

Possible large mass effects in direct determination of the CKM elements in top decays

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We discuss the possibility that mass effects, beyond phase space, may have substantial influence on the direct determination of V_{tq} in decays of top quarks. In principle, our considerations are valid both for singly produced top and for $t\bar{t}$ pair production. The mass effect is practically irrelevant for the extraction of V_{tb} from Fermilab Tevatron data, but it may have implications for higher energy multijet top decay.

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I. INTRODUCTION

Much effort is being invested, and it will intensify in the future, in searching for new physics beyond the standard model (SM). For example, if the values of any angle of the unitarity triangle [1] measured in two independent processes will turn out to be different from each other, then a new physics scenario is unavoidable.

Thus a thorough investigation of the elements of the Cabibbo-Kobayashi-Maskawa (CKM) [2,3] matrix is required. Up until now the three matrix elements involving the t quark were indirectly inferred from the contributions of t in various loop processes. In addition, $V_{tb} = 0.9990 - 0.9993$ can be deduced [4] indirectly from CKM unitarity assuming the existence of only three generations. It will be indeed exciting if V_{ti} , extracted indirectly, will attain values different from those that will be measured directly in top decays. In the present article we discuss the possibility that the mass effect, i.e., the large disparity in mass between the b and the s quarks, may significantly alter the results for $|V_{tb}|^2/|V_{ts}|^2$ obtained without considering the mass effect. Extra care should be therefore practiced in carrying out the extraction of CKM elements from top decays.

Lately, the Collider Detector at Fermilab (CDF) Collaboration has reported the measurement of the ratio of branching ratios,

$$\mathcal{R} \equiv \mathcal{B}(t \rightarrow Wb) / \mathcal{B}(t \rightarrow Wq), \quad (1)$$

from $p\bar{p}$ collisions at the Fermilab Tevatron with $\sqrt{s} = 1.8$ TeV [5] and deduced for the first time a direct value for V_{tb} . It was assumed that decays of the top quark to non- W final states can be safely neglected. Then, taking into account that the masses of the final state down type quarks can be ignored to a high accuracy (the relative effect of the b mass on the phase space is of the order of a few per mill), \mathcal{R} is related to the CKM elements of the top quark via

$$\mathcal{R} = \frac{|V_{tb}|^2}{|V_{tb}|^2 + |V_{ts}|^2 + |V_{td}|^2}. \quad (2)$$

Assuming three generations unitarity of the CKM matrix, the denominator is equal to unity and thus we can identify

$$\mathcal{R} = |V_{tb}|^2. \quad (3)$$

From the measured value $\mathcal{R} = 0.94_{-0.24}^{+0.31}$ they obtain [5] $|V_{tb}| = 0.97_{-0.12}^{+0.16}$, or $|V_{tb}| > 0.75 @ 95\%$ C.L. Now, without assuming three generations, one may rewrite Eq. (2) as

$$|V_{tb}| = \sqrt{\frac{\mathcal{R}}{1-\mathcal{R}} (|V_{ts}|^2 + |V_{td}|^2)}. \quad (4)$$

Before discussing the main issue of the present paper, let us make a rather trivial remark. One is tempted to use the measured \mathcal{R} and the central values for V_{ts} and V_{td} as given in the Particle Data Group (PDG) tables [4] to deduce $|V_{tb}|$. However, V_{ts} and V_{td} were obtained using data from the rare decays $b \rightarrow s(d)\gamma$ and from $B-\bar{B}$ mixing. But in these processes V_{ti} ($i=1,2$) enter in combination with V_{tb} . Therefore, in order to translate the experimental results for the above mentioned loop processes into values of V_{td} and V_{ts} as in the PDG tables, three generations unitarity is assumed. Consequently, such an approach to determine $|V_{tb}|$ from Eq. (4) is not free from the assumption of three generations unitarity of the CKM matrix.

In this paper we would like to dwell upon some other considerations related to Eq. (2). The data employed to determine V_{tb} [5] come from $t\bar{t}$ pair production at the Tevatron. They are classified in two disjoint sets according to the decay channels of the W boson emerging in the $t \rightarrow Wq$ transition. The final states used in the analysis are the “lepton + jets” and the “dilepton” samples with one or both of the W 's decaying leptonically. In both cases the selection criteria employ cuts on the phase space. In the following we would like to argue that such cuts on the phase space might produce some dependence of the rates on the mass of the down type quark in the $t \rightarrow Wq$ transition and hence spoil the simple relation of Eq. (2). In principle, this dependence stems from gluon radiation. In totally inclusive calculations at one loop order the mass effects can reach the few percent level [6]. Basically, the emission of additional gluons in the decay of the top quarks exhibits soft and collinear divergences that cancel when adding real and virtual contributions. However, at every order of perturbation theory logarithmic terms remain that depend on the relevant scales of the process, including scales which stem from cuts on the phase space of real gluon emissions. In double leading log approximation these logarithms can be resummed yielding an exponential. This result is, of course, the well-known Sudakov form fac-

tor [7], occurring for instance in jet physics [8]. Fortunately, the mass effect has no practical implications for the CDF extraction of V_{tb} [5].

