A new cosmological scenario in string theory

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We consider new cosmological solutions with a collapsing, an intermediate and an expanding phase. The boundary between the expanding (collapsing) phase and the intermediate phase is seen by comoving observers as a cosmological past (future) horizon. The solutions are naturally embedded in string and M theory. In the particular case of a two-dimensional cosmology, space-time is flat with an identification under boost and translation transformations. We consider the corresponding string theory orbifold and calculate the modular invariant one-loop partition function. In this case there is a strong parallel with the BTZ black hole. The higher dimensional cosmologies have a timelike curvature singularity in the intermediate region. In some cases the string coupling can be made small throughout all of space-time but string corrections become important at the singularity. This happens where string winding modes become light which could resolve the singularity. The new proposed space-time causal structure could have implications for cosmology, independently of string theory.

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I. INTRODUCTION

One of the central problems in our present view of the Universe has to do with the cosmological singularity. The observation of an expanding Universe leads us to believe that in the past the Universe was much denser. At the Planck scale general relativity breaks down where it is usually believed that a spacelike cosmological singularity develops. Despite the great advances of particle cosmology from the grand unified theory (GUT) scale to present times, the understanding of the cosmological singularity remains a challenge. The resolution of this problem is one of the main motivations to find a quantum theory of gravity.

It has long been understood that string theory, as a consistent theory of gravity, could be a good starting point to investigate the universe at the Planck scale (see [1] for a review and references). In the pre-big-bang scenario, the Universe starts in a contracting phase until string effects along a spacelike hypersurface become important. From this spacelike hypersurface the Universe will evolve to the present expanding phase. The main problem that has prevented a deeper understanding of the singularity problem is the understanding of the stringy phase. Recently, there has been a new proposal for a big-crunch–big-bang transition [2,3]. These authors considered a toy model in which space-time is seen as flat space with an identification along boost transformations, an orbifold earlier investigated in [4]. A problem in this string compactification is that there are closed timelike curves and space-time is not Hausdorff. So even if space-time is flat and there is no curvature singularity, there is still a singularity at the big-bang. This problem has motivated our research. We shall present a two-dimensional toy model where we manage to smooth this string orbifold and to hide the closed timelike curves behind a cosmological horizon. Related work, with a different kind of orbifold, appeared recently in [5].

The two-dimensional toy model may have a tractable string theory description as an orbifold of flat space, and we initiate its investigation. On the other hand, as a solution of the gravity low energy equations, one expects that the new space-time causal structure of the two-dimensional toy model can be generalized to higher dimensions. Indeed, we shall construct new cosmological solutions for open Universes in arbitrary dimensions with three phases: (1) A collapsing phase with a future cosmological horizon; (2) an intermediate phase where there is a timelike curvature singularity; (3) an expanding phase with a past cosmological horizon. In the context of string theory, we shall argue that near the timelike curvature singularity there are string winding modes that are becoming very light. This fact led us to conjecture that when these new light states are taken into account the singularity could be resolved, smoothing the geometry.

This paper is organized in the following way. In Sec. II we present the new space-time identifications that lead to a two-dimensional cosmological solution. Here we were inspired by an analogous construction for the Bañados-Teitelboim-Zanelli (BTZ) black hole [6,7]. We start with three-dimensional flat space and identify points in space-time under boost and translation transformations. After Kałuża-Klein reduction, we find a cosmological solution with the properties described in the previous paragraph. In this case, however, there is no singularity. The apparent singularity corresponds to the surface where the compactification circle becomes null. As a consequence, there will be causal timelike curves in the intermediate region that can be closed if one passes the causal singularity. The cosmological observ-

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ers in the expanding and collapsing regions will not intersect these closed timelike curves.

The geometrical construction of the two-dimensional model can be embedded in string theory. This topic is covered in Sec. III. We consider the case of a string orbifold and analyze first the region of validity of the corresponding gravity solution. It is seen that in the intermediate region, where there are closed timelike curves, string effects become important. We calculate the one-loop bosonic partition function for the orbifold and show that it is modular invariant. The introduction of a translation in the boost identification not only makes space-time smoother, but also introduces a regularization scheme in the calculation of the string partition function.

In Sec. IV we generalize the two-dimensional cosmological toy model to higher dimensions. We start with a theory of gravity with a scalar field and a form field strength. Then we compactify space-time on a (Ricci) flat manifold along which there is a flux of the form field. Comparison with the two-dimensional case led us to require the geometry to be smooth and invariant along a null hypersurface that we interpret as the cosmological horizon. These boundary conditions and the homogeneity requirement of the cosmological solution imply that the geometry has the form of the higher dimensional Milne Universe along this null hypersurface, leading to a negative curvature Universe. We analyze the resulting geometry, which has collapsing, intermediate and expanding regions as mentioned above. We finish this section by embedding the new solutions in string and M theory. Related work that considers a different type of time-dependent string theory solutions by fixing boundary conditions on spacelike hypersurfaces appeared very recently in [8].

In Sec. V we give our conclusions. We show that in the new cosmological solutions the usual cosmology horizon problem does not arise. Then we comment on the possible resolution of the cosmological timelike singularity. We argue that many new solutions can be found by imposing the boundary conditions using a different scalar potential. In particular, one can consider potentials of the type used in standard particle cosmology, avoiding the spacelike singularity of the big bang.

II. COSMOLOGICAL SOLUTION AS A QUOTIENT OF FLAT SPACE

Consider flat space-time in $2+1$ gravity with line element

$$ds^2 = -dT^2 + dX^2 + dY^2. \tag{2.1}$$

We shall identify points on this space along orbits of a subgroup of its isometry group, i.e. a subgroup of the three-dimensional Poincaré group. The situation is analogous to the BTZ black hole that is obtained from AdS_3 in $2+1$ gravity with a negative cosmological constant [6,7]. Later we shall embed the three-dimensional geometry here derived in string and M theory.

Let κ be the Killing vector

$$\kappa = 2\pi i(\Delta J + RP), \tag{2.2}$$

where

$$iJ = T\frac{\partial}{\partial X} + X\frac{\partial}{\partial T}, \quad iP = \frac{\partial}{\partial Y}, \tag{2.3}$$

are the generators of Lorentz boosts along the X direction and translations along the Y direction, respectively. The Killing vector κ defines a one parameter subgroup of isometries. We shall identify points Q along the orbits of this subgroup according to

$$Q \sim \exp(\kappa) Q. \tag{2.4}$$

The parameter Δ is related to the boost velocity by $v = \tanh(2\pi\Delta)$ and $2\pi R$ is the translation length. The resulting space is a smooth manifold with a flat metric, because the identifications are along an isometry of the initial space with no fixed points. However, now there are curves that join identified points and one needs to worry about the causal structure of the resulting space.

A necessary condition for the absence of timelike and null curves is $\kappa \cdot \kappa > 0$. The boundary of this region is the surface $\kappa \cdot \kappa = 0$ described by

$$-T^2 + X^2 = \frac{1}{E^2}, \tag{2.5}$$

where E has dimension length^{-1} and is defined by

$$E = \frac{\Delta}{R}. \tag{2.6}$$

Notice that the action induced by the Killing vector κ has no fixed points because the vector P always induces a translation along the Y direction. The orbits of κ in the quotient space are closed curves and cannot be continuously deformed to a point. These orbits are null on the surface defined by Eq. (2.5) and become timelike beyond it.

