Cabibbo allowed $D \to K\pi\gamma$ decays

S. Fajfer
Department of Physics, University of Ljubljana, Jadranska 19, 1000 Ljubljana, Slovenia
J. Stefan Institute, Jamova 39, P. O. Box 300, 1001 Ljubljana, Slovenia

A. Prapotnik
J. Stefan Institute, Jamova 39, P. O. Box 300, 1001 Ljubljana, Slovenia

P. Singer
Department of Physics, Technion - Israel Institute of Technology, Haifa 32000, Israel
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The weak radiative Cabibbo allowed decays $D^+ \to \bar{K}^0 \pi^+ \gamma$ and $D^0 \to K^- \pi^+ \gamma$ with nonresonant $K\pi$ are investigated by relying on the factorization approximation for the nonleptonic weak transitions and the model which combines the heavy quark effective theory and the chiral Lagrangian approach. The dominant contributions to the amplitudes come from the long-distance effects. The decay amplitude has both parity violating and parity conserving parts. The parity violating part also includes a bremsstrahlung contribution. The branching ratio obtained for the parity conserving part is of the order 10^{-4} for the $D^0 \to K^- \pi^+ \gamma$ decay and 10^{-5} for $D^+ \to \bar{K}^0 \pi^+ \gamma$, when the effect of light vector mesons is included, and smaller otherwise. The branching ratio for the parity violating part with a photon energy cut of 50 MeV is close to 10^{-3} for the D^0 decay and 4×10^{-4} for the D^+ decay. We present Dalitz plots and energy spectra for both transitions as derived from our model and we probe the role of the light vector mesons in these decays.

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I. INTRODUCTION

The investigation of radiative and dilepton weak decays of pseudoscalar charm mesons has been pursued rather vigorously in recent years, both theoretically and experimentally. To a certain extent, this activity has been fueled by the ongoing search for physics beyond the standard model, which might be of measurable consequence in certain charm radiative and dilepton decays [1–4]. To date, no radiative or dilepton weak decay of D has been detected. However, upper bounds have been established for a sizable number of these decays. The radiative decays $D^0 \to \rho^0,\ \omega^0,\ \phi,\ \bar{K}^{*0} + \gamma$ were recently bounded [5] to branching ratios in the 10^{-4} range, which is approaching the standard model expectations (see, e.g., Refs. [6,7] where additional previous works are mentioned). The dilepton decays $D \to P l^+ l^-$, $D \to V l^+ l^-$ are the subject of intensive searches at CLEO and Fermilab [8]. Here again, with upper bounds of $10^{-5} - 10^{-4}$ for branching ratios of the various modes one approaches the expectations of the standard model [2–4,9]. The situation should improve in the future, due to new possibilities for observation of charm meson decays at BELLE, BABAR, and Tevatron. Recently, upper limits in the $10^{-5} - 10^{-4}$ range were [10] also established for D^0 dilepton decays with two nonresonant pseudoscalar mesons in the final state $D^0 \to (\pi^+\pi^-, K^-\pi^+, K^+K^-)\ \mu^+\mu^-$, though no comparable results are available yet for similar photonic decays.

In the present work, we go beyond the existing treatments which deal with $D \to V\gamma$ only and we consider the three-body D radiative decays of type $D \to K\pi\gamma$, with nonresonant $K - \pi$. We undertake here the study of the Cabibbo allowed decays $D^+ \to \bar{K}^0 \pi^+ \gamma$ and $D^0 \to K^- \pi^+ \gamma$, which we consider to be the most likely candidates for early detection. These decays are the charm sector counterpart of the $K \to \pi\pi\gamma$ [11–13] decays, which have provided a wealth of information on meson dynamics. In the strange sector, the $K^+ \to \pi^+ \pi^0 \gamma$ and $K_L \to \pi^+ \pi^- \gamma$ are singled out as the most suitable ones for the investigation of the radiative decay mechanism; this, since the relative suppresion of the corresponding $K^+ \to \pi^+ \pi^0$ and $K_L \to \pi^+ \pi^-$ amplitudes leads to a situation where the direct radiative transition is not overwhelmed by the bremsstrahlung part.

In the $K \to \pi\pi\gamma$ decays, the long-distance contribution is dominant [13]. In the charm radiative decays, the theoretical studies show that likewise, the long-distance is the dominant feature of the decays [2,4,6,7,9]. The short-distance contribution realized by the penguin diagram $c \to u\gamma$ [14–16] might play a role in certain Cabibbo suppressed decays, which are not discussed in the present paper.

Our problem belongs to the sector of nonleptonic charm decays, which is known to represent a continuing theoretical challenge (see, e.g., [17,18], and references therein). The short distance effects are considered well understood but the perturbative techniques required for the evaluation of certain matrix elements are based on approximate models. Usually the factorization approximation is used (see, e.g., Ref. [17,19]), although the experimental data indicate the apparent need for the inclusion of nonfactorizable amplitudes in certain channels.

In this first treatment of $D \to K\pi\gamma$ decays we use the factorization approximation for the calculation of weak transition elements. We consider the use of this approach to be justified by the "near" success of the approach for the nonleptonic amplitudes. This will involve its use in the

$D(D^*)K\pi$ vertices as well as in $D(D^*)\to V$ and $D(D^*)\to P$ transitions, all of which required for the calculations of the $D\to K\pi\gamma$ amplitudes within our model. For the evaluation of the $(D,D^*)\to(P,V)$ transitions, we use the information obtained for these matrix elements from semileptonic decays (see, e.g., Ref. [20]). The general theoretical framework for our calculation is that of the heavy quark chiral Lagrangian [21,22]. In the $K\to\pi\pi\gamma$ decays, it has been shown that intermediate light vector mesons play an important role in the decay amplitude [11]. We shall investigate the role of intermediate light vector mesons also in the $D\to K\pi\gamma$ amplitude. In order to accomplish this, we use the extension of the formalism of Refs. [21,22] to include also the light vector mesons [23,24].

The present study of $D^+\to\overline{K}^0\pi^+\gamma$ and $D^0\to K^-\pi^+\gamma$ shows that the direct part of the radiative amplitude is not much smaller in strength than the bremsstrahlung part, rather similarly to the inhibited K decays mentioned above. If confirmed by experiments, it places these decays in the status of a most suitable ground for the investigation of the mechanisms involved in such nonleptonic D decays. Moreover, our calculation shows that the nonresonant $K\pi\gamma$ final states are dominant, having a partial width which is larger than the resonant $\overline{K}^*\gamma$ one by at least an order of magnitude. This outcome is mainly due to the contribution of the light vector mesons in the crossed channels.

In Sec. II we present the theoretical framework for our calculation. In Sec. III we display the explicit expressions of all the calculated decay amplitudes. Section IV contains the discussion and the summary.

II. THE THEORETICAL FRAMEWORK

The nonradiative two-body D decays, from which the bremsstrahlung part of the radiative decays originates, are $D^0\to K^-\pi^+$ and $D^+\to\overline{K}^0\pi^+$. The weak $\Delta I=1$ transition leads to two independent isospin amplitudes in the final state, $A_{1/2}$ and $A_{3/2}$ and the relations to the physical decays is [18]

$$A(D^+\to\overline{K}^0\pi^+)=A_{3/2};\quad A(D^0\to K^-\pi^+)=\frac{2}{3}A_{1/2}+\frac{1}{3}A_{3/2}. \tag{1}$$

From the determined branching ratios [25] of $\mathrm{BR}(D^+\to\overline{K}^0\pi^+)=(2.89\pm0.26)\%$ and $\mathrm{BR}(D^0\to K^-\pi^+)=(3.83\pm0.09)\%$ one learns that the relative size of the absolute values of the amplitudes $|A(D^0\to K^-\pi^+)|/|A(D^+\to\overline{K}^0\pi^+)|$ is 1.84. Using also the information from the third decay, $A(D^0\to\overline{K}^0\pi^0)=(\sqrt{2}/3)A_{1/2}-(\sqrt{2}/3)A_{3/2}$, the isospin analysis shows that $|A_{1/2}|/|A_{3/2}|\simeq2.7$, and their relative phase is 90° [26]. Despite this knowledge, there is still no complete interpretation for the mechanisms leading to the decays [26], although it is clear that the situation is different from the $K\to\pi\pi$ channels, where $\Delta I=1/2$ enhancement introduces a large disparity between the final state isospin amplitudes. The relevance of the above picture to the radiative decays will be discussed in the last section.

Since our problem of describing the $D\to K\pi\gamma$ decays involves transitions between heavy mesons and light pseudoscalars, we adopt the effective Lagrangian [21,22] which contains both the heavy flavor and the $SU(3)_L\times SU(3)_R$ chiral symmetry as the theoretical framework for our calculation. From the experience with $K\to\pi\pi\gamma$ decays, one knows [11–13] that the decay amplitude is largely determined by contributions from virtual vector mesons. Considering the possibilty that vector mesons would play a role in the $D\to K\pi\gamma$ decays as well (we remind the reader that we consider here nonresonant $K\pi$, the decays $D\to K^*\gamma$ having been treated separately [16,27]), we should complement the Lagrangian by introducing light vector mesons. For this we choose the generalization of the original Lagrangian [21,22] by Casalbuoni *et al.* [23] in which the original symmetry is broken spontaneously to diagonal $SU(3)_V$ [28] with the introduction of the light vector mesons. We present this formalism here in some detail (for more details see Ref. [27]) and use it as the main tool of our calculation. We shall also perform the calculations without vector mesons in the Lagrangian in which case the original heavy quark chiral Lagrangian [21,22] is used, in order to clarify their role in these decays.

