d\Omega_h \int_0^{v_m} dv \to \int_0^{v_m} dv \int d\Omega_h \to \int_0^{v_m} dv \frac{W^2}{\sqrt{\lambda_W}} \int \frac{dV_1 dV_2}{\sqrt{-D}}$$

$$\to \int dz_1 dV_1^s dV_2 \frac{W^2}{\sqrt{\lambda_W}} \frac{S'}{z_1\sqrt{-D}}$$

$$\to \int dz_1 d\Omega_h^s \frac{W^2}{\sqrt{\lambda_W}} \frac{S'}{z_1\sqrt{-D}} \frac{\sqrt{\lambda_{Ws}}\sqrt{-D_s}}{W_s^2}. \quad (71)$$

The final formulas for the transformation

$$\int d\Omega_h \int_0^{v_m} dv = \int_{z_1^m}^1 dz_1 \int d\Omega_h^s \frac{S'}{C_s} \quad (72)$$

are obtained if we use the equality that can be checked directly,

$$\frac{D_s}{D}=z_1^2, \qquad \frac{D_x}{D}=\frac{1}{z_2^2}. \quad (73)$$

Using Eq. (72), one can see that the exclusive radiative cross section transforms to the inclusive one after integration.

Let us show how to obtain a factorized part of the inclusive correction from the exclusive result. The subtracted part of Eqs. (58) can be rewritten as

$$\int_0^{v_m} \frac{dv}{S'(1-z_1)} = \int_{z_{1m}^1} \frac{dz_1}{1-z_1} - \int_{S'(v=v_m)}^{S'(v=0)} \frac{dS'}{S'}$$

$$= \int_{z_{1m}^1} \frac{dz_1}{1-z_1} - \log\left(\frac{S_0'W}{u_1(W-m_h)}\right). \quad (74)$$

After similar calculation for σ_X, we can see that inclusive RC is reproduced exactly.

VII. NUMERICAL ANALYSIS

Based on the derived analytical formulas, a FORTRAN code EXCLURAD was developed. This code computes RC to the fourfold cross section $(d^4\sigma/dWdQ^2d\cos\theta d\phi_h)$ and to polarization beam asymmetry for processes (1). Both exact and leading logarithm formulas obtained in the previous sections are included. Any value of the inelasticity cut can be optionally chosen. Once the computational algorithm for RC is established, there are two possible ways of incorporating RC into analysis of experimental data on electroproduction: (a) An iteration procedure similar to the one implemented in the RC code POLRAD for inclusive reactions [13] or (b) using realistic models for the structure functions (19) of coincidence electroproduction.

Although the former choice seems attractive due to its model independence, it requires, however, full and precise experimental mapping of the structure functions (19) in the entire range of the kinematic variables needed to compute the radiative process (33). Such a procedure is much more challenging and less efficient than for inclusive processes. Given that such mapping is not available at this time, the only choice left is (b). In this way, RC are applied to the model calculations, and then model parameters are fixed from available experimental data. Thus, RC appear as a necessary intermediate step in extracting model parameters from measurements.

We use the following models for calculation of structure functions (19):

MAID [34]. In this model, baryon resonances are described using Breit-Wigner forms, while background contributions are described using standard Born terms, mixed pseudovector-pseudoscalar πNN coupling and t-channel vector meson exchange. The final amplitudes are constrained by unitarity and gauge invariance. We use two versions of MAID: the earlier one, denoted MAID'98 and the most recent, quoted here as MAID2000.

AO (amplitude and observables) [35]. The amplitudes are parametrized as follows: S-channel resonances are parametrized with relativistic Breit-Wigner forms with momentum-dependent widths. This part of the amplitude is complex. In addition, the s-channel and u-channel pion Born terms are included. These are real numbers. Additional real background amplitudes are used with an energy dependence that has correct threshold behavior.