Recently, Khoury et al. [2] considered the cosmological solution arising from the identification of points in space-time that are related by a boost transformation. This case corresponds to setting $R=0$ in our model. In their work, the point $T=X=0$ is a fixed point of the orbifold and the light rays $|T|=|X|$ are mapped arbitrarily close to the origin. As a consequence, if one does not excise any region of space-time, there will be causal closed curves arbitrarily close to the point $T=X=0$. Also, the resulting manifold is not Hausdorff. In the model proposed here, the introduction of the translation in the space-time identification has resolved these problems. In particular, we also have causal closed curves but we shall argue that, from the point of view of the cosmological observer, these curves are not observable, and no inconsistencies should arise.

To understand the causal structure of the quotient space it is convenient to divide space in three different regions

$$\begin{aligned} &\text{I:} \quad |T| > |X| \ \text{ and } \ \kappa\cdot\kappa > 0, \\ &\text{II:} \quad |T| < |X| \ \text{ and } \ \kappa\cdot\kappa > 0, \end{aligned} \tag{2.7}$$

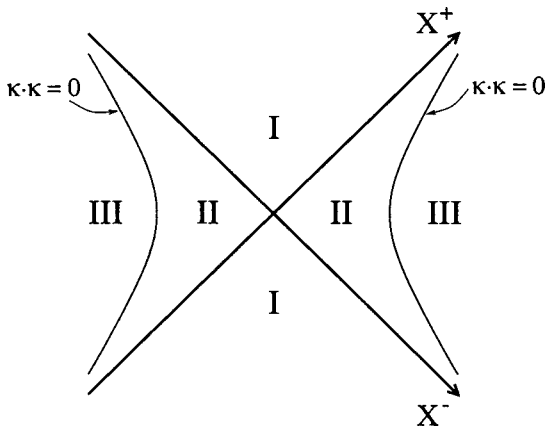

FIG. 1. The different regions in space-time in the light cone coordinates $X^{\pm}=(X\pm T)/\sqrt{2}$. The outer regions I will be interpreted as the cosmological collapsing and expanding phases. In regions III there are closed timelike curves that can be deformed to regions II, but always close in region III. The surface $|T|=|X|$ acts as a horizon because the closed timelike curves cannot be deformed to enter regions I.

$$\text{III:}\quad \kappa\cdot\kappa<0.$$

We shall call region I the outer region. For $T>0$ $(T<0)$ it describes an expanding (collapsing) universe. In this region, the above condition can be resumed to $\kappa\cdot\kappa>(2\pi R)^2$. Region II is the inside region, where $0<\kappa\cdot\kappa<(2\pi R)^2$. The region beyond which κ becomes timelike is called region III. In Fig. 1 the different regions are represented. The frontiers between regions I and II are null surfaces. This fact is very important because these surfaces become horizons and prevent causal closed curves from being extended to both the expanding and collapsing outer regions.

A. Compactification to two dimensions

We want to interpret this flat geometry from the two-dimensional point of view. In other words, we want to find the coordinate transformation that brings the Killing vector κ to the form

$$\kappa=2\pi R\frac{\partial}{\partial y}. \tag{2.8}$$

Then, to obtain the two-dimensional cosmological solution we consider the Kałuża-Klein compactification

$$ds_3^2=ds_2^2+e^{2\sigma}(dy+A_a dx^a)^2, \tag{2.9}$$

keeping in mind that one can always add extra spectator dimensions. The above Kałuża-Klein reduction of the three-dimensional Einstein-Hilbert action gives

$$S=\frac{1}{2\kappa_2^2}\int d^2x\sqrt{-g}e^{\sigma}\left[R-\frac{1}{4}e^{2\sigma}F^2\right], \tag{2.10}$$

where κ_2 is the two-dimensional gravitational coupling. The only dynamical degree of freedom is the scalar field.

Now let us analyze the geometry from the two-dimensional point of view. We consider the following coordinate transformations in both regions I and II:

$$\begin{aligned}&\text{I:}\begin{cases}T=t\cosh[E(x+y)],\\ X=t\sinh[E(x+y)],\\ Y=y,\end{cases}\\ &\text{II:}\begin{cases}T=t\sinh[E(x+y)],\\ X=t\cosh[E(x+y)],\\ Y=y.\end{cases}\end{aligned} \tag{2.11}$$

In the new coordinate system the killing vector κ is the form (2.8) required for compactification. We consider first the region I, where in the new coordinate system the line element becomes

$$ds^2=-dt^2+\frac{(Et)^2}{\Lambda(t)}dx^2+\Lambda(t)\left(dy+\frac{(Et)^2}{\Lambda(t)}dx\right)^2, \tag{2.12}$$

with

$$\Lambda(t)=1+(Et)^2. \tag{2.13}$$

From the above Kałuża-Klein ansatz it is straightforward to read the two-dimensional fields from the metric element. Both the t and x coordinates run from $-\infty$ to $+\infty$. The region $t<0$ describes a contracting universe while the region $t>0$ describes an expanding universe. The time coordinate t is the proper time for a comoving observer with the expansion (or collapse). We shall focus on the expanding region. The Kałuża-Klein 2-form field strength is given by

$$F=\frac{2E^2t}{\Lambda^2}dt\wedge dx, \tag{2.14}$$

which vanishes for large cosmological times, where the metric becomes flat. The radius of the compactification circle $R(t)=R\sqrt{\Lambda(t)}$, determined by the scalar field, is growing with time, and, therefore, at later times the Kałuża-Klein approximation is no longer valid. At earlier times, for $Et\ll 1$, the metric becomes

$$ds^2\sim -dt^2+(Et)^2dx^2. \tag{2.15}$$

At the surface $t=0$, $x=\pm\infty$ there is a harmless coordinate singularity. In fact, at this surface the metric has the usual form of the Milne universe obtained from a coordinate transformation of the usual flat space metric. This surface represents the past cosmological horizon for the observers in the expanding region I. The radius of the compact circle on the horizon is R.

The Kałuża-Klein form of the metric in region II can be obtained from the coordinate transformation (2.11). The final result is a line element that can be obtained from that of region I in Eq. (2.12) by the replacement $t\to it$:

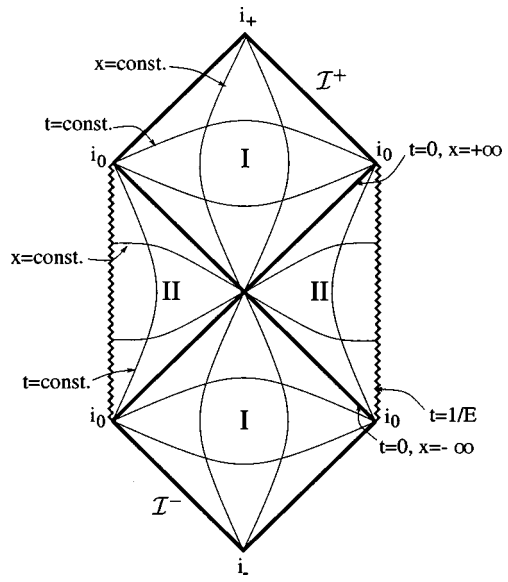

FIG. 2. Carter-Penrose diagram for the two-dimensional Kałuża-Klein cosmology. Spatial, future and past infinities are defined with respect to the expanding region I. The past (future) horizon of the expanding (collapsing) outer region is a Cauchy surface. The singularity at $t=1/E$ is an artifact of the compactification because this is the surface where κ becomes null.

$$ds^2=-\frac{(Et)^2}{\Lambda(t)}dx^2+dt^2+\Lambda(t)\left(dy-\frac{(Et)^2}{\Lambda(t)}dx\right)^2, \tag{2.16}$$

where

$$\Lambda(t)=1-(Et)^2. \tag{2.17}$$

Now the coordinates t and x have flipped their spatial and timelike characters. The surface $t=0$, $x=\pm\infty$ becomes a horizon for an observer in region II using this coordinate system. In fact, near this surface the metric becomes the Rindler metric. As we move away from the horizon the compactification radius decreases and vanishes at $Et=1$. At this surface the two-dimensional curvature and the invariant F^2 diverge. This is the surface where the killing vector κ becomes null, and the Kałuża-Klein approximation breaks down. Of course from the higher dimensional point of view this is not a curvature singularity because the metric is a quotient of flat space, but it is the locus where the orbits of κ become causal. Figure 2 contains the Carter-Penrose diagram for the geometry at a fixed position in the compact direction y. Similar two-dimensional space-time causal structures were considered in [9,10].

