# Testing of *CP*, *CPT,* and causality violation with light propagation in vacuum in the presence of uniform electric and magnetic fields

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We have considered the structure of the fundamental symmetry violating part of the photon refractive index in vacuum in the presence of constant electric and magnetic fields. This part of the refractive index can, in principle, contain $CPT$ symmetry breaking terms. Some of the terms violate Lorentz invariance, whereas the others violate locality and causality. Estimates of these effects, using laser experiments are considered.

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## I. INTRODUCTION

Recently, experiments on searching for the birefringence of a vacuum have been carried out and planned [1–3]. The BMV project [3] was proposed to achieve an accuracy sufficient for detection of vacuum birefringence predicted by QED. In addition, the search for exotic non-QED interactions is possible in such experiments. In this article we discuss what kinds of discrete, i.e., $P,T,C$, symmetry breaking terms can be present in the photon refractive index in vacuum in constant electric and magnetic fields.

$CP$ symmetry breaking in *K*-meson [4], and *B*-meson [5] decays, as well as time reversal symmetry violation in $K_0$-$\overline{K}_0$ oscillations [6], can be currently described by the standard model with the Cabibbo-Kobayashi-Maskawa matrix. It would be interesting to find $CP$ violation in other systems, different from the $K_0$ or $B_0$. It would be especially interesting to find some violation of the unconquerable $CPT$ symmetry. No signals for $CPT$ violation have been observed yet despite numerous experimental tests.

From the $CPT$ theorem [7] we know that $CPT$ invariance of some field theory follows from locality and invariance under Lorentz transforms. Usually $CPT$ violation is considered to be due to breaking of Lorentz invariance [8]. However, it is possible that locality is a less fundamental requirement than Lorentz invariance; therefore, experiments searching for $CPT$ violation of both types are of interest.

## II. ELECTROMAGNETIC WAVE IN VACUUM IN CONSTANT ELECTRIC AND MAGNETIC FIELDS

Let us consider the propagation of an electromagnetic wave in vacuum in the presence of uniform constant electric and magnetic fields. Since a photon has no electric charge, such a medium is a medium with constant refractive index. We will assume that the field of the electromagnetic wave obeys the Maxwell equation $\partial F^{\mu\nu}(x)/\partial x^{\nu}=-4\pi j^{\mu}(x)$, where $j(x)$ is the current of all the particles that can interact with the photons. The current arises due to vacuum polarization by the electromagnetic wave in the presence of external fields. Assuming that the wave field is weak and the vacuum in the homogeneous external field remains homogeneous, we can, in the general case, express the current linearly through the four-potential of the wave field:

$$j^{\mu}(x)=-\int \mathcal{P}^{\mu\nu}(x-x')A_{\nu}(x')d^4x', \tag{1}$$

where $\mathcal{P}^{\mu\nu}(x)$ is some tensor. We do not consider the case of strong external electric field, when vacuum instability [9] should be taken into account. After Fourier transforms of the four-current $j(x)=\int j(k)e^{-ikx}d^4k$ and four-potential of the electromagnetic field $A(x)=\int A(k)e^{-ikx}d^4k$ the Maxwell equation is rewritten as

$$k^2A^{\mu}(k)-k^{\mu}(kA)=-j^{\mu}(k), \tag{2}$$

where

$$j^{\mu}(k)=-\mathcal{P}^{\mu\nu}(k)A_{\nu}(k). \tag{3}$$

The four-tensor $\mathcal{P}^{\mu\nu}(k)=\int\mathcal{P}^{\mu\nu}(x)e^{ikx}d^4x$ must be constructed from the tensor of the external field $\mathcal{F}^{\mu\nu}$ and a photon wave vector $k$, since only they are available. By virtue of the gauge invariance and current conservation the relations $\mathcal{P}^{\mu\nu}k_{\nu}=k_{\mu}\mathcal{P}^{\mu\nu}=0$ must be imposed on $\mathcal{P}^{\mu\nu}$.

It is also necessary to emphasize that all the possible interactions are supposed to be small, so $k^2$ can be set to zero on the right-hand side of Maxwell's equation during evaluation of $\mathcal{P}^{\mu\nu}$; in addition, only that part of $\mathcal{P}^{\mu\nu}$ should be taken into account which does not become zero when acting on the four-vector polarization $e_{\mu}(k)$ of a real photon (for real photons $k^{\mu}e_{\mu}=0$). The structure of the polarization operator, including off mass shell terms, was considered in Ref. [10] for the case when all the symmetries are conserved and is considered in Appendix A for the case of symmetry violation.

In Eqs. (2) and (3) the gauge is not fixed yet. We shall choose the gauge with the null component of the four-potential being equal to zero: $\phi=0$. Then $\boldsymbol{E}=-\partial\boldsymbol{A}/\partial t$ and $\boldsymbol{E}(\boldsymbol{k},\omega)=i\omega\boldsymbol{A}$. In a given gauge we obtain from Eqs. (2) and (3)

$$\boldsymbol{k}^2E^i-k^i(\boldsymbol{kE})-\omega^2\left(\delta^{ij}+\frac{\mathcal{P}^{ij}}{\omega^2}\right)E^j=0, \tag{4}$$

where the three-dimensional tensor $\mathcal{P}^{ij}$ is the spatial part of the four-tensor $\mathcal{P}^{\mu\nu}$. Equation (4) shows that $\varepsilon^{ij}=\delta^{ij}+\mathcal{P}^{ij}/\omega^2$ plays the role of the product of the dielectric and magnetic constants of the vacuum in an external field. Further, for short, we shall simply call it the dielectric constant $\varepsilon^{ij}$ of the vacuum in the external fields.

Let us consider the structure of the four-tensor $\mathcal{P}^{\mu\nu}$ in detail. It can be presented as an expansion in orders of the external field. Provided the requirements $\mathcal{P}^{\mu\nu}k_\nu = k_\mu \mathcal{P}^{\mu\nu}=0$ [11] and $e_\mu^{*\prime}\mathcal{P}^{\mu\nu}e^\nu \neq 0$, $k^2=0$, are met, in the second order in the external field tensor $\mathcal{F}^{\mu\nu}$ we obtain

$$\mathcal{P}^{\mu\nu}=a_1\mathcal{F}^{\mu\alpha}k_\alpha\mathcal{F}^{\nu\sigma}k_\sigma + a_2 e^{\mu\lambda\varrho\sigma}\mathcal{F}_{\lambda\varrho}k_\sigma e^{\nu\phi\delta\alpha}\mathcal{F}_{\phi\delta}k_\alpha + ib_1 e^{\mu\nu\alpha\beta}k_\alpha\mathcal{F}_{\beta\varrho}\mathcal{F}^{\varrho\phi}k_\phi + c_1(e^{\mu\lambda\varrho\sigma}\mathcal{F}_{\lambda\varrho}k_\sigma\mathcal{F}^{\nu\delta}k_\delta + e^{\nu\lambda\varrho\sigma}\mathcal{F}_{\lambda\varrho}k_\sigma\mathcal{F}^{\mu\delta}k_\delta). \tag{5}$$

Equation (5) is valid when the external field is slowly varying with respect to the wavelength of the photon; further terms involving derivatives of the external field should be included.