Experimentally the phase space cuts are usually related to the jet measure definitions adopted in a given experiment. A simple and popular definition would be of the Sterman-Weinberg type [9], imposing cuts on the maximal energy ω of the emitted gluons and the angular opening of the jet ΔR (in the lab frame). In the particular experiment [5], ΔR is sufficiently large ($\Delta R \gg m_q/E_q$) and the leading double log effect is related to $\log(\omega/E_q)\log\Delta R$. In this case the mass effects are subleading nonexponentiable contributions of the type $m_q\log(m_q/E_q)$. However, the mass effects can potentially become important when the jet opening angle ΔR decreases toward the dead cone value m_q/E_q such that effectively the nonvanishing quark mass shields the collinear divergence. In the latter case large mass dependent logs arise. Experimentally such situations can be met in experiments with multijets where more severe phase space cuts are required. Indeed, mass effects up to 20% were observed in e^+e^- annihilation to $b\bar{b}$ in three jets, and significant effects are predicted in other environments [10]. Moreover, the effect becomes more pronounced when the number of jets increases.

II. SUDAKOV RESUMMATION OF SOFT GLUONS

A rigorous question of the mass effects in top decays should in principle be addressed within a Monte Carlo jet generator approach and will not be investigated here. In the present paper we only wish to illustrate a potential danger of a very high jet resolution. To this goal, let us consider an extreme (that is: unrealistic) case, imposing simple cuts on the maximal energy of the emitted gluons only ($\Delta R=0$).

The squared matrix element for emission of a real gluon (momentum k , $k^2=0$) in a $t \rightarrow Wq$ transition mediated by $\bar{q}\gamma^{\nu L}t$ reads

$$|\mathcal{M}_{\mathcal{R}}|^2 = (-ig_s)^2 C_F \left| \bar{q} \left[\gamma^\mu \frac{i(\not{p}_q + \not{k} + m_q)}{(p_q + k)^2 - m_q^2} \gamma^{\nu L} + \gamma^{\nu L} \frac{i(\not{p}_t - \not{k} + m_t)}{(p_t - k)^2 - m_t^2} \gamma^\mu \right] t \varepsilon_\mu \right|^2, \quad (5)$$

which becomes

$$|\mathcal{M}_{\mathcal{R}}|^2 = -g_s^2 C_F |\bar{q}\gamma^{\nu L}t|^2 \left(\frac{p_q^\mu}{p_q k} - \frac{p_t^\mu}{p_t k} \right)^2 \quad (6)$$

in the soft limit. Integrating over the gluon momentum in $D=4+2\epsilon$ dimensions, the real corrections in $\mathcal{O}(\alpha_s)$ are given by

$$\mathcal{F}_{\mathcal{R}} = -g_s^2 C_F \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} (2\pi) \delta(k^2) \theta(k_0) \left(\frac{p_q^\mu}{p_q k} - \frac{p_t^\mu}{p_t k} \right)^2$$

$$= -g_s^2 C_F \mu^{-2\epsilon} \int_0^\omega \frac{dk_0}{2\pi} \frac{k_0^{2+2\epsilon}}{2k_0} \frac{1}{k_0^2} \int \frac{d\Omega_{2+2\epsilon}}{(2\pi)^{2+2\epsilon}} \times \int_0^\pi d\theta \sin^{1+2\epsilon} \theta \left(\frac{\delta}{(1-\sqrt{1-\delta}\cos\theta)^2} - \frac{2}{(1-\sqrt{1-\delta}\cos\theta)} + 1 \right) \quad (7)$$

with $\delta \equiv m_q^2/E_q^2$. Integrating over the full angular region and bounding the gluon energy to be smaller than some maximal value ω we find

$$\mathcal{F}_{\mathcal{R}} = -\frac{\alpha_s C_F}{2\pi} \left\{ \frac{1}{\epsilon} \left[2 - \frac{1}{\sqrt{1-\delta}} \log \frac{1+\sqrt{1-\delta}}{1-\sqrt{1-\delta}} \right] + \left[\frac{1}{2} \log^2(\delta) + \log(\delta) \log \left(\frac{\omega^2}{\mu^2} \right) \right] \right\}, \quad (8)$$

where we have given the exact result for the $1/\epsilon$ terms and retained only the double leading logarithmic terms for $\delta \rightarrow 0$ in the finite part. The corresponding virtual corrections to the squared matrix element are of the form $2|\mathcal{M}^{(1)}\mathcal{M}^{*(0)}|$, where the matrix element to one loop order reads

$$2\mathcal{M}^{(1)} = 2(-ig_s)^2 C_F \cdot \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{-i}{k^2} \times \left[\bar{q} \gamma^\mu \frac{i(\not{p}_q - \not{k} + m_q)}{(p_q - k)^2 - m_q^2} \gamma^{\nu L} \frac{i(\not{p}_t - \not{k} + m_t)}{(p_t - k)^2 - m_t^2} \gamma_{\mu t} \right]. \quad (9)$$