The light degrees of freedom are described by the 3×3 Hermitian matrices

$$\Pi=\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}}+\frac{\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} & K^0 \\ K^- & \overline{K^0} & -\frac{2\eta_8}{\sqrt{6}}+\frac{\eta_0}{\sqrt{3}} \end{pmatrix} \tag{2}$$

and

$$\rho_\mu=\begin{pmatrix} \frac{\rho^0_\mu+\omega_\mu}{\sqrt{2}} & \rho^+_\mu & K^{*+}_\mu \\ \rho^-_\mu & \frac{-\rho^0_\mu+\omega_\mu}{\sqrt{2}} & K^{*0}_\mu \\ K^{*-}_\mu & \overline{K}^{*0}_\mu & \Phi_\mu \end{pmatrix} \tag{3}$$

for the pseudoscalar and vector mesons, respectively. They are usually expressed through the combinations

$$u=\exp\left(\frac{i\Pi}{f}\right), \tag{4}$$

where $f\simeq f_\pi=132$ MeV is the pion pseudoscalar decay constant and

$$\hat{\rho}_\mu=i\frac{g_v}{\sqrt{2}}\rho_\mu, \tag{5}$$

where $g_v=5.9$ was fixed in the case of exact flavor symmetry [28]. In the following we will also use the gauge field tensor $F_{\mu\nu}(\hat{\rho})$

$$F_{\mu\nu}(\hat{\rho})=\partial_\mu\hat{\rho}_\nu-\partial_\nu\hat{\rho}_\mu+[\hat{\rho}_\mu,\hat{\rho}_\nu]. \tag{6}$$

It is convenient to introduce two currents $\mathcal{V}_\mu=\frac{1}{2}(u^\dagger D_\mu u+uD_\mu u^\dagger)$ and $\mathcal{A}_\mu=\frac{1}{2}(u^\dagger D_\mu u-uD_\mu u^\dagger)$. The covariant derivative of u and $u^\dagger$ is defined as $D_\mu u=(\partial_\mu+\hat{B}_\mu)u$ and $D_\mu u^\dagger=(\partial_\mu+\hat{B}_\mu)u^\dagger$, with $\hat{B}_\mu=ieB_\mu Q$, $Q=\mathrm{diag}(2/3,-1/3,-1/3)$, B_μ being the photon field.

The light meson part of the strong Lagrangian can be written as [28]

$$\mathcal{L}_{\mathrm{light}}=-\frac{f^2}{2}\{\mathrm{tr}(\mathcal{A}_\mu\mathcal{A}^\mu)+a\,\mathrm{tr}[(\mathcal{V}_\mu-\hat{\rho}_\mu)^2]\}+\frac{1}{2g_v^2}\mathrm{tr}[F_{\mu\nu}(\hat{\rho})F^{\mu\nu}(\hat{\rho})]. \tag{7}$$

The constant a in Eq. (7) is in principle a free parameter. In the case of exact vector meson dominance (VDM) $a=2$ [28,29]. However, the photoproduction and decays data indicate [30] that the $SU(3)$ breaking modifies the VDM in

$$\mathcal{L}_{V-\gamma}=-eg_v f^2 B_\mu\left(\rho^{0\mu}+\frac{1}{3}\omega^\mu-\frac{\sqrt{2}}{3}\Phi^\mu\right). \tag{8}$$

Instead of the exact $SU(3)$ limit ($g_v=m_V/f$), we shall use the measured values, defining

$$\langle V(\epsilon_V,q)|V_\mu|0\rangle=i\epsilon_\mu^*(q)g_V(q^2). \tag{9}$$

The couplings $g_V(m_V^2)$ are obtained from the leptonic decays of these mesons. In our calculation we use $g_\rho(m_\rho^2)\simeq g_\rho(0)=0.17\ \mathrm{GeV}^2$, $g_\omega(m_\omega^2)\simeq g_\omega(0)=0.15\ \mathrm{GeV}^2$, and $g_\Phi(m_\Phi^2)\simeq g_\Phi(0)=0.24\ \mathrm{GeV}^2$.

Both the heavy pseudoscalar and the heavy vector mesons are incorporated in a 4×4 matrix

$$H_a=\frac{1}{2}(1+\not{v})(P_{a\mu}^*\gamma^\mu-P_a\gamma_5), \tag{10}$$

where $a=1,2,3$ is the $SU(3)_V$ index of the light flavors, and $P_{a\mu}^*$, P_a, annihilate a spin 1 and spin 0 heavy meson $Q\bar{q}_a$ of velocity v, respectively. They have a mass dimension 3/2 instead of the usual 1, so that the Lagrangian is in the heavy quark limit $m_Q\to\infty$ explicitly mass independent. Defining moreover

$$\bar{H}_a=\gamma^0H_a^\dagger\gamma^0=(P_{a\mu}^{*\dagger}\gamma^\mu+P_a^\dagger\gamma_5)\frac{1}{2}(1+\not{v}), \tag{11}$$

we can write the strong Lagrangian as [24]

$$\mathcal{L}_{\mathrm{even}}=\mathcal{L}_{\mathrm{light}}+i\mathrm{Tr}(H_av_\mu D^\mu\bar{H}_a)+ig\mathrm{Tr}[H_b\gamma_\mu\gamma_5(\mathcal{A}^\mu)_{ba}\bar{H}_a]+i\tilde{\beta}\mathrm{Tr}[H_bv_\mu(\mathcal{V}^\mu-\hat{\rho}_\mu)_{ba}\bar{H}_a], \tag{12}$$

where $D^\mu\bar{H}_a=(\partial_\mu+\mathcal{V}_\mu-ieQ'B_\mu)\bar{H}_a$, with $Q'=2/3$ for the c quark.

The coupling g can be fixed [31] by using the data [32] on the $D^*\to D\pi$ decay width. These data give $g=0.59$. The plus sign is taken to be in agreement with the quark model studies. The parameter $\tilde{\beta}$ is less known, but it seems that it can be safely neglected [17].

The electromagnetic field can couple to the mesons also through the anomalous interaction; i.e., through the odd parity Lagrangian. The contributions to this Lagrangian arise from terms of the Wess-Zumino-Witten kind, given by [29,33]

$$\mathcal{L}_{\mathrm{odd}}^{(1)}=-4\frac{C_{VV\Pi}}{f}\epsilon^{\mu\nu\alpha\beta}\mathrm{Tr}(\partial_\mu\rho_\nu\partial_\alpha\rho_\beta\Pi). \tag{13}$$

The coupling $C_{VV\Pi}$ can be determined in the case of the exact $SU(3)$ flavor symmetry following the hidden symmetry approach of Refs. [28,29] and it is found to be $C_{VV\Pi}=3g_v^2/32\pi^2=0.33$. In the actual calculation, we allowed for $SU(3)$ symmetry breaking and we used the $VP\gamma$ coupling as determined from experiment [25]. We will also need the odd-parity Lagrangian in the heavy sector. Such terms are required by the $D^*\to D\gamma$ transition, which cannot be generated from Eq. (12). There are two contributions [24,34] in it, characterized by coupling strengths λ and λ'. The first is given by

$$\mathcal{L}_1=i\lambda\mathrm{Tr}[H_a\sigma_{\mu\nu}F^{\mu\nu}(\hat{\rho})_{ab}\bar{H}_b]. \tag{14}$$

In this term the interactions of light vector mesons with heavy pseudoscalar or heavy vector mesons is described. The light vector meson can then couple to the photon by the standard VDM prescription. This term is of the order $1/\lambda_\chi$ with λ_χ being the chiral perturbation theory scale [35].

The second term gives the direct heavy quark-photon interaction and is generated by the Lagrangian

$$\mathcal{L}_2=-\lambda'\mathrm{Tr}[H_a\sigma_{\mu\nu}F^{\mu\nu}(B)\bar{H}_a]. \tag{15}$$

The parameter λ' is given in heavy quark symmetry limit by $\lambda'\simeq-1/(6m_c)$ [22] and it should be considered as a higher order term in $1/m_Q$ expansion [36].

In order to gain information on these couplings one has to use the existing data on $D^{*0}\to D^0\gamma$, $D^{*+}\to D^+\gamma$, and $D_s^{*+}\to D_s^+\gamma$ decays. Experimentally, the ratios $R_\gamma^0=\Gamma(D^{*0}\to D^0\gamma)/\Gamma(D^{*0}\to D^0\pi^0)$ and $R_\gamma^+=\Gamma(D^{*+}\to D^+\gamma)/\Gamma(D^{*+}\to D^+\pi^0)$ are known [25]. These data determine two possibilities [27]. One of them is $|\lambda/g|=0.839\ \mathrm{GeV}^{-1}$, $|\lambda'/g|=0.175\ \mathrm{GeV}^{-1}$. The second one does not agree with present data. With $g=0.59$ we obtain $\lambda=\pm0.49\ \mathrm{GeV}^{-1}$ and $\lambda'=\pm0.102\ \mathrm{GeV}^{-1}$.

The $\lambda'\simeq-1/(6m_c)$ would give with the mass of charm quark $m_c=1.4$ GeV that $\lambda'=-0.12\ \mathrm{GeV}^{-1}$, in good agreement with the above value. The simple quark model analysis indicates that λ' and λ are both negative [36]. In our numerical calculations we give the results using these parameters. In the literature (e.g., Refs. [31,36,37]) instead of λ the β parameter is often used. The value $\beta = 2.3\ \mathrm{GeV}^{-1}$ corresponds to $\lambda = -0.49\ \mathrm{GeV}^{-1}$, since $2\lambda(g_v/\sqrt{2})\ [g_\rho/m_\rho^2 + g_\omega/(3m_\omega^2)] = -(2/3)\beta$.

In addition to strong and electromagnetic interactions, we have to specify the weak one. The nonleptonic weak Lagrangian on the quark level for the Cabibbo allowed decays can be written as usual [19]

$$\mathcal{L}_{\mathrm{NL}}^{\mathrm{eff}}(\Delta c = \Delta s = 1) = -\frac{G_F}{\sqrt{2}} V_{ud} V_{cs}^* [a_1 O_1 + a_2 O_2], \quad (16)$$

where

$$O_1 = (\overline{u}d)_{V-A}^{\mu} (\overline{s}c)_{V-A,\mu}$$

and

$$O_2 = (\overline{u}c)_{V-A,\mu} (\overline{s}d)_{V-A}^{\mu},$$

V_{ij} are the CKM matrix elements, G_F is the Fermi constant, and $(\overline{\Psi}_1 \Psi_2)^\mu \equiv \overline{\Psi}_1 \gamma^\mu (1-\gamma^5) \Psi_2$. In our calculation we use $a_1 = 1.26$ and $a_2 = -0.55$ as found in Ref. [19].