As a remark let us note that there is a set of coordinates that extends the metric in the outer region I to the inner region II, and that this metric is well behaved throughout these regions. This is easily seen because the metric in region I near the horizon behaves exactly as that for the Milne universe. So the coordinate transformation

$$\tilde{T}=t\cosh(Ex),\quad \tilde{X}=t\sinh(Ex), \tag{2.18}$$

in the solution (2.12) also covers region II.

B. Geometry as a limit of the BTZ black hole

We have described, in the previous subsections, geometries derived as quotients of flat Minkowski three-dimensional space. In this subsection we show that these geometries have natural generalizations as quotients of AdS_3, which are nothing but the usual BTZ black hole solutions. Therefore we can regard the geometries considered previously as a limit of the BTZ black hole.

To make this relation more precise, we describe AdS_3, as usual, as the surface

$$-(Z^0)^2+(Z^1)^2+(Z^2)^2-(Z^3)^2=-L^2, \tag{2.19}$$

with the parametrization

$$\begin{aligned} Z^0+iZ^3&=-ie^{iT/L}\sqrt{L^2+X^2+Y^2},\\ Z^1+iZ^2&=X+iY. \end{aligned} \tag{2.20}$$

The symmetry group of AdS_3 is $SO(2,2)$, with generators $iJ_{\mu\nu}=Z_\mu\partial_\nu-Z_\nu\partial_\mu$. In the limit $L\to\infty$ we recover flat Minkowski space $\mathbb{R}^{1,2}$ with isometry group $ISO(1,2)$. Moreover, the generators $J_{\mu\nu}$ converge to the generators P_m, M_{mn} $(m,n=0,1,2)$ of $ISO(1,2)$ according to

$$J_{mn}\to M_{mn},\quad \frac{1}{L}J_{3m}\to P_m. \tag{2.21}$$

Therefore, the natural generalization of the construction considered in the last section is to consider the quotient of AdS_3 by the isometry e^κ, where

$$\kappa=2\pi i\left[\Delta J_{10}+\left(\frac{R}{L}\right)J_{32}\right]. \tag{2.22}$$

This corresponds [7] to the BTZ black hole geometry [6]

$$ds^2_{BTZ}=-N^2dv^2+N^{-2}dr^2+r^2(d\phi+N^\phi dv)^2, \tag{2.23}$$

where

$$\begin{aligned} N^2&=\frac{1}{L^2r^2}(r^2-r_+^2)(r^2-r_-^2),\\ N^\phi&=\frac{1}{L}\frac{r_+r_-}{r^2}, \end{aligned} \tag{2.24}$$

and

$$r_+=L\Delta,\quad r_-=R. \tag{2.25}$$

We assume that $L>E^{-1}$, so that $r_+>r_-$. Recall that this geometry describes a rotating black hole with inner and outer horizons at $r=r_-$ and $r=r_+$. In the regions $r>r_+$ and $r<r_-$ the coordinate r is spacelike, while in the region $r_-<r<r_+$ the coordinate r is timelike.

The Euclidean continuation of the time coordinate, $v \to \tau_E$, in the regions $r>r_+$ and $r<r_-$, yields quotients of the hyperbolic three-plane H_3 under an element of the symmetry group $SO(1,3)$ [11]. Recall that quotients of H_3 are parametrized by the upper complex half-plane as follows. If we write the metric of H_3 as

$$ds^2_{H_3} = L^2\left[(1+\rho^2)d\tau_E^2 + \frac{d\rho^2}{1+\rho^2} + \rho^2 d\phi^2\right], \quad (2.26)$$

then a quotient parametrized by $\tau = \tau_1 + i\tau_2$ is given by the identifications

$$\tau_E \sim \tau_E + 2\pi\tau_2, \quad \phi \sim \phi + 2\pi\tau_1. \quad (2.27)$$

This corresponds, from the point of view of the 2-dimensional CFT on the AdS boundary, to considering the CFT on a torus with modular parameter τ [12]. The Euclidean continuation of the regions $r>r_+$ and $r<r_-$ give quotients of H_3 with modular parameters $\tau_{\rm out}$ and $\tau_{\rm in}$, respectively, where

$$\tau_{\rm out} = \frac{1}{L}(r_- + ir_+) = \frac{R}{L} + i\Delta,$$
$$\tau_{\rm in} = \frac{1}{L}(r_+ + ir_-) = \Delta + i\frac{R}{L}. \quad (2.28)$$

Now we can easily connect the discussion above with the compactification considered in the previous subsections, which is obtained in the limit $L\to\infty$, with R, Δ and r fixed. In this limit clearly $r_- = R$ remains finite and

$$r_+ = \Delta L \to \infty, \quad (2.29)$$

so that the outer region $r>r_+$ is no longer part of the geometry. Then the intermediate region $r>r_-$ corresponds to region I in the previous subsections, and r, which is a timelike coordinate in this region, is related to the cosmological time. More precisely, in the limit $L\to\infty$ one obtains the geometry

$$ds^2_{BTZ} = -\frac{1}{\Delta^2}\left(\frac{r^2}{r^2-R^2}\right)dr^2 + \Delta^2\left(\frac{r^2-R^2}{r^2}\right)dv^2 + r^2\left(d\phi + \frac{\Delta R}{r^2}dv\right)^2, \quad (2.30)$$

which is nothing but the solution (2.12) under the change of coordinates

$$\left(\frac{r}{R}\right)^2 = 1 + (Et)^2, \quad \Delta v = -x,$$
$$R\phi = y + x. \quad (2.31)$$

In the above geometry one can consider the Euclidean continuation of the inner region $r<r_-$, which, as described above, gives the quotient of H_3 described by the modular parameter $\tau_{\rm in}$. The geometry (2.26) becomes (calling $L\tau_E = z$ and $L\rho = r$) the usual flat Euclidean metric $dz^2 + dr^2 + r^2 d\phi^2$ with identifications

$$z \sim z + 2\pi R, \quad \phi \sim \phi + 2\pi\Delta. \quad (2.32)$$

This is clearly the geometry of a Euclidean flux brane, as it is natural to expect.

C. Closed timelike curves?

Now we come to the delicate point of whether we should excise regions III, interpreting the surface $\kappa\cdot\kappa = 0$ as a causal singularity. We shall advocate that, from the point of view of an observer in the expanding (or collapsing) cosmological region I, there is no contradiction arising from the inclusion of those regions.

