Age constraints on brane models of dark energy

J. S. Alcaniz*
Astronomy Department, University of Washington, Seattle, Washington 98195-1580

Deepak Jain† and Abha Dev‡
Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India
(Received 25 June 2002; published 24 September 2002)

Inspired by recent developments in particle physics, the so-called brane world cosmology seems to provide an alternative explanation for the present dark energy problem. In this paper we use the estimated age of high-z objects to constrain the value of the cosmological parameters in some particular scenarios based on this large scale modification of gravity. We show that such models are compatible with these observations for values of the crossover distance between the 4 and 5 dimensions of the order of $r_c \leqslant 1.67 H_o^{-1}$.

DOI: 10.1103/PhysRevD.66.067301 PACS number(s): 98.80.Es, 04.50.+h, 95.35.+d

The revolutionary ideas associated with extra dimension brane world cosmologies have opened up new perspectives for a better understanding about the structure of the universe. The general principle behind such models is that our 4-dimensional Universe would be a surface or a brane embedded into a higher dimensional bulk space-time on which gravity can propagate [1–3] (see also [4] for a recent review). In the recent literature, different aspects of brane world cosmologies have been explored. For example, the issue related to the cosmological constant problem has been addressed [5] as well as cosmological perturbations [6], cosmological phase transitions [7], inflationary solutions [8], baryogenesis [9], stochastic background of gravitational waves [10], singularity, homogeneity, flatness and entropy problems [11], among others.

An interesting feature of some particular brane world scenarios is the possibility of an accelerated expansion at the late stages of the cosmic evolution with no need to invoke either a cosmological constant or a *quintessence* component [3,12]. Such models not only avoid the well known "cosmological constant problem" (Λ is set to be null) but also enable a description of the presently accelerated stage of the Universe within the string or M theory context [12,13] (see also [14] for a discussion on this topic). From the observational viewpoint, these particular scenarios seem to be in agreement with the most recent observations of cosmic microwave background and type Ia supernovae [15,16] (see, however, [17]) as well as with the measurements of the angular size of high-z radio sources [18] and the current gravitational lensing data [19].

In this paper we discuss new observational constraints on brane world cosmologies from age considerations due to the existence of some recently reported old high-redshift galaxies (OHRGs), namely, the LBDS 53W091, a 3.5-Gyr-old radio galaxy at $z=1.55$ [20] and the LBDS 53W069, a 4.0-Gyr-old radio galaxy at $z=1.43$ [21]. We focus our attention on models based on the framework of brane-induced gravity of Dvali *et al.* [2] that have been recently proposed in Refs. [3,12]. To be consistent with the inflationary flatness prediction as well as with the latest cosmic microwave background (CMB) measurements we restrict ourselves to flat models. In our analysis we assume that the bulk space-time is 5 dimensional.

The geometry of our 4-dimensional Universe is described by the Friedmann-Robertson-Walker (FRW) line element ($c=1$)

$$ds^2 = dt^2 - R^2(t)[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{1}$$

where r, θ, and ϕ are dimensionless comoving coordinates and $R(t)$ is the scale factor. The Friedmann equation for the kind of models we are considering is [12,15]

$$\dot{R}(t) = R(t)\left[\sqrt{\frac{\rho}{3M_{pl}^2} + \frac{1}{4r_c^2}} + \frac{1}{2r_c}\right], \tag{2}$$

where ρ is the energy density of the cosmic fluid, $r_c = M_{pl}^2/2M_5^3$ is the crossover scale defining the gravitational interaction among particles located on the brane and M_{pl} and M_5 are, respectively, the 4- and 5-dimensional Planck mass. As explained in [3], for distances smaller than r_c the force experienced by two punctual sources is the usual 4-dimensional gravitational $1/r^2$ force whereas for distances larger than r_c the gravitational force follows the 5-dimensional $1/r^3$ behavior.

Througout this paper we will consider only one component of nonrelativistic particles together with the bulk-induced term. In this case, the age-redshift relation as a function of the observable parameters is written as

$$t_z = \frac{1}{H_o}\int_0^{(1+z)^{-1}} \frac{dx}{x f(\Omega_{\rm m}, \Omega_{\rm r_c}, x)}$$

$$= \frac{1}{H_o} g(\Omega_{\rm m}, \Omega_{\rm r_c}, z). \tag{3}$$

*Electronic address: alcaniz@astro.washington.edu
†Electronic address: deepak@ducos.ernet.in
‡Electronic address: abha@ducos.ernet.in

In the above expression, x is a convenient integration variable and the dimensionless function $f(\Omega_{\rm m},\Omega_{{\rm r}_c},x)$ is given by

$$f(\Omega_{\rm m},\Omega_{{\rm r}_c},x)=\sqrt{\Omega_{{\rm r}_c}}+\sqrt{\Omega_{{\rm r}_c}+\Omega_{\rm m}\,x^{-3}}, \qquad (4)$$

where $\Omega_{\rm m}$ is the matter density parameter and

$$\Omega_{{\rm r}_c}=1/4r_c^2H_o^2 \qquad (5)$$

is the density parameter associated with the crossover radius r_c. From Eq. (2), it is possible to show that the flat condition implies

$$\Omega_{{\rm r}_c}=\left(\frac{1-\Omega_{\rm m}}{2}\right)^2. \qquad (6)$$

The total expanding age of the Universe is obtained by taking $z=0$ in Eq. (3). As one may check, in the limit $1/r_c \to 0$, the standard relation $[t_z=\frac{2}{3}H_o^{-1}(1+z)^{-3/2}]$ is recovered.