The quantity $e_\mu^{*\prime}\mathcal{P}^{\mu\nu}e_\nu$ is similar to the invariant forward photon scattering amplitude in the external field. Let us find its properties under $CPT$ transformation. Under $C$, $P$, and $T$ transforms the tensor of the external field, wave four-vector, and four-polarization of the photon are changed as [14]

$$T\mathcal{F}^{\mu\nu}\to -\mathcal{F}_{\mu\nu},\quad Tk^\mu\to k_\mu,\quad Te^\mu\to e_\mu^{*\prime},\quad Te_\mu^{*\prime}\to e^\mu,$$

$$C\mathcal{F}^{\mu\nu}\to -\mathcal{F}^{\mu\nu},\quad Ck^\mu\to k^\mu,\quad Ce^\mu\to -e^\mu,$$

$$P\mathcal{F}^{\mu\nu}\to \mathcal{F}_{\mu\nu},\quad Pk^\mu\to k_\mu,\quad Pe^\mu\to e_\mu. \tag{6}$$

Hence, $\mathcal{P}^{\mu\nu}$ should be symmetric to satisfy $CPT$ invariance. The term proportional to $b_1$ breaks $CPT$ invariance with parity breaking only. The term proportional to $c_1$ is $CPT$ invariant, but $P$, $CP$, and $T$ violating. The terms proportional to $a_1=(16/45)(\alpha^2/m^4)\approx 2.78\times 10^{-4}$ MeV$^{-4}$ and $a_2=(7/45)(\alpha^2/m^4)\approx 1.21\times 10^{-4}$ MeV$^{-4}$ arise in the framework of conventional QED [15,14]. Here $\alpha$ is fine structure constant and $m$ is the electron mass. From Eq. (5) follows the explicit form of the vacuum dielectric constant [16] in the stationary uniform electric $\boldsymbol{\mathcal{E}}$ and magnetic $\boldsymbol{\mathcal{B}}$ fields:

$$\varepsilon^{lj}=\delta^{lj}+a_1(\mathcal{E}^j\mathcal{E}^l+(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^l(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^j-\mathcal{E}^l(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^j-\mathcal{E}^j(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^l)+4a_2(\mathcal{B}^l\mathcal{B}^j+(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^l\mathcal{B}^j+\mathcal{B}^l(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^j+(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^l(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^j)$$
$$+ib_1e^{ljm}(n^m\mathcal{E}^2+n^m(\boldsymbol{\mathcal{E}}\times\boldsymbol{\mathcal{B}}\cdot\boldsymbol{n})+\mathcal{E}^m(\boldsymbol{\mathcal{E}}\boldsymbol{n})-(\boldsymbol{\mathcal{B}}\times\boldsymbol{\mathcal{E}})^m-[\boldsymbol{\mathcal{B}}\times(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})]^m)+2c_1(\mathcal{B}^l(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^j+\mathcal{B}^j(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^l$$
$$+(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^l(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^j+(\boldsymbol{\mathcal{B}}\times\boldsymbol{n})^l(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^j-(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^l\mathcal{E}^j-\mathcal{E}^l(\boldsymbol{\mathcal{E}}\times\boldsymbol{n})^j-\mathcal{B}^l\mathcal{E}^j-\mathcal{B}^j\mathcal{E}^l)+id_1e^{ljm}\mathcal{B}^m+id_2e^{ljm}\mathcal{E}^m+id_3e^{ljm}n^m, \tag{7}$$

where $\boldsymbol{n}=\boldsymbol{k}/|\boldsymbol{k}|$ and summation on the index $m$ is meant. Let us remark that to an accuracy up to the terms of second order in the external field the refractive index does not depend on the photon energy (except for the terms proportional to $d_1$, $d_2$, and $d_3$). In the Eq. (7) we have added by hand the terms involving $d_1$, $d_2$, and $d_3$, which should be absent owing to Lorentz invariance. Such terms as, for example, the Faraday effect $\sim e^{ljm}\mathcal{B}^m$ violate both $CPT$ and Lorentz invariance. The same is true for the term $\sim e^{ljm}\mathcal{E}^m$, which violates all the symmetries: $P$, $C$, $T$, and Lorentz, although it conserves $CP$. However, in the presence of a substance such as gas or plasma they are not Lorentz violating, because we have additional vector $u^\mu$ of the four-velocity of the substance. The vector $u^\mu$ allows us, for example, to construct the term $\mathcal{P}^{\mu\nu}\sim(\mathcal{F}^{\mu\nu}u^\eta-\mathcal{F}^{\eta\nu}u^\mu-\mathcal{F}^{\mu\eta}u^\nu)k_\eta$, responsible for the Faraday effect in a substance.

Therefore, experimental detection of the Faraday effect in vacuum means violation of both $CPT$ and Lorentz invariance.

## III. *CPT* THEOREM

According to the well known $CPT$ theorem [7] $CPT$ invariance follows from Lorentz invariance and locality; therefore, Lorentz invariant but $CPT$ violating terms should break locality. Let us consider this in more detail. A small perturbation of the vacuum in constant external fields by an electromagnetic wave can be described in the framework of the Schrödinger formalism (Appendix B). To second order in the current operator $\hat{j}(x)$ (without defining its particular form) one can obtain that

$$\mathcal{P}_{\mu\nu}(x)=4\pi i[\langle 0|\hat{j}_\mu(x)\hat{j}_\nu(0)|0\rangle-\langle 0|\hat{j}_\nu(0)\hat{j}_\mu(x)|0\rangle]\theta(t), \tag{8}$$

where $\theta(t)$ is a step function. Let us derive the requirement of $CPT$ invariance again, using a different method. For any $CPT$-odd or $CPT$-even operator $\hat{Z}(x)$ one can write $\Theta^{-1}\hat{Z}(x)\Theta=\pm\hat{Z}^+(-x)$ [7], where $\Theta$ is the operator of $CPT$ reflection, and $\hat{Z}^+$ is a Hermite conjugate operator. Applying this relation to the product $\hat{j}_\mu(x)\hat{j}_\nu(0)$ and taking into account the hermiticity of the current operator we obtain

$$\langle 0|\hat{j}_\mu(x)\hat{j}_\nu(0)|0\rangle=\langle 0|\hat{j}_\nu(0)\hat{j}_\mu(-x)|0\rangle, \tag{9}$$

where invariance of the vacuum under $CPT$ conjugation $\Theta|0\rangle=|0\rangle$ and $\langle 0|\Theta^{-1}=\langle 0|$ in a constant uniform field is used. The translational invariance of the vacuum in the homogeneous constant field $\langle 0|\hat{j}_\nu(0)\hat{j}_\mu(-x)|0\rangle=\langle 0|\hat{j}_\nu(x)\hat{j}_\mu(0)|0\rangle$ and Eq. (9) lead to the symmetry of the tensor $\mathcal{P}_{\mu\nu}(x)=\mathcal{P}_{\nu\mu}(x)$ as a condition of $CPT$ invariance, in agreement with the previous analysis. In a strong electric field the vacuum is unstable. It evolves from an "empty" state to a state with particle-antiparticle pairs and is no longer $T$ invariant as well as $CPT$ invariant. This leads to the appearance of antisymmetric terms in the polarization tensor [17]. The effect is suppressed by the multiplier $e^{-\pi m^2/e\mathcal{E}}$ and negligible for electrons and a laboratory electric field, but what about some unknown light particles? In the following we will treat vacuum as stable.

First we consider the case when the locality condition $\langle 0|\hat{j}^{\mu}(x)\hat{j}^{\nu}(0)-\hat{j}^{\nu}(0)\hat{j}^{\mu}(x)|0\rangle=0$ at $x^2<0$ is satisfied. This relation implies that events at points of four-space with spacelike separation are not connected in any way. It is a consequence of the limited velocity of an interaction propagation, or the absence of any tachyons that can transfer an interaction. Thus, $\mathcal{P}_{\mu\nu}(x)$ is not zero only in the future light cone. Transforms of the restricted real Lorentz group $L_+^{\dagger}$ map the future light cone $V^+$, consisting of points $x^2>0$, $x_0>0$, onto itself [7]. In other words, the presence of the step function does not spoil the Lorentz covariance of $\mathcal{P}_{\mu\nu}(x)$, as a point of the future with $x_0=t>0$, $x^2>0$ remains a point of the future with $t'>0$ for any system of reference. This means that the function

$$\mathcal{P}_{\mu\nu}(k)=\int_{t>0}\mathcal{P}_{\mu\nu}(x)e^{ikx}\,d^4x \tag{10}$$

is covariant under transforms of $L_+^{\dagger}$.

