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Positivity of high density effective theory

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We show that the effective field theory of low energy modes in dense QCD has a positive Euclidean path integral measure. The complexity of the measure of QCD at the finite chemical potential can be ascribed to modes which are irrelevant to the dynamics at sufficiently high density. Rigorous inequalities follow at asymptotic density. Lattice simulations of dense QCD should be possible using the quark determinant calculated in the effective theory.

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Quantum chromodynamics (QCD) with a nonzero chemical potential has a complex measure, which has thus far precluded lattice simulations [1]. Recent analytical work in color superconductivity [2] has demonstrated a rich phase structure at high density, and stimulated interest in QCD at a nonzero baryon density. Several experiments have been proposed to probe matter at a density of a few times the nuclear matter density [3]. Even rudimentary information about the behavior of dense matter would be useful to the experimental program, as well as to the study of compact astrophysical objects such as neutron stars. In this paper, we will show that QCD near a Fermi surface has a positive, semidefinite measure. The contribution of the remaining modes far from the Fermi surface can be systematically expanded, using a high density effective theory previously introduced by one of us [4]. This effective theory is sufficient to study phenomena such as color superconductivity, although quantities such as the equation of state are presumably largely determined by dynamics deep in the Fermi sea.

Let us recall why the measure of dense QCD is complex in Euclidean space. We use the following analytic continuation of the Dirac Lagrangian to Euclidean space:

$$x_0 \to -i x_E^4, \quad x_i \to x_E^i; \quad \gamma_0 \to \gamma_E^4, \quad \gamma_i \to i\gamma_E^i. \tag{1}$$

The Euclidean gamma matrices satisfy

$$\gamma_E^{\mu\dagger} = \gamma_E^\mu, \quad \{\gamma_E^\mu, \gamma_E^\nu\} = 2\delta^{\mu\nu}. \tag{2}$$

The Dirac-conjugated field, $\bar{\psi} = \psi^\dagger \gamma^0$, is mapped into a field, still denoted as $\bar{\psi}$, which is independent of ψ and transforms as $\psi^\dagger$ under $SO(4)$. Then, the grand canonical partition function for QCD is

$$Z(\mu) = \int dA_\mu \det(M) e^{-S(A_\mu)}, \tag{3}$$

where $S(A_\mu)$ is the positive semidefinite gauge action, and the Dirac operator

$$M = \gamma_E^\mu D_E^\mu + \mu\gamma_E^4, \tag{4}$$

where $D_E = \partial_E + iA_E$ is the analytic continuation of the covariant derivative. The Hermitian conjugate of the Dirac operator is

$$M^\dagger = -\gamma_E^\mu D_E^\mu + \mu\gamma_E^4. \tag{5}$$

The first term in Eq. (4) is anti-Hermitian, while the second is Hermitian, hence the generally complex eigenvalues. When $\mu = 0$, the eigenvalues are purely imaginary, but come in conjugate pairs (λ, λ^*) [6], so the resulting determinant is real and positive semidefinite:

$$\det M = \prod \lambda^* \lambda \geqslant 0. \tag{6}$$

In what follows we investigate the positivity properties of an effective theory describing only modes near the Fermi surface. A system of degenerate quarks with a net baryon number asymmetry is described by the QCD Lagrangian density with a chemical potential μ,

$$\mathcal{L}_{\rm QCD} = \bar{\psi} i \not{D} \psi - \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + \mu \bar{\psi} \gamma_0 \psi, \tag{7}$$

where the covariant derivative $D_\mu = \partial_\mu + iA_\mu$ and we neglect the quark mass for simplicity.

The energy spectrum of (free) quarks is given by an eigenvalue equation,

$$(\vec{\alpha} \cdot \vec{p} - \mu)\psi_\pm = E_\pm \psi_\pm, \tag{8}$$

where $\vec{\alpha} = \gamma_0 \vec{\gamma}$ and $\psi_\pm$ denote the energy eigenfunctions with eigenvalues $E_\pm = -\mu \pm |\vec{p}|$, respectively. At low energy $E < \mu$, the states ψ_+ near the Fermi surface, $|\vec{p}| \sim \mu$, are easily excited but ψ_-, which correspond to the states in the Dirac sea, are completely decoupled due to the presence of

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the energy gap μ provided by the Fermi sea. Therefore the appropriate degrees of freedom at low energy consist of gluons and ψ_+ only.

