Power-law inflation from the rolling tachyon

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Modeling the potential by an inverse square law in terms of the tachyon field $[V(T)=\beta T^{-2}]$ we find an exact solution for a spatially flat isotropic universe. We show that for $\beta>2\sqrt{3}/3$ the model undergoes power-law inflation. A way to construct other exact solutions is specified and exemplified.

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Several interesting attempts to reconcile the inflationary paradigm with string theories have resulted recently in much work (see, for example, [1]). These efforts, however, did not produce a completely successful union between string theory and cosmology. It is fair to say, at this stage, that string inspired cosmologies are not a match for standard inflation. Yet, many believe that superstrings or their generalizations provide an adequate description of fundamental interactions including gravity. It is therefore important to pursue exploring the connections between string theories and cosmology in order to find which, if any, relic or aspect of the fundamental theory may adequately account for the expansion of the universe.

Much has been written and emphasized about the role of the fundamental dilaton field in the context of string cosmology. Less is known about the tachyon component. This is mostly due to the fact that tachyons were considered rather a nuisance in string theory. Recent developments in the fluid description of the tachyon condensate in bosonic and supersymmetric string theory due to Sen [2,3] have resulted in enhancing our understanding of the role of the tachyon. Based on these works, several papers studying such a fluid in the cosmological context [5–8] have appeared very recently.

The main purpose of this article is to show that, under the assumption that the tachyon potential behaves as an inverse square in terms of the field, the Einstein equations lead to *simple* exact analytic solutions in the case of a spatially flat isotropic universe. These solutions undergo the so-called power-law inflationary expansion if the "slope" of the potential $\beta>2\sqrt{3}/3$, and decelerate otherwise. The fact that the rolling tachyon condensate can lead to power-law inflation may have important consequences in our understanding of cosmology.

We also indicate and exemplify how more general solutions can be constructed. The choice of the inverse square tachyon potential may at first sound artificial. Nevertheless, we will see that the potential behaves qualitatively similarly to some exact classical potentials derived in the context of open string field theory. Otherwise, there is no need to stress the importance of having a reasonable exact solution to the coupled tachyon-gravity equations; we just mention that further studies of qualitative behavior and numerical simulations, including the study of density perturbations, can be contrasted against such an exact solution.

To this end we consider a spatially flat Friedmann-Robertson-Walker (FRW) line element given by

$$ds^2=dt^2-a^2(t)(dx^2+dy^2+dz^2). \tag{1}$$

As shown by Sen [2,3] a rolling tachyon condensate may be described by an effective fluid with energy density and pressure given by

$$\rho=\frac{V(T)}{\sqrt{1-\dot{T}^2}}, \tag{2}$$

$$p=-V(T)\sqrt{1-\dot{T}^2}. \tag{3}$$

Here T is the tachyon field and $V(T)$ the tachyon potential. It is worth mentioning that this sort of model with an ordinary scalar field has been studied in cosmology on phenomenological grounds [4]. The pressure and the density of the fluid may be derived from the Lagrangian density $\mathcal{L}=p(\rho)=-V(T)\sqrt{1-\dot{T}^2}$.

The Friedmann equation and the entropy conservation equation take the forms [5,6]:

$$3H^2=\rho=\frac{V}{\sqrt{1-\dot{T}^2}} \tag{4}$$

and

$$\dot{\rho}=-3H(\rho+p). \tag{5}$$

We have set $8\pi G=1$, H is the Hubble parameter, and the overdot means differentiation with respect to time.

The entropy conservation equation is, in turn, equivalent to the equation of motion for the tachyon field T:

$$\frac{V\ddot{T}}{1-\dot{T}^2}+3H\,V\,\dot{T}+V'=0, \tag{6}$$

where $V'=dV/dT$.

Now, substituting V from the Friedmann equation together with the expressions for the pressure and the density into the entropy conservation equation, one gets the following interesting relation expressing the change of the tachyon in terms of the Hubble parameter and its derivative:

$$\dot{T}^2=-\frac{2\dot{H}}{3H^2}. \tag{7}$$

Below, we will show how this equation can be used to construct a variety of exact solutions, but first we derive solutions with the inverse square potential.

Equation (7) and the Friedmann equation may be written in yet different forms:

$$\dot{T} = -\frac{2}{3}H'/H(T)^2 \tag{8}$$

and

$$H'^2 - \frac{9}{4}H^4(T) + \frac{1}{4}V^2(T) = 0. \tag{9}$$

We now assume that the potential is an inverse square in terms of the tachyon field:

$$V(T) = \beta T^{-2}, \quad \beta > 0. \tag{10}$$

Although this potential diverges at $T=0$, it fairly mimics the behavior of a typical potential in the condensate of bosonic string theory. One expects the potential to have a maximum at $T \to 0$ and to die off for a large field. Recently, two groups [9,10] have given an exact (in string tension) form of the classical potential:

$$V(T) = V_0(1 + T/T_0)\exp(-T/T_0). \tag{11}$$

Apart from the unphysical divergence at vanishing tachyon in our toy model, the exact potential (11) has qualitatively similar behavior.