There are closed timelike curves in the region III where $\kappa\cdot\kappa < 0$. These curves can be deformed to the region II, resulting in causal closed curves that are partially in region II (one needs the region III to close the open causal curves in region II that start and end on the surface $\kappa\cdot\kappa = 0$). However, there are no causal closed curves passing through region I because of the cosmological horizon. Indeed the horizon is a Cauchy surface and therefore no causal curve intersects it more than once. Hence the comoving observer never intersects any closed causal curve. In this sense the existence of the closed timelike curves is harmless. Any set of initial conditions at cosmological time t_0 will evolve without contradiction. Also, the existence of closed timelike curves shielded by horizons appears in the BTZ black hole [6,7], as well as in other higher dimensional rotating black holes [13], both cases having string theory dual descriptions. Furthermore, near the causal singularity one expects new degrees of freedom to become important which could prevent the existence of the closed timelike curves (see [14] for a related discussion in the case of the BTZ black hole).

This geometry represents a smooth transition from a collapsing phase to an expanding phase, with the additional regions II and III in between. To understand how a set of initial boundary conditions propagates from the collapsing to the expanding phase, it is necessary to understand the evolution throughout the intermediate regions. Therefore it is important to investigate the modifications in field and string theories due to the boundary conditions here imposed. For example, in region II there is an increasing electric field and one would expect that pair production of Kałuża-Klein particles occurs. Also, the proper radius of the Kałuża-Klein circle is decreasing and therefore these particles become very massive as the electric field becomes very large. We can do an estimation of the pair production rate using the Schwinger formula $\Gamma \sim \exp(-\pi(m/\mathcal{E}))$, where $m = 1/R(t)$ is the particle's mass and $\mathcal{E}^2 = -F^2/2$ determines the electric field. This gives the following estimation for the pair production rate:

$$\Gamma \sim \exp\left(-\frac{\pi\Lambda(t)}{2ER}\right). \quad (2.33)$$

Asymptotically in region I this gives a very small rate. Along the light rays, it can be made small provided $ER = \Delta \ll 1$. In

region II the argument in the exponential becomes very small and the semi-classical approximation for the nucleation rate breaks down. Nevertheless, one expects that there will be Kałuża-Klein particles produced in region II that, for later cosmological times in region I, move at constant velocity and will be distributed homogeneously through the constant time sections of the comoving observer with the expansion. Note that this invariance under translations in x is nothing but the unbroken $SO(1,1)$ subgroup of the $ISO(1,2)$ left invariant by the compactification.

Clearly, the above example points to a better understanding of quantum processes in this space-time. In the next section, we start the investigation of these issues by embedding this construction in string theory.

III. STRING ORBIFOLD AS A COSMOLOGY

The above construction of a cosmological background by a quotient of flat space can be embedded in string and M theory by adding the appropriate number of flat spectator directions. In the case of ten-dimensional string theory one has the usual string in flat space but there is a twisted sector arising from the orbifold compactification. Before we analyze this string theory let us briefly describe the underlying geometry and its limits of validity.

Considering type II strings, the compactification on a circle to the nine-dimensional string frame is given by

$$ds_{10}^2 = ds_9^2 + e^{2\sigma}(dx^9 + A_a dx^a)^2, \tag{3.1}$$

where $\phi_9 \equiv \phi - \sigma/2$ is the nine-dimensional dilaton field. If one adds seven spectator flat directions to the construction of the last section and reduces along the y direction, one obtains

$$\begin{aligned} ds_9^2 &= -dt^2 + \frac{(Et)^2}{\Lambda(t)} dx^2 + ds^2(\mathbb{E}^7), \\ e^{2\sigma} &= 1 + (Et)^2 \equiv \Lambda(t), \quad A = \frac{(Et)^2}{\Lambda(t)} dx. \end{aligned} \tag{3.2}$$

The space-time causal structure of this geometry and its maximal extension was explained before. The conditions for the validity of the nine-dimensional description are as follows. First, the nine-dimensional string coupling has to be small

$$g_9 = \frac{g\alpha'^{1/4}}{\sqrt{R(t)}} \ll 1. \tag{3.3}$$

In region I the coupling decreases with time; therefore this condition always holds provided the ten-dimensional string coupling g is small. In region II, the condition fails near the causal singularity and one needs to use the ten-dimensional approximation. Indeed, since the ten-dimensional coupling g remains constant, the string orbifold analysis holds as long as g is kept small.

Second, the typical energy scale $E = \Delta/R$ in the geometry described by Eq. (3.2) should be much smaller than the massive string states scale

$$E \ll \frac{1}{\sqrt{\alpha'}} \Rightarrow \Delta \ll \frac{R}{\sqrt{\alpha'}}. \tag{3.4}$$

This condition can always be satisfied for Δ sufficiently small. Third, the string winding states should be very massive, i.e.

$$E \ll \frac{R(t)}{\alpha'}, \tag{3.5}$$

which in region I is compatible with the condition above. However, in region II the winding states become very light because the proper size of the circle converges to zero. This means that in this region new light string degrees of freedom become important and one needs to use the string orbifold description. Finally, space-time is effectively nine-dimensional provided the Kałuża-Klein modes are very massive

$$E \ll \frac{1}{R(t)}. \tag{3.6}$$

In region I, this condition will fail for large cosmological time. However, the region of validity can be made arbitrarily large by choosing Δ very small. In region II the condition is satisfied because the Kałuża-Klein modes become very massive. At last, one could worry that the curvature corrections become important in region II; however, since the solution is a flat space orbifold it seems reasonable to expect that such corrections vanish.

In the above analysis no assumption was made regarding the size of the compactification radius R compared with the string length. For some spacelike surface at time t in region I, provided Δ is sufficiently small, the typical energy scale $E = \Delta/R$ for phenomena on the cosmological solution is always much smaller than the Kałuża-Klein and string mass gaps. Also, the nine-dimensional string coupling is small. In region II the Kałuża-Klein states become very massive but the associated electric field becomes very large pointing to the Schwinger process, the winding states become very light and the nine-dimensional string coupling blows up. Clearly in region II the relevant description is in terms of the ten-dimensional string orbifold, to which we now turn.

A. String partition function

In this section we start to analyze the motion of a (bosonic) string in the quotient space described in the previous sections. We leave the generalization to the superstring to future work. Readers that are not familiar with string theory may wish to skip this subsection and move to the classical gravity analysis in Sec. IV (Refs. [15,16] provide the necessary background for the techniques presented here). Similar computations have been carried out in the case of open strings in electric fields [17], D-branes in relative motion [18] and closed strings in magnetic backgrounds [19,20]. We will limit ourselves to the computation

of the one-loop partition function, leaving to future work a more detailed analysis of the results. We will use units such that $\alpha'=2$.