At the hadronic level, the weak current transforms as $(\overline{3}_L, 1_R)$ under chiral $SU(3)_L \times SU(3)_R$, is linear in the heavy meson fields D^a and D_μ^{*a} and is taken as [20]

$$J_{Q_a}^{\mu} = \frac{1}{2} i\alpha Tr[\gamma^\mu (1-\gamma_5) H_b u_{ba}^\dagger] + \alpha_1 Tr[\gamma_5 H_b (\hat{\rho}^\mu - \mathcal{V}^\mu)_{bc} u_{ca}^\dagger] + \alpha_2 Tr[\gamma^\mu \gamma_5 H_b v_\alpha (\hat{\rho}^\alpha - \mathcal{V}^\alpha)_{bc} u_{ca}^\dagger] + \cdots, \quad (17)$$

where $\alpha = f_H \sqrt{m_H}$ [21], α_1 was first introduced by Casalbuoni *et al.* [23], while α_2 was introduced in Ref. [20]. It has to be included, since it is of the same order in the $1/m_Q$ and chiral expansion as the term proportional to α_1 [20].

The relevant matrix element is parametrized usually in $D \to V l \nu_l$ semileptonic decay as [16,19,20,38]

$$\begin{aligned}\langle V(p_V, \epsilon_V) | (V-A)^\mu | D(p) \rangle &= \frac{2V(q^2)}{m_D + m_V} \epsilon^{\mu\nu\alpha\beta} \epsilon_{V\nu}^* p_\alpha p_{V\beta} + i \epsilon_V^* q \frac{2m_V}{q^2} q_\mu \\ &\times [A_3(q^2) - A_0(q^2)] + i(m_D + m_V) \\ &\times [\epsilon_{V\mu}^* A_1(q^2)] - \frac{\epsilon_V^* q}{m_D + m_V} [(p + p_V)_\mu A_2(q^2)],\end{aligned} \quad (18)$$

where $q = p - p_V$. In order that these matrix elements should be finite at $q^2 = 0$, the form factors satisfy the relation [19]

$$A_3(q^2) - \frac{m_H + m_V}{2m_V} A_1(q^2) + \frac{m_H - m_V}{2m_V} A_2(q^2) = 0, \quad (19)$$

and $A_3(0) = A_0(0)$. We take the following expressions for the form factors at $q_{\max}^2$ [23] (we differ slightly from Ref. [23] which does not include α_2, but includes also heavy scalars as intermediate terms)

$$V(q_{\max}^2) = \frac{1}{\sqrt{2}} \lambda g_v f_D \frac{M+m}{M+\Delta}, \quad (20)$$

$$A_1(q_{\max}^2) = \frac{-\sqrt{2}\alpha_1 g_v \sqrt{M}}{M+m}, \quad (21)$$

and

$$A_2(q_{\max}^2) = -\frac{2g_v}{\sqrt{2}} \frac{M+m}{M^{3/2}} \alpha_2, \quad (22)$$

where Δ stands for the D^* and D mass difference. Assuming the pole dominance one can connect the value of form factors at $q_{\max}^2$ and 0 momentum transfer by $F(0) = F(q_{\max})[1 - (M-m)^2/M_p^2]$, where F stands for V, A_1, or A_2, M is the D meson mass; and m is the light vector meson mass. (For a discussion of a different approach which makes little numerical difference in our case, see Charles *et al.* [39].) Using the experimental data [25] $|V^{DK^*}(0)| = 1.02 \pm 0.12$, $|A_1^{DK^*}(0)| = 0.55 \pm 0.03$, and $|A_2^{DK^*}(0)| = 0.40 \pm 0.07$, we find for the couplings $\lambda = -0.56\ \mathrm{GeV}^{-1}$, $|\alpha_1| = 0.156\ \mathrm{GeV}^{1/2}$, $|\alpha_2| = 0.052\ \mathrm{GeV}^{1/2}$. The value of λ is in good agreement with results obtained from $D^* \to D\gamma$ data.

The light weak current is derived to be [24]

$$J_{ij}^{\mu} = i f^2 \{ u [\mathcal{A}^\mu - a(\mathcal{V}^\mu - \hat{\rho}^\mu)] u^\dagger \}_{ji}. \quad (23)$$

The photon emission is obtained by gauging the weak sector too. The important consequence of this procedure is that thereby the gauge invariance of the whole amplitude is achieved. This turns out to be equivalent to the usual procedure of achieving gauge invariance in bremsstrahlung processes with a momentum dependent strong vertex, as pointed out [40] for the somewhat similar process $V \to PP'\gamma$. Actually by gauging the weak sector we produce the same graphs, which were necessary to induce to satisfy the gauge invariance [40].

III. THE DECAY AMPLITUDES

The general Lorentz decomposition of the $D(P) \to K(p) \pi(q) \gamma(k, \epsilon)$ decay amplitude is given by

$$\mathcal{M} = -\frac{G_f}{\sqrt{2}} V_{du} V_{cs}^* \left(\overline{F}_1 \left[\frac{q \cdot \varepsilon}{q \cdot k} - \frac{p \cdot \varepsilon}{p \cdot k} \right] + F_2 \varepsilon^{\mu\alpha\beta\gamma} \varepsilon_\mu v_\alpha k_\beta q_\gamma \right). \quad (24)$$

The part of the amplitude containing the $\overline{F}_1$ form factor is parity violating (PV), while the one with F_2 is parity conserving (PC). Both of them are functions of scalar products of momenta as $k \cdot p$, $k \cdot q$. Note that $\overline{F}_1$ contains contributions which arise from bremsstrahlung part of the amplitude

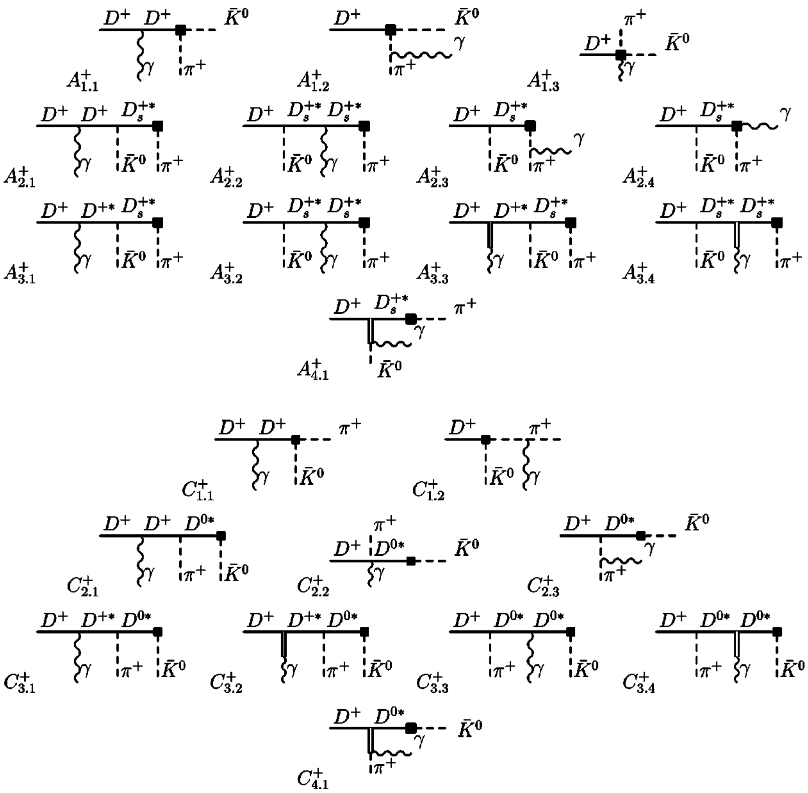

FIG. 1. Feynman diagrams contributing to the form factor $\overline{F}_1$ of the $D^+\to\overline{K}^0\pi^+\gamma$ decay. Diagrams denoted by $A^+_{i,j}$ ($C^+_{i,j}$) come from the operator O_1 (O_2). Sum of the contributions of each row is gauge invariant. In diagrams $A^+_{3,1}$, $A^+_{3,2}$, $C^+_{3,1}$, and $C^+_{3,3}$ the photon couples to the heavy mesons with strength λ'.

as well as a direct electric transition. On the other hand, F_2 corresponds to the magnetic transition.

In order to determine $\overline{F}_1$, F_2 we use the model described in the previous section. The diagrams contributing to these form factors are given in Figs. 1–4. In Figs. 1 and 2 are given Feynman diagrams contributing to the $D^+\to\overline{K}^0\pi^+\gamma$ decay amplitude while the contributions to the $D^0\to K^-\pi^+\gamma$ decay amplitude are presented in Figs. 3 and 4. Note that we denote heavy mesons by one full line, light pseudoscalar mesons by dashed lines, light vector mesons by two full lines, and photons by wavy lines. The weak vertex is denoted by a square box.

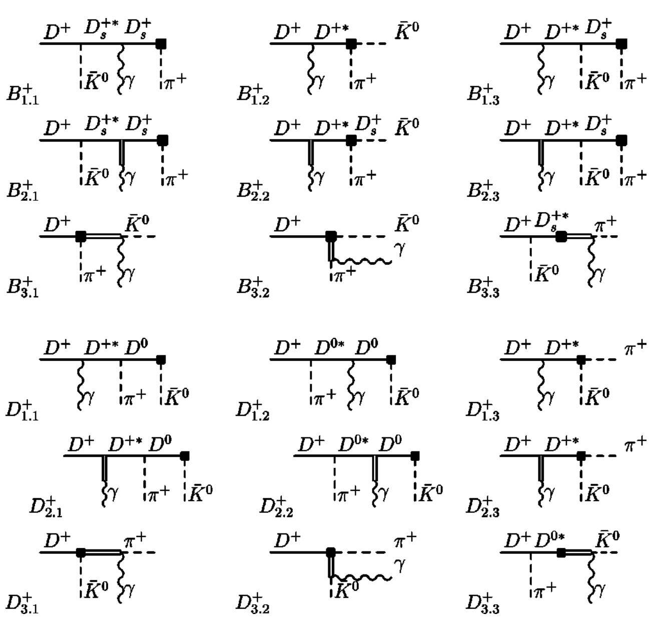

FIG. 2. Feynman diagrams contributing to the form factor F_2 of the $D^+\to\overline{K}^0\pi^+\gamma$ decay. Diagrams denoted by $B^+_{i,j}$ ($D^+_{i,j}$) come from the operator O_1 (O_2).