Another model that can be included into the code EXCLURAD in a simple and straightforward way is the dynamical model of Sato and Lee [36]. In this model, the off-shell nonresonant contributions to $\gamma^* p\to\Delta^+(1232)$ were calculated directly by applying reaction theory within the Hamiltonian formulation underlying "bare" photocoupling form factors. It should be noted that the Sato-Lee model does not include the contributions from higher resonances. Since the computational algorithm for RC does not depend on a particular choice of hadronic structure functions, the addition of any other model does not constitute a problem.

With the models at hand, we can now evaluate numerically the magnitude of RC as a function of various kinematics variables. Let us define the RC factor as follows:

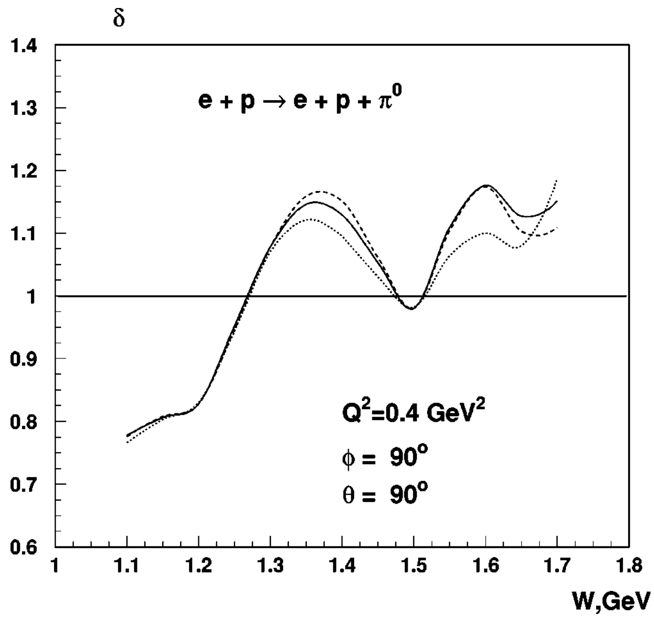

FIG. 3. *W* dependence of RC to the cross section of neutral pion production. The models used are MAID2000 (solid curve), MAID'98 [34] (dashed curve) and AO [35] (dotted curve).

$$\delta = \frac{\sigma_{obs}}{\sigma_0}. \tag{75}$$

For all the following plots, the electron beam energy is $E_{beam} = 1.645$ GeV and no cuts on inelasticity were used, except for Fig. 9. We choose representative kinematics for current experiments with the CLAS detector at Jefferson Lab Hall B. The RC factors to the pion electroproduction cross sections are presented in Figs. 3 and 4 as a function of *W*. In this region, the characteristic features of the cross section vs *W* are strong $\Delta(1232)$ and $S_{11}(1535)$ resonance peaks. One can see that RC is negative at the resonance peaks and positive in the dip regions between the peaks. Thus it tends to weaken the strength of the peaks and fill the dip regions above the given resonances. Also shown in Fig. 3 is the model dependence effect which appears to be noticeable at higher *W* away from the resonance peaks. At higher values of *W*, the magnitude of RC is larger in the charged pion production case. This is mainly due to the wider range of inelasticity v (24) associated with detection of a lighter hadron (i.e., the pion).

