Thermalization after inflation and production of massive stable particles

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We discuss thermalization through perturbative inflaton decay at the end of inflation. We find that a thermal plasma should form well before all inflatons have decayed, unless all gauge symmetries are badly broken during that epoch. However, before they thermalize, the very energetic inflaton decay products can contribute to the production of massive stable particles, either through collisions with the thermal plasma, or through collisions with each other. If such reactions exist, the same massive particles can also be produced directly in inflaton decay, once higher-order processes are included. We show that these new, nonthermal production mechanisms often significantly strengthen constraints on the parameters of models containing massive stable particles; for example, stable charged particles with mass below the inflaton mass seem to be essentially excluded.

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I. INTRODUCTION

According to inflationary models [1], which were first considered to address the flatness, isotropy, and (depending on the particle physics model of the early Universe) monopole problems of the hot big bang model, the Universe has undergone several stages during its evolution. During inflation, the energy density of the Universe is dominated by the potential energy of the inflaton and the Universe experiences a period of superluminal expansion. After inflation, coherent oscillations of the inflaton dominate the energy density of the Universe. At some later time these coherent oscillations decay to the fields to which they are coupled, and their energy density is transferred to relativistic particles; this reheating stage results in a radiation-dominated Friedmann-Robertson-Walker (FRW) universe. After the inflaton decay products thermalize, the dynamics of the Universe will be that of the hot big bang model.

Until a few years ago, reheating was treated as the perturbative, one particle decay of the inflaton with decay rate Γ_d (depending on the microphysics), leading to the simple estimate $T_R \sim (\Gamma_d M_{\rm Planck})^{1/2}$ for the reheat temperature [2].[1] The reheat temperature should be low enough so that the grand unified theory (GUT) symmetry is not restored and the original monopole problem is avoided. In many supersymmetric models there are even stricter bounds on the reheat temperature. Gravitinos (the spin-3/2 superpartners of gravitons) with a mass in the range of 100 GeV to 1 TeV (as expected for ''visible-sector'' superparticles) decay during or after big bang nucleosynthesis. They are also produced in a thermal bath, predominantly through $2 \rightarrow 2$ scatterings of gauge and gaugino quanta [3]. This results in the bound[2] $T_R \leqslant 10^7 - 10^9$ GeV, in order to avoid gravitino overproduction which would destroy the successful predictions of nucleosynthesis by its late decay [4].

It has been noticed in recent years that the initial stage of inflaton decay might occur through a complicated and nonperturbative process called parametric resonance,[3] leading to an out-of-equilibrium distribution of final state particles with energies considerably higher than the inflaton mass (the preheating stage) [9,10]. Parametric resonance is particularly efficient for decays to bosonic degrees of freedom. For an inflaton, ϕ, coupling to scalars χ via four-leg interactions $h^2\phi^2\chi^2$ or three-leg interactions $h m_\phi \phi \chi^2$, parametric resonance occurs if the coupling h satisfies [10]

$$h\bar{\phi} > m_\phi, \tag{1}$$

where $\bar{\phi}$ is the amplitude of the oscillations of the inflaton field, and m_ϕ is the frequency of oscillations, which is equal

[1]From now on $M_{\rm Planck} = 2.4 \times 10^{18}$ GeV represents the reduced Planck mass.

[2]This bound does not hold in models with gauge-mediated supersymmetry breaking, where the gravitino is much lighter than sparticles in the visible sector, so that the decay of the lightest visible sparticle into the gravitino occurs well before nucleosynthesis; nor does it hold in models with anomaly-mediated supersymmetry breaking, where the gravitino mass exceeds visible-sector sparticle masses by an inverse loop factor, so that gravitino decays are sufficiently rapid. We will comment on this second scenario later.

[3]Recently, nonthermal production of helicity $\pm 3/2$ gravitino [5], and helicity $\pm 1/2$ gravitino [6] from inflaton oscillations has been considered. For models with a single chiral supermultiplet, the helicity $\pm 1/2$ component of the gravitino is the superpartner of the inflaton known as an inflatino. The decay channels of the inflatino have been discussed in Ref. [7]. Also, it has been suggested [7], and explicitly shown [8] that in realistic models with several chiral multiplets helicity $\pm 1/2$ gravitino production is not a problem, so long as the inflationary scale is sufficiently higher than the scale of supersymmetry breaking in the hidden sector and the two sectors are coupled only gravitationally.

to the inflaton mass during oscillations.[4] The explosive decay of the inflaton oscillations via such couplings comes as a result of two effects. χ quanta with physical momentum $k \lesssim \sqrt{h\bar{\phi} m_\phi}$ are produced during a very short interval each time that ϕ passes through the origin. These quanta subsequently result in an even faster particle production once their occupation number exceeds unity. The combined effect destroys the coherent oscillations of the inflaton very rapidly and brings degrees of freedom which are coupled to the inflaton to local equilibrium with it and among themselves [11]. After some time the momentum of particles in this *preheat plasma* is redshifted below the inflaton mass and another stage of inflaton decay will occur. The nature of inflaton couplings determines what exactly happens at this stage. It is known that four-leg interactions $h^2\phi^2\chi^2$ alone do not result in complete decay of the inflaton and hence interactions which are linear in the inflaton field will be required. If such interactions have a large coupling, so that (1) is satisfied, the final stage of the inflaton decay will be very quick. This usually results in a high reheat temperature which could be problematic in supersymmetric models. Moreover, the bulk of the energy density may remain in coherent oscillations of the inflaton until the final decaying stage even if the inflaton coupling satisfies the condition in (1). This happens for resonant decay to fermions [12] where decay products cannot attain occupation numbers larger than 1. It can also be the case for bosonic parametric resonance once one goes beyond the simplest toy models. For example, in the *instant preheating* scenario [13] the χ quanta may not build up a large occupation number, if they quickly decay to other fields after each interval of production. Also, moderate self-interactions of final state bosons render resonant decay much less efficient [14,15].

It is therefore generally believed that an epoch of (perturbative) reheating from the decay of massive particles (or coherent field oscillations, which amounts to the same thing) is an essential ingredient of any potentially realistic cosmological model [16]. In what follows we generically call the decaying particle the ''inflaton,'' since we are (almost) certain that inflatons indeed exist. Note also that in a large class of well-motivated models, where the inflaton resides in a ''hidden sector'' of a supergravity theory [17], its couplings are suppressed by inverse powers of $M_{\rm Planck}$, and hence so weak that inflaton decays are purely perturbative. However, it should be clear that our results are equally well applicable to any other particle whose (late) decay results in entropy production.

Inflaton decays can only be described by perturbation theory in a trivial vacuum if the density n of particles produced from inflaton decay is less than the particle density in a thermal plasma with the same energy density ρ, i.e.

$$n^{1/3} < \rho^{1/4}. \tag{2}$$

[4]In this article we assume that m_ϕ is essentially constant between $t_{\rm I}$ and $t_{\rm d}$.

Even before all inflatons decay, the decay products form a plasma which, upon a very quick thermalization, has the instantaneous temperature

$$T \sim (g_*^{-1/2} H \Gamma_{\rm d} M_{\rm Planck}^2)^{1/4}, \tag{3}$$

where H is the Hubble parameter and g_* denotes the number of relativistic degrees of freedom in the plasma.[5] This temperature reaches its maximum $T_{\rm max}$ soon after the inflaton field starts to oscillate around the minimum of its potential, which happens for a Hubble parameter $H_I \leqslant m_\phi$. If inflaton decays at this early time are to be described by perturbation theory in a trivial vacuum, the constraint (2) should be satisfied already at H_I. This implies $T_{\rm max} < m_\phi$, i.e. $\Gamma_{\rm d} < m_\phi^3/M_{\rm Planck}^2$. Note that in chaotic inflation, where $\bar{\phi} \sim M_{\rm Planck}$ initially, the same bound follows from the constraint (1), since $\Gamma_{\rm d} \sim h^2 m_\phi$. We note in passing that requiring (2) to hold only at $H \sim \Gamma_{\rm d}$, where the bulk of inflaton decays take place, leads to the considerably weaker constraint $\Gamma_{\rm d} < m_\phi^2/M_{\rm Planck}$. Indeed, this weaker constraint is satisfied in all potentially realistic inflationary models we know of that satisfy (1). If $m_\phi^2/M_{\rm Planck} > \Gamma_{\rm d} > m_\phi^3/M_{\rm Planck}^2$, the dynamics of the Universe for $H \lesssim \Gamma_{\rm d}$ will be as in the standard hot big bang scenario. However, for some period of time with $H < m_\phi$, the thermalization of inflaton decay products would have to *reduce* the number of particles. This indicates that in fact one-particle decay may not have been the dominant mode for reducing the number of inflatons during that period. For the remainder of this article we will assume that $T_{\rm max}$ is indeed smaller than m_ϕ.

In addition to the thermalized plasma with temperature given by Eq. (3), there will be inflaton decay products that have not been thermalized yet, with energy $\simeq m_\phi/2$.[6] During this era the energy density of the Universe is still dominated by the (nonrelativistic) inflatons that have not decayed yet. The scale factor of the Universe a then varies as $a \propto T^{-8/3}$ [18]. The Universe remains in this phase as long as $H > \Gamma_{\rm d}$. This can have various cosmological implications, for example for Affleck-Dine baryogenesis [19,20] and electroweak baryogenesis [21]. Here we discuss the production of massive long-lived or stable particles [22–25].

Particles with mass $m_\chi \leqslant T_{\rm R}$, and not too small coupling to the thermal bath of inflaton decay products, are in thermal equilibrium and their abundance satisfies $n \sim T_{\rm R}^3$ at $H \simeq \Gamma_{\rm d}$. On the other hand, once all inflatons have decayed, stable particles with mass $m_\chi > T_{\rm R}$ can only be pair-produced by Wien's tail of the thermal spectrum; one might thus naively expect their abundance to be suppressed by the Boltzmann factor $\exp(-T_{\rm R}/m_\chi)$. This need not be true if these particles are produced at $H > \Gamma_{\rm d}$. Particles with mass $T_{\rm R} < m_\chi < T_{\rm max}$ and not too small coupling have an abundance $\propto T^3$ at early times. Once the plasma temperature drops below m, pair an-

[5]We assume that this number remains essentially constant for all $T > T_{\rm R}$.

[6]Here we consider two body decays of the inflaton. The average energy of decay products will be of the same order even if the inflaton dominantly decays to three body, or higher, final states.

nihilation to and pair creation from lighter species keeps the particle abundance at equilibrium so long as $\Gamma_{ann} \geqslant H$. Finally, at $T=T_f$, the comoving number density of the particle freezes at its final value. The physical number density of the particle at $T=T_R$ is redshifted by a factor of $(T_R/T_f)^8$. This suggests that the thermal abundance of particles with mass $T_R < m_\chi < T_{max}$ is in general only power-law suppressed. This has important consequences for the production of heavy (i.e. $m_\chi > T_R$) stable particles [22] in general, e.g. for the lightest supersymmetric particle (LSP) [23] and charged stable particles [24] in models with low reheat temperature.

To date, almost all studies have considered the creation of heavy particles from the scattering of two particles in the thermal distribution. On the other hand, particles produced from inflaton decay have an energy $E \simeq m_\phi/2$ before they thermalize; as already noted, such very energetic particles will exist until inflaton decay completes at $H \simeq \Gamma_d$. We argued above that for perturbative inflaton decay $m_\phi > T_{max}$. It is thus possible that heavy particles are efficiently produced from the scattering of such a ''hard'' particle (with energy $E \simeq m_\phi/2$) off ''soft'' particles in the thermal bath (with energy $E \sim T$), or off another ''hard'' particle. Note that this allows to produce particles with mass $m_\chi > T_{max}$. Of course, the hard particles eventually come into equilibrium with the thermal bath; the competition among different interactions thus determines the abundance of heavy particles produced through this nonthermal mechanism. Another possibility is that the heavy particles are themselves produced directly in inflaton decays [25], in which case their abundance can be even higher.

In this article we study these issues. We begin by a review of thermalization of the perturbative inflaton decay products, and verify that a thermal plasma can indeed build up well before the completion of inflaton decay, if the inflaton decay products couple to some light (or massless) gauge bosons. We then turn to particle production from hard-soft and hard-hard scatterings, and from inflaton decay, estimating the abundance of produced particles in each case. It is shown that heavy particles may be produced more abundantly than previously thought, in particular directly from inflaton decay. We will finally close with some concluding remarks.