Before proceeding to the actual calculation, we note the following complication. As well known, the leading terms of the expansion of the radiative amplitude in the photon momentum (k) are determined [41] by the original amplitude ($D\to K\pi$ in our case). However, the nonleptonic $D\to K\pi$ amplitude cannot be calculated accurately in the factorization approximation from the diagrams provided by our model. Such a calculation gives a rather good result for the $D^+\to\overline{K}^0\pi^+$ channel but is less successful for the $D^0\to K^-\pi^+$ decay. In order to overcome this problem and to be able to present accurately the bremsstrahlung component of the radiative transition, we shall use an alternative approach for its derivation. This approach then is to use the values of the experimental amplitudes $D\to K\pi$, assumed to have no internal structure, for the calculation of the bremsstrahlung component. In order to accommodate this we rewrite the decay amplitude (24) as

$$\mathcal{M}=-\frac{G_f}{\sqrt{2}}V_{du}V^*_{cs}\bigg(F_0\bigg[\frac{q\cdot\varepsilon}{q\cdot k}-\frac{p\cdot\varepsilon}{p\cdot k}\bigg]+F_1[(q\cdot\varepsilon)(p\cdot k)-(p\cdot\varepsilon)(q\cdot k)]+F_2\varepsilon^{\mu\alpha\beta\gamma}\varepsilon_\mu v_\alpha k_\beta q_\gamma\bigg), \qquad (25)$$

where F_0 is the experimentally determined $D\to K\pi$ amplitude and F_1, F_2 are the form factors of the electric and

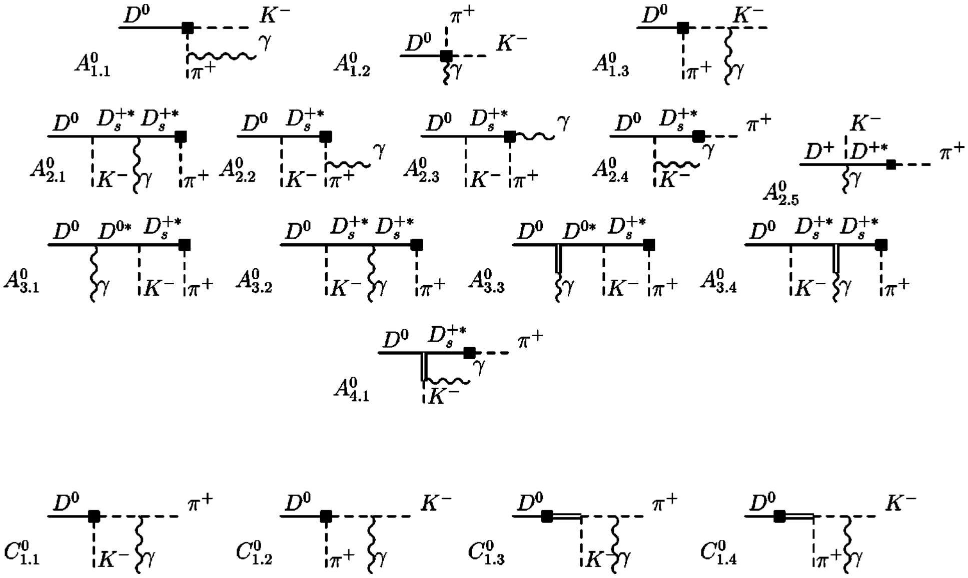

FIG. 3. Feynman diagrams contributing to the form factor $\overline{F}_1$ of the $D^0 \to K^- \pi^+ \gamma$ decay. Diagrams denoted by $A^+_{i,j}$ ($C^+_{i,j}$) come from the operator O_1 (O_2). Sum of the contributions of each row is gauge invariant. In diagrams $A^0_{3,1}$ and $A^0_{3,2}$ the photon couples to the heavy mesons with strength λ'.

magnetic direct transitions which we calculate with our model. At this point, it is worth remarking that our procedure introduces a certain inequality between the treatments for the bremsstrahlung and the direct emission. By using the experimental amplitude of $D \to K\pi$ we employ the result of the "full theory" for this part, while the direct emission is a first order calculation. Nevertheless, our procedure ensures the satisfaction of the Low theorem and we believe it to be optimal at this stage of our model-dependent calculational ability. When intermediate states appear to be on the mass shell, we use Breit-Wigner formula. However, we remark at this point that since we are interested in the $D \to (K\pi)_{\rm nonres}\gamma$ transitions, we delete the region of the K^* resonance appearing in diagram $D^0_{1,2}$ and we retain only the region in $(p+q)^2$ which is beyond $m_{K^*} \pm \Gamma_{K^*}$.

In Appendix A we present explicitly expressions for the form factors for the decay $D^+ \to \overline{K}^0 \pi^+ \gamma$ using the notations A_i^+, B_i^+, etc. The amplitude A_i^+, B_i^+, etc., is obtained as a sum of the amplitudes presented by the corresponding diagrams in the ith row in Figs. 1–4. Each A_i^+ (or B_i^+, etc.) is gauge invariant. For the electric parity - violating transition, we define both the total amplitude provided by the model $\overline{F}_1$, as well as the direct part only, F_1, obtained after deleting the bremsstrahlung diagrams. Then the amplitudes for $D^+ \to \overline{K}^0 \pi^+ \gamma$ are

$$\overline{F}_1(D^+ \to \overline{K}^0 \pi^+ \gamma) = \sum_{i=1}^{4} (A_i^+ + C_i^+), \tag{26}$$

$$F_1(D^+ \to \overline{K}^0 \pi^+ \gamma) = \frac{1}{(p\cdot k)(q\cdot k)} \sum_{i=3}^{4} (A_i^+ + C_i^+), \tag{27}$$

$$F_2(D^+ \to \overline{K}^0 \pi^+ \gamma) = \sum_{i=1}^{3} (B_i^+ + D_i^+). \tag{28}$$

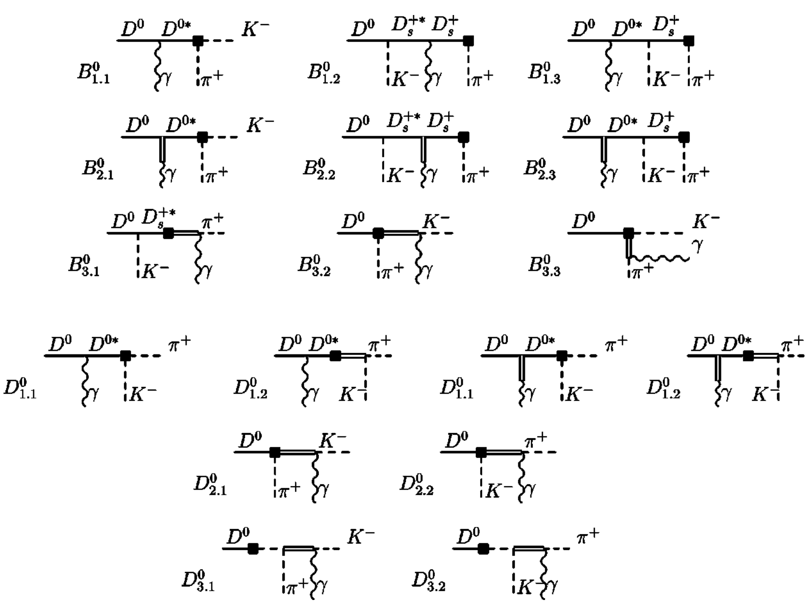

FIG. 4. Feynman diagrams contributing to the formfactor F_2 of the $D^0 \to K^- \pi^+ \gamma$ decay. Diagrams denoted by $B^0_{i,j}$ ($D^0_{i,j}$) come from the operator O_1 (O_2).

In the case of $D^0\to K^-\pi^+\gamma$ we have

$$\overline{F}_1(D^0\to K^-\pi^+\gamma)=\sum_{i=1}^{4} A_i^0+C_1^0, \tag{29}$$

$$F_1(D^0\to K^-\pi^+\gamma)=\frac{1}{(p\cdot k)(q\cdot k)}\sum_{i=3}^{4} A_i^0, \tag{30}$$

$$F_2(D^0\to K^-\pi^+\gamma)=\sum_{i=1}^{3} (B_i^0+D_i^0), \tag{31}$$

where A_i^0, B_i^0, etc., are gauge invariant sums of the amplitudes arising from the graphs in the ith row. In Appendices B we present the form factors for the $D^0\to K^-\pi^+\gamma$ decay. We denoted by $A_i^{+,0}$, $B_i^{+,0}$ contributions which are created by O_1 operator and by $C_i^{+,0}$, $D_i^{+,0}$ contributions caused by O_2.

As we mentioned, in the calculation we used the experimental value of $A(D\to K\pi)$ to calculate the bremsstrahlung part. The differential cross section of the decays is given by

$$d\Gamma=\frac{1}{(2\pi)^3}\frac{1}{32M^3}|\mathcal{M}|^2dm_{12}^2dm_{23}^2, \tag{32}$$

where $\mathcal{M}$ is the decay amplitude, given by Eq. (24) or (25), $m_{12}^2=(P-k)^2$, and $m_{23}^2=(P-p)^2$, where P, k, p are, respectively, the four-momenta of the D meson, photon, and K meson. The total decay width is written as $\Gamma=\Gamma_{\rm PC}+\Gamma_{\rm PV}$, where $\Gamma_{\rm PC}$ contains the contribution of F_2 and $\Gamma_{\rm PV}$ contains the contributions of F_0, F_1 (the PC and PV amplitudes do not interfere in the total width). Before giving numerical results, we make a few comments.