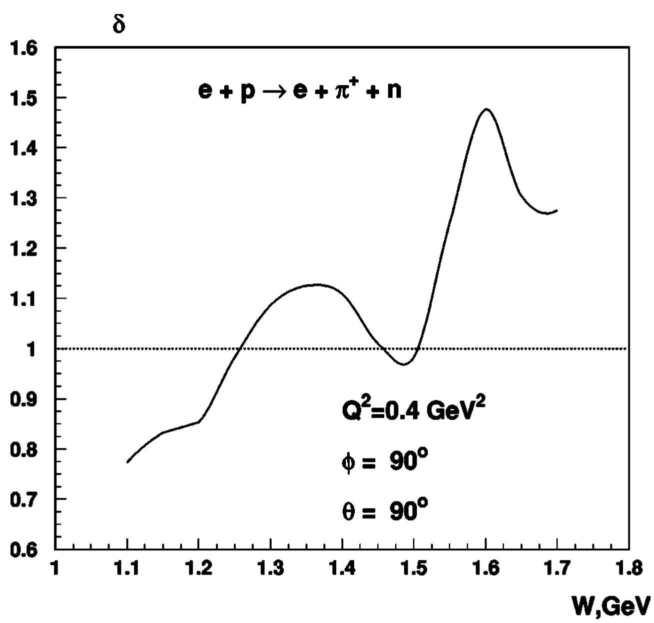

FIG. 4. *W*-dependence of RC to cross section of charged pion production. Kinematics and notation are as in Fig. 3.

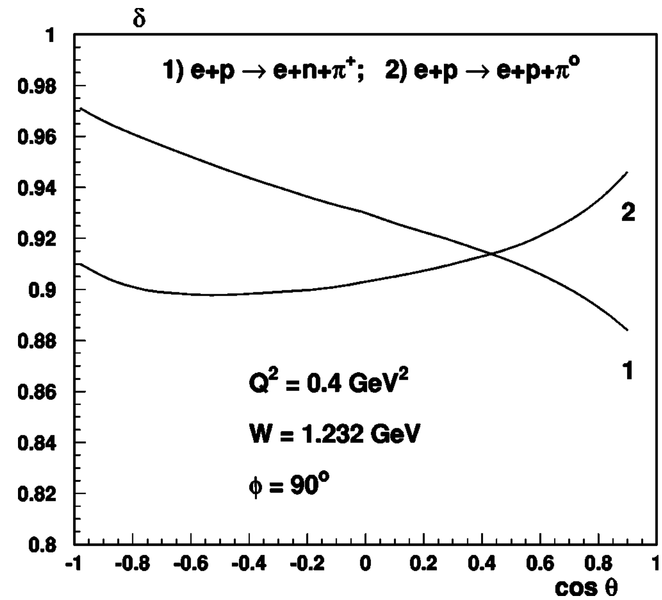

FIG. 5. RC to the cross section as a function of cos *θ*.

Before discussing the angular dependence of RC, let us first comment on the definitions of hadronic angles in order to avoid possible ambiguities. The cms angles θ and ϕ are

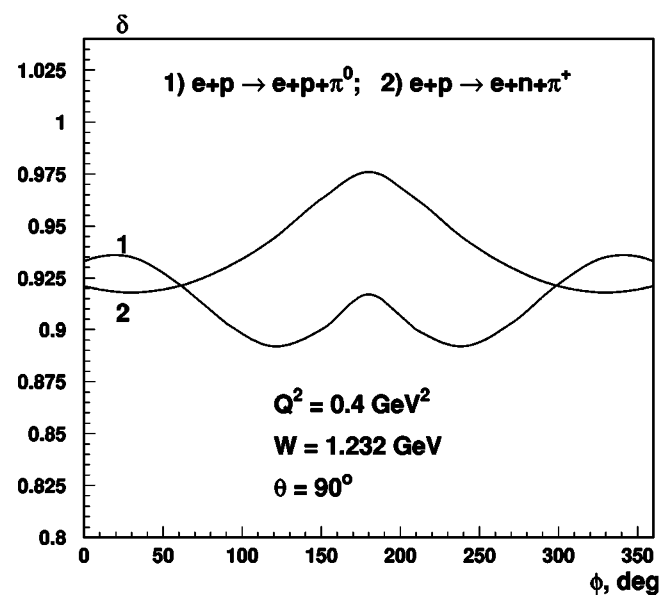

FIG. 6. Dependence of RC to the cross section on the azimuthal angle *ϕ*.