Now, we wish to construct an effective theory describing the dynamics of ψ_+ by integrating out modes whose energy is greater than μ. Consider a quark near the Fermi surface, whose momentum is close to $\mu\vec{v}_F$. Without loss of generality, we may decompose the momentum of a quark into a Fermi momentum and a residual momentum as

$$p_\mu = \mu v_\mu + l_\mu, \tag{9}$$

where $v^\mu = (0, \vec{v}_F)$. Since the quark energy is given as

$$E = -\mu + \sqrt{(l_\parallel + \mu)^2 + l_\perp^2}, \tag{10}$$

the residual momentum should satisfy $(l_\parallel + \mu)^2 + l_\perp^2 \leq 4\mu^2$ with $\vec{l}_\parallel = \vec{v}_F \vec{l} \cdot \vec{v}_F$ and $\vec{l}_\perp = \vec{l} - \vec{l}_\parallel$.

To describe the small excitations of the quark with Fermi momentum, $\mu\vec{v}_F$, we decompose the quark fields as

$$\psi(x) = e^{i\mu\vec{v}_F\cdot\vec{x}}[\psi_+(\vec{v}_F, x) + \psi_-(\vec{v}_F, x)], \tag{11}$$

where

$$\psi_\pm(\vec{v}_F, x) = P_\pm(\vec{v}_F) e^{-i\mu\vec{v}_F\cdot\vec{x}}\psi(x)$$

with

$$P_\pm(\vec{v}_F) \equiv \frac{1 \pm \vec{\alpha}\cdot\vec{v}_F}{2}. \tag{12}$$

The quark Lagrangian in Eq. (7) then becomes

$$\begin{aligned}\bar{\psi}(i\not{D} + \mu\gamma^0)\psi = {} & [\bar{\psi}_+(\vec{v}_F, x) i\gamma_\parallel^\mu D_\mu \psi_+(\vec{v}_F, x) \\ & + \bar{\psi}_-(\vec{v}_F, x)\gamma^0(2\mu + i\bar{D}_\parallel)\psi_-(\vec{v}_F, x)] \\ & + [\bar{\psi}_-(\vec{v}_F, x) i\not{D}_\perp \psi_+(\vec{v}_F, x) + \text{H.c.}],\end{aligned} \tag{13}$$

where $\gamma_\parallel^\mu \equiv (\gamma^0, \vec{v}_F \vec{v}_F \cdot \vec{\gamma})$, $\gamma_\perp^\mu = \gamma^\mu - \gamma_\parallel^\mu$, $\bar{D}_\parallel = \bar{V}^\mu D_\mu$ with $V^\mu = (1, \vec{v}_F)$, $\bar{V}^\mu = (1, -\vec{v}_F)$, and $\not{D}_\perp = \gamma_\perp^\mu D_\mu$.

At low energy, we integrate out all the "fast" modes ψ_- and derive the low energy effective Lagrangian by matching all the one-light particle irreducible amplitudes containing gluons and ψ_+ in loop expansion. The effects of fast modes will appear in the quantum corrections to the couplings of low energy interactions. At the tree level, the matching is equivalent to eliminating ψ_- in terms of equations of motion:

$$\begin{aligned}\psi_-(\vec{v}_F, x) &= -\frac{i\gamma^0}{2\mu + iD_\parallel}\not{D}_\perp \psi_+(\vec{v}_F, x) \\ &= -\frac{i\gamma^0}{2\mu}\sum_{n=0}^{\infty}\left(-\frac{iD_\parallel}{2\mu}\right)^n \not{D}_\perp \psi_+(\vec{v}_F, x).\end{aligned} \tag{14}$$

Therefore, the tree-level Lagrangian for ψ_+ becomes

$$\mathcal{L}_{\text{eff}}^0 = \bar{\psi}_+ i\gamma_\parallel^\mu D_\mu \psi_+ - \frac{1}{2\mu}\bar{\psi}_+ \gamma^0 (\not{D}_\perp)^2 \psi_+ + \cdots, \tag{15}$$

where the ellipsis denotes terms with higher derivatives.