The expressions for the amplitudes given in the Appendices contain several constants. A few are well determined (we use values given in Ref. [25]) and require no further explanation; as to the rest for f_D we use the lattice result, $f_D=207$ MeV [42] and for $f_{D*}=1.13f_D$. The couplings g, λ, λ' are determined as previously explained and we use $g=0.59$, $\lambda=-0.49$ GeV^{-1}, and $\lambda'=-0.102$ GeV^{-1}. The masses of D and D_s mesons are denoted by M and M_s, respectively.

Some of the amplitudes, such as $A_{2,1}^0$, $A_{2,4}^0$, $A_{2,5}^0$, all $A_{3,i}^0$ etc., contain the weak transition D^* to π. The D^* meson is off-shell, though heavy quark effective theory (HQET) requires it not to be too much off. This feature is reflected in its propagator as well. Since there is a delta function of momenta at the weak vertex, this requires the pion to have essentially a momentum of order Mv, i.e., the mass of the heavy meson. When applying the D^* propagator function, one finds that such graphs vanish in the heavy quark limit, thus giving a very small contribution.

Concerning the masses of light mesons (K,π) we keep the physical values in phase space and we approximate them by zero in the heavy quark propagators. In the explicit expressions for amplitudes given in Appendixes, all the indicated masses are taken at their physical values.

Turning now to the presentation of the results we have to start with a discussion of the bremsstrahlung contribution (IB). In our model IB is given by diagrams $(\mathrm{IB})^0=\Sigma_i(A_{1,i}^0+A_{2,i}^0+C_{1,i}^0)$ for the $D^0\to K^-\pi^+\gamma$ decay (the first two rows and the fifth row of Fig. 3) and by diagrams $(\mathrm{IB})^+=\Sigma_j(A_{1,j}^++A_{2,j}^++C_{1,j}^++C_{2,j}^+)$, i.e., the first two rows and rows 5 and 6 of Fig. 1 for the $D^+\to\overline{K^0}\pi^+\gamma$ decay. Now, in the limit of vanishing photon energy, the first two terms in the expansion of the IB amplitude in terms of the photon energy, obey the Low theorem [41]. Although this is satisfied theoretically, the question arises wheather the $D\to K\pi$ amplitude, as derived from our model, describes correctly the observed $D^+\to\overline{K^0}\pi^+$, $D^0\to K^-\pi^+$ decays. We calculated the amplitudes of these decays using our model and we find that the branching ratios obtained with the factorization approximation are 4.1 and 17%, respectively, compared with observed branching ratios of 2.9 and 3.9% [25]. It appears that although the model is reasonable for $D^+\to\overline{K^0}\pi^+$ (the $\Delta I=3/2$ amplitude), it misses the amplitude of $D^0\to K^-\pi^+$ by a factor of 2. On the one hand, this gives us a certain reassurance on the suitabilty of the model we use for calculating the radiative amplitudes. On the other hand, we shall perform also an alternative calculation, whereby the bremsstrahlung amplitudes of the model are deleted from the total radiative amplitude and replaced by the "experimental amplitude." This procedure is undertaken in order to enforce the fulfilment of the Low theorem for our radiative amplitudes. Thus, we assume constant $D\to K\pi$ amplitude of correct magnitude to reproduce the observed rates of $D^+\to\overline{K^0}\pi^+$ and $D^0\to K^-\pi^+$, from which we calculate the bremsstrahlung (IB) amplitudes. These have the form of the first term in Eq. (25) with constant F_0. To this we add the F_2 terms of the magnetic transition, which is not affected by this procedure, as well as the parity-violating F_1 terms not belonging to $(\mathrm{IB})^0$ and $(\mathrm{IB})^+$ diagrams. These F_1 terms then represent the direct electric transition of the radiative amplitude. We present results for both these alternative procedures. Although, the procedure based on the experimental $D\to K\pi$ amplitudes is apparently more reliable, we consider the "model" calculation to be of intrinsic value, setting out the ground for future calculations.

There is one more item to be explained. We are interested in the role played by the vector mesons in these decays; obviously not in the direct $K\pi$ channel, which belongs to $D^0\to\overline{K}^{*0}\gamma$ and was treated separately [6,7,16], rather as they appear as intermediate particles in VDM (e.g., diagrams $A_{3,4}^+$, $C_{3,4}^+$, $D_{2,3}^+$, $A_{3,3}^+$, and others) or in the crossed channels (e.g., diagrams $A_{4,1}^+$, $D_{3,1}^+$, $D_{3,2}^+$, $C_{1,3}^0$, $B_{3,2}^+$, and others). This is the main reason for our using an effective Lagrangian which contains the light vector mesons [23]. However, we calculate the radiative transitions also without including vector mesons in the Lagrangian, i.e., we drop all diagrams containing a double line in Figs. 1–4 (gauge invariance is maintained), which allows to elucidate their role in these decays.

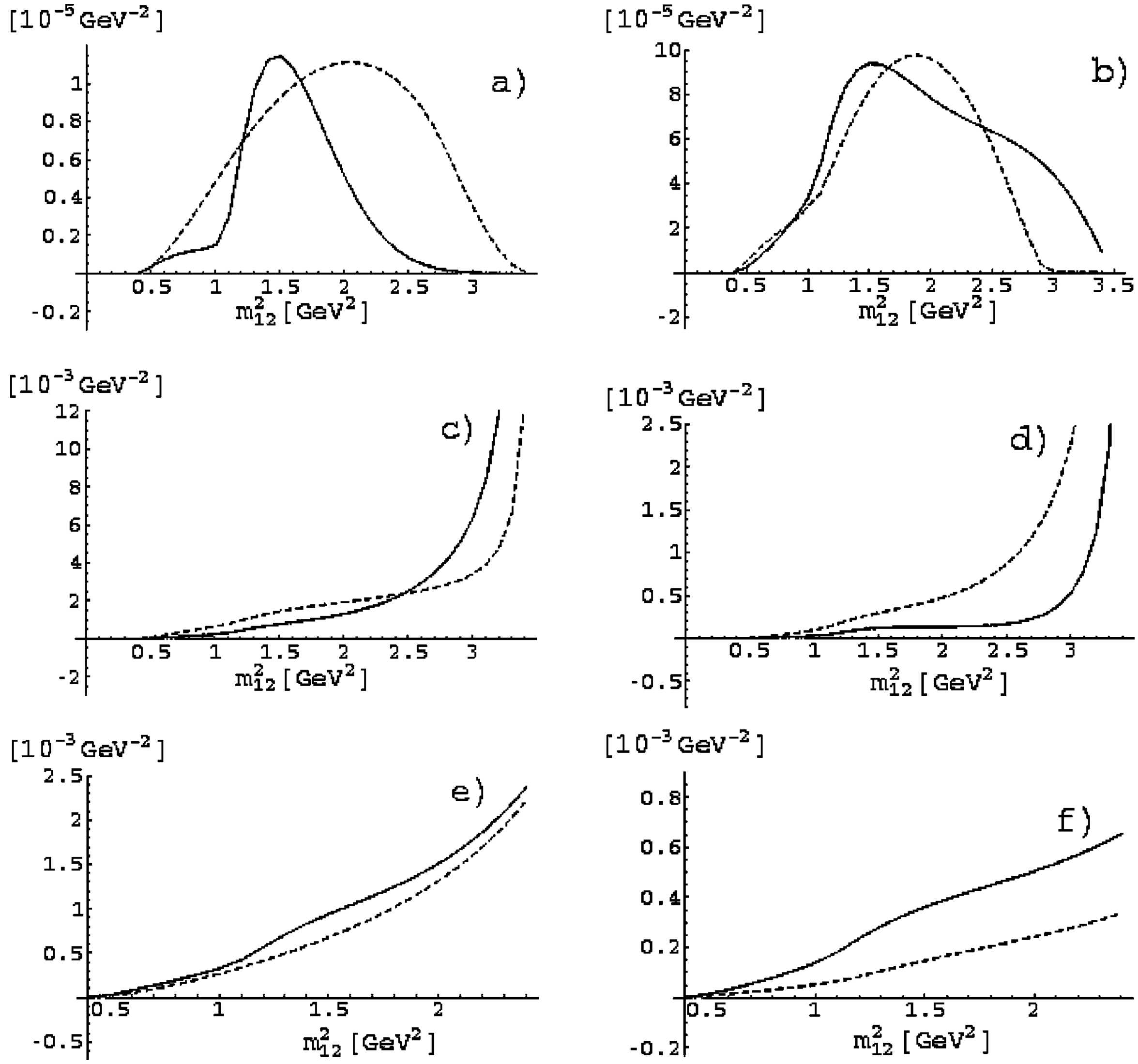

FIG. 5. $(1/\Gamma_{\rm total})(d\Gamma/dm_{12})$ for the decay $D^+\to\overline{K}^0\pi^+\gamma$ (left) and $D^0\to K^-\pi^+\gamma$ (right). Above: direct parity-conserving (dashed line) and parity-violating (putting $F_0=0$) (full line) terms. Middle: $(1/\Gamma_{\rm total})(d\Gamma/dm_{12})$ with Γ containing the full decay amplitudes, for model (dashed line) and model+exp. (full line). For the latter, maximal F_0, F_1 interference is exhibited. Below: $(1/\Gamma_{total})(d\Gamma/dm_{12})$ with Γ for the radiative decay calculated from model+exp. (full line) compared to pure bremsstrahlung emission (dashed line).