II. THERMALIZATION AFTER INFLATION

After inflation the inflaton field starts to oscillate coherently around the minimum of its potential; at some later time it will decay to the fields to which it is coupled. In the perturbative regime the decay occurs over many oscillations of the inflaton field, since $\Gamma_d \ll m_\phi$. This means that the oscillating inflaton field behaves like nonrelativistic matter consisting of a condensate of zero-mode bosons with mass m_ϕ. The decay rate Γ_d of the oscillating inflaton field is then identical to the total decay rate of free on-shell inflaton quanta [2]. Most inflatons have decayed by $t_d = (2/3)\Gamma_d^{-1}$. By that time the bulk of the energy density in the coherent oscillations of the inflaton field has thus been transferred to relativistic particles with an initial energy of order $m_\phi/2$. From then on the Universe is radiation-dominated, and the energy of relativistic particles is redshifted as $a^{-1} \propto t^{-1/2}$, where a is the scale factor in the FRW metric. The thermalization of the inflaton decay products then sets the stage for the familiar hot big bang universe.

However, inflaton decay does not suddenly happen at t_d. Rather, it is a prolonged process which starts once the inflaton field oscillations commence at $t=t_I$, when $H=H_I \sim m_\phi$. The comoving number density of zero-mode inflaton quanta at time t, $n_c(t)$, obeys the relation $n_c(t) = n_I \exp[-\Gamma_d(t-t_I)]$, where n_I is the inflaton number density at t_I. For $t_I \leqslant t \leqslant t_d$ the Universe is matter-dominated, which implies $a \propto t^{2/3}$, i.e. $H^{-1} = (3/2)t$. In the time interval between t and $t + (2/3)H^{-1} = 2t$, decay products with energy $\simeq m_\phi/2$ are produced and their physical number density $\overline{n_h}$ at the end of this interval is (assuming $t_I \ll t_d$):

$$\overline{n_h} \simeq n_I [\exp(-\Gamma_d t) - \exp(-2\Gamma_d t)] \left(\frac{H}{2H_I} \right)^2, \qquad (4)$$

where the last factor comes from the expansion of the physical volume. Particles which were produced earlier have a redshifted energy and number density. The spectrum of inflaton decay products has been derived in [26,27] and it has been shown that, if the inflaton decay products do not interact with each other, the number density and energy density of the plasma are dominated by particles with energy $\sim m_\phi/2$ in the spectrum.

Thermalization is a process during which the energy density ρ of a distribution of particles remains constant, while their number density n changes in such a way that the mean energy $\overline{E}$ of particles reaches its equilibrium value $\sim T$. For a distribution of relativistic particles which consists of n_B bosonic and n_F fermionic degrees of freedom in thermal equilibrium, and with negligible chemical potential, we have $\rho = \pi^2/30 (n_B + \frac{7}{8} n_F) T^4$ and $n = \zeta(3)/\pi^2 (n_B + \frac{3}{4} n_F) T^3$ [18]. Therefore, the ratios $\rho^{1/4}/\overline{E}$ and $n^{1/3}/\overline{E}$ are measures for the deviation from thermal equilibrium. For perturbative inflaton decay these ratios are initially less than 1, so that thermalization increases the number density and reduces the mean energy. Complete thermalization therefore requires interactions which change the number of particles to be in equilibrium.

There are three types of interactions which help to build up and maintain full (i.e. both kinetic and chemical) equilibrium: $2 \to 2$ scatterings, $2 \to N$ scatterings ($N \geqslant 3$), and particle decays. Elastic $2 \to 2$ scatterings redistribute the energy between the scattered particles, but play no role in achieving chemical equilibrium. Inelastic $2 \to 2$ (annihilation) reactions can help to maintain *relative* chemical equilibrium between different particle species, but again leave the total number of particles unchanged. For particles with energy $E \gg T$ and number density $n(E)$, the rate of $2 \to 2$ reactions with an energy exchange $\Delta E \sim E$ is typically given by $\Gamma \sim \alpha^2 n(E)/E^2$, where α is the relevant coupling constant. On the other hand, $2 \to N$ scatterings and particle decays increase the number of particles and hence play a crucial role in reaching full equilibrium. The rate of $2 \to N$ reactions is suppressed by additional powers of α. Therefore inelastic scatterings with a large squared 4-momentum exchange $|t| \sim E^2$

come to equilibrium later than $2\rightarrow 2$ reactions do. This would result in a late thermalization and a low reheat temperature, if these were the most important reactions which increase the total number of particles [28,29]. One possibility to achieve chemical equilibrium more quickly is through the ''catalyzed thermalization'' scenario [26]. As shown in Refs. [26,27], in the absence of interactions $n(E)/E^2$ increases with decreasing E in the spectrum of inflaton decay products, due to redshifting of ''early'' inflaton decay products. This suggests that a seed of particles with energy $E \ll m_\phi$, which constitute a tiny fraction of the number density and the energy density of the plasma, may thermalize much earlier than the bulk of the plasma. The large number of created soft particles can then act as targets triggering a rapid thermalization of the bulk. However, as pointed out in [27], elastic $2\rightarrow 2$ scattering of particles in the bulk off particles in the seed is efficient; it destroys the seed by bringing it into kinetic equilibrium with the bulk. Nevertheless, catalyzed thermalization can still take place if inflaton decay products themselves decay before coming into kinetic equilibrium with the bulk [27]. However, it does not seem very probable that decays of on-shell particles alone will be enough for building the chemical equilibrium.

In general, inelastic scatterings which produce relatively soft particles are the most important processes leading to full equilibrium. This has recently been illustrated in Ref. [30] where the t-channel scattering of two matter fermions with energy $\simeq m_\phi/2$ (from inflaton decay) to two fermions, plus one gauge boson with typical energy $E \ll m_\phi$, has been considered. The key observation is that (for $T=0$ and in a flat space-time) t-channel scattering is divergent as $\Delta E \rightarrow 0$. One thus has to choose a physical infrared cutoff in order to estimate the thermalization rate. A reasonable choice is the inverse of the average separation between two particles in the plasma, $\overline{r} \propto n^{-1/3}$, where n is the number density of the plasma. This leads to a thermalization rate for hard particles (with $E \sim m_\phi/2$) of the order[7]

$$\Gamma_{\rm in} \sim n \cdot \sigma(2\rightarrow 3) \sim \alpha^3 n^{1/3}. \tag{5}$$

Once $\Gamma_{\rm in} > H$, relatively soft gauge bosons with energy $\gtrsim T$ are efficiently created. This increases n, and hence the thermalization rate (5). Note that the increased target density over-compensates the increase of the cutoff $|t|_{\rm min}$. From then on the number of particles with energy $\sim T$ increases faster than exponentially and quickly reaches its final value, as pointed out in Ref. [31].[8] Inelastic $2\rightarrow 2$ reactions then rapidly build up full kinetic equilibrium. This suggests that $\Gamma_{\rm in}$ can in fact be considered as the thermalization rate, with n being the original (prethermalization) density, $n \sim g_* T^4/(3m_\phi)$, where g_* is the effective number of relativistic degrees of freedom; here T is the temperature (3) which the plasma would have if it were thermalized.

This argument assumes that $2\rightarrow 3$ scattering can efficiently produce ''soft'' particles with energy $\gtrsim T$. The production of even softer particles would not help in thermalizing the plasma, since it would not slow down the parent particle appreciably, and might even lead to a density of such very soft particles which exceeds their thermal density. $2\rightarrow 3$ reactions with all virtual particles (in propagators) having virtualities of order $n^{1/3}$ can indeed produce ''soft'' particles with energy $E_s \gg n^{1/3}$, if these ''soft'' particles are nearly collinear with an incident hard particle, the emission angle being of order $\sqrt{n^{2/3}/(m_\phi E_s)}$. One might wonder whether the emission of such a very collinear particle is in fact physically distinguishable from no emission at all. We believe this is the case. Note first of all that the lower cut on the virtualities of all propagators implies that the ''collinear'' particle is in fact not that close to the emitted one in full phase space. Moreover, after a time of order $1/\Gamma_{\rm in}$ there will be many such soft particles. Scattering of an almost collinear ''soft+hard'' pair on a soft particle can yield a final state with soft particles being emitted at a large angle only if the ''soft'' particle in the initial state participates; the ''hard'' particle in the initial state can only scatter at a very small angle. In other words, even before full thermalization, the plasma allows to physically detect the presence of soft, collinear particles in the ''beam'' of hard particles; the process that ''detects'' these particles also removes them from the ''beam.'' This happens at a time scale significantly shorter than $\Gamma_{\rm in}$, since one only needs $2\rightarrow 2$ reactions here.

In order to check whether the inflaton decay products can thermalize before the completion of inflaton decay, we compute the maximum temperature $T_{\rm max}$ of the thermal plasma; thermalization occurs before inflaton decay completes if $T_{\rm max} > T_{\rm R}$. We use Eq. (3), with $H=\Gamma_{\rm in}$ from Eq. (5), and $\Gamma_{\rm d} = g_*^{1/4} T_{\rm R}^2 / M_{\rm Planck}$, resulting in

$$T_{\rm max} \sim T_{\rm R} \left[\alpha^3 \left(\frac{g_*}{3} \right)^{1/3} \frac{M_{\rm Planck}}{m_\phi^{1/3} T_{\rm R}^{2/3}} \right]^{3/8}, \tag{6}$$

[7]We ignore factors of order $\log E/T \sim \log m_\phi/T$ where T is the temperature of the plasma, which could increase the rate by a factor of a few. Note also that the authors of Ref. [30] took $|t|_{\rm min} \sim T^2$, but estimated the target density n from the density of particles *before* thermalization, $n \sim T^4/m_\phi$. This results in a lower estimate of the thermalization rate.

[8]The situation is slightly more subtle if the soft gauge boson belongs to an Abelian gauge group (e.g. photon). As we will show shortly, only inelastic scatterings with gauge boson exchange in the t or u channel results in the estimate in Eq. (5). Such diagrams exist for scattering of a fermion (e.g. quark) from a soft non-Abelian gauge boson (e.g. gluon) [31]. On the other hand, in inelastic electron-photon scattering the t-channel diagram has an electron as exchanged particle, and hence has a rate much smaller than that of electron-electron scattering. However, soft photons with $E \sim T$ annihilate into soft $e^+ e^-$ pairs at a rate which exceeds (5) by a factor of $1/\alpha$. These soft $e^\pm$ can then serve as targets for subsequent scatterings of hard electrons. The thermalization rate, which is set by the rate of the slowest relevant reaction, is then still approximately given by Eq. (5).

which is somewhat higher[9] than the estimate in Ref. [30]:

$$T_{\max} \sim T_{\rm R} \left(\alpha^3 g_*^{3/4} \frac{M_{\rm Planck}}{3 m_\phi} \right)^{1/2}. \tag{7}$$

In any case it is reasonable to expect that the largest temperature of the Universe after inflation is between the values estimated in Eq. (6) and Eq. (7). An interesting observation is that $T_{\max}$, from Eq. (6), grows $\propto T_{\rm R}^{3/4}$. Even if m_ϕ is near its Cosmic Background Explorer (COBE) derived upper bound [32] of $\sim 10^{13}$ GeV, for a chaotic inflation model, and $T_{\rm R}$ is around 10^9 GeV (which saturates the gravitino bound), $T_{\max}$ will exceed $T_{\rm R}$ if the coupling $\alpha^3 \gtrsim 10^{-8}$. This is easily accommodated for particles with gauge interactions as α will be the gauge fine structure constant. For inflationary models with smaller m_ϕ, early thermalization can be realized with even weaker couplings. Recall also that we assume perturbative inflaton decays, which require $T_{\max} < m_\phi/2$. Together with Eq. (6), taking $\alpha \lesssim 0.1$, this implies $T_{\max} \lesssim 10^{11}$ (10^5) GeV for $T_{\rm R} = 10^9$ (1) GeV; this bound is saturated for $m_\phi = 2T_{\max}$. This implies in particular that there will be no "wimpzilla" ($m_\chi \sim 10^{12}$ GeV) [22] production from *thermalized* inflaton decay products, even if we use the higher estimate (6) for $T_{\max}$.