The function of Eq. (10) can also be defined for the complex $k=\kappa+i\mathcal{K}$, with $\mathcal{K}$ belonging to the future light cone ($\mathcal{K}\in V^+$), since in any frame of reference $\mathrm{Im}\,k_0>0$ and the integral in Eq. (10) converges. Let us define the forward tube $\mathcal{F}$ as the set of complex $k=\kappa+i\mathcal{K}$, where $\mathcal{K}$ belongs to $V^+$ [7]. Then the extended tube $\mathcal{F}'$ is the set of complex $k$, obtained as a result of all the complex Lorentz transforms $L_+(C)$ [7] with determinant $+1$ to the points of $\mathcal{F}$. Due to analytical continuation to $\mathcal{F}'$ the function $\mathcal{P}_{\mu\nu}(k)$ becomes covariant under transformations from the complex Lorentz group $L_+$. The value of $\mathcal{P}_{\mu\nu}(k)$ at real $k$ is a boundary value of $\mathcal{P}_{\mu\nu}(\kappa)=\lim_{\mathcal{K}\to 0,\mathcal{K}\in V^+}\mathcal{P}_{\mu\nu}(k)$. The reflections of all four axes are included in $L_+$ and, therefore, $\mathcal{P}_{\mu\nu}(k)=\mathcal{P}_{\mu\nu}(-k)$. We cannot, however, pass to real $k$ in this equality, as, if in the left-hand side $\mathrm{Im}\,k\to 0$ belonging to $V^+$, in the right-hand side of the equality $\mathrm{Im}(-k)\in V^-$. It is known that the extended tube also contains real points (Yost points) [7]. All Yost points are spacelike. Let us show that the relation $\mathcal{P}_{\mu\nu}(k)=\mathcal{P}_{\nu\mu}(-k)$ is satisfied at the Yost points. Using the relation $\langle 0|\hat{j}_{\mu}(x)|n\rangle=e^{iP_nx}\langle 0|\hat{j}_{\mu}(0)|n\rangle$ we rewrite $\mathcal{P}_{\mu\nu}(x)$ as

$$\mathcal{P}_{\mu\nu}(x)=4\pi i\sum_n\,[\langle 0|\hat{j}_{\mu}(0)|n\rangle\langle n|\hat{j}_{\nu}(0)|0\rangle e^{-iP_nx}-\langle 0|\hat{j}_{\nu}(0)|n\rangle\langle n|\hat{j}_{\mu}(0)|0\rangle e^{iP_nx}]\theta(t), \tag{11}$$

where $P_n\equiv\{\varepsilon_n,\boldsymbol{p}_n\}$ is the four-momentum of the particle-antiparticle states in the external field. Multiplying Eq. (11) by $e^{-\epsilon t}$, where $\epsilon$ is an infinitesimal number, does not spoil convergence of the integral in Eq. (10) and allows us to write $\mathcal{P}_{\mu\nu}(k)$ as

$$\mathcal{P}_{\mu\nu}(k)=2(2\pi)^4\sum_n\left(\frac{\langle 0|\hat{j}_{\mu}(0)|n\rangle\langle n|\hat{j}_{\nu}(0)|0\rangle}{\varepsilon_n-\omega-i\epsilon}\delta^{(3)}(\boldsymbol{p}_n-\boldsymbol{k})+\frac{\langle 0|\hat{j}_{\nu}(0)|n\rangle\langle n|\hat{j}_{\mu}(0)|0\rangle}{\varepsilon_n+\omega+i\epsilon}\delta^{(3)}(\boldsymbol{p}_n+\boldsymbol{k})\right). \tag{12}$$

For the spacelike $k$ we can consider everything in the system of reference where $\omega=0$; then

$$\mathcal{P}_{\mu\nu}(k)=2(2\pi)^4\sum_n\frac{\varepsilon_n}{\varepsilon_n^2+\epsilon^2}\times[\langle 0|\hat{j}_{\mu}(0)|n\rangle\langle n|\hat{j}_{\nu}(0)|0\rangle\delta^{(3)}(\boldsymbol{p}_n-\boldsymbol{k})+\langle 0|\hat{j}_{\nu}(0)|n\rangle\langle n|\hat{j}_{\mu}(0)|0\rangle\delta^{(3)}(\boldsymbol{p}_n+\boldsymbol{k})]. \tag{13}$$

From Eq. (13) it follows that the relation $\mathcal{P}_{\mu\nu}(k)=\mathcal{P}_{\nu\mu}(-k)$ is valid. By virtue of analytic continuation this relation is valid for complex $k$ of the extended tube $\mathcal{F}'$ (although it is violated on passing to the limit of timelike real $k$, as then in the left-hand side $\mathrm{Im}\,k\to 0$ belongs to the future light cone, while on the right-hand side $\mathrm{Im}(-k)$ approaches zero in the past light cone). Consequently, we have throughout the extended tube

$$\mathcal{P}_{\mu\nu}(k)=\mathcal{P}_{\nu\mu}(-k)=\mathcal{P}_{\nu\mu}(k). \tag{14}$$

At the end and beginning of the equality we can turn to the limit of real $k$: $\mathrm{Im}\;k\to 0$, $\mathrm{Im}\,k\in V^+$, and find that $\mathcal{P}^{\mu\nu}(k)$ obeys $CPT$ invariance. Thus we have proved the $CPT$ theorem for our special case, showing that locality, Lorentz invariance, and field-theoretic Schrödinger equation lead to the $CPT$ invariance of $\mathcal{P}^{\mu\nu}(k)$.

Let us assume now that local commutativity does not hold for the operator $\hat{j}(x)$. The current operator can be nonlocal and, for example, may be expressed as $\hat{j}_{\mu}(x)=\int K_{\mu\nu}(x-x')\hat{\mathcal{J}}^{\nu}(x')$, where the operator $\hat{\mathcal{J}}^{\nu}(x)$ is local (expressed through the fields and their derivatives) and $K_{\mu\nu}(x-x')$ is a function describing nonlocality. Then, in the general case, the expression of Eq. (8) is distinct from zero at spacelike points. Therefore, due to the presence of the $\theta$ function Lorentz invariance has been lost. To maintain the Lorentz invariance we must "remove" the $\theta$ function in some way, which will mean violation of causality. Certainly, we cannot simply remove the $\theta$ function and are forced to abandon the field-theoretic Schrödinger equation. Modification of the Schrödinger equation to the case of nonlocal theories is offered in Ref. [18]; however, most likely it is not a unique possibility and we will not consider it here.

Thus, experimental detection of terms of the Faraday effect type $\sim e^{ijm}\mathcal{B}^m$ and $\sim e^{ijm}\mathcal{E}^m$ in vacuum would mean $CPT$ and Lorentz invariance violation. If we do not detect such terms, but do detect the $CPT$ violating term proportional to $b_1$, it means violation of locality and causality, but

Lorentz invariance. Locality and causality may be violated through the presence of tachyons (particles with superluminal velocities). Tachyons arise in a number of Lorentz invariant theories. Even in the Rarita-Schwinger theory of a particle of spin 3/2 interacting with an electromagnetic field a tachyonlike solution appears [19]. However, no tachyons have been detected experimentally.

## IV. EVOLUTION OF LIGHT POLARIZATION UNDER *CP* AND *CPT* VIOLATION

One of the traditional ways to describe light polarization is to use Stokes parameters $\zeta_1,\zeta_2,\zeta_3$ [20] which can be measured by experimentalists. We can describe evolution of the Stokes parameters when the electromagnetic wave propagates in a medium with a tensor refractive index. Because of the small difference of the refractive index from unity we can consider the electromagnetic wave to be transverse. Nontransversal terms will give the next order of smallness in the constants $a_1,a_2 \ldots$ . Thus, the dispersion equation for the wave vector can be written as

$$(n^2-\hat{\epsilon})\boldsymbol{E}=0, \tag{15}$$

where $n=k/\omega$, and the wave strength vector $\boldsymbol{E}$ is perpendicular to $\boldsymbol{k}$ and has only $x,y$ components if the wave propagates in the $z$ direction. Because of the smallness of $n^2-1$, Eq. (15) can be rewritten as $(2n-\hat{\epsilon}-1)\boldsymbol{E}=0$. Putting to zero a determinant of the equation we can find eigenvectors $\boldsymbol{e}_l$ belonging to the eigenvalues $k_l$. Expanding the initial strength vector of the wave $\boldsymbol{E}_0=\Sigma_l\alpha_l\boldsymbol{e}_l$ allows one to find the evolution of the strength vector under photon propagation through the volume occupied by the external fields:

$$\boldsymbol{E}(z)=\sum e^{ik_lz}\alpha_l e_l=e^{i\omega\hat{n}z}\boldsymbol{E}_0. \tag{16}$$