Consider the first term in our effective Lagrangian, which when continued to Euclidean space yields the operator

$$M_{\text{eft}} = \gamma_\parallel^E \cdot D(A). \tag{16}$$

M_{eft} is antiHermitian and it anti-commutes with γ_5, so it leads to a positive semidefinite determinant. However, note that the Dirac operator is not well defined in the space of $\psi_+(\vec{v}_F, x)$ (for fixed v_F), since it maps $\psi_+(\vec{v}_F, x)$ into $\psi_+(-\vec{v}_F, x)$:

$$i\not{D}_\parallel P_+\psi = P_- i\not{D}_\parallel \psi. \tag{17}$$

Since $P_-(\vec{v}_F) = P_+(-\vec{v}_F)$, $i\not{D}\psi_+(\vec{v}_F, x)$ are $\psi_+(-\vec{v}_F, x)$ modes, or fluctuations of a quark with momentum $-\mu\vec{v}_F$.

We can demonstrate the necessity of including both $\psi_+(\vec{v}_F, x)$ and $\psi_+(-\vec{v}_F, x)$ modes in our effective theory by considering charge conservation in a world with only $+\vec{v}_F$ quarks. The divergence of the quark current at one loop is

$$\langle \partial_\mu J^{a\mu}(\vec{v}_F, x)\rangle = g_s \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot x} p^\mu \Pi_{\mu\nu}^{ab}(p) A_\parallel^{b\nu}(-p), \tag{18}$$

where $A_\parallel = (A_0, \vec{v}_F \vec{v}_F \cdot \vec{A})$ and $\Pi_{\mu\nu}^{ab}$ is the vacuum polarization tensor in the effective theory given as [4]

$$\Pi_{ab}^{\mu\nu}(p) = i\mu^2 \delta_{ab} \frac{\vec{p}\cdot\vec{v}_F V^\mu V^\nu}{p\cdot V + i\epsilon\vec{p}\cdot\vec{v}_F}. \tag{19}$$

The polarization tensor has to be transverse to maintain gauge invariance. We find that if we have both fields $\psi_+(\vec{v}_F, x)$ and $\psi_+(-\vec{v}_F, x)$ the current is conserved for scalar potentials and the gauge symmetry is not anomalous:

$$\langle \partial_\mu J_a^\mu(\vec{v}_F, x) + \partial_\mu J_a^\mu(-\vec{v}_F, x)\rangle = 0. \tag{20}$$

Therefore, we need to introduce quark fields with opposite momenta. The Dirac operator is well defined on this larger space.

This anomaly can be understood in terms of spectral flow, since the Fermi surface is (in a certain sense) not gauge-invariant. Under a gauge transformation, $U(x) = e^{i\vec{q}\cdot\vec{x}}$, the Hamiltonian changes and the energy spectrum of free modes of residual momentum $\vec{l}$ shifts to $E = \vec{l}\cdot\vec{v}_F + \vec{q}\cdot\vec{v}_F$. Quarks near the Fermi surface with $\vec{v}_F\cdot\vec{q} > 0$ flow out of the Fermi sea, creating charge. This charge creation is compensated by quarks with opposite $\vec{v}_F$; their energy decreases and they flow into the Fermi sea. However, unless modes with opposite velocities (i.e., both sides of the Fermi sphere) are included, charge is not conserved.

Thus far we have considered the quark velocity as a parameter labelling different sectors of the quark field. This is similar to the approach of heavy quark effective theory (HQET) [5], in which the velocity of the heavy charm or bottom quark is almost conserved due to the hierarchy of scales between the heavy quark mass and the QCD scale. However, this approach contains an ambiguity often referred to as "reparameterization invariance," related to the nonuniqueness of the decomposition (9) of quark momenta into a large and residual component. In the dense QCD case, two $\psi(v_F,x)$ modes whose values of v_F are not very different may actually represent the same degrees of freedom of the original quark field. In what follows we give a different formulation which describes *all* velocity modes of the quark field, and is suitable for defining the quasiparticle determinant.