For $\Gamma_{\rm in} \geqslant H \geqslant \Gamma_{\rm d}$, fresh inflaton decays will keep producing particles with energy $E \simeq m_\phi/2$, but these particles quickly thermalize by scattering off a large number of soft particles in the thermal plasma. The time scale for this process is again given by Eq. (5), where now $n^{1/3} \sim T$. Inflaton decay effectively completes at $H \simeq \Gamma_{\rm d}$ when the plasma temperature is $T_{\rm R}$. From then on the Universe is radiation-dominated and its temperature will be redshifted as a^{-1}.

Since the estimate for $T_{\max}$ crucially depends on the rate for inelastic scatterings it is instructive to explore this issue a little more deeply. As mentioned earlier, for processes with a t-channel singularity, e.g. $e^+e^- \to e^+e^-\gamma$ via the t-channel exchange of a photon, one finds $\sigma_{\rm in} \propto |t|^{-1}$ rather than $\sigma_{\rm in} \propto s^{-1}$. In fact this is the very same singularity which exists in $e^+e^- \to e^+e^-$ scattering in the t channel. The appearance of $|t|^{-1}$ in the cross section can be understood in the following way. σ receives a factor of $|t|^{-2}$ from the photon propagator while the contribution from the phase space integration is $\propto |t|$, thus resulting in $\sigma \propto |t|^{-1}$. This implies that boson exchange in the t channel is required in order to have $\sigma \propto |t|^{-1}$ since the contribution of diagrams with a fermion propagator will be $\propto s^{-1}$ (after phase space integration). Moreover, it can be shown that for essentially massless external particles the contribution from scalar boson exchange is also suppressed. For example, consider the t-channel scattering of two fermions via scalar exchange, as shown in Fig. 1(a). A fermion-fermion-scalar vertex naturally arises from a Yukawa coupling or, in supersymmetric models, from the D-term part of the action. Note that scalar interactions flip the chirality of the fermion line. However, for massless fermions a flip of chirality also implies a flip of helicity, i.e. a spin flip; this is forbidden by angular momentum conservation for forward scattering, where $t \to 0$. As a result, the diagram of Fig. 1(a) has no t-channel singularity at all.[10]

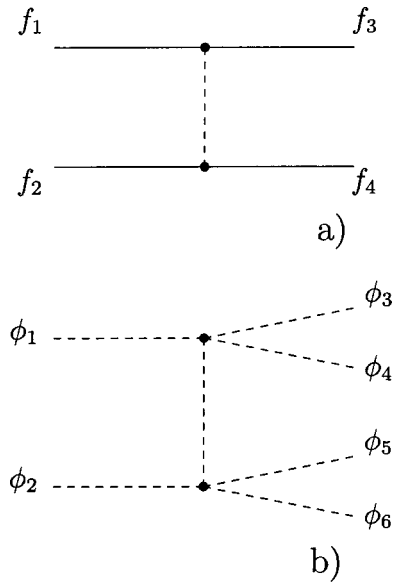

FIG. 1. Typical scattering diagrams with scalar boson exchange in the t-channel for (a) fermions or antifermions in the initial and final state, (b) inelastic $2 \to 4$ scattering involving only scalar particles.

Finally, it is also possible to have inelastic $2 \to 4$ scatterings, shown in Fig. 1(b), with only scalar particles in external and internal lines. A vertex comprising four scalars can for example arise from the Higgs self-coupling in the standard model (SM), or from the F- and D-term parts of the scalar potential in supersymmetric models. However, in this case integration over the total allowed phase space only leads to logarithmic divergences in the limit of vanishing external masses. Including the squared t-channel propagator in Fig. 1(b), the integral in question can be written as $\int dt dM_{34}^2 dM_{56}^2 1/t^2$, where $M_{ij}^2 = (p_i + p_j)^2$ is the squared invariant mass of the pair $i,j = 3,4$ or 5,6. $|t|$ reaches its minimum for small M_{ij}^2, in which case $|t|_{\min} \simeq (M_{34}^2 M_{56}^2)/s$; the integrals over M_{34}^2 and M_{56}^2 then only give rise to logarithmic singularities, as advertised. Ignoring such logarithmic factors, the total cross section will thus again behave as $\sigma \propto s^{-1}$.

The fact that only diagrams with light gauge bosons as internal and/or external lines give rise to $\sigma_{\rm in} \propto |t|^{-1}$ can have interesting implications for thermalization, in scenarios where the gauge group is completely broken in the early Universe. This may for example happen in supersymmetric models where flat directions in the scalar potential [33] can acquire a large vacuum expectation value (VEV) during

[9]The two estimates coincide if $\Gamma_{\rm d}$ saturates its upper bound of $m_\phi^3/M_{\rm Planck}^2$.

[10]This can also be seen easily by computing the relevant Dirac traces. Each fermion line gives rise to a separate trace, which is proportional to the scalar product of the 4-momenta of the two fermions involved. For massless particles these exactly cancel the $1/t^2$ of the squared scalar propagator.

inflation[11] [34]. If gauge bosons have a mass m_g such that $\alpha^{-1}n^{1/3}<m_g<m_\phi$, then the most important diagrams are those with resonant gauge boson exchange in the s channel, with $\sigma_{\rm in}\sim\alpha m_g^{-2}$. Moreover, for $m_g>m_\phi$ we will have $\sigma_{\rm in}\sim\alpha^3 m_\phi^{-2}$. This affects the estimate for the thermalization rate $\Gamma_{\rm in}$ and may indeed result in a (much) smaller $T_{\rm max}$. As one consequence, thermal effects on the flat direction dynamics [19,20] may be alleviated. Once flat directions start oscillating, their VEV is redshifted, due to the expansion of the Universe, and the induced mass for the gauge bosons rapidly decreases. The flat direction oscillations start when $H\sim V''$, the second derivative of the scalar potential. In models where supersymmetry breaking is communicated to the visible sector through gauge interactions at a relatively low scale (of order tens of TeV) the potential at high field values is exceedingly flat. It is therefore conceivable that the flat direction induced masses for the gauge bosons, if they completely break the gauge group, could delay thermalization until late times. This might allow to construct models with low reheat temperature even if the inflaton decay width $\Gamma_{\rm d}$ is not very small. In the remainder of this article we ignore such possible effects and use the (rather generous) estimate in Eq. (6) for $T_{\rm max}$. This allows us to study the most efficient production of massive particles which is possible from the plasma of inflaton decay products.

III. HEAVY PARTICLE PRODUCTION

We now move on to the issue of heavy particle production. In recent years several mechanisms have been put forward for creating very heavy, even superheavy, particles in cosmologically interesting abundances. For instance particle production could take place in a time-varying gravitational background during the phase transition from inflationary to the radiation-dominated or matter-dominated phase [35]. Another possibility is to create supermassive particles from preheating [36]. Here we will focus on the production of very massive particles from various processes, including a thermal bath, during perturbative reheating. Note that particle production from other sources, if present, would further strengthen the bounds which we will derive as they simply add to production from mechanisms discussed here.

The situation for a weakly interacting massive particle (WIMP) χ with mass $m_\chi\lesssim T_{\rm R}$ is very well established [37]. Such a species is at equilibrium with the thermal bath for $T>m_\chi$ and its number density follows $n_\chi\propto T^3$. Once the temperature drops below m_χ, the particle becomes nonrelativistic and its equilibrium number density becomes $n_\chi^{\rm eq}\sim(Tm_\chi)^{3/2}\exp(-m_\chi/T)$. Pair annihilation of χ's to light particles, occurring at a rate approximately given by $\Gamma_{\rm ann}\sim\alpha_\chi^2 n_\chi m_\chi^{-2}$, preserves its chemical equilibrium with the bath so long as $\Gamma_{\rm ann}\geqslant H$. Here α_χ is the effective coupling constant of χ to particles in the plasma. The annihilation rate eventually drops below the Hubble expansion rate, mostly due to the fact that $n_\chi^{\rm eq}$ is exponentially suppressed. Then the comoving number density of χ will be frozen at its final value $n_\chi\sim(T_{\rm f}m_\chi)^{3/2}\exp(-m_\chi/T_{\rm f})$, where $T_{\rm f}$ denotes the freeze-out temperature. In the post-inflationary era, i.e. for $H<\Gamma_{\rm d}$, typically $T_{\rm f}\sim(m_\chi/20)$ if $\alpha_\chi\sim\alpha$, up to logarithmic corrections [37]. Taking $\alpha_\chi\simeq\alpha(m_\chi/M_Z)^2$ gives a lower bound of a few GeV on the mass of heavy, stable neutrinos, known as the Lee-Weinberg bound. In the other extreme, the unitarity bound $\sigma(\chi\chi\to\text{anything})<4\pi m_\chi^{-2}$ provides a firm upper bound of about 100 TeV [38] on any stable particle that was in thermal equilibrium at any temperature $T<T_{\rm R}$.[12]

The upper bound on m_χ might be relaxed considerably if the initial thermal equilibrium condition for χ is relaxed, i.e. if $T_{\rm f}>T_{\rm R}$. Even in that case the χ particles can have been in thermal equilibrium with the plasma of SM particles, whose temperature T can significantly exceed $T_{\rm R}$ at sufficiently early times $t<1/\Gamma_{\rm d}$. However, we saw above that at those times the thermal bath did not dominate the energy density of the Universe. This gives rise to a significant difference from the freeze-out situation discussed above, due to the different T dependence of the redshift factor. In this case χ pair annihilation reactions freeze out at a higher temperature, since $H\sim\sqrt{g_*}T^4/(T_{\rm R}^2M_{\rm Planck})>\sqrt{g_*}T^2/M_{\rm Planck}$ for $H>\Gamma_{\rm d}$. Also, n_χ is now redshifted $\propto T^8$, which is much faster than in a radiation-dominated Universe. The situation in this case has been investigated in detail in Refs. [22–25] where the relevant Boltzmann equations governing the production and annihilation of χ's are solved both numerically and analytically. In Ref. [22] out of equilibrium production of χ from scatterings in the thermal bath is studied and the final result is found to be (the superscript ''ss'' stands for χ production from ''soft-soft'' scattering; see below)

$$\Omega_\chi^{\rm ss}h^2\sim\left(\frac{200}{g_*}\right)^{3/2}\alpha_\chi^2\left(\frac{2000T_{\rm R}}{m_\chi}\right)^7\quad(\chi\ \text{not in equilibrium}).\tag{8}$$

Here Ω_χ is the χ mass density in units of the critical density, and h is the Hubble constant in units of 100 km/(s Mpc). We have taken the cross section for χ pair production or annihilation to be $\sigma\simeq\alpha_\chi^2/m_\chi^2$. Most χ particles are produced at $T\simeq m_\chi/4$ [23]. The density of earlier produced particles is strongly redshifted, while χ production at later times is suppressed by the Boltzmann factor. It is important to note that Ω_χ is only suppressed by $(T_{\rm R}/m_\chi)^7$ rather than by $\exp(-m_\chi/T_{\rm R})$. We thus see that a stable particle with mass $m_\chi\gg100$ TeV might act as the dark matter in the Universe (i.e. $\Omega_\chi\simeq0.3$), if $m_\chi\sim2000\ T_{\rm R}\cdot\alpha_\chi^{2/7}$. Recall that for $m_\chi<20T_{\rm R}$ the standard analysis holds as the freeze-out occurs in a radiation-dominated Universe.

Equation (8) predicts a relic density that increases with the χ coupling strength. However, this is true only if the χ density is always smaller than its equilibrium density, which requires [23]

[11]For a perturbative decay, inflaton couplings to other fields are so small that the mass induced by the inflaton VEV (if any) will be much smaller than the inflaton mass. Therefore scalar fields can indeed acquire a large VEV during inflation.

[12]Provided the comoving entropy of the Universe remained essentially constant for $T<T_{\rm f}$.

$$\alpha_\chi^2 \leqslant \overline{\alpha_\chi^2} \simeq 200\left(\frac{g_*}{200}\right)^{1/2}\frac{m_\chi^3}{M_{\rm Planck}T_{\rm R}^2}. \quad (9)$$

If this condition is violated, today's χ relic density is given by

$$\Omega_\chi^{\rm ss}h^2 \sim \left(\frac{200}{g_*}\right)^{1/2}\frac{T_{\rm R}x_{\rm f}^{4+a}}{m_\chi\alpha_\chi^2}\left(\frac{T_{\rm R}}{8\times 10^5\ \ {\rm GeV}}\right)^2 \quad (\chi \text{ in equilibrium}), \quad (10)$$

where the exponent $a=0(1)$ if $\chi\chi$ annihilation proceeds from an $S(P)$ wave initial state. The freeze-out temperature is now given by $x_{\rm f}\equiv m_\chi/T_{\rm f} \simeq \log(0.08 g_*^{-1/2}\alpha_\chi^2 x_{\rm f}^{2.5-a} M_{\rm Planck}T_{\rm R}^2/m_\chi^3)$, as compared to $x_{\rm f} \sim \log(0.2 g_*^{-1/2}\alpha_\chi^2 x_{\rm f}^{0.5-a} M_{\rm Planck}/m_\chi)$ if freeze-out occurs at $T<T_{\rm R}$.