Taking into account appropriate counter terms to restore the Ward identities, one is therefore left with

$$2|\mathcal{M}^{(1)}\mathcal{M}^{*(0)}| = ig_s^2 C_F |\bar{q}\gamma^{\nu L}t|^2 \mu^{4-D} \int \frac{d^D k}{(2\pi)^D} \frac{1}{k^2} \times \left(\frac{2p_q^\mu}{k^2 - 2p_q k} - \frac{2p_t^\mu}{k^2 - 2p_t k} \right)^2, \quad (10)$$

in the soft limit $|k| \rightarrow 0$. Keeping again the exact results for the $1/\epsilon$ parts and the double leading logarithms for the finite part only, the virtual corrections are given by

$$\mathcal{F}_{\mathcal{V}} = \frac{\alpha_s C_F}{2\pi} \left\{ \frac{1}{\epsilon} \left[2 - \frac{1}{\sqrt{1-\delta}} \log \frac{1+\sqrt{1-\delta}}{1-\sqrt{1-\delta}} \right] + \left[\frac{1}{2} \log^2(\delta) + \log(\delta) \log \left(\frac{E_q^2}{\mu^2} \right) \right] \right\}, \quad (11)$$

demonstrating the cancellation of soft and collinear divergences.

Resumming the real and virtual corrections and neglecting recoil effects on the energy of the quark we find

$$\exp[\mathcal{F}_{\mathcal{R}+\nu}^q] = \exp\left[-\frac{\alpha_s C_F}{2\pi} \log\left(\frac{m_q^2}{E_q^2}\right) \log\left(\frac{\omega^2}{E_q^2}\right)\right] \quad (12)$$

for quark flavor q . This Sudakov form factor represents, to double-log accuracy, the probability that *no* gluon with energy larger than ω has been emitted during the $t \rightarrow Wq$ transition. Taking into account that $|V_{td}|$ is much smaller than $|V_{ts}|$, Eq. (2) translates into

$$\begin{aligned} \frac{|V_{ts}|^2}{|V_{tb}|^2} &\propto \frac{1-\mathcal{R}}{\mathcal{R}} \frac{\exp[\mathcal{F}_{\mathcal{R}+\nu}^b]}{\exp[\mathcal{F}_{\mathcal{R}+\nu}^s]} \\ &= \frac{1-\mathcal{R}}{\mathcal{R}} \exp\left[-\frac{\alpha_s C_F}{2\pi} \log\left(\frac{m_b^2}{m_s^2}\right) \log\left(\frac{\omega^2}{E_q^2}\right)\right]. \end{aligned} \quad (13)$$

Obviously, this ratio becomes greatly enhanced when going to more and more exclusive measures, i.e., to lower and lower maximal energies permitted for the gluons radiated off the quarks. For instance, if we assume the following values:

$$\frac{m_b}{m_s} = 45, \quad \frac{\omega}{E_q} = 0.1, \quad \alpha_s = 0.1, \quad (14)$$

we end up with a large enhancement factor of $\mathcal{O}(2)$ for the ratio $|V_{ts}|^2/|V_{tb}|^2$.

The considered case of only soft gluon cut without imposing any cuts on collinear gluons is certainly not physical. It merely serves us as a calculable “toy case” to illustrate potential pitfalls in the direct determination of $|V_{tq}|$ via top

decays. For any real jet measure the mass effect will be smaller and should be the subject of a full Monte Carlo computation.

III. CONCLUSIONS

Although we do not dispute the CDF result for V_{tb} [5], for which a b quark acts, practically, as a massless quark, we believe that our results show that extra care should be practiced in extracting the CKM elements involving the top quark. For very high jet resolutions we can get a larger mass effect as the mass of the down type quark decreases. This, by the way, may enhance the prospects for “tagging” jets originating from light quarks. Mass effects, in particular those present in multijets, have already been observed and discussed in various environments, not including the top quark [10]. Thus, it is of essential importance for the direct measurement of $|V_{tq}|$ to look as inclusively as possible for q quarks stemming from top decays. Finally, note that the effect discussed here is independent of the production mechanism of the t quark, in particular whether the top is produced singly or in a $t\bar{t}$ pair. Moreover, although the motivation of this paper originated from the CDF measurement, our point can be of worth to other CKM measurements especially at the next generation of hadron and lepton collider experiments.

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