We use, in the following, light cone coordinates

$$X^{\pm}=\frac{1}{\sqrt{2}}(X\pm T), \tag{3.7}$$

so that the basic identifications introduced in Sec. II are given by

$$X^{\pm}\sim e^{\pm 2\pi\Delta}X^{\pm}, \quad Y\sim Y+2\pi R. \tag{3.8}$$

Let us then focus on the winding sector with winding number w. First of all, it is clear that the mode expansion of the field $Y(z,\bar{z})$ is the usual one of a compact boson. The only difference with the standard S^1 compactification is given by a modified constraint on the total momentum P, which must be compatible with the identification (3.8) and must therefore satisfy

$$\exp[2\pi i(RP+\Delta J)]=1,$$
$$P=\frac{1}{R}(n-\Delta J), \tag{3.9}$$

where n is an integer and J is the boost operator. The left and right momenta for Y are then given by

$$p_{L,R}=P\pm\frac{wR}{2}. \tag{3.10}$$

The mode expansions of the fields $X^{\pm}(z,\bar{z})$ are, on the other hand, modified and are given explicitly by

$$X^{\pm}(z,\bar{z})=i\sum_n\left(\frac{1}{n\pm i\nu}\frac{a_n^{\pm}}{z^{n\pm i\nu}}+\frac{1}{n\mp i\nu}\frac{\tilde{a}_n^{\pm}}{\bar{z}^{n\mp i\nu}}\right), \tag{3.11}$$

where $\nu=w\Delta$ and where the oscillators satisfy the commutation relations

$$[a_m^{\pm},a_n^{\mp}]=(m\pm i\nu)\delta_{m+n},$$
$$[\tilde{a}_m^{\pm},\tilde{a}_n^{\mp}]=(m\mp i\nu)\delta_{m+n}, \tag{3.12}$$

and the Hermitianity conditions $(a_m^{\pm})^{\dagger}=a_{-m}^{\pm}$, $(\tilde{a}_m^{\pm})^{\dagger}=\tilde{a}_{-m}^{\pm}$. The contribution to the Virasoro generators from the fields $X^{\pm}$ has been computed previously in [18,19] and is given by ($\cdots$ denotes contributions from other fields)

$$L_0=\cdots+\frac{1}{2}i\nu(1-i\nu)+\sum_{n\geqslant 1}a_{-n}^{+}a_n^{-}+\sum_{n\geqslant 0}a_{-n}^{-}a_n^{+},$$
$$\tilde{L}_0=\cdots+\frac{1}{2}i\nu(1-i\nu)+\sum_{n\geqslant 0}\tilde{a}_{-n}^{+}\tilde{a}_n^{-}+\sum_{n\geqslant 1}\tilde{a}_{-n}^{-}\tilde{a}_n^{+}, \tag{3.13}$$

where implicitly we have chosen to consider a_0^{-} and $\tilde{a}_0^{+}$ as creation operators.[1] Finally, the boost operator $J=J_L+J_R$ defined by $i[J,X^{\pm}]=\pm X^{\pm}$, is given explicitly by

$$J_L=-i\sum_{n\geqslant 1}N_n^{+}+i\sum_{n\geqslant 0}N_n^{-},$$
$$J_R=-i\sum_{n\geqslant 0}\tilde{N}_n^{+}+i\sum_{n\geqslant 1}\tilde{N}_n^{-}, \tag{3.14}$$

where $N_n^{\pm}=(n\mp i\nu)^{-1}a_{-n}^{\pm}a_n^{\mp}$ and $\tilde{N}_n^{\pm}=(n\pm i\nu)^{-1}\tilde{a}_{-n}^{\pm}\tilde{a}_n^{\mp}$ are the usual occupation numbers. It is then clear that one can rewrite the *total* Virasoro generators for the three bosons T, X, and Y in terms of the usual *integral* level numbers $\mathbb{L},\tilde{\mathbb{L}}$ and the boost operator as

$$L_0=\frac{1}{2}i\nu(1-i\nu)+\nu J_L+\frac{1}{2}p_L^2+\mathbb{L},$$
$$\tilde{L}_0=\frac{1}{2}i\nu(1-i\nu)-\nu J_R+\frac{1}{2}p_R^2+\tilde{\mathbb{L}}. \tag{3.15}$$

We are now ready to compute the partition function Z_3 for the three bosons T, X and Y. We have

$$Z_3=(q\bar{q})^{-1/8}\sum_{w,n}\mathrm{Tr}q^{\mathbb{L}}\bar{q}^{\tilde{\mathbb{L}}}\left(\frac{q}{\bar{q}}\right)^{(1/2)nw}$$
$$\times(q\bar{q})^{(1/2)[(wR/2)^2+(n-\Delta J/R)^2]}$$
$$\times(q\bar{q})^{(1/2)\nu(J_L-J_R)}(q\bar{q})^{(1/2)i\nu(1-i\nu)}, \tag{3.16}$$

where, as customary, $q=e^{2\pi i\tau}$ and $\tau=\tau_1+i\tau_2$. Performing the usual Poisson resummation on n brings the above expression to the simpler form

$$Z_3=(q\bar{q})^{-1/8}\frac{R}{\sqrt{2\tau_2}}\sum_{w,w'}\exp\left[-\frac{\pi R^2}{2\tau_2}T\bar{T}\right.$$
$$\left.-2\pi\tau_2\Delta^2w^2\right]q^{(1/2)i\nu}\mathrm{Tr}_L(e^{2\pi iT\Delta J_L}q^{\mathbb{L}})\bar{q}^{(1/2)i\nu}$$
$$\times\mathrm{Tr}_R(e^{2\pi i\bar{T}\Delta J_R}\bar{q}^{\tilde{\mathbb{L}}}), \tag{3.17}$$

where

$$T=w\tau-w'. \tag{3.18}$$

In the above sum the term with $w=w'=0$, which is by itself modular invariant, gives the usual partition function of the uncompactified theory. We will therefore focus, in the following, only on the other terms in the sum (we will denote

[1]The correct treatment of the zero modes is quite subtle. In fact they form two copies of the Heisenberg algebra $[a_0^{+},a_0^{-}]=[\tilde{a}_0^{-},\tilde{a}_0^{+}]=i\nu$. We choose to quantize the zero modes as creation and annihilation operators in order to preserve the conformal invariance of the partition function.

the restricted sum by Σ'). With this in mind, the computation of the holomorphic trace Tr_L is simple. Defining $c = e^{2\pi i(i\Delta T)} = q^{i\nu} e^{2\pi w'\Delta}$, one has

$$\mathrm{Tr}_L(e^{2\pi i T\Delta J_L} q^{\mathbb{L}}) = \frac{1}{1-c}\prod_{n\geqslant 1}\frac{1}{(1-q^n)(1-q^n c)(1-q^n c^{-1})} = iq^{1/8}c^{-1/2}\frac{1}{\theta_1(i\Delta T|\tau)}. \tag{3.19}$$

Similarly, the antiholomorphic trace is given by $\mathrm{Tr}_R = \overline{c}\,\overline{\mathrm{Tr}_L}$. Therefore, the partition function Z_3 is given by the final expression (reinserting α')

$$Z_3 = \frac{R}{\sqrt{\alpha'\tau_2}}\sum_{w,w'}{}' e^{-(\pi R^2/\alpha')(T\overline{T}/\tau_2) - 2\pi\tau_2\Delta^2 w^2} \times |\theta_1(i\Delta T|\tau)|^{-2}. \tag{3.20}$$

Using the fact that $\partial_\zeta\theta_1(\zeta|\tau)|_{\zeta=0} = 2\pi\eta^3(\tau)$, we see that, as $\Delta\to 0$, the partition function diverges. This is due to the fact that, in the limit, the rotational symmetry in the $X-T$ plane is restored and the partition function is then proportional to the space-time volume. Moreover, the modular properties of θ_1 insure modular invariance of the full expression.