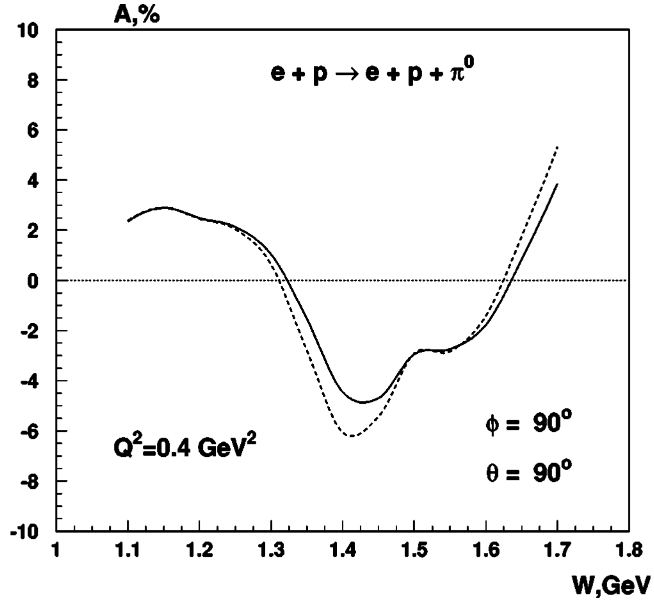

FIG. 7. *W* dependence of the beam polarization asymmetry in neutral pion production. The solid (dashed) curve denote the asymmetry with (without) RC. MAID2000 was used to compute the structure functions.

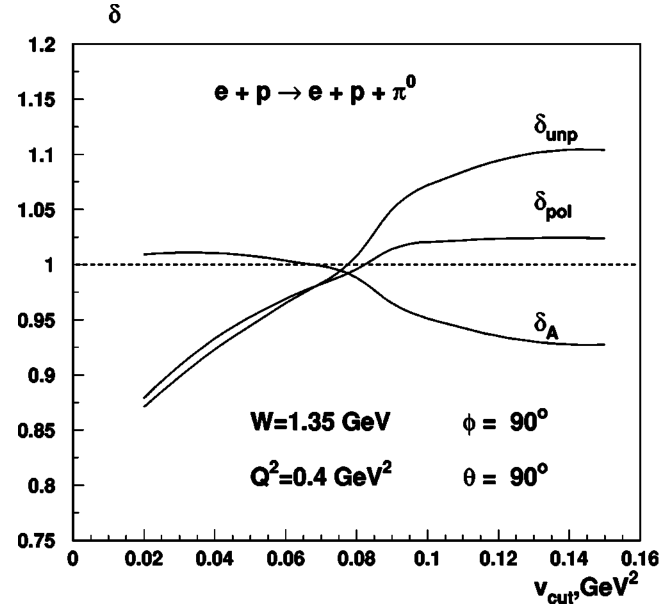

FIG. 9. Dependence of RC to cross section and polarization beam asymmetry on the inelasticity cut v for neutral pion production. The quantity δ_{unp} is RC to the unpolarized part of the cross section, δ_{pol} is RC to the polarized part of the cross section, and $\delta_A = \delta_{unp}/\delta_{pol}$, i.e., it is RC to the beam polarization asymmetry. MAID2000 was used for structure functions.

between the direction of momentum lost by electrons ($\vec{q} = \vec{k}_1 - \vec{k}_2$) and momentum of the final pion $\vec{p}_h$, provided that the pion is detected. For the neutral pion production case, the convention is to also use the direction of pion momentum reconstructed from kinematics. In order to follow the convention for the neutral pion production, we define θ and ϕ with respect to the direction of $-\vec{p}_h$ *opposite* to the final proton momentum, while keeping in mind that it is different from the final pion momentum for the radiative processes Figs. 1(b) and 1(c).

The angular dependence of RC is shown in Figs. 5 and 6, where it is plotted as a function of cos θ and ϕ, respectively. The kinematics corresponds to the $\Delta(1232)$ peak, where RC leads to suppression of the cross section. It can be seen from Fig. 5 that approaching the forward direction at the given value of ϕ, the magnitude of RC as a function of cos θ for the charged-pion channel smoothly increases from about 3% to 12%, while for the neutral pions it decreases from 10% to 5%. A common feature for both the channels is that RC is larger in magnitude for the parallel kinematics, when the detected hadron moves along the transferred momentum. The RC factor varies as a function of azimuthal angle ϕ as well (Fig. 6). Dependence of RC on the angle ϕ has important implications for the super-Rosenbluth separation of electroproduction structure functions, and the θ dependence would affect the partial-wave analysis, resulting in corrections to electroexcitation parameters of baryon resonances.