Here we have introduced an operator of the refractive index according to the formula $2(\hat{n}-1)=\hat{\epsilon}-1$. To describe partially polarized light the density $2\times 2$ matrix $\rho_{ij}=\overline{E_iE_j^*}/\overline{|\boldsymbol{E}|^2}$ is used. From Eq. (16) it follows that

$$\frac{d\boldsymbol{E}(z)}{dz}=i\omega\hat{n}\boldsymbol{E}(z). \tag{17}$$

Equation (17) gives the evolution of the density matrix:

$$\frac{d\rho}{dz}=i\omega[\hat{n}\hat{\rho}-\hat{\rho}\hat{n}^+-\hat{\rho}\,\mathrm{Tr}\{\hat{\rho}(\hat{n}-\hat{n}^+)\}]. \tag{18}$$

Generally, the refractive index operator can be expanded via the unit basis vectors $\boldsymbol{e}_x$ and $\boldsymbol{e}_y$ in the following way:

$$\hat{n}-1=A\boldsymbol{e}_x\otimes\boldsymbol{e}_x+B\boldsymbol{e}_y\otimes\boldsymbol{e}_y+C(\boldsymbol{e}_x\otimes\boldsymbol{e}_y+\boldsymbol{e}_y\otimes\boldsymbol{e}_x)+iD(\boldsymbol{e}_x\otimes\boldsymbol{e}_y-\boldsymbol{e}_y\otimes\boldsymbol{e}_x), \tag{19}$$

or in the matrix form

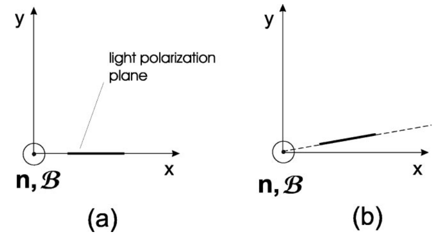


FIG. 1. (a) Linearly polarized light with $\zeta_3=1$, $\zeta_1=\zeta_2=0$; (b) light with $\zeta_3\approx 1$, $\zeta_1\neq 0$, $\zeta_2=0$.

$$\hat{n}-1=A\begin{pmatrix}1&0\\0&0\end{pmatrix}+B\begin{pmatrix}0&0\\0&1\end{pmatrix}+C\begin{pmatrix}0&1\\1&0\end{pmatrix}+D\begin{pmatrix}0&i\\-i&0\end{pmatrix}, \tag{20}$$

where the quantities $A,B,C,D$ should be expressed in terms of $a_1,a_2,\ldots$ for a concrete external field configuration.

The density matrix can be parametrized by the Stokes parameters [20]

$$\hat{\rho}=\frac{1}{2}\begin{pmatrix}1+\zeta_3&\zeta_1-i\zeta_2\\ \zeta_1+i\zeta_2&1-\zeta_3\end{pmatrix}. \tag{21}$$

Distinguishing the real and imaginary parts in the coefficients $A=A'+iA''\cdots$ we find from Eq. (18) that

$$\begin{aligned}\frac{1}{\omega}\frac{d\zeta_1}{dz}&=(A'-B')\zeta_2+(A''-B'')\zeta_1\zeta_3-2C''(1-\zeta_1^2)\\&\quad+2D'\zeta_3-2D''\zeta_1\zeta_2,\\ \frac{1}{\omega}\frac{d\zeta_2}{dz}&=(B'-A')\zeta_1+(A''-B'')\zeta_2\zeta_3+2C'\zeta_3\\&\quad+2C''\zeta_1\zeta_2+2D''(1-\zeta_2^2),\\ \frac{1}{\omega}\frac{d\zeta_3}{dz}&=-(A''-B'')(1-\zeta_3^2)-2C'\zeta_2+2C''\zeta_1\zeta_3\\&\quad-2D'\zeta_1+2D''\zeta_2\zeta_3.\end{aligned} \tag{22}$$

The Faraday effect can be measured, if we choose the magnetic field to be parallel to the wave vector of the photon as shown in Figs. 1(a) and 1(b). Then in Eq. (20) only the term proportional to $D=\frac{1}{2}(d_1\mathcal{B}+d_3)$ remains. As light passes through the volume occupied by the magnetic field, the light with the only Stokes parameter initially distinct from zero, $\zeta_3$ [20], will gain polarization corresponding to the parameter $\zeta_1$ and, in contrast, light with the only initially nonzero parameter $\zeta_1$ gains polarization corresponding to $\zeta_3$. Thus the light polarization rotates as shown in Fig. 1.

Let us recall that $\zeta_3=1$ and $\zeta_3=-1$ correspond to the light polarized along the $x$ and $y$ axes, respectively. The parameter $\zeta_1=\pm 1$ describes polarization at 45° to the $y$ axis. The light ellipticity $\Psi$ is expressed through $\zeta_2=2\Psi$ for fully polarized light. Partially polarized light can be expanded as a sum of natural light and elliptically polarized light. In this case $\zeta_2=2\Psi P$, where $P$ is the light polarization and $\Psi$ is the ellipticity of the polarized part.

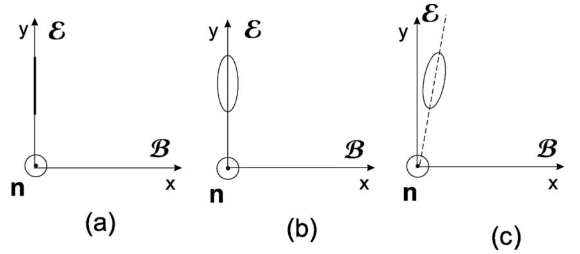


FIG. 2. (a) Linearly polarized light with $\zeta_3=-1$, $\zeta_1=\zeta_2=0$; (b) light with $\zeta_3\approx-1$, $\zeta_2\neq0$, and $\zeta_1=0$; (c) light with $\zeta_3\approx-1$, $\zeta_2\neq0$, and $\zeta_1\neq0$ (ellipse of polarization is slightly rotated).

In the BMV project [3] it is planned to achieve an accuracy sufficient for measurements of the vacuum birefringence predicted by QED, i.e., $\Delta n\sim10^{-21}$ at $\mathcal{B}\sim25$ T. Earlier, $\Delta n\sim1.3\times10^{-20}$ was measured in the BNL experiment [1] and $\Delta n\sim6.7\times10^{-20}$ by PVLAS [2]. Thus, from measurements of $\Delta n$ corresponding to Faraday rotation at the level $\Delta n_{CPTL}=D\sim10^{-21}$ in a magnetic field of 25 T (we will use the system of units $e^2/4\pi=\alpha$,1 T $=195$ eV$^2$,1 V/m$=6.5\times10^{-7}$ eV$^2$) one will be able to obtain the restriction $d_1=2D/\mathcal{B}\sim4\times10^{-13}$ MeV$^{-2}$.

The presence of a residual pressure in the resonator imposes a restriction on the measurement of $\Delta n$ of the vacuum. Assuming the residual pressure in the equipment to be $10^{-11}$ Torr, we find that $\Delta n$ of the Faraday effect for helium at this pressure is $\Delta n\sim10^{-22}$. Thus, the $CPT$ violating Faraday effect can be measured with this accuracy.

For measurement of the terms proportional to $b_1$ we may choose a magnetic field perpendicular to the photon wave vector. The electric field should be chosen perpendicular to both the photon wave vector and the magnetic field strength vector. Thus the photon wave vector, the direction of the magnetic field, and the direction of the electric field form a triplet of mutually orthogonal vectors as shown in Figs. 2 and 3. The refractive index contains terms which are of odd or even order in the vector $\boldsymbol{n}$. For a laser experiment only the terms of even order in the wave vector are of interest, because these effects accumulate under the passage of a photon back and forth between the resonator mirrors [21]. Considering only these terms we find that all the coefficients $A=(a_1/2)(\mathcal{E}^2+\mathcal{B}^2)$, $B=2a_2(\mathcal{E}^2+\mathcal{B}^2)$, $C=-c_1\mathcal{E}\mathcal{B}$, and $D=-b_1\mathcal{E}\mathcal{B}+d_3/2$ are different from zero. First, assume that all the coefficients are approximately of the same order of magnitude. Then the light with the initial polarization $\zeta_3$ receives polarization $\zeta_1$. The polarization $\zeta_1$ can also arise due to the imaginary part of the coefficient $C$; however, this contribution does not depend on the sign of the initial polarization $\zeta_3$ and can be separated by changing the sign of $\zeta_3$ during the experiment. Taking the electric field strength $\mathcal{E}\sim10^6$ V/m we obtain a restriction on $CPT$ and the causality violation constant $b_1=\Delta n'_{CPT}/\mathcal{E}\mathcal{B}=D/\mathcal{E}\mathcal{B}\sim0.31$ MeV$^{-4}$ if $\Delta n'_{CPT}$ is measured with accuracy $10^{-21}$. To measure the $CP$ violating constant $c_1$ we have to search for the ellipticity parameter $\zeta_2$ when the light was initially linearly polarized with $\zeta_3=1$.