For the parity conserving part of the decays, representing the magnetic transition, we obtain

$$\mathrm{BR}(D^+\to\overline{K}^0\pi^+\gamma)_{\rm PC}=2.0\times10^{-5}, \tag{33}$$

$$\mathrm{BR}(D^0\to K^-\pi^+\gamma)_{\rm PC}=1.4\times10^{-4}. \tag{34}$$

If we disregard the contribution of vector mesons, the rates are reduced to $\mathrm{BR}(D^+\to\overline{K}^0\pi^+\gamma)_{\rm PC}^{\rm no\,VM}=\ 3.0\times10^{-6}$ and $\mathrm{BR}(D^0\to K^-\pi^+\gamma)_{\rm PC}^{\rm no\,VM}=\ 6.6\times10^{-7}$. The decrease is sharper for the D^0 decay, since in this case the light vector mesons gave the dominant contribution to the rate, this is not the case for D^+ where such a contribution is doubly Cabibbo suppressed. The differential distribution for these transitions, as a function of $m_{12}^2=(P-k)^2$, is given as the dashed line distribution in Figs. 5(a), 5(b), for these two decays. The distribution is mainly symmetrical, with the peak occuring at $k\simeq400$ MeV. Thus, this is the region in which the effect of the direct transition has best chance for detection.

Turning to the parity-violating transitions we start with the procedure whereby we enforce the Low theorem by using Eq. (25). Here we face the question of unknown phase between F_0 and F_1. We give therfore the results in terms of a range, limited by minimal and maximal interference between F_0 and F_1.

Thus, we get for the branching ratios of the electric transitions, with $|F_0|$ determined experimentally,

$$\mathrm{BR}(D^+\to\overline{K}^0\pi^+\gamma)_{\rm PV,ex}^{k>50\ \rm MeV}=(3.6-3.8)\times10^{-4}, \tag{35}$$

$$\mathrm{BR}(D^+\to\overline{K}^0\pi^+\gamma)_{\rm PV,ex}^{k>100\ \rm MeV}=(2.3-2.5)\times10^{-4}. \tag{36}$$

For the D^0 radiative decay we get

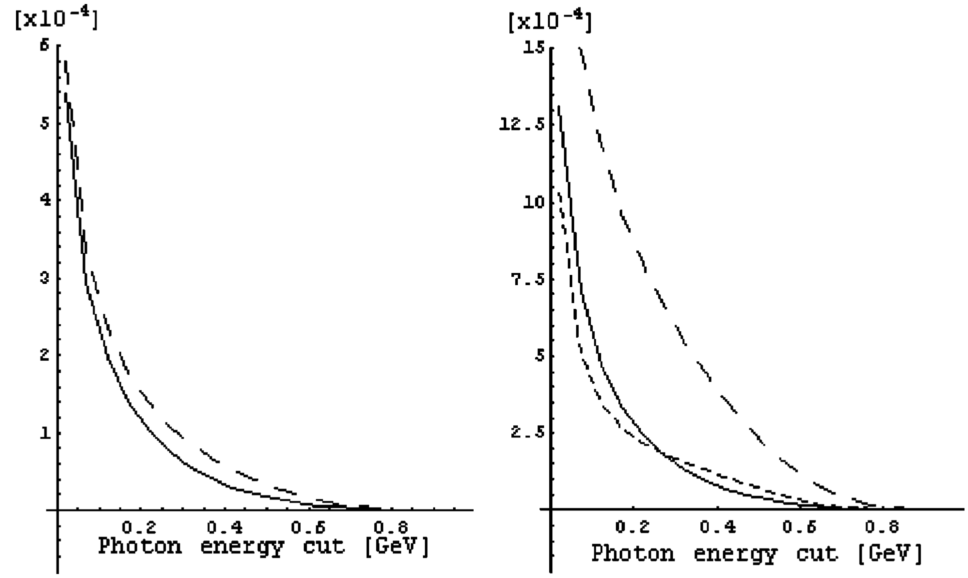

FIG. 6. Branching ratio of radiative decays. Left: decay $D^+\to\bar{K}^0\pi^+\gamma$, right: decay $D^0\to K^-\pi^+\gamma$, full line: bremsstrahlung only. Long dashed line: all contributions included and positive F_0, F_1 interference; short dashed line: negative interference.

$$\mathrm{BR}(D^0\to K^-\pi^+\gamma)^{k>50\ \mathrm{MeV}}_{\mathrm{PV,ex}}=(5.0-15)\times 10^{-4}, \quad (37)$$

$$\mathrm{BR}(D^0\to K^-\pi^+\gamma)^{k>100\ \mathrm{MeV}}_{\mathrm{PV,ex}}=(2.6-11)\times 10^{-4}. \quad (38)$$

The uncertainty in the F_0/F_1 phase is less of a problem in $D^+\to\bar{K}^0\pi^+\gamma$ than in $D^0\to K^-\pi^+\gamma$. If we take the bremsstrahlung amplitude alone as determined from the knowledge of $|F_0|$, disregarding the direct electric F_1 term, the above numbers are replaced by 3.6×10^{-4} and 2.3×10^{-4} for D^+ decay and 8.6×10^{-4} and 5.5×10^{-4} for the D^0 decay. In Fig. 6 we also show the dependence of the branching ratio of the bremsstrahlung amplitude on the lower energy bound, for both decays. The contribution of the direct parity violating term (putting $F_0=0$), is $\mathrm{BR}(D^+\to\bar{K}^0\pi^+\gamma)_{\mathrm{dir,PV}}=1.0\times 10^{-5}$ and $\mathrm{BR}(D^0\to K^-\pi^+\gamma)_{\mathrm{dir,PV}}=1.64\times 10^{-4}$.

We also checked the effect of the vector mesons for the PV transition. Using again the formalism presented in Sec. II, we found that in the PV transition the effect of vector mesons is rather negligible; there is practically no change in Eqs. (35), (36) and only a a narrowing of the range in Eqs. (37) and (38), to bring it essentially to the values of pure bremsstrahlung we indicated after Eq. (38).

We have calculated the decay rates also by using our model, for the whole radiative amplitudes i.e., using all graphs of Figs. 1–4. Comparing these results with those of Eqs. (35)–(38) gives an indication of the possible uncertainty of our model. We obtain

$$\mathrm{BR}(D^+\to\bar{K}^0\pi^+\gamma)^{k>50\ \mathrm{MeV}}_{\mathrm{PV,model}}=3.0\times 10^{-4}, \quad (39)$$

$$\mathrm{BR}(D^+\to\bar{K}^0\pi^+\gamma)^{k>100\ \mathrm{MeV}}_{\mathrm{PV,model}}=2.5\times 10^{-4}. \quad (40)$$

For the D^0 radiative decay we get

$$\mathrm{BR}(D^0\to K^-\pi^+\gamma)^{k>50\ \mathrm{MeV}}_{\mathrm{PV,model}}=2.3\times 10^{-3}, \quad (41)$$

$$\mathrm{BR}(D^0\to K^-\pi^+\gamma)^{k>100\ \mathrm{MeV}}_{\mathrm{PV,model}}=1.5\times 10^{-3}. \quad (42)$$

In Figs. 5(c), 5(d), we compare the rate of the decays $d\Gamma=d\Gamma_{\mathrm{PC}}+d\Gamma_{\mathrm{PV}}$ for the two alternative calculations concerning the PV part. In Fig. 5(e), 5(f) we compare the rate of decay, $d\Gamma=d\Gamma_{\mathrm{PC}}+d\Gamma_{\mathrm{PV}}$ calculated from Eq. (25) to the bremsstrahlung rate, to emphasize the feasibilty of detecting the direct emission. Finally, in Fig. 7 we present Dalitz plots for these decays.

In concluding this section, we wish to reemphasize that in all the figures presented we include the contribution of non-virtual vector mesons only in the crossed channels (i.e., $\rho\to\pi\gamma$, $K^*\to K\gamma$, etc.). Such contributions are visible in the Dalitz plots. On the other hand, we do not include the direct channel contribution of $K^*\to K\pi$, since what we calculate here is the nonresonant $D\to K\pi\gamma$ mode. Nor do we include any other form of final state $K\pi$ interaction. In a comparison with data, a possible $K\pi$ peak arising from $\bar{K}^*$ should be deleted. We add that we have calculated the contribution of $\bar{K}^*$ to the partial width in the direct channel and we find it to be more than ten times smaller than the nonresonant rate (which includes the vector mesons only in crossed channels), in agreement with previous estimates [8,10] for the $D\to V\gamma$ modes.

IV. DISCUSSION AND SUMMARY

The calculation we presented is the first attempt to formulate a theoretical framework for decays of type $D\to K\pi\gamma$, with nonresonant $K\pi$. The calculational framework is the strong Lagrangian (12)–(15) used in the tree approximation, and factorization for the weak matrix elements. In the present article we treat the Cabibbo allowed decays and among these, only channels which have both inner bremsstrahlung and direct radiation components $D^+\to\bar{K}^0\pi^+\gamma$ and $D^0\to K^-\pi^+\gamma$, i.e., those which are the most likely ones for