First, a more precise definition of the breakup of the quark field into Fermi surface modes. Using the momentum operator in a position eigenstate basis: $\vec{p}=-i\vec{\partial}$, we construct the Fermi velocity operator:

$$\vec{v}=\frac{-i}{\sqrt{-\nabla^2}}\frac{\partial}{\partial\vec{x}}, \tag{21}$$

which is Hermitian, and a unit vector.

Using the velocity operator, we define the projection operators $P_\pm$ as before and break up the quark field as, $\psi(x)=\psi_+(x)+\psi_-(x)$, with $\psi_\pm=P_\pm\psi$. By leaving $\vec{v}$ as an operator we can work in coordinate space without introducing the HQET-inspired velocity Fourier transform which introduces v_F as a parameter. If we expand the quark field in the eigenstates of the velocity operators, we recover the previous formalism with all Fermi velocities summed up.

The leading low-energy part of the quark action is given by

$$\mathcal{L}_+=\bar{\psi}P_-(v)(i\partial\!\!\!/-A\!\!\!/+\mu\gamma_0)P_+(v)\psi. \tag{22}$$

As before, we define the fields ψ_+ to absorb the large Fermi momentum:

$$\psi_+(x)=e^{-i\mu\vec{x}\cdot\vec{v}}P_+(v)\psi(x). \tag{23}$$

Let us denote the eigenvalue v obtained by acting on the field ψ (which has momentum of order μ) as v_l (or v "large"), whereas eigenvalues obtained by acting on the effective field theory modes ψ_+ are denoted v_r (or v "residual"). If the original quark mode had momentum p with $|p|>\mu$ (i.e., was a particle), then v_l and v_r are parallel, whereas if $|p|<\mu$ (as for a hole) then v_r and v_l are antiparallel. In the first case, we have $P_+(v_l)=P_+(v_r)$ whereas in the second case $P_+(v_l)=P_-(v_r)$. Thus, the residual modes ψ_+ can satisfy either of $P_\pm(v_r)\psi_+=\psi_+$, depending on whether the original ψ mode from which it was derived was a particle or a hole. In fact, ψ_+ modes can also satisfy either of $P_\pm(v_l)\psi_+=\psi_+$ since they can originate from ψ modes with momentum $\sim+\mu v$ as well as $-\mu v$ (both are present in the original measure: $D\bar{\psi}D\psi$). So, the functional measure for ψ_+ modes contains all possible spinor functions—the only restriction is on the momenta: $|l_0|,|\vec{l}|<\Lambda$, where Λ is the cutoff.

In light of the ambiguity between v_l and v_r, the equation $\psi=e^{+i\mu x\cdot v}\psi_+$ must be modified to

$$\begin{aligned}\psi&=\exp(+i\mu x\cdot v\quad\alpha\cdot v)\psi_+\\&=\exp(+i\mu x\cdot v_r\quad\alpha\cdot v_r)\psi_+,\end{aligned} \tag{24}$$

where the factor of $\alpha\cdot v_r$ corrects the sign in the momentum shift if v_r and v_l are anti-parallel. In general, any expression with two powers of v is unaffected by this ambiguity. For notational simplicity we define a non-local operator

$$X\equiv\mu\quad x\cdot v\quad\alpha\cdot v=\mu\frac{\alpha^i x^j}{\nabla^2}\frac{\partial^2}{\partial x^i\partial x^j}. \tag{25}$$

Taking this into account, we obtain the following action:

$$\mathcal{L}_+=\bar{\psi}_+e^{-iX}(i\partial\!\!\!/-A\!\!\!/+\mu\gamma_0)e^{+iX}\psi_+. \tag{26}$$

We treat the $A\!\!\!/$ term separately from $i\partial\!\!\!/+\mu\gamma_0$ since the former does not commute with X, while the latter does. Continuing to Euclidean space, and using the identity $P_-\gamma_\mu P_+=\gamma_\mu^{\parallel}P_+$, we obtain

$$\mathcal{L}_+=\bar{\psi}_+\gamma_{\parallel}^{\mu}(\partial^\mu+iA_+^\mu)\psi_+, \tag{27}$$

where

$$A_+^\mu=e^{-iX}\quad A^\mu\quad e^{+iX}, \tag{28}$$

and all γ matrices are Euclidean. The term containing A cannot be fully simplified because $[v,A]\neq 0$. Physically, this is because the gauge field carries momentum and can deflect the quark velocity. The redefined ψ_+ modes are functions only of the residual momenta l, and the exponential factors in the A term reflect the fact that the gluon originally couples to the quark field ψ, not the residual mode ψ_+.