In the case when $|A-B|\gg|D|,|C|$ [but $|(A'-B')\omega z|\ll1$] ‘‘mixing’’ of the polarizations $\zeta_1$ and $\zeta_2$ occurs. Still, the light initially polarized with $\zeta_3$ will gain polarizations $\zeta_1$ and $\zeta_2$ only in the case when $C$ or $D$ differs from zero. But we will not know $C$ or $D$. Fortunately, we have a possibility to avoid this difficulty. The sign of the Cotton-Mouton effect for nitrogen is opposite to the sign of the vacuum Cotton-Mouton effect [23]; therefore, using nitrogen at a residual pressure of about $10^{-7}$ Torr we can compensate for the difference $A'-B'$ and distinguish $D$ from $C$.

Apparently, the possibility exists to measure much smaller $\Delta n$. Baryshevsky offers the interesting idea of using laser amplifiers [24], which do not change the polarization properties of light, but, at the same time, will stop photon beam damping. Ideally, the amplifier should be combined with a mirror, as shown in Fig. 3, to obtain an ‘‘amplified’’ mirror with reflectivity 1 or more than 1. Light can be localized in such a trap for several hours. Assuming, for example, that we can measure an angle of polarization rotation $\Delta\theta=\Delta n\omega z\sim10^{-10}$ and the lifetime of a photon in the trap is 1 h, we find the minimum measured $\Delta n\sim10^{-27}$, for $\omega=2.4$ eV. However, a number of technical problems can arise in this scheme. For instance, we need the amplifier to remain isotropic after multiple passage of the polarized light through it.

Finally, it may be possible to obtain restrictions on this $CPT$ violating term by examining the polarization of light from distant galaxies. It is necessary to separate the vacuum effects from the Faraday rotation in the magnetic field and the substance of the galaxies. Earlier, such an analysis yielded the restriction $\Delta n_{CS}=d_3\sim10^{-33}$ ($\omega=2.4$ eV) [25] for the term $\varepsilon^{ij}\sim e^{ijm}n^m$ (Chern-Simons term).

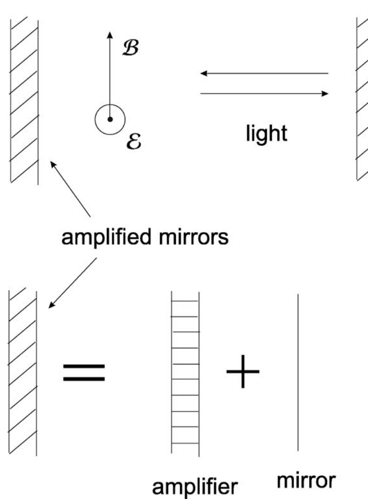


FIG. 3. Scheme of a photon trap.

## V. COMPARISON OF THE LASER EXPERIMENT TESTS WITH SOME OTHER KNOWN TESTS

Let us estimate $CPT$ violation of the Faraday type $\sim e^{ijm}\mathcal{B}^m$. Certainly, we cannot be sure of the applicability of the Feynman diagram technique in the case of $CPT$ invariance violation. But it can still be suitable for heuristic estimates. In the framework of QED, the refractive index, proportional to $a_1,a_2$, is evaluated using the square diagram

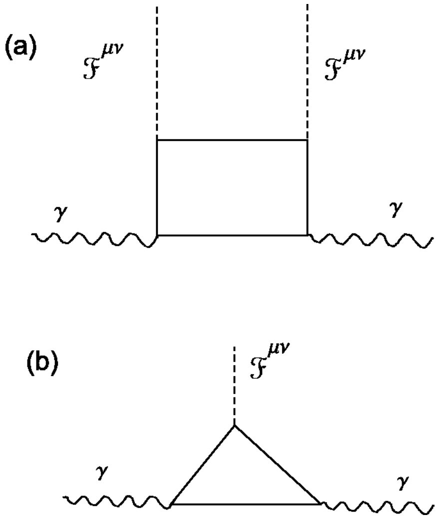


FIG. 4. (a) QED graph of vacuum polarization contribution to the photon refractive index in a static field. (b) Analogous graph of *C*-odd vacuum polarization.

shown in Fig. 4. Each vertex with the external electromagnetic field corresponds to the factor $(e/\sqrt{4\pi})(\mathcal{B}/m^2)$ or $(e/\sqrt{4\pi})(\mathcal{E}/m^2)$ in $\Delta n$, where $m$ is the electron mass. Each of the remaining vertices corresponds to the factor $e/\sqrt{4\pi}$. We can, in the same way, estimate the $CPT$ and Lorentz violating term $\sim e^{ijm}\mathcal{B}^m$, considering the triangle diagram shown in Fig. 4. The triangle diagram cannot appear in standard QED, as the diagram is not invariant under $C$ conjugation. Let us assume the most remarkable possibility, that the violation of $C$, $CPT$, and Lorentz invariance is induced by some unknown particles interacting with the photons with $C$ violation of the order of unity. By analogy with the calculation of the standard square diagram we assume that the vertex with the external field corresponds to the factor $(g/\sqrt{4\pi})(\mathcal{B}/\mu^2)$ in $\Delta n$ and the other vertices correspond to the factor $g/\sqrt{4\pi}$ in $\Delta n$, where $g$ is the coupling of the particle with photons and $\mu$ is the particle mass. As a result, we have

$$\Delta n_{CPTL} \approx \frac{g^2}{4\pi}\frac{g\mathcal{B}}{\sqrt{4\pi}\mu^2}. \tag{23}$$

In the BMV project it is planned to reach an accuracy sufficient for a measurement of $\Delta n$ predicted by QED, i.e., $\Delta n \sim 10^{-21}$ (strength of the magnetic field is 25 T). A measurement of $\Delta n_{CPTL}$ with this accuracy gives a restriction on the coupling $g$. It is interesting to compare this restriction with what follows from the $CPT$ test, based on a comparison of the $g$ factors of electrons and positrons: $\alpha_{CPT} = (g_{e^+} - g_{e^-})/g_{avr} < 10^{-12}$ [26]. Under the assumption of the same mechanism of $C$ parity violation, $\alpha_{CPT}$ arises from the diagram shown in Fig. 5(b). For the sake of simplicity, we again make very heuristic estimates of the diagram shown in Fig. 5(b). First, we remark that the relative contribution of the diagram shown in Fig. 5(a) (the usual vacuum polarization) to the $g$ factor of the electron is $\sim[e^4/(4\pi)^2](m^2/\mu^2)$ if the virtual particle mass $\mu \gg m$, and is $\sim[e^4/(4\pi)^2]\ln(m/\mu)$

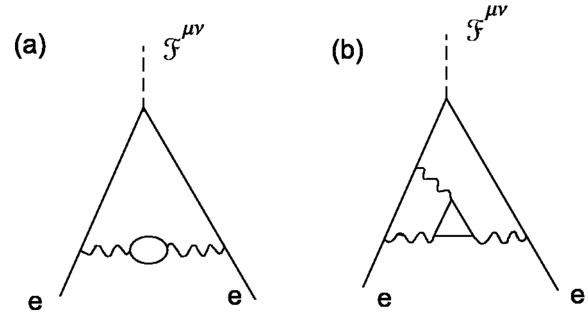


FIG. 5. (a) Graph of vacuum polarization contribution to the electron or positron $g$ factor. (b) Analogous graph of *C*-odd vacuum polarization.