We observe that by fixing the product $H_o t_z$ from observations, the limits on the parameter $\Omega_{{\rm r}_c}$ can be readily obtained from Eq. (3). Note also that the age parameter depends only on the product of the two quantities H_o and t_z, which are measured from completely different methods. To clarify these points, in the following we briefly outline our main assumptions for this analysis. Our approach is based on Ref. [22].

Firstly, we take for granted that the age of the Universe at a given redshift is bigger than or at least equal to the age of its oldest objects. As one may conclude, in the brane world scenarios discussed here, the comparison of these two quantities implies a lower bound for $\Omega_{{\rm r}_c}$, since the age of the Universe increases for larger values of this quantity. As is well known, for the dark and baryonic components, the age of the Universe increases when $\Omega_{\rm m}$ decreases, thereby implying the existence of an upper bound for the matter density parameter [24]. In order to quantify these qualitative arguments, it is convenient to introduce the ratio

$$\frac{t_z}{t_g}=\frac{g(\Omega_{\rm m},\Omega_{{\rm r}_c},z)}{H_o t_g}\geqslant 1, \qquad (7)$$

where t_g is the age of an arbitrary object, say, a galaxy at a given redshift z and $g(\Omega_{\rm m},\Omega_{{\rm r}_c},z)$ is the dimensionless factor defined in Eq. (3). For each extragalatic object, the denominator of the above equation defines a dimensionless age parameter $T_g=H_o t_g$. In particular, the 3.5-Gyr-old galaxy (53W091) at $z=1.55$ yields $T_g=3.5H_o$ Gyr which, for the most recent determinations of the Hubble parameter, $H_o=70\pm7$ km s^{-1} Mpc^{-1} [23], takes values on the interval $0.22\leqslant T_g\leqslant 0.27$. It thus follows that $T_g\geqslant 0.22$. Therefore, for a given value of H_o, only models having an expanding age bigger than this value at $z=1.55$ will be compatible with the existence of this object. In particular, taking $1/r_c\to 0$ (the standard flat case) in Eq. (3), one obtains $T_g\leqslant 0.16$, which means that the Einstein–de Sitter model is formally ruled out by this test [22].

In order to assure the robustness of the limits, two conditions have been systematically adopted in our computations, namely, the minimal value of the Hubble parameter, $H_o=63$ km s^{-1} Mpc^{-1}, and the underestimated age of the galaxies. Such conditions are almost self-explanatory when interpreted in the spirit of inequality (7). First, because the smaller the value of H_o, the larger the age that is predicted by the model, and, second, because objects with smaller ages are more easily accommodated by the cosmological models, thereby guaranteeing that conservative bounds are always favored in the estimates presented here. Naturally, similar considerations may also be applied to the 4.0-Gyr-old galaxy (53W069) at $z=1.43$. In this case, one obtains $T_g\geqslant 0.26$.

In Figs. 1 and 2 we show the diagrams of the dimensionless age parameter $H_o t_z$ as a function of the redshift for

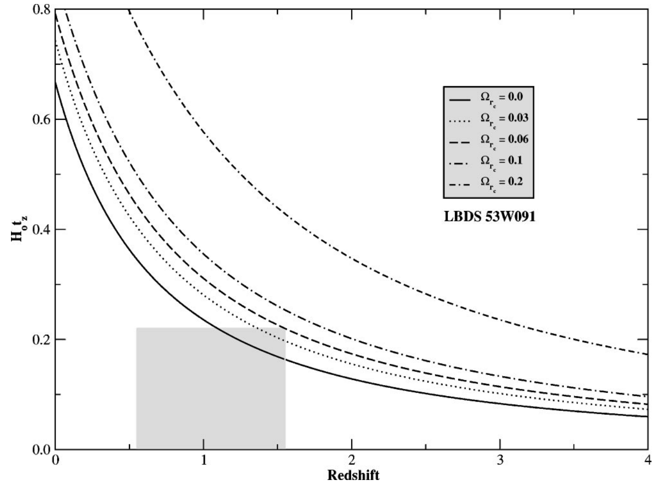

FIG. 1. Dimensionless age parameter as a function of redshift for brane world comologies. As explained in the text, all curves crossing the shadowed area yield an age parameter smaller than the minimal value required by the galaxy LBDS 53W091 reported in Ref. [20].