Of course, Eqs. (8) and (10) are only applicable if $m_\chi \lesssim 2T_{\rm max}$, cf. Eq. (6) and the subsequent discussion. This requires

$$\frac{m_\chi}{T_{\rm R}} < 4\alpha^{9/8}\left(\frac{g_*}{200}\right)^{1/8}\frac{M_{\rm Planck}^{3/8}}{T_{\rm R}^{1/4}m_\phi^{1/8}} \leqslant 4\alpha\left(\frac{g_*}{200}\right)^{1/9}\left(\frac{M_{\rm Planck}}{T_{\rm R}}\right)^{1/3}; \quad (11)$$

in the second step we have used the condition $T_{\rm max}\leqslant m_\phi/2$. For example, for $\alpha=0.05$, $T_{\rm max}\gtrsim 1000T_{\rm R}$ is only possible if $T_{\rm R}<2\times 10^{-12}M_{\rm Planck}$. Equation (9) shows that equilibrium is more difficult to achieve for larger m_χ; this is not surprising, since the cross section for χ pair production scales like m_χ^{-2}. This means that Eq. (8) will usually be applicable in models with large m_χ and comparatively smaller $T_{\rm R}$, while in the opposite case Eq. (10) may have to be used. We will see shortly that both situations can arise in potentially interesting scenarios. Finally, for fixed $T_{\rm R}$ and m_χ, χ production from the thermal plasma is maximized if the condition (9) is saturated. This gives

$$\Omega_\chi^{\rm ss,max}h^2 \sim 3\times 10^{25}\frac{200}{g_*}\frac{T_{\rm R}^5}{m_\chi^4 M_{\rm Planck}}. \quad (12)$$

So far we have only discussed χ production from the scattering of ''soft'' particles in the thermal distribution. However, ''hard'' particles with energy $E\simeq m_\phi/2 \gg T$ are continuously created by (two body) inflaton decay for $H \geqslant \Gamma_{\rm d}$. These particles eventually thermalize with the bath, but this takes a finite amount of time. The presence of hard inflaton decay products can therefore affect heavy particle production in two ways. Firstly, χ's can be produced from $2\to 2$ scatterings of a hard particle off either soft particles in the thermal bath (if kinematically allowed), or off other hard particles. Moreover, χ's might be directly produced from inflaton decay. In both cases χ production may be enhanced and this is the issue to which we now turn.

A. Particle production from hard-soft scatterings

It is seen from Eq. (4) that the energy density in inflaton decay products created in the time interval $[(3/2)H^{-1}, 3H^{-1}]$, for $H>\Gamma_{\rm d}$, is comparable to the energy density in the existing thermal plasma which has a temperature $T\sim(H\Gamma_{\rm d}M_{\rm Planck})^{1/4}$. This implies that thermalization of these hard particles will increase the comoving number density of the thermal bath, roughly speaking by a factor of 2, in addition to bringing the decay products into kinetic equilibrium. To the accuracy we are working in, we can therefore set the density of ''hard'' inflaton decay products produced during that time interval[13] to $\overline{n_h}\sim\rho/m_\phi\sim 0.3 g_* T^4/m_\phi$. In order to estimate the rate of heavy particle production from these ''hard'' inflaton decay products, we also have to know the time needed to reduce the energy of these hard particles from a value $\sim m_\phi/2$ to a value near T. As shown in Ref. [30], $2\to 2$ scattering reactions are *not* very efficient. The reaction rate is large, but the average energy loss per scattering is only $\mathcal{O}(T^2/m_\phi)$, giving a slow-down time of order $[\alpha^2 T^2/m_\phi]^{-1}$ (up to logarithmic factors). On the other hand, we saw at the end of Sec. II that inelastic $2\to 3$ reactions allow large energy losses (in nearly collinear particles) even if all virtual particles only have virtuality of order T. The slow-down rate is thus again given by Eq. (5), where the ''target'' density is now $n\simeq 0.2 g_* T^3$:[14]

$$\Gamma_{\rm slow} \simeq 3\alpha^3 T\left(\frac{g_*}{200}\right)^{1/3}. \quad (13)$$

Next let us estimate the rate for χ pair production from hard-soft scatterings. This process is kinematically allowed so long as $ET \geqslant 4m_\chi^2$, where E is the energy of the hard particle so that the square of the center-of-mass energy is typically a few times ET. The hard particle initially has energy $E\simeq m_\phi/2$ and average (see above) number density $\overline{n_h} \sim g_* T^4/(3m_\phi)$, just after its production from inflaton decay. It loses its energy at the rate given in Eq. (13). Note that the time required for the hard particle to drop below the kinematical threshold $E_{\rm min}\simeq m_\chi^2/T$ depends only logarithmically on this threshold. We ignore this logarithmic factor, and simply estimate the slow-down time as $1/\Gamma_{\rm slow}$. On the other hand, the rate for χ production from hard-soft scatterings is approximately given by

$$\Gamma_\chi^{\rm hs} \sim \left(\frac{\alpha_\chi^2}{Tm_\phi}+\frac{\alpha\alpha_\chi^2}{m_\chi^2}\right)0.2T^3, \quad (14)$$

where we conservatively assume that χ's can be produced from scatterings of just one species (e.g. electrons). The two

[13] More exactly, $\overline{n_h}=\int_t^{2t} dn_h/dt$, where only the *production* of hard particles from inflaton decay is included in dn_h/dt, i.e. $dn_h/dt=\Gamma_{\rm d}n_\phi(t)$.

[14] We note in passing that the logarithmic factors cancel in Eq. (13). We included the g_* factor in the cutoff $|t|_{\rm min}$ since (almost) all particles in the plasma have non-Abelian interactions with the exchanged particles in the $2\to 3$ scattering diagram.

contributions in Eq. (14) describe $2\to2$ reactions with squared center-of-mass energy $\sim m_\phi T$ and "radiative return" $2\to3$ reactions, respectively; in the latter case the hard particle emits a collinear particle prior to the collision, thereby reducing the effective cms energy of the collision to a value near m_χ. This results in the estimate[15]

$$n_\chi^{\rm hs}(T)\sim \overline{n}_h\cdot\frac{\Gamma_\chi^{\rm hs}}{\Gamma_{\rm slow}} \sim 4\left(\frac{g_*}{200}\right)^{2/3}\frac{\alpha_\chi^2}{\alpha^2}\left(\frac{T^5}{\alpha m_\phi^2}+\frac{T^6}{m_\phi m_\chi^2}\right), \qquad (15)$$

for χ's produced at temperature T. It is clear that because of the redshift factor $(T_{\rm R}/T)^8$ production close to $T_{\rm thr}$ makes the dominant contribution, where $T_{\rm thr}\equiv 4m_\chi^2/m_\phi$. In order to make a safe (under)estimate we choose the temperature $T_0=2T_{\rm thr}$ for presenting our results; note that the χ pair production cross section at threshold, $s=4m_\chi^2$, is suppressed kinematically. If $T_0<T_{\rm R}$, the physical χ density at $H=\Gamma_{\rm d}$, i.e. at $T=T_{\rm R}$, is simply given by Eq. (15) with $T=T_{\rm R}$. In this case the maximal cms energy at $T_{\rm R}$ is still above $8m_\chi^2$. This means that the total χ production cross section will be dominated by $2\to3$ "radiative return" reactions, i.e. the second, T^6 contribution in Eq. (15) will usually dominate. On the other hand, $T_0>T_{\rm R}$ leads to a physical χ density at $H=\Gamma_{\rm d}$ of order

$$n_\chi^{\rm hs}(T_R)\sim 10^{-2}\left(\frac{g_*}{200}\right)^{2/3}\frac{\alpha_\chi^2}{\alpha^3}\frac{T_{\rm R}^8 m_\phi}{m_\chi^6}. \qquad (16)$$

In this case the contribution from $2\to3$ processes is suppressed (by an extra power of α).

In order to translate the χ density at $T=T_{\rm R}$ into the present χ relic density, we use the relation [18]

$$\Omega_\chi h^2=\frac{m_\chi n_\chi(T_{\rm R})}{\rho_R(T_{\rm R})}\cdot\frac{T_{\rm R}}{T_{\rm now}}\cdot(\Omega_R h^2)_{\rm now} =6.5\times10^{-7}\cdot\frac{200}{g_*}\cdot\frac{m_\chi n_\chi(T_{\rm R})}{T_{\rm R}^3 T_{\rm now}}, \qquad (17)$$

where ρ_R is the energy density in radiation, and we have used $(\Omega_R h^2)_{\rm now}=4.3\times10^{-5}$ [22]. Our final results for the contribution of hard-soft collisions to the χ relic density are thus as follows: for $T_0<T_{\rm R}$,

$$\Omega_\chi^{\rm hs}h^2\sim\left(\frac{200}{g_*}\right)^{1/3}\frac{\alpha_\chi^2}{\alpha^2}\left(\frac{T_{\rm R}}{10^4\ {\rm GeV}}\right)^2\frac{10^{13}\ {\rm GeV}}{m_\phi}\times\frac{100T_{\rm R}}{m_\chi}\left(1+\frac{m_\chi^2}{\alpha T_{\rm R}m_\phi}\right), \quad (T_0<T_{\rm R}) \qquad (18)$$

where the second term in the last round parentheses comes from $2\to2$ processes. In the opposite situation, we have

$$\Omega_\chi^{\rm hs}h^2\sim\left(\frac{200}{g_*}\right)^{1/3}\frac{\alpha_\chi^2}{\alpha^3}\frac{m_\phi}{10^{13}\ {\rm GeV}}\left(\frac{3000T_R}{m_\chi}\right)^5, \quad (T_0>T_{\rm R}) \qquad (19)$$

where we have again ignored the contribution from $2\to3$ processes.

Two conditions have to be satisfied for our estimates (15)–(19) to be applicable. First, we need $T_0<T_{\rm max}$. Note that this constraint is weaker by a factor m_χ/m_ϕ than the analogous constraint for the applicability of Eqs. (8),(10). Second, at temperature $T=\max(T_{\rm R},T_0)$, n_χ should be sufficiently small that $\chi\chi$ annihilation reactions can be ignored. This is true if the $\chi\chi$ annihilation rate is smaller than the expansion rate, $\alpha_\chi^2 n_\chi/m_\chi^2<H(T)\simeq\sqrt{g_*}T^4/(M_{\rm Planck}T_{\rm R}^2)$. If $T_0<T_{\rm R}$, $\chi\chi$ annihilation is thus negligible if

$$\left(\frac{m_\chi}{T_{\rm R}}\right)^4>\frac{1}{3}\left(\frac{g_*}{200}\right)^{1/6}\left(\frac{\alpha_\chi^2}{\alpha}\right)^2\frac{M_{\rm Planck}}{m_\phi} \quad (T_0<T_{\rm R}). \qquad (20)$$

Recall that we need $m_\chi>20T_{\rm R}$, since otherwise χ's would have been in equilibrium at $T_{\rm R}$. Moreover, typically $\alpha_\chi^2/\alpha\lesssim10^{-2}$ for weakly interacting particles. The condition (20) will therefore always be comfortably satisfied in chaotic inflation, where $m_\phi\sim10^{-5}M_{\rm Planck}$. If $T_0>T_{\rm R}$, the condition for $\chi\chi$ annihilation to be negligible is

$$\frac{1}{3}\left(\frac{g_*}{200}\right)^{1/6}\frac{\alpha_\chi^4}{\alpha^3}<\frac{m_\phi^3}{T_{\rm R}^2M_{\rm Planck}} \quad (T_0>T_{\rm R}). \qquad (21)$$

Again, this condition can only be violated if $m_\phi\ll10^{13}$ GeV. We thus conclude that in most scenarios with rather heavy inflatons the estimates (15)–(19) are indeed applicable.