when $\mu \ll m$ [14]. This fact is a reflection of a more general rule. The contribution of the virtual particle loop connected by the photon lines to the electrons is proportional to some degrees of $m/\mu$ when $\mu \gg m$, and to some degree of $\ln(m/\mu)$ when $\mu \ll m$. Thus, the contribution of the diagram shown in Fig. 5(b) can be estimated as

$$\alpha_{CPT} \approx \frac{e^3 g^3}{(4\pi)^3} F\left(\frac{m^3}{\mu^3}\right), \tag{24}$$

where the Spens function $F(x)$ [14] has the asymptotic $F(x) \approx x$ at $x \ll 1$ and $F(x) \approx \pi^2/6 + \frac{1}{2}\ln^2(x)$ when $x \gg 1$. Figure 6 shows restrictions on the coupling $g$ of $C$, $CP$, $CPT$, and Lorentz violating interactions following from the inequalities $\Delta n_{CPTL} < 10^{-21}$ and $\alpha_{CPT} < 10^{-12}$. As we can see, measurement of the Faraday effect in vacuum gives much more stringent restrictions on $g$ in the above model of $CPT$ violation than the traditional comparison of the electron and positron $g$ factors. Certainly, it happens because we have chosen the model with $CPT$ violation in the

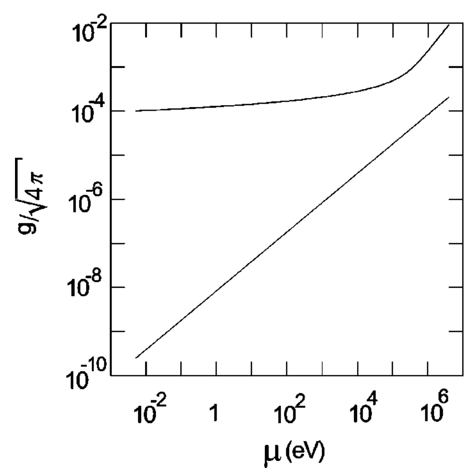


FIG. 6. Restrictions on the dimensionless coupling $g$ of some particles interacting with photons, with the relative $C$, $CP$, $CPT$, and Lorentz invariance violation of order unity. The straight curve relates the Faraday effect in vacuum and corresponds to the inequality $\Delta n_{CPTL} < 10^{-21}$. The bent curve arises from the difference of the electron and positron $g$ factors and corresponds to the inequality $(g_{e^+} - g_{e^-})/g_{avr} < 10^{-12}$.

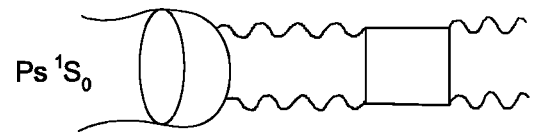


FIG. 7. Graph of $P$ violating photon interaction in final state in two-photon para-positronium decay.

photon sector; therefore, the experiments dealing directly with photons have an advantage.

Let us now consider the terms proportional to $b_1$, which break $P$, $CP$, $CPT$ and causality, but at the same time, are Lorentz invariant and do not break $C$ parity. To estimate the appropriate $\Delta n'_{CPT}$ we should consider the square diagram of Fig. 4(a). In the same way we find

$$\Delta n'_{CPT} \sim \frac{g'^2}{4\pi}\left(\frac{g'\mathcal{B}}{\sqrt{4\pi}\mu^2}\right)\left(\frac{g'\mathcal{E}}{\sqrt{4\pi}\mu^2}\right). \tag{25}$$

The restriction on $g'$ obtained from a measurement of $\Delta n'_{CPT}$ with the accuracy $10^{-21}$ can be compared, for example, with the restriction on $CP$ violation in para-positronium decay into two photons. The positronium $^1S_0$ state has negative spatial parity [14]; therefore, the probability of decay into two polarized photons should be proportional to $(\boldsymbol{e}_1 \times \boldsymbol{e}_2 \cdot \boldsymbol{k})$ [27], where $\boldsymbol{e}_1$ and $\boldsymbol{e}_2$ are the photon polarizations and $\boldsymbol{k}$ is the momentum of one of the photons (the other photon has opposite momentum). The presence of $P$-even $(\boldsymbol{e}_1 \cdot \boldsymbol{e}_2)$ correlation is a signal of $P$ and $CP$ violation ($C$ parity is conserved in para-positronium two-photon decay). For this process we can say nothing about $T$ invariance, because we do not compare it with the reverse process of $\gamma+\gamma \to \text{Ps}$. The branching ratio $\alpha'_{CP}$ of the decay with $(\boldsymbol{e}_1 \cdot \boldsymbol{e}_2)$ can be estimated from the diagram shown in Fig. 7 and is given by

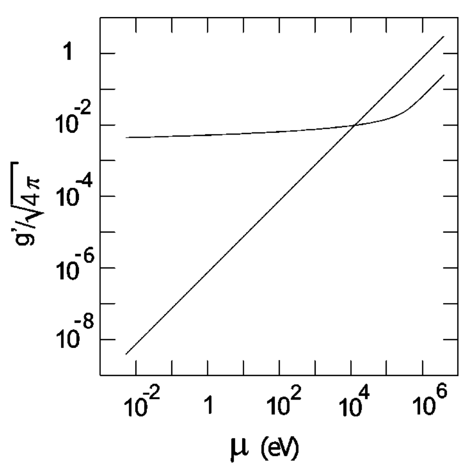


FIG. 8. Restrictions on the dimensionless coupling $g'$ of particles interacting with photons with violation of parity, $CP$, $CPT$, and causality of order unity, but conserving Lorentz invariance. The restrictions follow from the inequality $\Delta n'_{CPT} < 10^{-21}$ (straight curve) and the inequality $\alpha'_{CP} < 10^{-6}$ (bent curve).

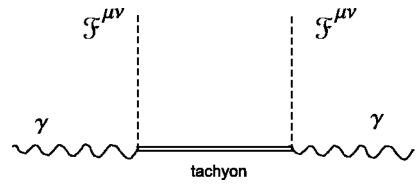


FIG. 9. $CPT$ violation due to exchange of causality violating particle (tachyon) of axion type.

$$\alpha'_{CP} \sim \frac{g'^4}{(4\pi)^2} F\left(\frac{m^4}{\mu^4}\right). \tag{26}$$

The restrictions on $g'$ following from the inequalities $\Delta n'_{CPT} < 10^{-21}$ and $\alpha'_{CP} < 10^{-6}$ are shown in Fig. 8. We have taken the electric field strength $10^6$ V/m, so that the dimensionless parameter $(e\mathcal{E}/m^2) \sim 10^{-12}$. The vertices we considered are of the type one photon–two particles; however, it is possible to consider a vertex of the type two photons–one particle. In this case for evaluation of $\Delta n''_{CPT}$ we should consider the diagram shown in Fig. 9. For reasons of dimensionality we obtain

$$\Delta n''_{CPT} \sim \left(\frac{g''\mathcal{B}}{\sqrt{4\pi}\mu^2}\right)\left(\frac{g''\mathcal{E}}{\sqrt{4\pi}\mu^2}\right). \tag{27}$$

$CP$ violation in positronium decay can be estimated from the diagram shown in Fig. 10 as

$$\alpha''_{CP} \sim \frac{g''^2}{4\pi} F\left(\frac{m^2}{\mu^2}\right). \tag{28}$$

Unfortunately, due to the weakness of the electric field possible in a laser experiment, compared to the magnetic one, the restrictions on such $CPT$ and causality breaking tachyon coupling are $\sqrt{\mathcal{E}/\mathcal{B}}$ times weaker than the restrictions on the usual axion coupling for which $\Delta n \sim (g_a \mathcal{B}/\sqrt{4\pi}\mu_a^2)^2$. For the strength of the electric and magnetic fields used in our work, this gives about 100 times difference (Fig. 11). For the usual axion the very rigid restriction $g_a/\sqrt{4\pi}\mu_a \sim 10^{-9}$–$10^{-10}$ GeV$^{-1}$ follows from astrophysics [23,28]. However, laser experiments can be considered irrespective of the models as independent tests of $CPT$ invariance.

## VI. CONCLUSION

To summarize, laser experiments searching for $CPT$, Lorentz invariance, and causality violation for photons in vacuum, in the presence of constant uniform magnetic and electrical fields, are competitive with tests using positron and electron $g$ factor comparison and searching for $CP$ violation

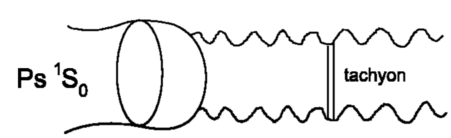


FIG. 10. Tachyon induced $CP$ violation in the two-photon decay of para-positronium.