The kinetic term in Eq. (27) can be simplified to

$$\gamma_{\parallel}^{\mu}\partial^\mu=\gamma^\mu\partial^\mu \tag{29}$$

since $v\cdot\partial v\cdot\gamma=\partial\cdot\gamma$. The action (27) is the most general dimension 4 term with the rotational, gauge invariance[1] and projection properties appropriate to quark quasiparticles. Therefore, it is a general consequence of any Fermi liquid description of quarklike excitations.

The operator in Eq. (27) is anti-Hermitian and leads to a positive, semidefinite determinant since it anticommutes with γ_5. The corrections given in Eq. (15) are all Hermitian, so higher orders in the $1/\mu$ expansion may reintroduce complexity. The structure of the leading term plus corrections is

[1]If we simultaneously gauge transform A_+ and ψ_+ in Eq. (27) the result is invariant. There is a simple relation between the gauge transform of the + fields and that of the original fields: $U_+(x)=U(x)e^{iX}$. Of course, the momentum-space support of the + gauge transform must be limited to modes below the cutoff Λ.

anti-Hermitian plus Hermitian, just as in the original QCD Dirac Lagrangian with chemical potential. The leading terms in the effective action for gluons (these terms are generated when we match our effective theory, with energy cutoff Λ, to QCD) also contribute only real, positive terms to the partition function:

$$S_{\rm eff}(A)=\int d^4x_E\left(\frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu}+\frac{M^2}{16\pi}\sum_{\vec{v}_F} A^a_{\perp\mu}A^a_{\perp\mu}\right)\geqslant 0, \tag{30}$$

where $A_\perp=A-A_\parallel$ and the Debye screening mass is $M=\sqrt{N_f/(2\pi^2)}g_s\mu$.

Matching of hard gluon effects also leads to four-quark operators in the effective theory. At asymptotic density, we can neglect these operators, since forward scattering dominates Cooper pairing interactions (due to Landau damping [2]). However, at lower densities hard operators may be important. Matching effects due to hard gluon exchange still lead to a positive action for attractive channels, since they arise from quasiparticle-gluon interactions which are originally positive.[2] Only interactions involving virtual antiquarks lead to nonpositive interactions, and these are always suppressed by powers of μ.

Positivity of the measure allows for rigorous QCD inequalities at asymptotic density. For example, inequalities among masses of bound states can be obtained using bounds on bare quasiparticle propagators. One subtlety that arises is that a quark mass term does not lead to a quasiparticle gap (the mass term just shifts the Fermi surface). Hence, for technical reasons the proof of nonbreaking of vector symmetries [7] must be modified. [Naive application of the Vafa-Witten theorem would preclude the breaking of baryon number that is observed in the color-flavor-locked (CFL) phase [8].] A quasiparticle gap can be inserted by hand to regulate the bare propagator, but it will explicitly violate baryon number. However, following the logic of the Vafa-Witten proof, any symmetries which are preserved by the regulator gap cannot be broken spontaneously. One can, for example, still conclude that isospin symmetry is never spontaneously broken. In the case of three flavors, one can use the CFL gap as a regulator to show rigorously that none of the symmetries of the CFL phase are broken at asymptotic density. On the other hand, by applying anomaly matching conditions [9], we can prove that the axial symmetries *are* broken. We therefore conclude that the CFL phase is the true ground state for three light flavors at asymptotic density.