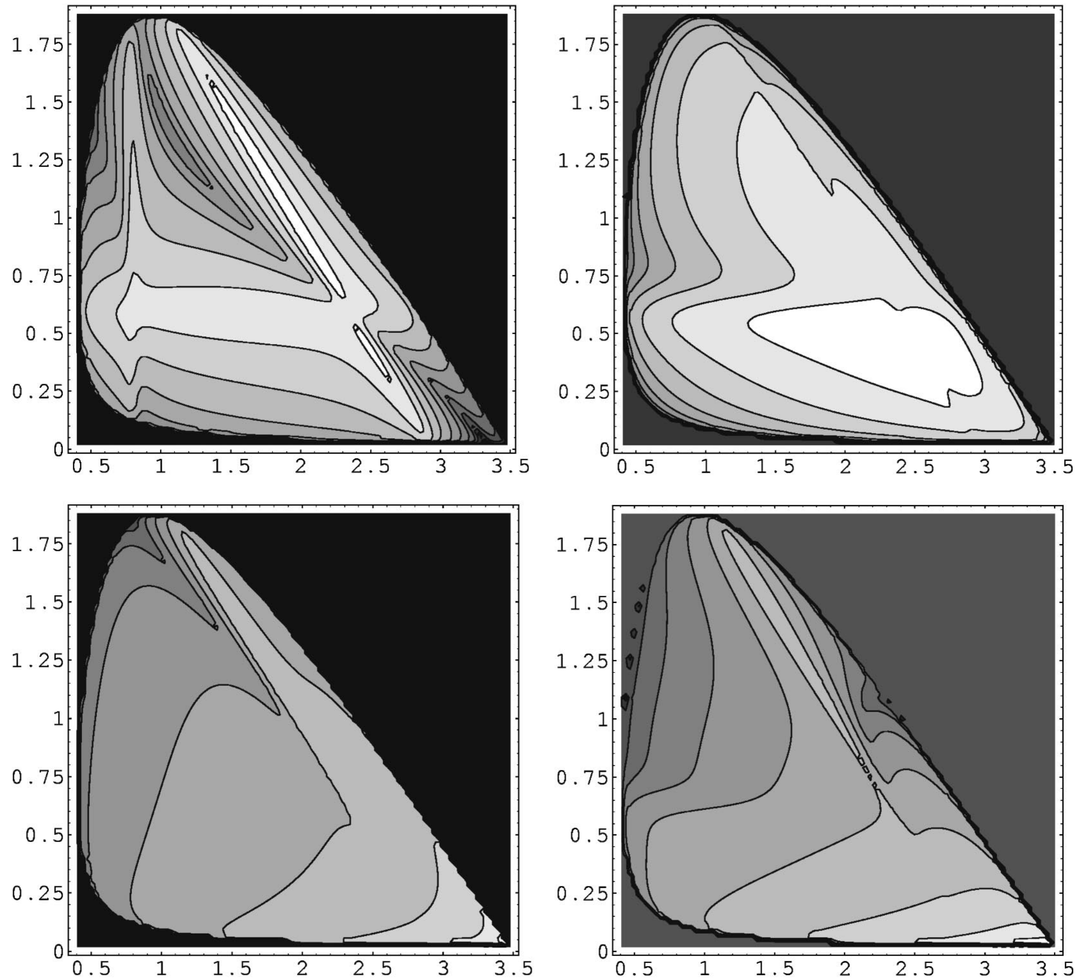

FIG. 7. Dalitz plot of the parity conserving (above) and parity violating part (below) of the $D^0 \to K^- \pi^+ \gamma$ (left) and $D^+ \to \overline{K^0} \pi^+ \gamma$ decay (right). With gray levels on contour plot (left) and on z axis on the 3D plot (right) we present $(2/G_F) d\Gamma/(dm_{12} dm_{23})$ in the logarithmic scale. Invariant mass $m_{12} = \sqrt{(P-k)^2}$ is plotted on the x axis and $m_{23} = \sqrt{(P-p)^2}$ on the y axis of contour plot. The x and y axes of the three-dimensional plot are labeled.

early detection. There is a third channel in this class, $D^0 \to \overline{K^0} \pi^0 \gamma$, which has only a direct component in the radiative decay and will be discussed separately.

Our results show that the relative expected strengths of the direct and bremsstrahlung components are of a magnitude which would permit the experimental determination of both, in next generation of experiments. This is important, since the direct amplitude provides information on the decay mechanism. In the radiative decay of D^+, the magnetic direct component amounts to about 6% of the total rate [see Eqs. (33), (35)] and together with direct electric component which is of comparable magnitude [see Fig. 5(a)], dominate the decay spectrum in the region of high photon energies, say above $k \simeq 250$ MeV [see Fig. 5(f)]. A similar situation occurs in the $D^0 \to K^- \pi^+ \gamma$ transition, where the direct radiative decay containing both electric and magnetic parts which are of nearly equal magnitude, amounts to over 30% of the total radiative decay rate [see Eqs. (34) and (37)]. The numbers we mentioned are for $k > 50$ MeV, but as we stressed the region of high photon momenta beyond 200 MeV is where the direct transition even dominates. As to the reasons for the rather large direct amplitudes, we consider it to be the outcome of the various constants appearing in the model used, especially the rather large value of $|g| = 0.59$ as recently determined.

We have checked the sensitivity of our results to various parameters we used. The uncertainty in the strong coupling $g = 0.59 \pm 0.07$, may change our results for direct branching ratios by at most 15%. On the other hand, the uncertainty in λ and λ', and changing of sign of $\alpha_{1,2}$ is comparably negligible. As to the values of f_D, if we vary it by a reasonable amount we can induce changes in the direct amplitudes by a few tens of percent.

As we explained in the text, the results (35)–(38) are obtained by using the experimental $D \to K\pi$ amplitudes to calculate the inner bremsstrahlung. If we use the model for doing it, we get the result exhibited in Eq. (39)–(42), which do not differ from Eqs. (35), (36), i.e., the $D^+ \to \overline{K^0} \pi^+ \gamma$ decay, but are larger by a factor of about 2 in the amplitude in the case of $D^0 \to K^- \pi^+ \gamma$ decay. This is apparently related to the known difficulty of calculating the $D^0 \to K^- \pi^+$ amplitude in the factorization approximation; therefore, we consider the results given in Eqs. (37), (38) to be on a safer ground.

If we disregard the contribution of vector mesons to the direct part of the radiative decays, the parity-conserving part of the amplitude is considerably decreased, by one order of magnitude in the rate in $D^+\to\bar{K}^0\pi^+\gamma$ decay and by two orders of magnitude in $D^0\to K^-\pi^+\gamma$. On the other hand, their contribution is not felt in a significant way in in the parity-violating part of the amplitudes. In any case, the detection of the direct part of these decays at the predicted rates, will constitute a proof of the important role of the light vector mesons. The contribution of the vector mesons in the crossed channels is evident in the Dalitz plots of Fig. 7. We point out that the contribution of the vector mesons in F_1, as evidenced from the relevant graphs, is wholy determined by the λ, λ' couplings.

Figures 5(a), 5(b) give the expected spectra for the direct component, which would be detectable in the region of high photon energies. For the D^+ decay, the direct electric and magnetic transitions are of comparable strength. This prediction of the model should be testable, as it shiffts the peak of the spectrum to $E_\gamma=480$ MeV, while if the magnetic transition is dominant it should peak $E_\gamma=400$ MeV. In the $D^0\to K^-\pi^+\gamma$ decay the magnetic and electric components are likewise of nearly equal size, again testable in the spectrum. It is worthwhile to point out that the relative values of the parity-conserving amplitudes is rather large $|A(D^0\to K^-\pi^+\gamma)|^2/|A(D^+\to\bar{K}^0\pi^+\gamma)|^2\simeq 7$. The main reason for it are certain contributions (such as $D^0_{1,1}$), which appear in D^0 decay but are doubly Cabibbo forbidded in the D^+ decay. Also, as we pointed out, the $\Delta I=1/2$ amplitude is larger than the $\Delta I=3/2$ one in the $D\to K\pi$ channels.

Finally, we wish to emphasize a most interesting implication of our calculation. When one compares the results obtained here for the radiative decays to nonresonant $\bar{K}\pi$, with those previously obtained for the $D\to\bar{K}^*\gamma$ [4,6,7,16] it emerges that the nonresonant channel is the more frequent one. The direct decay $D^+\to\bar{K}^0\pi^+\gamma$ is expected to have a BR of $\simeq 3\times10^{-5}$ in our model. To this one should add the IB component, which brings its BR to about 4×10^{-4} for $k>50$ MeV. The radiative decay $D^0\to\bar{K}^{*0}\gamma$ is expected with BR of 0.5×10^{-4}. The nonresonant direct $D^0\to K^-\pi^+\gamma$ decay we investigated here, has a BR of $\simeq 3\times10^{-4}$ and including the IB component will occur with a rate 8×10^{-4} for $k>50$ MeV. The experimental verification of this systematics will provide a check for the suitabilty of the theoretical methods employed. We should remark, however, at this point that we did not address the possibility of the decays $D\to R+\gamma$, where R is a higher $K\pi$ or $K\pi\pi$ resonance. To our knowledge, there is no calculation available on this topic. Our expectation is that such modes are at most comparable in strength to the $D\to\bar{K}^*\gamma$ decay; preliminary data from BELLE [43] indicate that this is the case in B decays. We conclude by expressing the hope that the interesting features which these decays provide and were analyzed in this paper, will bring to an experimental search in the near future.

ACKNOWLEDGMENTS

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APPENDIX A: THE DECAY AMPLITUDES FOR $D^+\to\bar{K}^0\pi^+\gamma$

Here we give the expressions for the sum of amplitudes in each row presented in Fig. 1. The contributions which arise due to O_1 operator are

$$A_1^+=-ie\frac{f_D f_\pi}{f_K}\frac{(v\cdot q+v\cdot k)p\cdot k}{v\cdot k},$$

$$A_2^+=-ie\sqrt{\frac{M_s}{M}}\frac{f_{Ds}f_\pi}{f_K}g\frac{p\cdot q-(v\cdot q)(v\cdot p)+v\cdot k(M-v\cdot p)}{v\cdot p+\Delta}\frac{p\cdot k}{v\cdot k},$$

$$A_3^+=ieg\sqrt{\frac{M_s}{M}}\frac{f_{Ds}f_\pi}{f_K}\frac{(v\cdot k)(q\cdot k)(p\cdot k)}{v\cdot k+v\cdot p+\Delta}\left\{\frac{-1}{v\cdot k+\Delta}\left[2\lambda'-\frac{\sqrt{2}}{2}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}-\frac{q_\rho}{m_\rho^2}\right)\right]+\frac{1}{v\cdot p+\Delta}\left(2\lambda'+\frac{\sqrt{2}}{3}\lambda g_v\frac{q_\phi}{m_\phi^2}\right)\right\},$$

$$A_4^+=i\sqrt{2}f_{Ds}f_\pi g_{K^{0*}\bar{K}^0\gamma}g_v\lambda e\sqrt{M_sM}\frac{(v\cdot k)(q\cdot k)^2}{[v\cdot(p+k)+\Delta][(p+k)^2-m_{K*}^2+i\Gamma_{K*}m_{K*}]}.$$