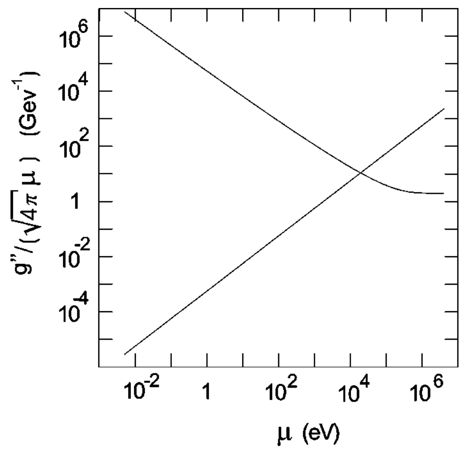


FIG. 11. Restrictions on the tachyon coupling arising from the inequality $\Delta n''_{CPT}<10^{-21}$ (straight curve) and from the inequality $\alpha''_{CP}<10^{-6}$ (bent curve).

in positronium decay, provided that $CPT$ is broken in the photon sector. It is essential that in the case of unbroken Lorentz invariance we have the possibility of testing causality and locality.

## ACKNOWLEDGMENTS

The authors are grateful to Professor Vladimir Baryshevsky for valuable discussions.

## APPENDIX A

In this appendix we consider the structure of the tensor $\mathcal{P}^{\mu\nu}$ in the external field, including off mass shell terms. Despite the above classical consideration the expansion of the tensor $\mathcal{P}^{\mu\nu}$ in the orders of $\mathcal{F}^{\mu\nu}$ and $k^\mu$ is not only valid for soft photons. In fact we can deduce it by considering the one-photon retarded Green's function in the external stationary uniform electromagnetic field $\mathcal{D}_{\mu\nu}(x-x')=i\langle 0|[\hat{A}_\mu(x),\hat{A}_\nu(x')]|0\rangle\theta(t-t')$. The photon propagation modes can be described by the poles of the Fourier transform $\mathcal{D}_{\mu\nu}(k)=\int\mathcal{D}_{\mu\nu}(x)e^{ikx}\,d^4x$ of the Green's function. The dispersion relation for the propagation modes reads

$$\det|\mathcal{D}^{-1}_{\mu\nu}(k)|=0. \tag{A1}$$

The photon Green's function is expressed through the Green's function of the free photon $D_{\mu\nu}(k)$ and the polarization operator as

$$\mathcal{D}_{\mu\nu}(k)=D_{\mu\nu}(k)+D_{\mu\alpha}(k)\mathcal{P}^{\alpha\beta}(k)\mathcal{D}_{\beta\nu}(k).$$

Thus $\mathcal{D}^{-1}_{\mu\nu}=D^{-1}_{\mu\nu}-\mathcal{P}^{\mu\nu}$. Taking $D^{-1}_{\mu\nu}(k)=(k^2g_{\mu\nu}-k_\nu k_\mu)$ we come to a dispersion relation like Eqs. (2) and (3).

The $4\times4$ tensor $\mathcal{P}^{\mu\nu}$ contains 16 independent components. Thus it can be expanded over 16 independent tensors. Ten of them are symmetric and six are antisymmetric. The gauge invariance condition $\mathcal{P}^{\mu\nu}k_\nu$, $\mu\{0,1,2,3\}$, reduces the number of symmetric terms to six. It also reduces the number of antisymmetric terms to three, because for any antisymmetric tensor $k_\mu\mathcal{P}^{\mu\nu}k_\nu=0$ are automatically valid and only three of four gauge conditions are independent. Independent tensors should be expressed through the tensor of an external field $\mathcal{F}^{\mu\nu}$ and the photon wave vector *k*. The expansion can be written as

$$\begin{aligned}\mathcal{P}^{\mu\nu}=&a_0(k^2g^{\mu\nu}-k^\mu k^\nu)+a_1\mathcal{F}^{\mu\alpha}k_\alpha\mathcal{F}^{\nu\sigma}k_\sigma+4a_2\tilde{\mathcal{F}}^{\mu\alpha}k_\alpha\tilde{\mathcal{F}}^{\nu\sigma}k_\sigma+2c_1(\tilde{\mathcal{F}}^{\mu\alpha}k_\alpha\mathcal{F}^{\nu\sigma}k_\sigma+\mathcal{F}^{\mu\alpha}k_\alpha\tilde{\mathcal{F}}^{\nu\sigma}k_\sigma)\\&+c_2[(k^2\mathcal{F}^{\mu\alpha}\mathcal{F}_{\alpha\lambda}k^\lambda-k^\mu k^\delta\mathcal{F}_{\delta\beta}\mathcal{F}^{\beta\sigma}k_\sigma)\mathcal{F}^{\nu\rho}k_\rho+(k^2\mathcal{F}^{\nu\alpha}\mathcal{F}_{\alpha\lambda}k^\lambda-k^\nu k^\delta\mathcal{F}_{\delta\beta}\mathcal{F}^{\beta\sigma}k_\sigma)\mathcal{F}^{\mu\rho}k_\rho]\\&+c_3[(k^2\mathcal{F}^{\mu\alpha}\mathcal{F}_{\alpha\lambda}k^\lambda-k^\mu k^\delta\mathcal{F}_{\delta\beta}\mathcal{F}^{\beta\sigma}k_\sigma)\tilde{\mathcal{F}}^{\nu\rho}k_\rho+(k^2\mathcal{F}^{\nu\alpha}\mathcal{F}_{\alpha\lambda}k^\lambda-k^\nu k^\delta\mathcal{F}_{\delta\beta}\mathcal{F}^{\beta\sigma}k_\sigma)\tilde{\mathcal{F}}^{\mu\rho}k_\rho]\\&+b_1e^{\mu\nu\alpha\beta}k_\alpha\mathcal{F}_{\beta\eta}\mathcal{F}^{\eta\phi}k_\phi+b_2e^{\mu\nu\lambda\alpha}\mathcal{F}_{\lambda\sigma}k^\sigma k_\alpha+b_3(k^\mu\mathcal{F}^{\nu\lambda}k_\lambda-k^\nu\mathcal{F}^{\mu\lambda}k_\lambda+k^2\mathcal{F}^{\mu\nu}),\end{aligned} \tag{A2}$$

where $\tilde{\mathcal{F}}^{\mu\nu}=\frac{1}{2}e^{\mu\nu\eta\delta}\mathcal{F}_{\eta\delta}$. The coefficients $a_0,a_1,\ldots$ are functions of four independent scalars $k^2$, $k_\mu\mathcal{F}^{\mu\nu}\mathcal{F}_{\nu\lambda}k^\lambda$, $\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu}$, $\mathcal{G}=\mathcal{F}_{\mu\nu}\tilde{\mathcal{F}}^{\mu\nu}$ [10]. The terms involving $a_0,c_2,c_3,b_2,b_3$ do not lead to observable effects at first order in the constants, because evaluation of these terms on the photon mass shell gives zero. For instance, the quantity $e^{*\mu}(k^2g_{\mu\nu}-k_\mu k_\nu)e^\nu$ equals zero because the free photon satisfies $k^2=0$ and $ek=0$. The symmetry properties of all the terms are given in Table I. Let us remark that the scalar $\mathcal{G}$ is *P* and *T* violating so if the coefficients $a_0,a_1,\ldots$ contain odd orders of $\mathcal{G}$ its symmetry properties change. Conventional QED allows the terms proportional to $a_0,a_1,a_2$ and also the term involving $c_1$, which appears only with the odd degree of $\mathcal{G}$. In a pure magnetic field $\mathcal{G}=0$ and the aforementioned term disappears.