It may be possible to simulate dense QCD using the effective field theory determinant in place of the usual quark determinant. We know that this is a good approximation at very high density, and it should remain a good approximation as long as $\Lambda_{\rm QCD}$, the characteristic scale of the dominant physics, is smaller than μ. At lower densities the weak coupling approximation no longer holds, so Eq. (14) is no longer a good guide to the higher order corrections, which are only constrained by symmetry and projection requirements. However, we can use naive dimensional analysis [10] to estimate the coefficients of the higher dimension operators.

An estimate of the size of corrections to the determinant from higher orders can be obtained by considering the Euclidean relation

$$\det M=e^{\rm Tr\ ln\ M}=e^{-\epsilon_0 V}, \tag{31}$$

where ϵ_0 is the vacuum energy density and V the volume of the system. Corrections to the vacuum energy can be estimated using conventional diagrammatic methods and naive dimensional analysis. We find that corrections are roughly suppressed by powers of $(\alpha_s/2\pi)(\Lambda/\mu)$.

To realize the effective theory (27) directly on the lattice, one can replace the plaquette $U_{n,n+\mu}\sim 1+iaA_\mu(n)$ by $U_+\equiv e^{-iX}Ue^{+iX}$ in the fermion action, but not in the gauge action. In effect, one computes the fermion determinant in the usual way, but as a function of U_+. The momenta of the quasiparticle ψ_+ modes is simply the residual momenta l, which is unrestricted except that its magnitude must be small compared to μ. One can impose this condition on ψ_+ by choosing a lattice spacing $a_{\rm det}\gg\mu^{-1}$ in the quark determinant. However, a challenging aspect of this determinant is that it is computed on the geometry of a spherical shell rather than a ball:

$$d^3p=dp\ \ p^2\ \ d\Omega=dl(\mu+l)^2\ \ d\Omega\neq d^3l=dl\ \ l^2 d\Omega. \tag{32}$$

To obtain the $e^{\pm iX}$ operators, one must first realize (and presumably diagonalize) the velocity operator (21). Since the momentum operator has a simple representation in coordinate space, the most challenging aspect of $\vec{v}$ is the normalization factor [the square root of the Laplacian in Eq. (21)], which is nonlocal. (A similar problem arises in lattice models of chiral fermions.) One should probably investigate a simpler method for directly enforcing the normalization condition on $\vec{v}$. Note, however, that $\vec{v}$ and X are independent of the gauge field, so they need be computed only once for each lattice.

Probably the most direct way to utilize our results on the lattice is as follows. Imagine coupling dense quark matter to a background gauge field A whose magnitude and derivatives are characterized by a scale $\Lambda\ll\mu$. The leading part of the low-energy effective theory (describing only Fermi surface modes) has a real and positive determinant. The quark determinant (or equivalently, its logarithm which is the effective action) can be expanded in powers of $1/\mu$, with the leading term real and positive. Since the determinant is a functional of the gauge field and its derivatives, the expansion will effectively be in powers of Λ over μ. This means that the ordinary lattice determinant $\det[\gamma_\mu D_\mu+\mu\gamma_4]$ computed in such backgrounds should be real and positive to leading order in $1/\mu$. Physically, the low-momentum gauge fields cannot excite modes deep within the Fermi sea (such as anti-

[2]A simple way to study the positivity of four-quark operators is to replace them by a vector field with trivial quadratic term V_μ^2 which couples to quarks like the original gluon: $V_\mu\bar{\psi}\gamma^\mu\psi$. Completing the square, we see that the resulting path integral is positive if the four-quark interactions are attractive.

quarks) which lead to complex contributions to the determinant. If we further take $\Lambda \gg \Lambda_{QCD}$, then the neglected effects of higher momentum gluon modes are suppressed by powers of $\alpha_s(\Lambda) \ll 1$.

In order to restrict ourselves to gauge fields satisfying $\Lambda \ll \mu$ we need to couple the quark determinant living on a lattice with spacing $a \sim 1/\mu$ to a set of gauge fields living on a coarser lattice of spacing $a' \sim 1/\Lambda$. Ordinary calculations of the dense matter quark determinant on lattices with $a = a'$ will not yield a real, positive result since the subleading terms in the $1/\mu$ expansion of the determinant are as large as the leading term.

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