In order to compute the total partition function, we can use the usual trick [21] of extending the integration region from the fundamental domain $|\tau|>1$, $|\tau_1|<\frac{1}{2}$ to the strip $\Gamma = \{\tau\in\mathbb{C}|\tau_2>0, |\tau_1|<\frac{1}{2}\}$, while at the same time restricting the sum in Eq. (3.20) to $w=0, w'\geqslant 1$. Then we have

$$Z = \int_\Gamma \frac{d\tau d\overline{\tau}}{\tau_2^2} Z_g Z_b^{23}\widetilde{Z}_3, \tag{3.21}$$

where Z_g and Z_b are the usual partition functions for the b-c ghosts and for a noncompact boson, and where

$$\widetilde{Z}_3 = \frac{R}{\sqrt{\alpha'\tau_2}}\sum_{w'\geqslant 1} e^{-(\pi R^2/\alpha')(w'^2/\tau_2)}|\theta_1(i\Delta w'|\tau)|^{-2}. \tag{3.22}$$

Let us note that the above expression has strong similarities with the expression for the Euclidean BTZ black hole partition function found in [22], as expected from the analysis in Sec. II B. The expression for $\widetilde{Z}_3$ in fact exhibits a similar structure of poles in the region Γ, which are in correspondence with the zeros of $\theta_1(i\Delta w'|\tau)$. The function $\theta_1(\zeta|\tau)$, as a function of ζ, has in fact a simple zero at the points $\zeta = a\tau - b$ $(a,b\in\mathbb{Z})$. More precisely

$$\theta_1(\zeta|\tau)\sim(-)^{a+b}2\pi\eta^3(\tau)e^{-\pi i a(a+1)\tau}(\zeta - a\tau + b). \tag{3.23}$$

Then the poles of $\widetilde{Z}_3$ are determined by the equation $i\Delta w' = a\tau - b$ and are located at

$$\tau = \frac{1}{a}(b + i\Delta w'). \tag{3.24}$$

In order for the poles to be in Γ, we must have $a\geqslant 1$ and $|b|\leqslant 2a$. For fixed w', the structure of the poles is the same as those found in [22].

Finally let us comment on the origin of the poles. They arise from the zero modes of the winding sector. In fact, stretched winding strings have a length which is $2\pi R$ when they sit at the origin $X=T=0$, but which decreases as they approach the causal singularity, where they become almost massless. One can easily analyze the classical dynamics of these winding strings. In particular, in region I these strings move from the collapsing to the expanding regions, either passing through the origin where they have minimal length or passing through the intermediate region. At large cosmological times these strings become very long and stretched. Such states could be the origin of the pole structure of Z_3. In particular, the physical meaning of Z_3 could be related to nucleation processes of such long strings.

We leave to future work a more detailed analysis of the integral (3.21) and of the precise physical meaning of the poles in $\widetilde{Z}_3$, and the study of the possible relation of the winding string modes with the long strings studied in [22–25].

B. M theory and 9-11 flip

In this subsection we comment on the M-theory compactification. Consider the construction of Sec. II and add eight spectator flat directions. Then reducing to the type IIA theory one obtains the following background fields:

$$ds_{10}^2 = \Lambda^{1/2}[-dt^2 + ds^2(\mathbb{E}^8)] + \frac{(Et)^2}{\Lambda^{1/2}}dx^2, \tag{3.25}$$

$$e^{4\phi/3} = \Lambda(t), \quad A = \frac{(Et)^2}{\Lambda}dx.$$

By compactifying one of the spectator directions on a circle, this solution is related to the previous nine-dimensional solution by a 9-11 flip. Also, one can analyze the validity of the gravity approximation, finding that this description is appropriate for a large expansion period in region I provided that the boost parameter Δ is sufficiently small, and that the compactification radius R satisfies $R\ll\sqrt{\alpha'}$ (and therefore $g\ll 1$). In region II, the string coupling becomes small since the proper length of the compactification circle is decreasing. Near the causal singularity the string coupling becomes zero and the compactification radius becomes lightlike. Hence one expects that the correct description of the apparent ten-dimensional singularity is given by matrix theory. We think this issue deserves further investigation because it would give a resolution of the singularity in the above cosmological solution.

IV. COSMOLOGICAL SOLUTIONS IN ARBITRARY DIMENSIONS

So far we have considered geometries that arise from Kałuża-Klein compactifications. Naturally, one expects that such geometries can be generalized to arbitrary dimensions and arbitrary degrees of the form gauge field. These new space-times should share many properties with the case where space-time is a quotient of flat space. The situation is analogous to the BTZ black hole, which is simply a quotient of three-dimensional anti–de Sitter space but retains the standard properties of higher dimensional black holes.

A short cut to constructing such a generalization is to realize the similarity between the dilatonic Melvin solution [26] and the cosmological solution of Sec. II. In the case of the Kałuża-Klein Melvin solution, the geometry is simply flat space with an identification along the orbits of the isometry subgroup generated by rotations on a plane together with translations [27]. Hence, replacing the boost by a rotation in our construction one recovers the Kałuża-Klein Melvin solution. For example, starting with the cosmological solution (3.25) and making the analytic continuation $t\to ir$, $E\to iE$, $\mathbb{E}^8\to\mathbb{M}^8$, one obtains the flux 7-brane solution of the type IIA theory [28,29]. This suggests that the generalization of the two-dimensional cosmological solution to higher dimensions can be found by an analytic continuation of the flux brane geometries [30–32]. Earlier work on cosmological string backgrounds can be found in [33–36].

A. The action and basic ansatz

To keep our discussion general we shall consider a D-dimensional space-time with a $\tilde{d}$-form field strength $F=dA$ and a scalar field ϕ. The corresponding gravitational action is

$$S=\frac{1}{2\kappa_D^2}\int d^Dx\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-\frac{1}{2\tilde{d}!}e^{\alpha\phi}F^2\right], \tag{4.1}$$

where κ_D is the gravitational coupling. Of course this action can be regarded as a consistent truncation of either string or M-theory low energy actions, where F represents any of the field strengths or electromagnetic dual in these theories. We shall reduce the theory to $d+1=D-\tilde{d}$ dimensions according to the ansatz

$$\begin{aligned} ds_D^2&=e^{-[2\tilde{d}/(d-1)]\lambda}ds^2+e^{2\lambda}ds^2(\mathbb{E}^{\tilde{d}}),\\ F&=E\,\epsilon(\mathbb{E}^{\tilde{d}}). \end{aligned} \tag{4.2}$$

Then the action (4.1) becomes effectively

$$S=\frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left[R-\frac{1}{2}(\partial\phi)^2-\frac{\tilde{d}(\tilde{d}+d-1)}{d-1}(\partial\lambda)^2-V(\phi,\lambda)\right], \tag{4.3}$$

where κ is the $(d+1)$-dimensional gravitational coupling and we conveniently use the Einstein metric. The potential $V(\phi,\lambda)$ has the form

$$V(\phi,\lambda)=\frac{E^2}{2}\exp\left(\alpha\phi-2\frac{d\tilde{d}}{d-1}\lambda\right). \tag{4.4}$$

Before deriving the equations of motion for the action it is convenient to define the scalar fields ρ and ψ by the relations

$$\begin{aligned} \rho&=\alpha(\tilde{d}+d-1)\lambda+d\phi,\\ \psi&=\frac{2d\tilde{d}}{d-1}\lambda-\alpha\phi. \end{aligned} \tag{4.5}$$