Finally, unless m_χ is quite close to m_ϕ, in which case T_0 is likely to exceed $T_{\rm max}$, T_0 is well below m_χ; if $m_\chi>20T_{\rm R}$, T_0 is then also well below the freeze-out temperature $T_{\rm f}$ even if χ's once were in thermal equilibrium with the plasma. The χ density from hard-soft scattering can thus simply be added to the contribution from soft-soft scattering.

For a first assessment of the importance of the contribution from hard-soft scattering, we compare Eq. (18) with the maximal contribution (12) from soft-soft scattering. We find that the contribution from hard-soft scattering would dominate if $m_\phi/m_\chi<5\times10^{-5}\cdot(\alpha_\chi/\alpha)^2\cdot(m_\chi/T_{\rm R})^2$, where we have taken $g_*\simeq200$ [as in the minimal supersymmetric standard model (MSSM)]. This condition can only be satisfied if $m_\chi\gg T_{\rm R}$ is not too far below m_ϕ, in which case Eq. (12) is

[15]More precisely, this is the density of χ particles produced between t and $2t$, i.e. during one Hubble time, during which T remained approximately constant.

not applicable anyway, since $T_{\max} < m_\chi$. In other words, whenever the soft-soft contribution is near its maximum, it will dominate over the hard-soft contribution. On the other hand, the hard-soft contribution will often dominate over the soft-soft one if χ never was in thermal equilibrium. For $T_0 < T_R$, this is true if $\alpha^2 m_\phi / m_\chi < (m_\chi / T_R)^4 (m_\chi / 10^{16}\ \mathrm{GeV})$, which is satisfied unless T_R and m_χ are both rather small. Similarly, for $T_0 > T_R$, Eq. (19) will dominate over Eq. (8) if $m_\phi > 10^{13}$ GeV $\alpha^3 (700 T_R / m_\chi)^2$. Recall that the last term must be $\lesssim 0.1$, since otherwise the soft-soft contribution by itself would overclose the Universe. In this case the hard-soft contribution will thus dominate in models with rather large inflaton mass. Of course, this means that cosmological constraints on the model parameters T_R and m_χ will often be considerably stronger than previously thought. We will come back to this point when we present some numerical examples.

B. Particle production from hard-hard scatterings

If $T_0 > T_R$, we should also consider χ production from scattering of two hard particles. Recall that "hard" particles are continuously created as long as $H > \Gamma_d$. Collisions of these particles with each other can produce χ pairs if $m_\chi < m_\phi/2$. Note that this constraint is independent of the temperature. On the other hand, once a thermal plasma has been established, the density of "hard" particles will always be much smaller than the density of particles in the plasma. If hard-soft scattering at $T = T_R$ can still produce χ pairs, it will certainly dominate over hard-hard production of χ's. On the other hand, it is also possible that $T_0 > T_{\max}$, in which case hard-soft scattering (and soft-soft scattering) does not produce any χ particles.

Note that the rate of χ production from hard-hard scattering is quadratic in the density of hard particles. We can no longer use our earlier approximate solution of the Boltzmann equation in terms of the density $\overline{n_h}$ of hard particles produced in the time interval from t to $2t$, since the actual density $n_h(t)$ at any given time will be much smaller than this. Let us first consider the era after thermalization, where a plasma with temperature T already exists. As noted earlier, a hard particle will then only survive for a time $\sim 1/\Gamma_{\mathrm{slow}}$, see Eq. (13). During that era, i.e. for $T_{\max} > T > T_R$, the production of hard particles from inflaton decays and their slow-down will be in equilibrium, i.e. the instantaneous density $n_h(t) = 2\Gamma_d n_\phi(t)/\Gamma_{\mathrm{slow}}$, where n_ϕ is the density of inflatons. This latter quantity is given by $2\Gamma_d n_\phi \sim H g_* T^4/(3m_\phi)$, where we have made use of the fact that T does not change too much over one Hubble time. This leads to a physical χ density at $T = T_R$, for χ particles produced during one Hubble time after thermalization, of order

$$n_\chi^{\mathrm{hh}}(T_R) \sim \frac{2000 T^2 T_R^6 \sigma_{\chi\chi}}{\alpha^6 M_{\mathrm{Planck}} m_\phi^2} \left(\frac{g_*}{200} \right)^{11/6}. \tag{22}$$

We make the following ansatz for the χ production cross section:

$$\sigma_{\chi\chi} \sim \frac{\alpha_\chi^2}{m_\phi^2} + \min\left[\frac{\alpha \alpha_\chi^2}{m_\chi^2},\ \alpha_\chi^2 \alpha n_{\mathrm{plasma}}^{-2/3} \right]. \tag{23}$$

The first term is again the perturbative $2 \to 2$ production cross sections, whereas the second term cuts off the t-channel propagator at the appropriate power of the density of relativistic particles. This second term is needed since Eq. (22) shows that χ production from hard-hard scattering is dominated by *early* times, i.e. *high* temperatures. In particular, $T > m_\chi$ is possible, in which case no propagator should be allowed to be as large as $1/m_\chi^2$.

The fact that the highest temperatures give the biggest contribution in Eq. (22) raises the question of what happened before thermalization. In this epoch the hard particles by definition did not have time to slow down appreciably (except by redshifting). Moreover, the co-moving inflaton density remained essentially constant during that period. Hence now $n_h(t) \simeq 2\Gamma_d (t - t_I) n_\phi(t)$, with $n_\phi(t) = n_\phi(t_I)(t_I/t)^2 \simeq n_\phi(T_R)/(t\Gamma_d)^2 \simeq 1/t^2 \cdot M_{\mathrm{Planck}}^2/(6m_\phi)$. In the last step we have used $n_\phi(T_R) \simeq g_* T_R^4/(3m_\phi)$ and $\Gamma_d \simeq \sqrt{g_*} T_R^2 / M_{\mathrm{Planck}}$. Introducing $X_\chi = t^2 n_\chi$ (so that X_χ is not affected by the Hubble expansion), the production of χ particles before thermalization is described by

$$\frac{dX_\chi}{dt} \simeq \sigma_{\chi\chi} \frac{g_* M_{\mathrm{Planck}}^2 T_R^4}{m_\phi^2} \cdot \frac{(t - t_I)^2}{t^2}, \tag{24}$$

where the factor T_R^4 comes from the factor Γ_d^2 contained in $n_h^2(t)$. For $t \gg t_I$, X_χ will thus grow linearly with t, which means that the physical density $n_\chi \propto 1/t$. Of course, this behavior persists only until the onset of thermalization, i.e. for $t \lesssim 1/\Gamma_{\mathrm{in}}$, see Eq. (5). Using Eq. (6) together with the requirement $T_{\max} \lesssim m_\phi/2$, it is easy to see that the solution to Eq. (24) at $t = 1/\Gamma_{\mathrm{in}}$ always exceeds the result (22). In other words, χ production through hard-hard scattering is most efficient *before* thermalization. This is perhaps not so surprising, since during this early epoch, the hard particles have a higher physical density (once $t \gg t_I$) and survive longer than after thermalization has occurred. Including the redshift from $T_{\max}$ to T_R and using Eq. (17) we arrive at our final estimate

$$\Omega_\chi^{\mathrm{hh}} h^2 \sim 6 \times 10^{27} \cdot \left(\frac{g_*}{200} \right)^{1/2} \sigma_{\chi\chi} \frac{m_\chi T_R^7}{m_\phi^2 T_{\max}^4}, \tag{25}$$

where $\sigma_{\chi\chi}$ is given by Eq. (23) with $n_{\mathrm{plasma}} \sim g_* T_{\max}^4/(3m_\phi)$. We have checked that for m_ϕ near 10^{13} GeV the χ density from hard-hard scattering always stays sufficiently small for χ annihilation reactions to be negligible. It is also easy to see that for $T_0 < T_R$ the contribution (25) is less than the hard-soft contribution (18). However, the hard-hard contribution will exceed the hard-soft contribution (19) if $T_0 > T_R$ and $m_\chi \gtrsim 0.1 (m_\phi^5 T_R^2 M_{\mathrm{Planck}})^{1/8}$; this could indeed be the case if m_χ is rather close to m_ϕ but well above T_R.

C. Numerical examples

As is well known, any stable particle must satisfy $\Omega_\chi < 1$, since otherwise it would "overclose" the Universe. However, in some cases other considerations give stronger constraints on the abundance of χ. This can happen, for example, for unstable massive particles whose lifetime τ_χ is comparable to the age of the Universe τ_U. Radiative decays of such relics which take place after recombination (i.e. at $t \geqslant 10^{13}$ s) are tightly constrained by astrophysical bounds on the gamma-ray background [39]. The tightest bound arises for decays around the present epoch [39]:

$$\Omega_\chi h^2 \leqslant 10^{-8}. \tag{26}$$

Another interesting example is that of charged stable particles whose abundance is severely constrained from searches for exotic isotopes in sea water [24]. The most stringent bound on the abundance of such particles with electric charge -1 is derived for masses $m_\chi \simeq 100 - 10000$ GeV [24]:

$$\Omega_\chi h^2 \leqslant 10^{-20}; \tag{27}$$

for heavier particles this bound becomes weaker.

Having mentioned the different cosmological and astrophysical constraints on long-lived or stable massive particles, in Fig. 2 we present three numerical examples to compare the significance of hard-soft and hard-hard scatterings with that of soft-soft scatterings. We plot $\Omega_\chi h^2$ as a function of m_χ for $\alpha_\chi = 0.01$ and $\alpha = 0.05$. The parameters (T_R, m_ϕ) are chosen as $(10^8$ GeV$, 10^{13}$ GeV$)$ (a), $(10^5$ GeV$, 10^{13}$ GeV$)$ (b) and $(3$ MeV$, 10^8$ GeV$)$ (c), respectively.

In Fig. 2a, $T_{max} \simeq 2 \times 10^{10}$ GeV; this explains the very rapid fall of soft-soft contributions (which are never in equilibrium for the range of m_χ shown) for $m_\chi > 3 \times 10^{10}$ GeV. We have multiplied the result of Eq. (8) with $\exp(-2m_\chi/T_{max}) \cdot \exp(2)$ if $m_\chi > T_{max}$, since this contribution needs two factors of the "soft" particle density. The exact form of this exponential cutoff is debatable, but it is clear that the soft-soft contribution should decrease exponentially, and hence quickly become irrelevant, once $m_\chi > T_{max}$. In the present case, the soft-soft contribution is overwhelmed by the hard-soft one even prior to this sharp decrease (which makes the exact form of the cutoff irrelevant). This is because $T_0 < T_R$ as long as $m_\chi < 1.1 \times 10^{10}$ GeV, in which case hard-soft scatterings which occur at the very last stage of inflaton decay are the dominant source of χ production. However, the hard-soft contribution itself drops very quickly for $m_\chi > 10^{11}$ GeV, since T_0 exceeds T_{max} in this mass range. We have multiplied Eq. (19) by $\exp(-T_0/T_{max}) \cdot \exp(1)$ if $T_0 > T_{max}$. Once again the exact form of this cutoff is not very important, since the hard-hard contribution becomes dominant above this point until it is kinematically suppressed, and then forbidden, for $m_\chi \approx 5 \times 10^{12}$ GeV. It is interesting to note that hard-hard scatterings can very efficiently produce "wimpzillas" up to this kinematical cutoff. In fact, for this choice of T_R, m_ϕ and α_χ, the overclosure bound (for a strictly stable χ) and the astrophysical bound (for a late decaying χ) require that m_χ must be bigger than $m_\phi/2$, otherwise hard-hard scatterings will produce χ in unacceptably large abundances. Recall, however, Eq. (6) and Eq. (25) which show that $\Omega_\chi^{hh} \propto T_R^4$ for fixed m_ϕ; reducing the reheat temperature by a factor of 2