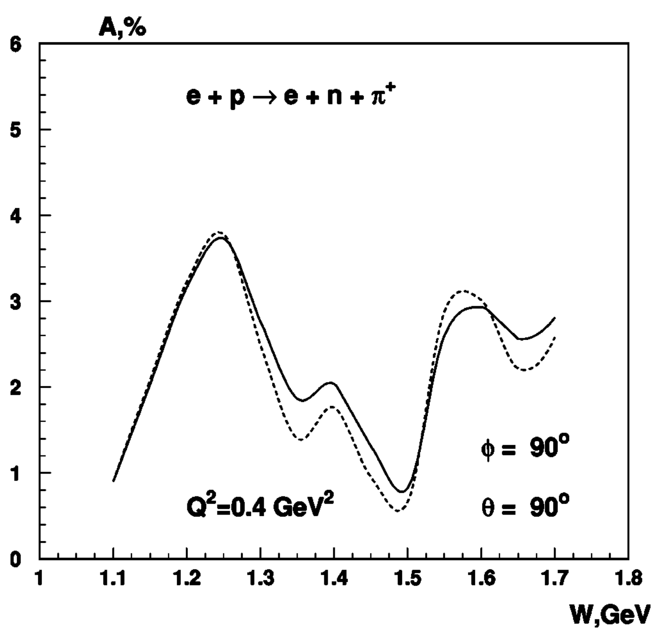

FIG. 8. The beam polarization asymmetry in charged pion production as a function of *W* with (solid curve) and without (dashed curve) RC. The notation is as in Fig. 7.

The beam polarization asymmetry is plotted in Figs. 7 and 8 for the neutral and charged channels, respectively. One can see that for the asymmetry, RC changes from enhancement to suppression when passing across the Δ and S_{11} resonance regions. The RC factor is most substantial in the dip regions between resonances. Figure 9 demonstrates the dependence on inelasticity cut v_{cut} (24). It can be seen that for the smaller values of the cut, resulting in the selection of softer bremsstrahlung photons, RC to both polarization-dependent and polarization-independent parts of the cross section are almost the same, resulting in a small correction to polarization asymmetry. As v_{cut} increases, the correction to the asymmetry also increases due to the hard-photon emission coming into play.

Any experimental spectrum for electroproduction contains a radiative tail that, due to the finite energy resolution, cannot be experimentally separated from Born contribution. In practice, detected events are included into data analysis up to a certain (cut) limit and are interpreted as radiative events. The contribution of radiative events should be calculated theoretically using the same inelasticity cut (v_{cut}). The resulting contribution of the radiative events is subtracted from the integrated experimental spectrum in order to obtain the value of Born cross section that, naturally, should be cut independent.

The FORTRAN code EXCLURAD can be downloaded from http://www.jlab.org/RC or obtained directly from the authors.

VIII. DISCUSSION AND CONCLUSION

In this paper we obtain explicit formulas for the lowest-order QED radiative correction to cross section and polarization beam asymmetry in the exclusive pion electroproduction. Analytic formulas are tested in several ways. Apart from traditional cross-checks like soft-photon and leading-logarithm limits, it is found that integration with respect to the hadronic angles reproduces the inclusive radiative correction.

A FORTRAN code EXCLURAD is developed on the basis of the analytic formulas. Numerical analysis carried out for Jefferson Lab kinematic conditions shows the following.

Radiative correction to the cross section of electroproduction is very sensitive to the cut on inelasticity. The harder cut eliminates contributions from the higher-energy part of the bremsstrahlung spectrum, thus leading to the smaller magnitude of RC to the polarization asymmetry.