With this field redefinition, $\rho=\rho_0$ constant solves the equation of motion for this scalar, which decouples from the remaining field equations. Then the gravitational action coupled to the scalar ψ becomes

$$S=\frac{1}{2\kappa^2}\int d^{d+1}x\sqrt{-g}\left[R-\frac{\beta}{2}(\partial\psi)^2-V(\psi)\right], \tag{4.6}$$

where the potential $V(\psi)$ has the form

$$V(\psi)=\frac{E^2}{2}\exp(-\psi), \tag{4.7}$$

and β is the numerical factor

$$\beta^{-1}=\alpha^2+\frac{2d^2\tilde{d}}{(\tilde{d}+d-1)(d-1)}. \tag{4.8}$$

B. Region I

To find the metric and scalar field in the region that is analogous to the region I of the Kałuża-Klein case, consider the Robertson-Walker space-time metric for an open universe

$$\begin{aligned} ds^2&=-dt^2+a^2(t)ds^2(H_d)\\ &=-dt^2+a^2(t)(d\chi^2+\sinh^2\chi d\Omega_{d-1}^2), \end{aligned} \tag{4.9}$$

together with the time-dependent scalar field $\psi(t)$. The equation of motion for the scalar is

$$\beta\left(\ddot{\psi}+d\frac{\dot{a}}{a}\dot{\psi}\right)=-\frac{\partial V}{\partial\psi}=V(\psi), \tag{4.10}$$

while Einstein equations take the usual form

$$\begin{aligned} \left(\frac{\dot{a}}{a}\right)^2-\frac{1}{a^2}&=\frac{1}{d(d-1)}\left[\frac{\beta}{2}\dot{\psi}^2+V(\psi)\right],\\ \frac{d}{dt}\left(\frac{\dot{a}}{a}\right)+\frac{1}{a^2}&=\frac{\beta}{2(1-d)}\dot{\psi}^2, \end{aligned} \tag{4.11}$$

where dots represent derivatives with respect to the coordinate t.

In analogy with the two-dimensional case we look for a geometry with similar behavior around $t=0$. In particular, we require the geometry to have the Milne form along the light rays. This is only possible provided the homogeneous space is the hyperboloid H_d, justifying our choice for an open universe cosmology. Then, one can find a solution to the above system of differential equations as a power expansion in the dimensionless quantity (Et). A straightforward calculation gives for the first terms in this expansion

$$\begin{aligned} a(t) &= t\left(1+\frac{e^{-\psi_0}}{3d(d-1)}(Et)^2+\cdots\right), \\ \psi(t) &= \psi_0+\frac{e^{-\psi_0}}{4\beta}(Et)^2+\cdots. \end{aligned} \tag{4.12}$$

Next we want to find the asymptotics for this geometry. As it is the case with cosmological solutions in an open universe, at later times we have curvature domination. Hence, the correct ansatz for the asymptotic solution is to set $a=a_0 t$, and then solve for the scalar field $\psi(t)$ and use the Freedman equation to fix the constant a_0. This results in the asymptotic behavior

$$a(t)=a_0 t, \quad e^{\psi(t)}=\frac{(Et)^2}{4(d-1)\beta}, \tag{4.13}$$

with the constant a_0 given by

$$a_0^2=\left(1-\frac{2\beta}{d-1}\right)^{-1}. \tag{4.14}$$

The picture we have in region I is exactly the same as for the Kałuża-Klein case. An observer that is comoving with the expansion could think that at an earlier time the matter density blows up and there is a cosmological spacelike singularity. This is the usual understanding in cosmology. However, in the picture we are proposing here, there is a past cosmological horizon where the geometry is perfectly smooth. In fact, we have fixed the boundary conditions on this horizon to evolve into the future. Of course, there is also a collapsing region I, where the comoving observer sees a future horizon where the boundary conditions are imposed. We shall give a more detailed discussion of this proposal in the conclusion.

C. Region II

The boundary conditions imposed above on the surface $t=0$, $\chi=+\infty$ are that the geometry looks like the Milne universe, which in fact is just flat space. The nontrivial assumption we are making is that the scalar field and the conformal factor have the same behavior throughout the whole surface $t=0$, $\chi=+\infty$. This is essential to have a homogeneous cosmology.

We can pass from the expanding (or collapsing) region I to a region II by a suitable change of coordinates. Again there is a close analogy with the two-dimensional case. First we change to a coordinate system well behaved around the coordinate singularity at $t=0$

$$\tilde{T}=t\cosh\chi, \quad \tilde{X}=t\sinh\chi. \tag{4.15}$$

In these coordinates the metric and scalar fields around the light rays $|\tilde{T}|=|\tilde{X}|$ are well behaved and there is no coordinate singularity, which allows us to continue the solution to region II where $|\tilde{T}|<|\tilde{X}|$. To make the symmetries of space manifest it is convenient to define new (t,χ) coordinates in region II by

$$\tilde{T}=t\sinh\chi, \quad \tilde{X}=t\cosh\chi. \tag{4.16}$$

Then the metric ansatz takes the form

$$\begin{aligned} ds^2 &= +dt^2+a^2(t)ds^2(dS_d) \\ &= dt^2+a^2(t)(-d\chi^2+\cosh^2\chi d\Omega_{d-1}^2). \end{aligned} \tag{4.17}$$

The $SO(1,d)$ symmetry of the original ansatz in region I, realized on the constant time d-dimensional hyperboloids, is now the symmetry of the constant t space-time slices. In fact, these slices are simply the d-dimensional de Sitter space. Notice that, in region II, the coordinate t becomes spacelike and the coordinate χ timelike. Along the horizon the $SO(1,d)$ symmetry acts as translations justifying the boundary conditions we have imposed.

The form of the solution near the $t=0$ surface can be obtained simply by the analytic continuation of the solution in region I

$$a_{II}(t)=-ia_I(it), \quad \psi_{II}(t)=\psi_I(it). \tag{4.18}$$

This gives the following expansion:

$$\begin{aligned} a(t) &= t\left(1-\frac{e^{-\psi_0}}{3d(d-1)}(Et)^2+\cdots\right), \\ \psi(t) &= \psi_0-\frac{e^{-\psi_0}}{4\beta}(Et)^2+\cdots. \end{aligned} \tag{4.19}$$

To find the asymptotics of the solution in region II, notice that if $a(t)=\cos t$ the metric describes $(d+1)$-dimensional de Sitter space and the surface $t=\pi/2$ is a coordinate singularity. In fact, if this was the case for this solution we would have a cosmological solution without any singularity. Starting from region II at $t=\pi/2$ we would have a de Sitter phase, which would then evolve to the Robertson-Walker cosmology in region I. Unfortunately this is not the case for the potential $V(\psi)$ that results from the Kałuża-Klein compactification. We have done some numerics and verified that the solution develops a singularity around some $t=t_0$, exactly as in the two-dimensional case. The correct asymptotics can be found by doing the ansatz

$$\begin{aligned} a(t) &= a_0(t_0-t)^\gamma+\cdots, \\ \psi(t) &= \eta\log(\theta E(t_0-t))+\cdots. \end{aligned} \tag{4.20}$$

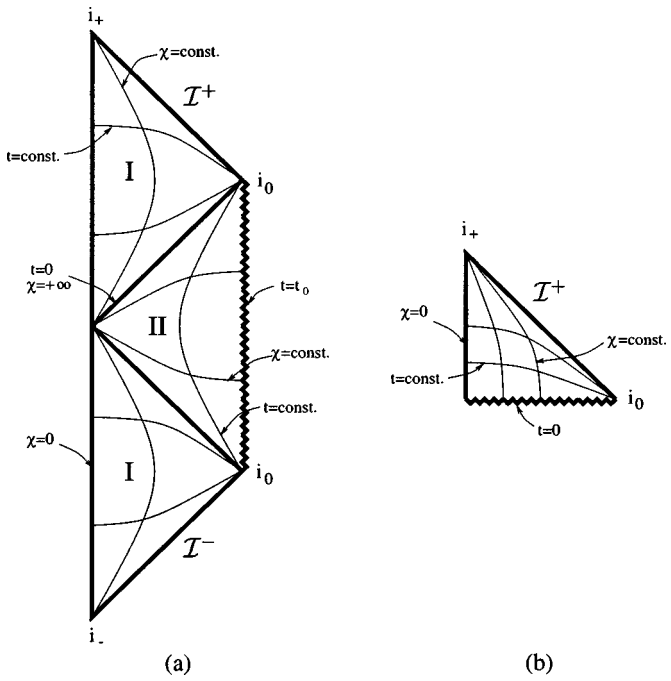

FIG. 3. Carter-Penrose diagrams for open Universe cosmologies. In both diagrams each point represents a $(d-1)$ sphere and $\chi=0$ is a coordinate singularity. The standard diagram in (b) is presented for comparison with our proposal.