## APPENDIX B

Here we deduce Eq. (8) from the field-theoretic Schrödinger equation. The classical four-current $j_\mu(\boldsymbol{r},t)$ corresponds to some Schrödinger operator $\hat{j}_\mu(\boldsymbol{r})$, so that perturbation of the vacuum in a constant field by an electromagnetic wave can be described by the interaction Hamiltonian $\hat{V}(t)=\int\hat{j}^\mu(\boldsymbol{r})A_\mu(x)d^3r$. Let us recall that $A(x)$ represents the four-potential of the wave. We will assume that the wave rises adiabatically from zero value at infinity. A

TABLE I. Symmetry properties of the terms of the tensor $\mathcal{P}^{\mu\nu}$ allowed by Lorentz and gauge invariance.

| Term | Symmetry Base $CPT$ | Symmetry Modified by $\mathcal{G}^{2n+1}$ $CPT$ | Observability with real photons |
|---|---|---|---|
| $a_0$ | $+++$ | $+--$ | invisible |
| $a_1$ | $+++$ | $+--$ | visible |
| $a_2$ | $+++$ | $+--$ | $\langle\rangle$ |
| $c_1$ | $+--$ | $+++$ | $\langle\rangle$ |
| $c_2$ | $-+-$ | $--+$ | invisible |
| $c_3$ | $--+$ | $-+-$ | $\langle\rangle$ |
| $b_1$ | $+-+$ | $++-$ | visible |
| $b_2$ | $-++$ | $---$ | invisible |
| $b_3$ | $---$ | $-++$ | $\langle\rangle$ |

perturbed state of the system is described by the Schrödinger equation:

$$\frac{d}{dt}|t\rangle=(\hat{H}_0+\hat{V})|t\rangle. \tag{B1}$$

Vacuum states in constant external fields are eigenstates of the Hamiltonian $\hat{H}_0$ in the absence of the wave: $\hat{H}_0|n\rangle=\varepsilon_n|n\rangle$. Expansion of the state $|t\rangle$ to states $|n\rangle$ gives

$$|t\rangle=|0\rangle+\sum_{n\neq 0}a_n(t)|n\rangle e^{-i\varepsilon_n t}. \tag{B2}$$

Substituting the given expression in Eq. (B1) we obtain

$$i\frac{da_n(t)}{dt}=\langle n|\hat{V}(t)|0\rangle e^{i\varepsilon_n t}. \tag{B3}$$

Using the Fourier transform of the wave four-potential $A_\mu(x)=\int A_\mu(k)e^{-ikx}d^4x$ and the translational invariance of the vacuum $\langle n|\hat{j}_\mu(\boldsymbol{r})|0\rangle=\langle n|\hat{j}_\mu(0)|0\rangle e^{-i\boldsymbol{p}_n\boldsymbol{r}}$, we obtain

$$\begin{aligned}\langle n|\hat{V}(t)|0\rangle&=\int\langle n|\hat{j}^\mu(\boldsymbol{r})|0\rangle e^{i\varepsilon_n t-ikx}A_\mu(k)d^4k d^3\boldsymbol{r}\\&=\int\langle n|\hat{j}^\mu(0)|0\rangle e^{ip_n x-ikx}A_\mu(k)d^4k d^3\boldsymbol{r}\\&=(2\pi)^3\langle n|\hat{j}^\mu(0)|0\rangle\int\delta^{(3)}(\boldsymbol{p}_n-\boldsymbol{k})\\&\quad\times e^{i\varepsilon_n t-i\omega t}A_\mu(k)d^4k.\end{aligned} \tag{B4}$$

The solution of Eq. (B3) can be written as

$$\begin{aligned}a_n(t)&=-i\int_{-\infty}^{t}\langle n|\hat{V}(\tau)|0\rangle e^{i\varepsilon_n\tau}d\tau\\&=(2\pi)^3\langle n|\hat{j}^\mu(0)|0\rangle\\&\quad\times\int\delta^{(3)}(\boldsymbol{p}_n+\boldsymbol{k})\frac{e^{i\varepsilon_n t-i\omega t}}{\omega-\varepsilon_n+i0}A_\mu(k)d^4k.\end{aligned} \tag{B5}$$

Then we can find the average value of $\hat{j}(\boldsymbol{r})$. Evaluation of $j(\boldsymbol{r},t)=\langle t|\hat{j}(\boldsymbol{r})|t\rangle$ with the help of $a_n(t)$ given by Eq. (B5) leads to

$$\begin{aligned}j_\mu(x)&=-(2\pi)^3\int\sum_{n\neq 0}\Bigg(\langle n|\hat{j}^\nu(0)|0\rangle\delta^{(3)}(\boldsymbol{p}_n-\boldsymbol{k})\frac{e^{i(\varepsilon_n-\omega)t}}{\varepsilon_n-\omega-i0}\\&\quad\times A_\nu(k)\langle 0|\hat{j}_\mu(x)|n\rangle+\langle 0|\hat{j}^\nu(0)|n\rangle\delta^{(3)}(\boldsymbol{p}_n-\boldsymbol{k})\\&\quad\times\frac{e^{-i(\varepsilon_n-\omega)t}}{\varepsilon_n-\omega+i0}A^*_\nu(k)\langle n|\hat{j}_\mu(x)|0\rangle\Bigg)d^4k\\&=-(2\pi)^3\int\sum_n\Bigg(\langle 0|\hat{j}_\mu(0)|n\rangle\\&\quad\times\langle n|\hat{j}^\nu(0)|0\rangle\frac{\delta^{(3)}(\boldsymbol{p}_n-\boldsymbol{k})}{\varepsilon_n-\omega-i0}+\langle 0|\hat{j}^\nu(0)|n\rangle\\&\quad\times\langle n|\hat{j}_\mu(0)|0\rangle\frac{\delta^{(3)}(\boldsymbol{p}_n+\boldsymbol{k})}{\varepsilon_n+\omega+i0}\Bigg)A_\nu(k)e^{-ikx}d^4k.\end{aligned} \tag{B6}$$

From Eq. (B6), in view of the definition given by Eq. (3), we obtain Eq. (12), which is the Fourier transform of Eq. (8). Let us note, that the Fourier transform of the causal polarization operator

$$\Pi_{\mu\nu}(x)=4\pi i\langle 0|T\hat{j}_\mu(x)\hat{j}_\nu(0)|0\rangle \tag{B7}$$

differs from Eq. (12) by the sign before $i0$ in the second term. For the photon refractive index it is necessary to use just the delayed polarization operator $\mathcal{P}_{\mu\nu}$, as in this case $\mathcal{P}^{\mu\nu}(k)$ given by Eq. (12) has the right properties $\mathcal{P}^*_{\mu\nu}(k)=\mathcal{P}_{\mu\nu}(-k)$ required by the reality of the field $A(x)$.


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[11] Transversity of $\mathcal{P}^{\mu\nu}$ following from current conservation strongly restricts the $\mathcal{P}^{\mu\nu}$ structure, forbidding, for example, the terms $\mathcal{P}^{\mu\nu}\sim\mathcal{F}^{\mu\nu}$ and $\mathcal{P}^{\mu\nu}\sim e^{\mu\nu\delta\eta}\mathcal{F}_{\delta\eta}$, which could lead to the three-dimensional terms $\varepsilon^{ij}\sim e^{ijm}\mathcal{B}^m$ and $\varepsilon^{ij}\sim e^{ijm}\mathcal{E}^m$ in the dielectric constant. For massless photons current conservation arises automatically when we multiply Eq. (2) by $k^\mu$. Some models suggest existence of massive photons, violating gauge invariance. This can be described by the introduction of an effective "mass term" $\mathfrak{m}^2A^\mu(k)$ into Eq. (2). Current conservation no longer arises automatically. Still we may demand it together with the Lorentz gauge $k_\mu A^\mu(k)=0$. For theories that do not suggest this, the gauge nontransversal part of $\mathcal{P}^{\mu\nu}$ can be estimated by multiplying Eq. (2) with the mass term by $k_\mu$. We find that the nontransversal part of $\mathcal{P}^{\mu\nu}\sim\mathfrak{m}^2$. From Eq. (4) we find the corresponding part of the refractive index $\Delta n\sim\mathfrak{m}^2/\omega^2$. Geomagnetic data lead to the limit $\mathfrak{m}<3\times10^{-24}$ GeV [12], while observations of the galactic magnetic field give $\mathfrak{m}<3\times10^{-36}$ GeV [13]; thus $\Delta n\sim10^{-30}$–$10^{-54}$ for $\omega\sim3$ eV. Therefore the transversity of $\mathcal{P}^{\mu\nu}$ that we have used is justified in any case.

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