RC to cross sections can be as high as several tens of percent.

RC may vary depending on the chosen model for electroproduction structure functions. The proposed RC procedure may be viewed as a necessary intermediate step in interpretation of experimental results in terms of model parameters. An iteration procedure may be required for the regions where model dependence is substantial.

RC have a nontrivial angular dependence in $\cos\theta$ and ϕ. This is particularly significant, as these are often used as input to extract structure functions and partial wave amplitudes.

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APPENDIX

In this appendix we give formulas for θ_i^F [Eq. (32)]. In the unpolarized case,

$$\theta_i^F=\theta_{i2}+R_w\theta_{i3},$$

$$\theta_{12}=4F_{IR}\tau,$$

$$\theta_{13}=-4F-2F_d,\tau^2,$$

$$2\theta_{22}=-4M^2F_{IR}\tau-F_dS_p^2\tau+F_{1+}S_pS_x+2F_{2-}S_p+2F_{IR}S_x,$$

$$2\theta_{23}=4M^2F+2M^2F_d\tau^2-F_dS_x\tau-F_{1+}S_p,$$

$$2\theta_{32}=-4m_h^2F_{IR}\tau-F_d\tau V_+^2+F_{1+}V_-V_++2F_{2-}\mu V_++2\mu F_{IR}V_-, \tag{A1}$$

$$2\theta_{33}=4m_h^2F+2m_h^2F_d\tau^2-F_d\mu\tau V_--F_{1+}\mu V_+,$$

$$2\theta_{42}=-2F_dS_p\tau V_++F_{1+}S_pV_-+F_{1+}S_xV_++2F_{2-}\mu S_p+2F_{2-}V_++2\mu F_{IR}S_x-4F_{IR}S_m\tau+2F_{IR}V_-,$$

$$2\theta_{43}=4FS_m-F_d\mu S_x\tau+2F_dS_m\tau^2-F_d\tau V_--F_{1+}\mu S_p-F_{1+}V_+.$$

Here $V_\pm=V_1\pm V_2$,

$$F_d=\frac{1}{\kappa_1\kappa_2}, \tag{A2}$$

$$F_{1+}=\frac{1}{\kappa_1}+\frac{1}{\kappa_2}, \tag{A3}$$

$$F_{2\pm}=\left(\frac{m^2}{\kappa_2^2}\pm\frac{m^2}{\kappa_1^2}\right), \tag{A4}$$

$$F_{IR}=F_{2+}-Q^2F_d, \tag{A5}$$

where $\kappa_{1,2}$ are defined in Eq. (27).

$$\theta_{52}=4\left\{2F_{11}E_2+2F_{22}E_1+F_d[E_{12}\tau-Q^2(E_1+E_2)]+\frac{2}{S}(E_1S-E_{12}\tau+E_{12}-E_2S)\right\},$$

$$\theta_{53}=2\left(F_{1+}(E_2-E_1)-F_d(E_1+E_2)\tau+\frac{4}{S}F_{11}E_2(\tau-1)\right), \tag{A6}$$

$$E_{12}=\epsilon(k_1,k_2,p_1,p_h)=\frac{1}{4}[-(V_1X-Q^2S_m+SV_2)^2+4(SX-M^2Q^2)(Q^2m_h^2-V_1V_2)]^{1/2},$$

$$E_1=\epsilon(k,k_1,p_1,p_h)=\frac{1}{4}\{-[R_wV_1(\tau-1)-R_w\mu S+S_m\kappa_1]^2+4[\kappa_1M^2+R_w(\tau-1)S](-\kappa_1m_h^2+V_1R_w\mu)\}^{1/2}, \tag{A7}$$

$$E_2=\epsilon(k,k_2,p_1,p_h)=\frac{1}{4}\{-[R_wV_2(\tau-1)-R_w\mu X+S_m\kappa_2]^2+4[\kappa_2M^2+R_w(\tau-1)X](-\kappa_2m_h^2+V_2R_w\mu)\}^{1/2}.$$

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