With this assumption we can see that Eq. (9) has a solution $H \sim T^{-1}$. Substituting this solution into Eq. (8) we find that the tachyon field is linear in time. After some simple algebra we have

$$a = t^n \tag{12}$$

along with

$$n = \frac{1}{3} + \frac{1}{6}\sqrt{4 + 9\,\beta^2}. \tag{13}$$

The tachyon field has the form

$$T = \sqrt{2/3n}\,t. \tag{14}$$

The condition for inflation for these solutions is that $n > 1$. These are the so-called power-law inflationary solutions [11,12] and in the inflaton driven models are associated with an exponential potential. Limitations on n come from the reality of Eq. (7). This imposes the positivity of n, and this is why the negative branch of Eq. (13) was discarded. The same equation prohibits, in general, the so-called super- or pole inflation, by imposing a nonpositive sign on the time derivative of the Hubble parameter. From Eq. (7) we immediately find that the inflationary solutions with $n > 1$ correspond to $\dot{T}^2 < 2/3$ (cf. [5]). These models actually inflate forever.

In terms of the potential parameter β, inflation occurs whenever $\beta > 2\sqrt{3}/3$. The range of the parameter β for decelerating models is $0 < \beta < 2\sqrt{3}/3$. In terms of the power n, the last inequality translates into $2/3 < n < 1$.

We were lucky, of course, in choosing the form of the potential to solve Eq. (9) and the rest of the equations exactly. Given a different potential, solving the coupled differential equations happens to be a rather difficult task. One may, however, approach this problem from a different perspective. Rather then starting with a given potential, one can start with a given expansion factor $a(t)$. The Hubble parameter $H(t)$ is then readily found and the tachyon field $T(t)$ can be read from Eq. (7). The potential in the form of $V(t)$ is then found from the equation

$$V = 3H^2\sqrt{1-\dot{T}^2} = 3H^2\sqrt{1 + \frac{2\dot{H}}{3H^2}}. \tag{15}$$

Inverting $T(t)$ to get $t(T)$, we finally find $V(T)$. The main drawback in this scheme is that one may often end up with an unphysical tachyon potential; the advantage, on the other hand, is that, given a known cosmological expansion, one can figure out the tachyon potential and the field itself.

Alternatively, one can start with "favorable" behavior for the tachyon. Equation (7) is then easily integrated to give one the Hubble scale $H^{-1} = \frac{3}{2}\int \dot{T}^2 dt$. From here, to find the scale factor and the potential $V(T)$ is just a matter of algebra.

Let us see how it works. The case of exponential expansion with constant Hubble parameter corresponds to a constant tachyon and constant potential. The limiting $t^{2/3}$ behavior corresponds to pressureless dust. To exemplify the procedure for less straightforward cases we assume that the scale factor behaves as

$$a(t) = \exp(mt^n). \tag{16}$$

After some algebra we find

$$H = mn\,t^{n-1} \tag{17}$$

together with

$$\dot{T}^2 = \frac{2(1-n)}{3mn}t^{-n} \Rightarrow T = \gamma t^{(2-n)/2}. \tag{18}$$

Here $\gamma = 2\sqrt{6(1-n)/mn}/3(2-n)$, and the integration constant in Eq. (18) has been ignored. We can now invert $T(t)$ to get $t(T) = (T/\gamma)^{2/2-n}$. In terms of $T(t)$, the potential becomes

$$V(T) = AT^{4(n-1)/(2-n)}\sqrt{B + CT^{-2n/(2-n)}}, \tag{19}$$

where A, B, and C are constants expressed in terms of m and n. The special case $n=2$ should be treated separately, and leads to a logarithmically divergent tachyon at $t=0$.

The reality condition for the tachyon derivative imposes $0 < n < 1$ for $m > 0$, and either $n < 0$ or $n > 1$ for $m < 0$. The sign of the acceleration depends basically on the form $mn\,t^n\,(n - 1 + mn\,t^n)$; depending therefore on m and n one can have a variety of models with different behaviors with respect to inflation. For $n > 1$ (negative m) the expansion is regular at $t=0$ but rather singular at $t\to\pm\infty$. These models lead, however, to a singular tachyon at $t=0$. One can have both a nonsingular tachyon and expansion at $t=0$ for $1<n<2$. Let us choose, for example, $n=4/3$; then for negative times the model expands (due to negative m and therefore positive H) from the singularity at $t\to-\infty$, starts contracting for $t>0$, and continues to contract toward the singularity at $t\to\infty$. As far as inflation is concerned this model accelerates initially for large negative time, stops the acceleration, and decelerates near $t=0$. It finally has an accelerated collapse for large positive times.

To sum up: Assuming a toy model potential for the tachyon field, we have shown that this choice leads to potentially very interesting power-law inflationary solutions. We have found a simple equation (7) which allows one by starting with a given expansion to find the exact form of the tachyon field and its potential.

Some interesting questions remain open. It is well known that in the standard inflaton driven cosmology the power-law inflation represents a late time attractor when the potential is exponential [12]. It would be interesting to see whether the power-law inflationary solutions driven by the inverse square potential of the tachyon have similar status. We also are aware that in the inflaton driven expansion one may reconstruct the potential starting from the behavior of the density perturbations [13], where the so-called $H(\phi)$ formalism due to Lidsey [14] is used. Here one can obviously undertake a similar task using $H(T)$ as an analogue of $H(\phi)$, with Eq. (7) being the input for such a study. Finally, it would be interesting to study the effects of anisotropies and inhomogeneities in this setting. We hope to be able to address these questions in the near future.

Note added. After this paper was submitted an overlapping paper by T. Padmanabhan, Phys. Rev. D **66**, 021301 (2002), also appeared.

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