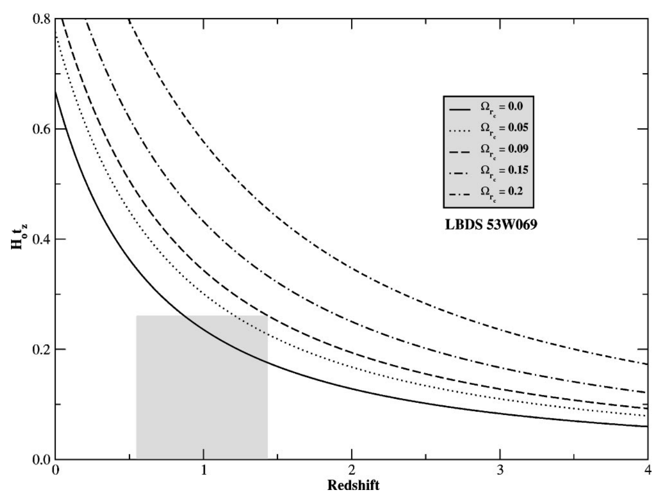

FIG. 2. The same as in Fig. 1 for the LBDS 53W069.

several values of Ω_{r_c} for the LDBS 53W091 and LBDS 53W069, respectively. The forbidden regions in the graphs have been determined by taking the values of T_g for each galaxy separately. All curves crossing the shadowed rectangle yield an age parameter smaller than the minimal value required by these objects. From this analysis we see that the curves intersecting the right upper corner of the rectangle correspond to $\Omega_{r_c}=0.06$ (53W091) and $\Omega_{r_c}=0.09$ (53W069). These values establish the lower limits on Ω_{r_c} allowed by these two galaxies and provide a minimal total age of the Universe of 12.3 and 13.1 Gyr, respectively. Substituting these values of Ω_{r_c} into Eq. (5), it is possible to estimate the crossover distance between the 4-dimensional and 5-dimensional gravities. In this case, for the LBDS 53W091 and LBDS 53W069 bounds, we obtain, respectively,

$$r_c \leqslant 2.04 H_o^{-1} \tag{8a}$$

and

$$r_c \leqslant 1.67 H_o^{-1}. \tag{8b}$$

At this point, it is interesting to compare our estimates of the crossover radius r_c with some other recent determinations of this quantity from independent methods. Recently, Deffayet *et al.* [15] using type Ia supernovae (SNe Ia)+ cosmic microwave background data found $r_c \simeq 1.4 H_o^{-1}$ for a flat model with $\Omega_m = 0.3$. Avelino and Martins [17], using a large sample of 92 supernova, obtained an approximate relation for the degeneracy in the Ω_{r_c}-Ω_m plane given by $\Omega_{r_c} \simeq \frac{2}{5}\Omega_m + \frac{1}{10}$. This approximate best fit line intersects the flat universe curve in such a way that we can use Eq. (6) to find $\Omega_{r_c} \simeq 0.12$ (for $\Omega_m = 0.3$) or, equivalently, $r_c \simeq 1.4 H_o^{-1}$. More recently, it was shown that measurements of the angular size of high-z sources requires a slightly closed universe with a crossover radius of the order of $\sim 0.94 H_o^{-1}$ [18] whereas the current gravitational lensing data implies $r_c = 1.76 H_o^{-1}$ [19]. These results, together with the main estimates of the present paper, are summarized in Table I.

TABLE I. Recent estimates of the crossover radius r_c.

Method	Reference	r_c [a]
SNe Ia + CMB	[15]	1.4
SNe Ia	[17]	1.4
Angular size	[18]	0.94
Gravitational lenses	[19]	1.76
OHRGs:		
$z = 1.55\ldots$	This paper	$\leqslant 2.04$
$z = 1.43\ldots$	This paper	$\leqslant 1.67$

[a] In units of H_o^{-1}.

We have investigated new observational constraints for some particular scenarios of brane world cosmology. Such models, inspired by recent developments in particle physics, constitute an interesting arena in which cosmological tests can play the important role of establishing a more solid conection between fundamental physics and astronomical observations. We have shown that these scenarios are compatible with the most recent observations of old galaxies at high redshifts for values of the crossover distance $r_c \leqslant 1.67 H_o^{-1}$. To assure the robustness of the limits derived here we have adopted in our computations the minimal value of the Hubble parameter given by [23], i.e., $H_o = 63\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$, as well as the underestimated age of the galaxies. Our results indicate that, similarly to models with a relic cosmological constant, there is no "age of the Universe problem" in the context of these brane world scenarios. Naturally, only with a new and more precise set of observations will it be possible to show whether or not this class of Superstring-M inspired models constitutes a viable possibility for the present dark energy problem.

The authors are grateful to R. Silva for helpful discussions and a critical reading of the manuscript. J.S.A. is supported by the Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq, Brazil) and CNPq (62.0053/01-1-PADCT III/Milenio).

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