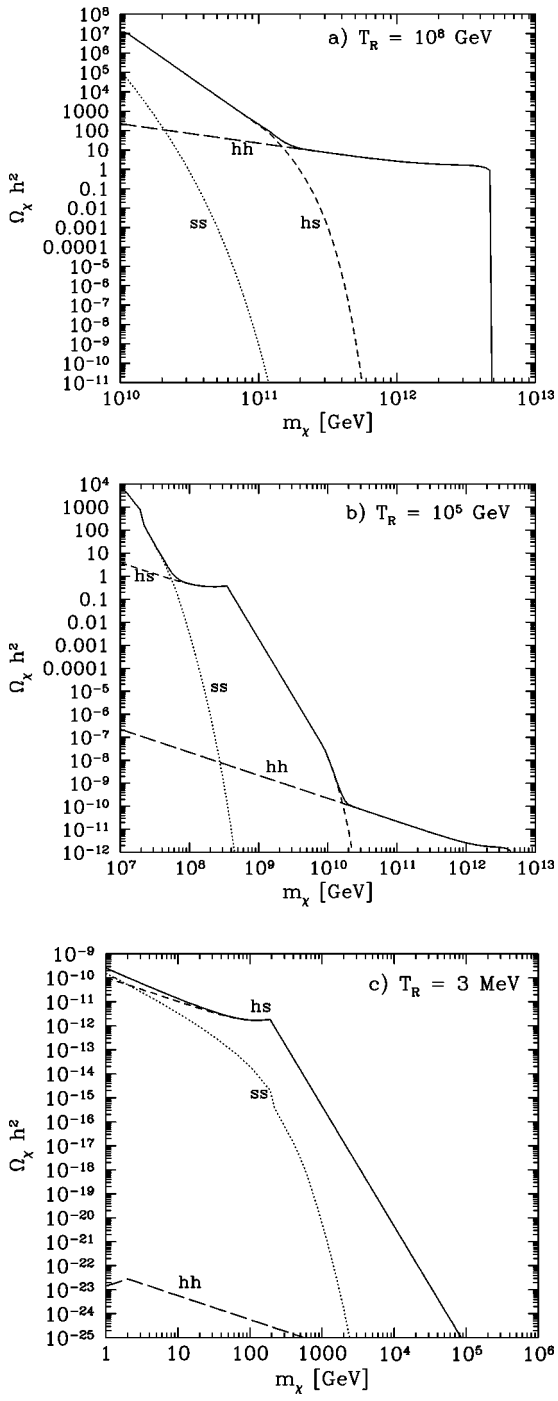

FIG. 2. The relic density χ particles would currently have if they are absolutely stable is shown as a function of m_χ for couplings $\alpha = 0.05$, $\alpha_\chi = 0.01$, and (a) $(T_R, m_\phi) = (10^8$ GeV$, 10^{13}$ GeV$)$, (b) $(10^5$ GeV$, 10^{13}$ GeV$)$, (c) $(3$ MeV$, 10^8$ GeV$)$. The soft-soft, hard-soft and hard-hard contributions are shown by the dotted, short dashed and long dashed curves, respectively, while the solid curves show the sum of all three contributions.

would therefore result in an acceptable χ relic density if $T_0 > T_{\max}$. In contrast, even without the exponential cutoff, the soft-soft contribution would have satisfied the overclosure bound for $m_\chi > 5\times10^{10}$ GeV, i.e. for the parameters of Fig. 2(a) the hard-hard contribution raises the bound on m_χ by two orders of magnitude.

The lower reheat temperature in Fig. 2(b) leads to a lower value of $T_{\max}\simeq 6\times10^7$ GeV. In this case soft-soft production of χ pairs actually was in equilibrium for $m_\chi < 2\times10^7$ GeV, but this contribution again falls very sharply once $m_\chi > T_{\max}$, where the hard-soft contribution becomes dominant. A kink in the hard-soft contribution appears at $m_\chi \simeq 4\times10^8$ GeV, where T_0 starts exceeding T_R,[16] The curve flattens out just before the kink since here the $2\to2$ contribution to the production cross section become important. The hard-soft contribution decays rapidly for $m_\chi > 8\times10^9$ GeV, where $T_0 > T_{\max}$. Then the hard-hard contribution naturally takes over and quickly becomes the only source until the kinematical cutoff at 5×10^{12} GeV. The overclosure bound now requires $m_\chi \geqslant 10^8$ GeV (by a funny coincidence the soft-soft and hard-soft contributions are comparable around this lower limit). On the other hand, for an unstable χ the astrophysical bound in Eq. (26) requires $T_0 > T_{\max}$, i.e. $m_\chi > 10^{10}$ GeV, if $\tau_\chi \sim \tau_U$ (hard-soft and hard-hard contributions are again coincidentally comparable around this limit). Notice that the hard-soft contribution has increased this limit by about a factor of 10. It is also observed that now the hard-hard contribution does not suffice to make χ an interesting dark matter particle for any value of m_χ. This again follows from the T_R^4 behavior of this contribution.

Finally, Fig. 2(c) displays the results for $T_R = 3$ MeV, near the lower limit required for successful nucleosynthesis; we also chose a smaller inflaton mass $m_\phi = 10^8$ GeV so that this very low reheat temperature will be achieved more naturally. In this case $T_{\max}\simeq 600$ GeV. A main difference from the previous cases is that now the soft-soft contribution is very small even for masses as low as $m_\chi \simeq 1$ GeV. The reason is that the abundance of χ particles, produced from soft-soft scatterings, is determined by their annihilation which is at equilibrium in this case if $m_\chi \leqslant 200$ GeV. As a result the soft-soft contribution is subdominant for almost all values of m_χ; as usual, it cuts off sharply once $m_\chi > T_{\max}$. Again a kink in the curve depicting the hard-soft contribution is recognized at $m_\chi \simeq 200$ GeV, above which $T_0 > T_R$. Even the hard-soft contribution essentially vanishes for $m_\chi > 10^5$ GeV, where T_0 exceeds $T_{\max}$. The hard-hard contribution is now completely negligible since T_R is very small. The only meaningful constraint in this case is that from Eq. (27). It is seen that the hard-soft contribution to χ production again increases the lower bound on m_χ by about one order of magnitude, to about 10^4 GeV, compared to the bound derived in Ref. [24], which only includes the soft-soft contribution. Naturalness arguments indicate that the mass of the lightest superparticle (LSP) should not exceed 1 TeV; this argument is independent of whether or not the LSP is charged. The constraint of Fig. 1(c) is not compatible with this bound. Indeed, we find that once the hard-soft contribution is included, even for $T_R = 1$ MeV, $m_\chi = 1$ TeV is compatible with the bound (27) only if $m_\phi \leqslant 5\times10^5$ GeV. This indicates that scenarios with a charged LSP would require a model of low scale inflation.

One comment is in order before moving on to the next subsection. The production of χ particles from soft-soft scatterings is independent of the main channel through which inflaton decays, so long as the decay products thermalize sufficiently rapidly (i.e. such that $m_\chi < T_{\max}$). For example, assume that (for some reason) the inflaton mostly decays to leptons while χ can only be produced from the scattering of quarks. In this case Ω_χ^{ss} remains essentially unchanged since electroweak gauge interactions bring quarks into equilibrium with a thermal bath created by leptons. On the other hand, Ω_χ^{hs} and Ω_χ^{hh} will be suppressed since a hard lepton cannot produce χ from $2\to2$ scattering off soft particles in the thermal bath and/or off another hard lepton. In this scenario χ production from hard-soft or hard-hard processes can still occur via $2\to4$ scattering reactions; if $q\bar{q}\to\chi\bar{\chi}$ is allowed,[17] so are $lq\to lq\bar{\chi}\chi$ and $l^+l^-\to q\bar{q}\bar{\chi}\chi$. The corresponding contributions Ω_χ^{hs} and Ω_χ^{hh} will then be smaller than our estimates in Eqs. (18), (19) and (25) by several orders of magnitude. This possibility may be relevant to the production of exotic electroweak gauge singlet particles with large masses, thus relaxing constraints on the model parameters in that case. However, the estimates for the LSP and charged stable particle examples will remain unaffected since these species can be produced from $2\to2$ scatterings of all standard model particles.

D. Particle production from inflaton decay

We now discuss the direct production of χ particles in inflaton decay whose importance has recently been noticed [40]. Most inflatons decay at $T\simeq T_R$; moreover, the density of χ particles produced in earlier inflaton decays will be greatly diluted. Since inflaton decay conserves energy, the density of inflatons can be estimated as $n_\phi \simeq 0.3 g_* T_R^4/m_\phi$. Let us denote the branching ratio for $\phi\to\chi$ decays (more accurately, the average number of χ particles produced in each ϕ decay) by $B(\phi\to\chi)$. The χ density from ϕ decay can then be estimated as [25]

$$\Omega_\chi^{\rm decay} h^2 \simeq 2\times10^8 B(\phi\to\chi)\frac{m_\chi}{m_\phi}\frac{T_R}{1\ \text{GeV}}. \tag{28}$$

Equation (28) holds if the χ annihilation rate is smaller than the Hubble expansion rate at $T\simeq T_R$, which requires

$$\frac{m_\phi}{M_{\rm Planck}} > 5B(\phi\to\chi)\alpha_\chi^2\left(\frac{T_R}{m_\chi}\right)^2\left(\frac{g_*}{200}\right)^{1/2}. \tag{29}$$

[16] This kink is also barely visible in Fig. 2(a) at $m_\chi \simeq 10^{10}$ GeV.

[17] $\chi = \bar{\chi}$ if χ is a Majorana particle.

This condition will be satisfied in chaotic inflation models with $m_\phi \sim 10^{-5} M_{\rm Planck}$, if m_χ is large enough to avoid overclosure from thermal χ production alone. It might be violated in models with a light inflaton. In that case the true χ density at $T_{\rm R}$ can be estimated by equating the annihilation rate with the expansion rate:

$$\Omega_\chi^{\max} \simeq \frac{5\times 10^7}{\alpha_\chi^2} \frac{m_\chi^3}{(1\ \ {\rm GeV})\cdot M_{\rm Planck} T_{\rm R}} \left(\frac{200}{g_*}\right)^{1/2}. \quad (30)$$

This maximal density violates the overclosure constraint $\Omega_\chi < 1$ badly for the kind of weakly interacting ($\alpha_\chi \lesssim 0.1$), massive ($m_\chi \gg T_{\rm R}$ and $m_\chi \gtrsim 1$ TeV) particles we are interested in.[18] For the remainder of this article we will therefore estimate the χ density from inflaton decay using Eq. (28).

We now discuss estimates[19] of $B(\phi \to \chi)$. This quantity is obviously model dependent, so we have to investigate several scenarios. The first, important special case is where χ is the LSP. If m_ϕ is large compared to typical visible-sector superparticle masses, ϕ will decay into particles and superparticles with approximately equal probability.[20] This can be illustrated for two possible cases: when the main inflaton decay mode is via a superpotential coupling, and for a gravitationally decaying inflaton. In the first case consider the inflaton superfield Φ, comprising the inflaton ϕ and its superpartner inflatino $\tilde{\phi}$, and a chiral supermultiplet Ψ, which comprises a complex scalar field $\tilde{\phi}$ and its fermionic partner ψ, with the superpotential coupling

$$W \supset \frac{1}{2} m_\phi \Phi^2 + \frac{1}{2} h_\chi \Phi \Psi^2. \quad (31)$$

This superpotential generates the following terms:

$$h_\chi \phi \bar{\psi} \psi \quad ; \quad h_\chi m_\phi \phi^* \tilde{\psi} \tilde{\psi}, \quad (32)$$

in the Lagrangian. It is easily verified that ϕ decays to both of ψ and $\tilde{\phi}$ (which in turn decay to matter particles plus the LSP) at a rate given by $\Gamma = (h_\chi^2/8\pi) m_\phi$.

Now let us turn to the case of a gravitationally decaying inflaton. As a simple example, consider minimal supergravity where the scalar potential is given by [47]

$$V = e^G \left(\frac{\partial G}{\partial \varphi_i} \frac{\partial G}{\partial \varphi_i^*} - \frac{3}{M_{\rm Planck}^2} \right) M_{\rm Planck}^6. \quad (33)$$

Here G is the Kähler function defined by

$$G = \frac{\varphi_i \varphi_i^*}{M_{\rm Planck}^2} + \log\left(\frac{|W|^2}{M_{\rm Planck}^6} \right), \quad (34)$$

where φ_i are the scalar fields in the theory. The inflation sector superpotential looks like $W \sim (1/2) m_\phi (\Phi - v)^2$ around the minimum of the potential where v denotes the inflaton VEV at the minimum.[21] The superpotential also includes the familiar Yukawa couplings for the matter sector, e.g. $h_u {\rm H}_u {\rm Qu}$ where Q and u denote the superfields containing the doublet of left-handed and the singlet of right-handed (s)quarks, respectively, and $\rm H_u$ is the superfield which contains the Higgs doublet giving mass to the up-type quarks. Then it is easy to see that Eq. (33) leads to the following term (among others):

$$h_u \frac{v m_\phi}{M_{\rm Planck}^2} \phi^* H_u \tilde{Q} \tilde{u} \quad (35)$$

in the Lagrangian for inflaton decay to three scalars, including two superparticles.[22] This implies that the rate for production of particles and sparticles through inflaton decays into three light scalars is approximately the same.