The contributions coming from O_2 operator are

$$C_1^+ = -ie\frac{f_D f_K}{f_\pi}\frac{(v\cdot p)(p\cdot k)}{v\cdot k},$$

$$C_2^+ = ie\frac{f_D f_K}{f_\pi} g \frac{p\cdot q-(v\cdot p)(v\cdot q)+v\cdot k(M-v\cdot p)}{v\cdot q+v\cdot k+\Delta}\frac{p\cdot k}{v\cdot k},$$

$$C_3^+ = -ieg\frac{f_D f_K}{f_\pi}\frac{(v\cdot k)(q\cdot k)(p\cdot k)}{v\cdot k+v\cdot q+\Delta}\left\{\frac{-1}{v\cdot k+\Delta}\left[2\lambda'-\frac{\sqrt{2}}{2}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}-\frac{q_\rho}{m_\rho^2}\right)\right]+\frac{1}{v\cdot q+\Delta}\left[2\lambda'-\frac{\sqrt{2}}{2}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}+\frac{q_\rho}{m_\rho^2}\right)\right]\right\},$$

$$C_4^+ = -i\sqrt{2}f_D f_K g_{\rho\pi\gamma}g_v\lambda eM\frac{(v\cdot k)(p\cdot k)^2}{[v\cdot(q+k)+\Delta][(q+k)^2-m_\rho^2+i\Gamma_\rho m_\rho]}.$$

Next we give the expressions for the sum of amplitudes in each row presented in Fig. 2. The contributions arising from the O_1 operator

$$B_1^+ = 2eM\frac{f_D f_\pi}{f_K}\lambda\left[\frac{1}{v\cdot k+\Delta}\right.$$
$$\left.+g\frac{f_{Ds}\sqrt{M_s}}{f_D\sqrt{M}}\frac{v\cdot q}{v\cdot p+v\cdot k}\left(\frac{1}{v\cdot k+\Delta}+\frac{1}{v\cdot p+\Delta}\right)\right],$$

$$B_2^+ = -\frac{1}{\sqrt{2}}e\frac{f_\pi}{f_K}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}-\frac{q_\rho}{m_\rho^2}\right)\frac{1}{(v\cdot k+\Delta)}$$
$$\times\left(Mf_D+\sqrt{MM_s}f_{Ds}\frac{gv\cdot q}{v\cdot k+v\cdot p}\right)$$
$$+\frac{\sqrt{2}}{3}\sqrt{MM_s}e\frac{f_{Ds}f_\pi}{f_K}g\lambda g_v$$
$$\times\frac{q_\phi}{m_\phi^2}\frac{gv\cdot q}{(v\cdot p+\Delta)(v\cdot k+v\cdot p)},$$

$$B_3^+ = -\sqrt{2M}eg_v g_{K^{0*}\bar{K}^0\gamma}f_\pi\frac{\alpha_1 M-\alpha_2 v\cdot q}{(p+k)^2-m_{K*}^2+i\Gamma_{K*}m_{K*}}$$
$$+eg_{\rho\pi\gamma}g_\rho\frac{f_D}{f_K}\sqrt{M}\frac{\sqrt{M}+\frac{f_{Ds}}{f_D}\sqrt{M_s}g\frac{M-v\cdot p}{v\cdot p+\Delta}}{(k+q)^2-m_\rho^2+i\Gamma_\rho m_\rho}.$$

The operator O_2 gives the following contributions:

$$D_1^+ = -2eM\frac{f_D f_K}{f_\pi}\lambda'\left(\frac{1}{v\cdot k+\Delta}+g\frac{v\cdot p}{(v\cdot q+v\cdot k)(v\cdot k+\Delta)}\right.$$
$$\left.+g\frac{v\cdot p}{(v\cdot q+v\cdot k)(v\cdot q+\Delta)}\right),$$

$$D_2^+ = \frac{1}{\sqrt{2}}Me\frac{f_D f_K}{f_\pi}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}-\frac{q_\rho}{m_\rho^2}\right)\frac{1}{(v\cdot k+\Delta)}$$
$$\times\left(1+g\frac{v\cdot p}{v\cdot k+v\cdot q}\right)$$
$$+\frac{1}{\sqrt{2}}Me\frac{f_D f_K}{f_\pi}g\lambda g_v\frac{v\cdot p}{(v\cdot q+\Delta)(v\cdot k+v\cdot q)}$$
$$\times\left(\frac{q_\omega}{3m_\omega^2}+\frac{q_\rho}{m_\rho^2}\right),$$

$$D_3^+ = \sqrt{2M}eg_v g_{\rho\pi\gamma}f_K\frac{\alpha_1 M-\alpha_2 v\cdot p}{(q+k)^2-m_\rho^2+i\Gamma_\rho m_\rho}$$
$$-eg_{\bar{K}^{0*}\bar{K}^0\gamma}g_{K*}\frac{f_D}{f_\pi}M\frac{1+g\frac{M-v\cdot q}{v\cdot q+\Delta}}{(k+p)^2-m_{K*}^2+i\Gamma_{K*}m_{K*}}.$$

APPENDIX B: THE DECAY AMPLITUDES FOR $D^0\to K^-\pi^+\gamma$

The expressions for the sum of amplitudes (the O_1 operator) in each row exhibited in Fig. 3 are

$$A_1^0 = -iMe\frac{f_D f_\pi}{f_K}(v\cdot q+v\cdot k),$$

$$A_2^0 = ie\sqrt{MM_s}\frac{f_{Ds}f_\pi}{f_K}g\left(\frac{p\cdot k(v\cdot p-M)}{M(v\cdot p+\Delta)}-\frac{p\cdot k[p\cdot q-(v\cdot p)(v\cdot q)]}{M(v\cdot p+\Delta)(v\cdot p+v\cdot k+\Delta)}+\frac{Mq\cdot k-Mp\cdot q+M(v\cdot q)(v\cdot p)-(v\cdot q)(q\cdot k)}{M(v\cdot p+v\cdot k+\Delta)}\right),$$

$$A_3^0 = ie\sqrt{M_s/M}\,\frac{f_{Ds}f_\pi}{f_K}g\frac{(v\cdot k)(p\cdot k)(q\cdot k)}{(v\cdot k+v\cdot p+\Delta)}\left(\frac{2\lambda' + \frac{\sqrt{2}}{3}\lambda g_v\frac{q_\phi}{m_\phi^2}}{(v\cdot p+\Delta)} - \frac{2\lambda' - \frac{\sqrt{2}}{2}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}+\frac{q_\rho}{m_\rho^2}\right)}{v\cdot k+\Delta}\right),$$

$$A_4^0 = i\sqrt{2}f_{Ds}f_\pi g_{K^*K\gamma}g_v\lambda e\sqrt{M_sM}\frac{(v\cdot k)(q\cdot k)^2}{(v\cdot(p+k)+\Delta)((p+k)^2-m_{K^*}^2+i\Gamma_{K^*}m_{K^*})}.$$

The sum of amplitudes coming from operator O_2, shown in Fig. 3 as C_1^0, is vanishing.

Next we present the expressions for the sums of amplitudes in each row shown in Fig. 4. The results for the operator O_1 are

$$B_1^0 = 2eM\frac{f_Df_\pi}{f_K}\lambda'\left[\frac{1}{v\cdot k+\Delta} + g\frac{f_{Ds}\sqrt{M_s}}{f_D\sqrt{M}}\frac{v\cdot q}{v\cdot p+v\cdot k}\left(\frac{1}{v\cdot k+\Delta}+\frac{1}{v\cdot p+\Delta}\right)\right],$$

$$B_2^0 = -\frac{1}{\sqrt{2}}e\frac{f_\pi}{f_K}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}+\frac{q_\rho}{m_\rho^2}\right)\frac{1}{(v\cdot k+\Delta)}\left(Mf_D+\sqrt{MM_s}f_{Ds}\frac{gv\cdot q}{v\cdot k+v\cdot p}\right)$$
$$+\frac{\sqrt{2}}{3}\sqrt{MM_s}e\frac{f_{Ds}f_\pi}{f_K}g\lambda g_v\frac{q_\phi}{m_\phi^2}\frac{gv\cdot q}{(v\cdot p+\Delta)(v\cdot k+v\cdot p)},$$

$$B_3^0 = -\sqrt{2M}eg_vg_{K^*\bar{K}\gamma}f_\pi\frac{\alpha_1M-\alpha_2v\cdot q}{(p+k)^2-m_{K*}^2+i\Gamma_{K^*}m_{K^*}} + eg_{\rho\pi\gamma}g_\rho\frac{f_D}{f_K}\frac{M+\frac{f_{Ds}}{f_D}\sqrt{M_sM}g\frac{M-v\cdot p}{v\cdot p+\Delta}}{(k+q)^2-m_\rho^2+i\Gamma_\rho m_\rho}.$$

Finally, the sums of amplitudes in each row due to the operator O_2:

$$D_1^0 = Mef_D\frac{1}{v\cdot k+\Delta}\left(1+\frac{m_{K^*}^2}{(p+q)^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}}\right)\left[2\lambda' - \frac{\sqrt{2}}{2}\lambda g_v\left(\frac{q_\omega}{3m_\omega^2}+\frac{q_\rho}{m_\rho^2}\right)\right],$$

$$D_2^0 = Me\frac{f_D}{f_\pi}g_Kg_{KK^*\gamma}\frac{1}{(p+k)^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}} + Me\frac{f_D}{f_K}g_\rho g_{\rho\pi\gamma}\frac{1}{(q+k)^2-m_\rho^2+im_\rho\Gamma_\rho},$$

$$D_3^0 = -\frac{e}{2}f_Df_K\frac{M^3}{(M^2-m_{K^0}^2)}\left(\frac{m_{K^*}^2}{g_{K^*}}\frac{g_{K^*K\gamma}}{(p+k)^2-m_{K^*}^2+im_{K^*}\Gamma_{K^*}} + \frac{m_\rho^2}{g_\rho}\frac{g_{\rho\pi\gamma}}{(q+k)^2-m_\rho^2+im_\rho\Gamma_\rho}\right).$$

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