Then the equations of motion give the following values for the dimensionless constants η, γ and θ:

$$\eta=2, \quad \gamma=\frac{2\beta}{d-1}, \quad \theta^{-2}=4\beta(1-d\gamma), \qquad (4.21)$$

while the value of a_0 is a constant of integration, that is fixed by the value of the scalar field at the light rays.

We have verified that the analytic behavior for the different asymptotics in both regions I and II exactly matches the numerical analysis. In Fig. 3(a) the Carter-Penrose diagram for the cosmological solutions is shown. For comparison we included in Fig. 3(b) the usual Robertson-Walker open Universe diagram. The pre-big-bang string cosmology scenario [1] considers the latter diagram and glues a collapsing phase to an expanding phase along the spacelike singularity. For the new space-time global structure here presented $t=0$ is a cosmological horizon. For small t the universe undergoes an acceleration and, when it becomes curvature dominated, it expands linearly with time.

D. String and M-theory cosmologies

As mentioned before the above solutions are naturally embedded in string and M theory. So we would like to understand where the gravity approximation is valid and where space-time and world-sheet string effects become important. The behavior of the solution is essentially determined by the constant α in the coupling between the dilaton field and the gauge field. For the RR gauge fields of the type II theories we have $\alpha=(5-\tilde{d})/2$, for the NS-NS 2-form gauge field $a=-1$ and for its dual 6-form gauge field $a=1$. If $\alpha>0$ ($\alpha<0$) the string coupling will become very large (small) near the singularity and it will become very small (large) in the asymptotics of regions I. In all cases the volume of the compact space becomes very small near the curvature singularity, which is telling us that the string winding states become very light. Asymptotically space-time decompactifies.

Let us analyze in more detail the cases with $\alpha=0$. Consider first the type IIB theory with the self-dual 5-form field strength. As for the D3-brane the string coupling can be made small throughout the whole solution. There will be curvature corrections near the singularity where string winding states become very light and the gravity approximation breaks down. Two other cases with $\alpha=0$ correspond to the M-theory 4-form field strength or its dual 7-form. The latter case is particularly interesting. For $\tilde{d}=7$, $\alpha=0$ and $D=11$, we have a four-dimensional cosmology for an open universe, and the compact manifold is seven-dimensional. Throughout this paper we considered flat manifolds in the internal directions, but more generally one can consider any Ricci flat manifold. In this case, one could consider a G_2 manifold for which there has been a great deal of interest for flat space compactifications, which yield $\mathcal{N}=1$ SUSY in $D=4$ (see [37] and references therein). Here, supersymmetry is broken by the form flux [38].

As this work was being completed an interesting paper [8] appeared, also considering new time dependent string theory backgrounds. These backgrounds are different from the ones introduced here. They analyzed the case where the boundary conditions for the scalar field are set on a spacelike hypersurface, while we have considered the case of a null hypersurface. We have interpreted this null hypersurface as the cosmological horizon in an open Universe cosmology. This fact allowed us to continue the geometry in both directions from the horizon. A natural question is whether one can also continue the other new geometries presented in [8] to a region similar to region II herein described. For the $D=4$ Einstein-Maxwell solution of [8], it is indeed possible to extend the geometry to a region II where there will be a timelike singularity. The Penrose diagram is similar to the diagram of Fig. 2 with each point representing a two-dimensional hyperbolic plane.

V. CONCLUSION: TOWARDS A SOLUTION OF THE HORIZON AND SINGULARITY PROBLEMS?

In this paper we have proposed a new cosmological scenario that tries to evade the cosmological singularity problem. The essential point was to consider collapsing and expanding phases with future and past cosmological horizons, and to fix the boundary conditions along these null hypersurfaces. From these horizons, we constructed the space-time geometry for specific examples that arise quite naturally in string theory. In the cosmological solution that we have studied, and more generally in cosmological solutions that have identical space-time causal structure, the usual horizon problem of standard cosmology does not arise. In fact, consider two points of the hyperboloid at constant cosmological time t in the expanding phase, that have an arbitrarily large spacelike separation. Then from the Carter-Penrose diagram, be-

cause there is a past cosmological horizon, the past light cones of these points always intersect.

Another aspect of this proposal is that it raises the hope of resolving the cosmological singularity problem. First, in the two-dimensional toy model presented here we were able, by embedding the geometry in string theory, to interpret the geometry as arising from a string theory orbifold from which one can at least do some calculations. In particular, we have started the analysis of the one-loop partition function which is modular invariant. The analogy with strings in thermal AdS_3 [22] is definitely worth pursuing. This toy model was inspired by the recent proposal in [2], and provides a regulator for their proposal. Indeed, with the identifications by a boost and a translation, at small translation parameter R space-time is still smooth (in particular Hausdorff) and there is a horizon. So this looks like a good regulator for the singular space-time one obtains when setting $R=0$ from the beginning. Secondly, in the case of the higher dimensional solutions, there are cases where the string coupling can be kept small throughout the whole space-time. There is a timelike singularity in the region II of space-time, where the compact space is shrinking and where winding string states become important. It would be very interesting to resolve these singularities within string theory. A reasonable conjecture is that the new light degrees of freedom could acquire some vacuum expectation value (VEV) resolving the singularity.

There is possibly another way of avoiding the cosmological singularity. This could happen if the potential for the scalar field had a different shape. Then one would start from a pure de Sitter phase in region II; as the fields evolve into the horizon one would be able to smoothly pass to the expanding region I. We have tried some numerics with different types of potentials but so far have not succeeded. Essentially, there is a fine-tuning problem. As one starts from the de Sitter phase, it seems very difficult to evolve the differential equations such that we reach the horizon with the exact behavior for the scale factor ($\dot{a}=1$), and scalar field ($\dot{\psi}=0$), such that the transition can be achieved. On the other hand, there may exist solitons that interpolate between different vacua in regions I and II (like domain walls). We think this issue deserves further attention because, if successful, it would give a completely smooth cosmological solution. Let us note that this is not in contradiction with the singularity theorems because in the de Sitter phase the strong energy condition is not satisfied.

Finally, independently of string theory, one should reexamine many issues in cosmology by considering different types of potentials, as those used in inflationary and particle cosmology, and by imposing the boundary conditions at the cosmological horizon. From these boundary conditions the universe can evolve into the usual cosmological epochs. Another important issue that we plan to analyze is the question of thermal radiation seen by the cosmological observer. Consider, for example, a scalar field and fix its boundary conditions to have only positive frequency modes on the cosmological horizon. Then one can study the spectrum at later times. Note that the same reasoning cannot be applied in the presence of a spacelike big-bang singularity.

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