Once one goes beyond minimal supergravity, the inflaton can also decay to gauge fields and gauginos. Consider the case where the gauge superfields have nonminimal kinetic terms, as a result of $M_{\rm Planck}$ suppressed couplings to the inflaton, in the following form:

$$f_{\alpha\beta} = \left[1 + a\left(\frac{\phi}{M_{\rm Planck}}\right) + b\left(\frac{\phi}{M_{\rm Planck}}\right)^2 + \ldots \right] \delta_{\alpha\beta}. \quad (36)$$

Then to the leading order one finds the term

$$a \frac{\phi}{M_{\rm Planck}} F^\alpha_{\mu\nu} F^{\mu\nu,\alpha}, \quad (37)$$

for the inflaton coupling to gauge fields, where α represents the relevant gauge group index. This results in inflaton decay to a pair of gauge quanta at the rate $\Gamma \sim a^2 m_\phi^3 / M_{\rm Planck}^2$. The corresponding term from the kinetic energy of the gauginos

[18] Equation (30) describes the maximal χ density if χ decouples at $T \sim T_R$. It is not applicable to WIMPs decoupling at $T < T_{\rm R}$.

[19] In Ref. [25] simultaneous production of χ from the thermal bath and from direct inflaton decay has been studied and a bound on the branching ratio for such a decay mode is derived. Our aim here is to show that higher order processes naturally provide a decay mode, even if χ is not directly coupled to the inflaton.

[20] The nonthermal production of LSP has also been considered in other contexts. For a neutralino LSP these include its production from the decay of Q-balls [41], in cosmic string decays as a possible solution to the observational conflicts of WIMP cold dark matter [42], in decay of the Polonyi field [43], and in moduli decay in connection with models of anomaly-mediated supersymmetry breaking [44] and models with intermediate scale unification [45]. Nonthermal production of axino LSP, as dark matter, from neutralino decay has been considered in [46].

[21] In the absence of any superpotential coupling to other multiplets which provides a linear term in ϕ, a nonzero v is required for the inflaton decay to take place.

[22] The main mode for decay to three scalars is to H_u and two squarks of the third generation, because of the large top Yukawa coupling.

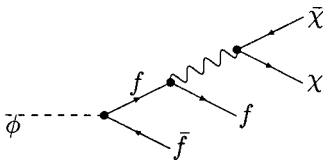

FIG. 3. Sample diagram for χ production in four-body inflaton decay.

results in a derivative coupling and hence a decay rate which is suppressed by a factor $m_{\tilde{g}}^2/m_\phi^2$, where $m_{\tilde{g}}$ is the gaugino mass [44]. However, there exists another term in the Lagrangian [47] which is responsible for gaugino mass from supersymmetry breaking by the inflaton energy density:

$$F_\phi \frac{\partial f^*_{\alpha\beta}}{\partial \phi^*} \lambda^\alpha \lambda^\beta, \tag{38}$$

where F_ϕ is the F term associated with the inflaton superfield, given by $\simeq m_\phi \phi$. It is easy to see that the term in (38), to the leading order, results in inflaton decay to two gauginos at the rate $\Gamma \sim a^2 m_\phi^3/M_{\rm Planck}^2$, which is comparable to that for inflaton decay to two gauge quanta.[23]

Moreover, all superparticles will quickly decay into the LSP and some standard particle(s). As long as $m_\chi > T_{\rm R}$, the time scale for these decays will be shorter than the superparticle annihilation time scale even if $\alpha_\chi \simeq 0.1$. As a result, if χ is the LSP, then $B(\phi \to \chi) \simeq 1$, independently of the nature of the LSP.

Another possibility is that the inflaton couples to all particles with more or less equal strength, e.g. through non-renormalizable interactions. In that case one expects $B(\phi \to \chi) \sim 1/g_* \sim 1/200$. However, even if ϕ has no direct couplings to χ, the rate (28) can be large. The key observation is that χ can be produced in ϕ decays that occur in higher order in perturbation theory whenever χ can be produced from annihilation of particles in the thermal plasma. In most realistic cases, $\phi \to f\bar{f}\chi\bar{\chi}$ decays will be possible if χ has gauge interactions, where f stands for some gauge nonsinglet with tree-level coupling to ϕ. A diagram contributing to this decay is shown in Fig. 3. Note that the part of the diagram describing $\chi\bar{\chi}$ production is identical to the diagram describing $\chi\bar{\chi} \leftrightarrow f\bar{f}$ transitions. This leads to the following estimate:

$$B(\phi \to \chi)_4 \sim \frac{C_4 \alpha_\chi^2}{96\pi^3} \left(1 - \frac{4m_\chi^2}{m_\phi^2}\right)^2 \left(1 - \frac{2m_\chi}{m_\phi}\right)^{5/2}, \tag{39}$$

where C_4 is a multiplicity (color) factor. The phase space factors have been written in a fashion that reproduces the correct behavior for $m_\chi \to m_\phi/2$ as well as for $m_\chi \to 0$. This estimate provides a lower bound on $B(\phi \to \chi)$ under the conditions assumed for our calculation of $\Omega_\chi^{\rm hs}$ and $\Omega_\chi^{\rm hh}$; whenever a primary inflaton decay product can interact with a particle in the thermal plasma, or with another primary decay product, to produce a $\chi\bar{\chi}$ pair, $\phi \to \chi$ four-body decays must exist. It is easy to see that the contribution (28) to the χ density from inflaton decay will exceed the hard-hard contribution (25), if $T_{\rm max}$ can be estimated from Eq. (6). If $T_0 > T_{\rm R}$, the decay contribution will exceed the hard-soft contribution if $m_\chi^6 > 0.3 m_\phi^2 T_{\rm R}^4/\alpha^3$, which is true in almost all models that avoid overclosure from thermal χ production alone. Even if $T_0 < T_{\rm R}$, the decay contribution will dominate over the hard-soft contribution if

$$\alpha^2 > 150\left[\left(\frac{T_{\rm R}}{m_\chi}\right)^2 + \frac{T_R}{\alpha m_\phi}\right], \tag{40}$$

where the first and second term in the square brackets describe χ pair production from $2 \to 3$ and $2 \to 2$ processes, respectively; see Eq. (14). This condition can be mildly violated, i.e. in some cases the hard-soft contribution may exceed the decay contribution. For example, in the scenarios considered in Fig. 2, as long as $m_\chi \ll m_\phi$ we find $\Omega_\chi^{\rm decay} \sim 6 C_4 \times 10^5 m_\chi/(10^{10}\ {\rm GeV})$ in (a); $\sim 0.6 C_4 m_\chi/(10^7\ {\rm GeV})$ in (b); and $\sim 2 \times 10^{-10} C_4 m_\chi/{\rm GeV}$ in (c). For $C_4 = 1$, there is a narrow range of m_χ in Figs. 2(a) and 2(b) where $\Omega_\chi^{\rm decay} < \Omega_\chi^{\rm hs}$; however, in this range Ω_χ exceeds the upper bound of 1 significantly. We thus conclude that the decay contribution to Ω_χ will usually dominate over nonthermal χ production from inflaton decay products if four-body $\phi \to \chi$ decays exist.

Occasionally one has to go to even higher order in perturbation theory to produce χ particles from ϕ decays. For example, if χ has only strong interactions but ϕ only couples to $SU(3)$ singlets, $\chi\bar{\chi}$ pairs can only be produced in six body final states, $\phi \to f\bar{f}q\bar{q}\chi\bar{\chi}$. A representative diagram can be obtained from the one shown in Fig. 3 by replacing the χ lines by quark lines, attaching an additional virtual gluon to one of the quarks which finally splits into $\chi\bar{\chi}$. The branching ratio for such six body decays can be estimated as

$$B(\phi \to \chi)_6 \sim \frac{C_6 \alpha_\chi^2 \alpha^2}{1.1 \times 10^7} \left(1 - \frac{4m_\chi^2}{m_\phi^2}\right)^4 \left(1 - \frac{2m_\chi}{m_\phi}\right)^{9/2}. \tag{41}$$

Another example where $\chi\bar{\chi}$ pairs can only be produced in ϕ decays into six body final states occurs if the inflaton only couples to fields that are singlets under the SM gauge group, e.g. right-handed (s)neutrinos ν_R [48]. These (s)neutrinos can emit a virtual Higgs boson, which can split into a top quark-antiquark pair; one of which can emit a virtual gluon, which in turn splits into a (strongly interacting) $\chi\bar{\chi}$ pair. In this scenario the factor α^2 in Eq. (41) would have to be replaced by the combination of Yukawa couplings $h_{\nu_R}^2 h_t^2/(16\pi^2)$. If $2m_\chi < m_{\nu_R}$, $\chi\bar{\chi}$ pairs can already be produced in four body final states from ν_R decay. The effective $\phi \to \chi$ branching

[23]Note that moduli decay to gauginos through the term in (38) will be suppressed [44]. The reason is that the superpotential is (at most) linear in the moduli superfield, so that the corresponding F term does not contain the moduli field. In contrast, in almost all inflationary models the dependence of the superpotential on the inflaton superfield is quadratic or higher.

ratio would then again be given by Eq. (39), with m_ϕ replaced by m_{ν_R} in the kinematical factors.

Finally, in supergravity models with an explicit (supersymmetric) χ mass term there in general exists a coupling between ϕ and either χ itself or, for fermionic χ, to its scalar superpartner, of the form $a(m_\phi m_\chi / M_{\rm Planck})\phi^* \chi\chi + {\rm H.c.}$ in the scalar potential;[24] this term is completely analogous to the one shown in (35).[25] A reasonable estimate for the coupling strength is [49] $a \sim v/M_{\rm Planck}$, unless an R symmetry suppresses a. Assuming that most inflatons decay into other channels, so that $\Gamma_{\rm decay} \sim \sqrt{g_*} T_{\rm R}^2 / M_{\rm Planck}$ remains valid, this gives

$$B(\phi \to \chi) \sim \frac{a^2 m_\chi^2 m_\phi}{16\pi \sqrt{g_*} M_{\rm Planck} T_{\rm R}^2} \left(1 - \frac{4m_\chi^2}{m_\phi^2}\right)^{1/2}. \quad (42)$$

The production of χ particles from inflaton decay will be important for large m_χ and large ratio $m_\chi / T_{\rm R}$, but tends to become less relevant for large ratio m_ϕ / m_χ. Even if $m_\chi < T_{\rm max}$, χ production from the thermal plasma (8) will be subdominant if

$$\frac{B(\phi \to \chi)}{\alpha_\chi^2} > \left(\frac{100 T_{\rm R}}{m_\chi}\right)^6 \frac{m_\phi}{m_\chi} \frac{1\ {\rm TeV}}{m_\chi}. \quad (43)$$

The first factor on the r.h.s. of (43) must be $\lesssim 10^{-6}$ in order to avoid over-production of χ from thermal sources alone. Even if $\phi \to \chi$ decays only occur in higher orders of perturbation theory, the l.h.s. of (43) will be of order 10^{-4} (10^{-10}) for four (six) body final states, see Eqs. (39),(41); if two-body tree-level $\phi \to \chi\bar{\chi}$ decays are allowed, the l.h.s. of (43) will usually be bigger than unity. We thus see that even for $m_\phi \sim 10^{13}$ GeV, as in chaotic inflation models, and for $m_\chi \simeq 10^3 T_{\rm R}$, χ production from decay will dominate if $m_\chi \gtrsim 10^7 (10^{10})$ GeV for four (six) body final states; this agrees with the numerical results shown in Fig. 2. As a second example, consider LSP production in models with very low reheat temperature. Naturalness arguments imply that the LSP mass should lie within a factor of 5 or so of 200 GeV. Recall that in this case $B(\phi \to \chi) = 1$. Taking $\alpha_\chi \sim 0.01$, we see that χ production from decay will dominate over production from the thermal plasma if $m_\phi < 6 \times 10^7$ GeV for $T_{\rm R} = 1$ GeV; this statement will be true for all $m_\phi \lesssim 10^{13}$ GeV if $T_{\rm R} \lesssim 100$ MeV.

In [40] we showed that the decay contribution (28) by itself leads to very stringent constraints on models with massive stable χ particles. In particular, charged stable particles with mass below ~ 100 TeV seem to be excluded, unless $m_\chi > m_\phi/2$. In case of a (neutral) LSP with mass around 200 GeV, the overclosure constraint implies $m_\phi / T_{\rm R} > 4 \times 10^{10}$, i.e. a very low reheat temperature, unless χ was in thermal equilibrium below T_R; recall that $B(\phi \to \chi) = 1$ in this case. Finally, if $m_\phi \sim 10^{13}$ GeV a "wimpzilla" with mass $m_\chi \sim 10^{12}$ GeV will be a good dark matter candidate only if it has a very low branching ratio, $B(\phi \to \chi) \sim 5 \times 10^{-8}$ GeV$/T_{\rm R}$, i.e. if its couplings to ordinary matter are very small.

Our calculation is also applicable to entropy-producing particle decays that might occur at very late times. If χ is lighter than this additional ϕ' particle all our expressions go through with the obvious replacement $\phi \to \phi'$ everywhere. More generally our result holds if ϕ decays result in a radiation-dominated era with $T_{\rm R} > m_{\phi'}$. If ϕ' is sufficiently long-lived, the Universe will eventually enter a second matter-dominated epoch. ϕ' decays then give rise to a second epoch of reheating, leading to a radiation-dominated Universe with final reheating temperature $T_{\rm R_f}$, and increasing the entropy by a factor $m_{\phi'} / T_{\rm R_f}$. This could be incorporated into Eq. (28) by replacing $T_{\rm R} \to T_{\rm R} T_{\rm R_f} / m_{\phi'} > T_{\rm R_f}$.

In a similar vein, consider the production of a neutral LSP from gravitino decay in gravity-mediated models of supersymmetry breaking; note that in this case the decaying particle does not dominate the energy density of the Universe. Gravitinos with mass $m_{3/2} \sim 100$ GeV-3 TeV have a lifetime $\tau > 1$ s, which can ruin the success of nucleosynthesis if gravitinos are produced in large abundances [4]. This, as is well known, leads to constraints on the reheating temperature of the Universe $T_{\rm R}$ [3]. On the other hand, for $m_{3/2} > 10$ TeV the gravitino decays before nucleosynthesis and has no effect on the light element abundances.[26] However, even in this case a significant upper limit on $T_{\rm R}$ can be derived from the following argument. If the LSP has a mass $\geqslant 100$ GeV, gravitinos (as heavy as 10^6 GeV) decay much after the freeze-out of LSP annihilation. The overclosure bound then results in a constraint on the gravitino number to entropy ratio, $n_{3/2}/s \leqslant 4 \times 10^{-11}$, even when the gravitinos decay before the onset of nucleosynthesis. For thermal gravitino production, where $n_{3/2}/s \simeq 10^{-11} (T_{\rm R}/10^{11}\ {\rm GeV})$ [3], this results in the limit $T_{\rm R} \leqslant 10^{11}$ GeV; including possible nonthermal production of gravitinos [6] will presumably sharpen this limit.

Finally, let us point out possible implications of deviating from two major assumptions which we made throughout this article: a significant branching ratio of primary inflaton decays into known SM particles (possibly including their superpartners), and allowed (pair-)production of χ particles in the scattering of matter particles. First, consider the case when the inflaton exclusively decays to exotic light particles while χ is produced through its coupling to matter particles. Assume that these exotic particles only couple with strength $\alpha' \ll \alpha$ to SM particles. This results in a smaller $T_{\rm max}$; depending on the details of the model, one has to replace the factor α^3 in Eq. (6) either by $\alpha\alpha'^2$ or by α'^2. We consider the latter possibility and assume for simplicity that $T_{\rm R}$ re-

[24] This term also induces an A term from supersymmetry breaking by the inflaton energy density.

[25] Note that a superpotential mass term for the χ multiplet will be allowed under a Z_2 discrete symmetry. For a continuous symmetry one requires two multiplets χ_1 and χ_2 with opposite charges.

[26] A gravitino mass which is much larger than superparticle masses can be naturally found, for example, in models of no-scale supergravity [50].

mains unchanged. The thermal contribution $\Omega_\chi^{\rm ss}$ will be significantly reduced if the new value of $T_{\rm max}$ is below m_χ by more than one order of magnitude or so. We therefore require that $T_{\rm max} \simeq m_\chi$. Let us find $\Omega_\chi^{\rm decay}$ by assuming that $\Omega_\chi^{\rm ss} \simeq 1$. The main difference is that χ production from inflaton decay now occurs through six body final state diagrams, with α^2 in Eq. (41) replaced by α'^2. The requirement that $T_{\rm max} \geqslant m_\chi$ results in a bound on α'^2, see Eq. (6). For $m_\chi = 2\times 10^3 \alpha_\chi^{2/7} T_{\rm R}$, which saturates $\Omega_\chi^{\rm ss}$, and after using the bound on α'^2 in Eq. (28), we find

$$\Omega_\chi^{\rm decay} \gtrsim 5\times 10^3 \alpha_\chi^{32/21} \frac{m_\chi^{5/3}}{m_\phi^{2/3} M_{\rm Planck}} \frac{m_\chi}{1\ {\rm GeV}}, \qquad (44)$$

for χ production in six body decay of the inflaton. Even for the most conservative choice, $m_\phi \simeq 10^{13}$ GeV, Eq. (44) requires $m_\chi < 7\times 10^9$ GeV for $\alpha_\chi \sim 0.01$. Note that χ production from hard-soft and hard-hard scatterings also only occurs through higher order processes, thus $\Omega_\chi^{\rm hs}$ and $\Omega_\chi^{\rm hh}$ are suppressed by a few orders of magnitude compared to expressions (18), (19) and (25). Moreover, $T_{\rm max} \ll m_\chi$ if α' is very small, implying that $\Omega_\chi^{\rm ss}$ will be exponentially suppressed. In this case χ production from inflaton decay will dominate, since it is only suppressed by a factor α'^2.

Second, consider the case where the inflaton decays to matter but χ particles can only be produced from scatterings of some intermediate particle χ' whose interaction with matter has a strength $\alpha' \ll \alpha$. If α' is not very small χ' will be in thermal equilibrium with matter and, provided $T_{\rm max} > m_\chi$, $\Omega_\chi^{\rm ss}$ will only be reduced by a statistics factor, since now only a small fraction of all soft-soft scatters can produce χ particles. On the other hand, $\Omega_\chi^{\rm hs}$ and $\Omega_\chi^{\rm hh}$ will decrease much more. The hard-soft and hard-hard scatterings now can produce χ's only in four-body final states, e.g. $\bar{f}f \to \bar{\chi}'\chi'\bar{\chi}\chi$, with cross section $\propto \alpha_\chi^2 \alpha'^2$. By using the expression in Eq. (3) a bound on α' is found in order for χ' to be at thermal equilibrium for $T \geqslant m_\chi$, which requires $\alpha'^2 m_\chi \geqslant H$. If we now assume $\Omega_\chi^{\rm ss} \simeq 1$, taking $g_* = 200$, we have

$$\Omega_\chi^{\rm decay} \gtrsim 10^5 \alpha_\chi^{12/7} \frac{m_\chi^2}{m_\phi M_{\rm Planck}} \frac{m_\chi}{1\ {\rm GeV}}, \qquad (45)$$

from six body decay of the inflaton. Again taking $\alpha_\chi = 0.01$ and $m_\phi = 10^{13}$ GeV, this results in the bound $m_\chi \leqslant 10^{10}$ GeV in order not to overclose the Universe. As a closing remark, we shall notice that such a rather contrived scenario might be realized for exotic χ particles (e.g. ''wimpzillas''), but not when χ is the LSP or a charged stable particle, which have electroweak gauge couplings to ordinary matter.

IV. SUMMARY AND CONCLUSIONS

In this article we studied the thermalization of perturbative inflaton decay products, with emphasis on applications to the production of massive stable or long-lived particles χ. We found that a thermal plasma should form well before inflaton decay is complete, if the theory contains light or massless gauge bosons; $2 \to 3$ reactions where a fairly energetic, nearly collinear particle is emitted in the initial or final state play a crucial role here. The existence of light gauge bosons is required since only gauge boson exchange in the t or u channel leads to cross sections that significantly exceed α^2/m_ϕ^2, where α is the relevant (gauge) coupling strength and m_ϕ the inflaton mass. This indicates that reheating might be delayed greatly if some scalar field(s) break all gauge symmetries during this epoch, which may naturally happen in the presence of supersymmetric flat directions.

Even if massless gauge bosons exist, thermalization takes a finite amount of time. As a result, the maximal temperature of the thermal plasma will usually be well below m_ϕ (but can exceed the reheat temperature $T_{\rm R}$ significantly); this limits the region of parameter space where *thermal* χ production can play a role. On the other hand, it allows the very energetic primary inflaton decay products to produce χ particles either in collisions with the thermal plasma (''hard-soft'' scattering), or with each other (''hard-hard'' reactions). We estimated the rate for these reactions in the simple approximation where the primary inflaton decay products have energy $m_\phi/2$ for one thermalization time, and then have energy T. If m_ϕ is rather close to m_χ this will probably overestimate the true rate, since then the energy of the primary decay products will drop below the production threshold faster than in our approximation. On the other hand, if $m_\phi \gg m_\chi$, our approximation will likely be an underestimate, since in the process of thermalization a single particle with $E \sim m_\phi/2$ can produce several (most likely nearly collinear) particles with $m_\phi/2 > E > m_\chi$, all of which can contribute to nonthermal χ production; note that this also allows χ production from the scattering of ''hard'' particles even if the primary inflaton decay products do not couple directly to χ. We found that the hard-soft contribution will dominate over the hard-hard one if it is still kinematically allowed at $T = T_{\rm R}$, but can otherwise be subdominant; either of these two new production mechanisms can exceed the rate from purely thermal χ production.

We also discussed χ production in inflaton decay. We pointed out that decays of this kind must be allowed, at least in four-body final states, if χ particles can be produced in collisions of primary inflaton decay products; in certain (somewhat contrived) scenarios one may have to consider six-body final states. In fact, in most cases this seems to be the most important nonthermal (but perturbative) mechanism producing massive particles with $m_\chi < m_\phi/2$; this contribution also often exceeds thermal χ production by several orders of magnitude, even if m_χ is below the maximal temperature of the thermal plasma. Nonthermal χ production therefore significantly sharpens limits on model parameters that follow from upper bounds on the χ relic density. For example, if m_ϕ is well above visible sector superparticle masses, each inflaton decay will produce the $\mathcal{O}(1)$ lightest superparticle (LSP). If these LSPs were not in equilibrium at $T_{\rm R}$, the bound $\Omega_{\rm LSP} < 1$ then implies that the reheat temperature must be at least ten orders of magnitude below the inflaton mass, i.e. the inflaton decay width must be at least 26 orders of magnitude smaller than m_ϕ. This does not seem to

be very plausible. As is well known, the requirement $\Omega_{\rm LSP} < 1$ imposes severe constraints on the parameter space of many supersymmetric models if the LSP was in equilibrium at T_R [51]. Our analysis indicates that it is quite difficult to evade these constraints by changing the cosmology. Similarly, we found that stable charged particles can only be tolerated if they are too heavy to be produced in inflaton decays.

Many of the results presented in this paper are only semiquantitative. Unfortunately in most cases significant improvements can only be made at great effort. For example, a more accurate treatment of thermalization would require a solution of the Boltzmann equations in the presence of a non-trivial, but non-thermal, background of relativistic particles. Once a thermal plasma has been established, a proper treatment of the slow-down of primary inflaton decay products would require a careful treatment of the full momentum dependence of the particle distribution functions. On the other hand, our estimates of inflaton decay branching ratios should be quite reliable if $m_\chi > T_{\rm R}$ (which is required for χ not to have been in thermal equilibrium at $T_{\rm R}$); even for many-body decays, details of the matrix elements should change our estimates only by $\mathcal{O}(1)$ factors. Fortunately we found that this is often also the most important of the new, nonthermal mechanisms for the production of massive particles at the end of inflation. We therefore conclude that cosmological constraints on models with stable or long-lived massive particles are (much) more severe than had previously been thought.

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