Neutrino emission from Goldstone modes in dense quark matter

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We calculate neutrino emissivities from the decay and scattering of Goldstone bosons in the color-flavor-locked (CFL) phase of quarks at high baryon density. Interactions in the CFL phase are described by an effective low-energy theory. For temperatures in the tens of keV range, relevant to the long-term cooling of neutron stars, the emissivities involving Goldstone bosons dominate over those involving quarks, because gaps in the CFL phase are ~ 100 MeV while the masses of Goldstone modes are on the order of 10 MeV. For the same reason, the specific heat of the CFL phase is also dominated by the Goldstone modes. Notwithstanding this, both the emissivity and the specific heat from the massive modes remain rather small, because of their extremely small number densities. The values of the emissivity and the specific heat imply that the time scale for the cooling of the CFL core is $\sim 10^{26}$ yr, which makes the CFL phase invisible as the exterior layers of normal matter surrounding the core will continue to cool through significantly more rapid processes. If the CFL phase appears during the evolution of a protoneutron star, neutrino interactions with Goldstone bosons are expected to be significantly more important since temperatures are high enough (~ 20–40 MeV) to admit large number densities of Goldstone modes.

I. INTRODUCTION

Neutrino emission and interactions in matter at high baryon density play crucial roles in astrophysical phenomena such as core collapse supernovae and neutron stars [1]. Neutrinos drive supernova dynamics from beginning to end: they become trapped within the star's core early in the collapse, forming a vast energetic reservoir, and their eventual emission from the protoneutron star is prodigious enough—containing nearly all the energy ($\sim 10^{53}$ ergs) released in the explosion—to dramatically control subsequent events [2]. The ν-luminosity and the time scale over which νs remain observable from a protoneutron star are also governed by charged and neutral current interactions involving matter at high baryon density [3]. The long-term cooling of a neutron star, up to a million years of age, is controlled by ν-emissivity and the specific heat of the densest parts of the star; during this time the star is observable through photon emissions, which may allow us to determine the star's mass, radius, and internal constitution [4]. The tabulation of surface temperatures and ages of neutron stars is currently one of the primary goals of x-ray astronomy.

Collins and Perry [5] noted that the superdense matter in neutron star cores might consist of weakly interacting quarks rather than of hadrons, due to the asymptotic freedom of QCD. Asymptotic freedom implies that, at very high baryon density, for which the baryon chemical potential $\mu_B \gg \Lambda_{QCD}$, QCD is amenable to perturbative techniques. However, recent studies have shown that the naive ground state of the system, a Fermi liquid of weakly interacting quarks, is unstable with respect to the formation of diquark condensates [6–9]. Attractive interactions induce the quarks

to pair which results in a gap in their excitation spectrum that may be as large as $\Delta \simeq 100$ MeV [10,11]. The quark Cooper pairs are antisymmetric in color. If only two flavors participate, the pair wave functions have the structure $\epsilon^{abc} u^b d^c$, where a,b, and c are color indices. This implies that color $SU(3)$ is broken to $SU(2)$ by a Higgs mechanism and that one type of quark and three types of gluons remain massless. These degrees of freedom will dominate the low energy excitations of the system.[1] If the baryon chemical potential μ_B is very large, we expect that all three light flavors participate. In this case, the wave function of the Cooper pairs is expected to be color-flavor locked [12–15]. Color-flavor locking implies that all quarks and gluons acquire a gap.

Because the critical temperature associated with color superconductivity is large, we expect that quark matter existing in the core of a neutron star will be in a superconducting state. This leads to the question whether one can infer the presence of a color superconducting core from astronomical observation, and whether such observations can distinguish among the different color superconducting phases.

Color superconductivity is a Fermi surface phenomenon and its effect on the equation of state is expected to be small. The impact of a diquark condensed phase on the long-term cooling of neutron stars was studied in [16,17]. These works explored the consequences of neutrino emission processes that involved gapped quarks. The cooling of a pure quark star, considered in [16], implied that cooling would occur very fast, indeed too fast to be consistent with the existing

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[1] If the temperature is very small, $T < 1$ keV, the ungapped quark may acquire a gap and the gluons will be confined into glueballs.

x-ray data. In [17], in which a hybrid star with a mixed phase of hadrons and quarks was considered, large quark gaps rendered quark matter invisible and vanishing quark gaps led to cooling behaviors which were indistinguishable from those of normal stars. A possible connection between the occurrence of quark matter in neutron stars and neutron star glitches was proposed in [18]. Neutrino diffusion in the two-flavor superconducting (2SC) condensed phase was investigated in [19].

In this paper, we focus our attention on the long-term cooling stage, which begins when the star's temperature has dropped to a few tenths of an MeV ($\sim 10^9$ K or $T_9 = 1$) and when most of its lepton content has been lost due to neutrino radiation during the so-called protoneutron star (PNS) evolution that lasts about a minute or so. The extent of a quark phase is maximized when the trapped neutrinos have left the star [20]. Thereafter, the star continues to lose its energy by radiating low-energy neutrinos until it enters a photon cooling epoch and its surface temperature may be estimated by the detection of x rays. Coupled with an independent estimate of the star's age, the cooling history of the star may be inferred.

In a phase in which all quarks are gapped, the neutrino emissivity from the direct Urca processes $f_1 + \ell \rightarrow f_2 + \nu_l, f_2 \rightarrow f_1 + \ell + \bar{\nu}_l$, where f_1 and f_2 are quarks and ℓ is either an electron or muon, are strongly suppressed due to the energy cost ($\sim \Delta$) involved in breaking the pair. (The modified Urca process, which contains an additional quark in both the entrance and exit channels, is additionally suppressed.) However, there are other mechanisms for neutrino emission from this phase. One such process is the pair breaking and recombination of quarks [21]. This process can dominate over conventional cooling modes just below the transition temperature $T_c \sim \Delta/2$. We demonstrate here that as the temperature falls below T_c, neutrino emission will be dominated by decay of massless or almost massless collective modes, hereafter denoted by ϕ, that are associated with the breakdown of a global symmetry. The low energy excitations of CFL quark matter are an exactly massless mode associated with the breakdown of the $U(1)$ of baryon number and a very light octet and singlet of Nambu-Goldstone bosons associated with the breaking of the exact $SU(3)_L \times SU(3)_R$ chiral and approximate $U(1)_A$ axial symmetry.

The purpose of this work is to provide benchmark estimates of the neutrino emissivity from the electroweak decays of Goldstone modes. In Sec. II, we employ an effective theory of CFL Goldstone modes coupled to electroweak gauge fields. Specifically, we calculate the neutrino emissivity from the processes:

(1) $\pi^\pm \rightarrow l^\pm + \bar{\nu}_l$, $K^\pm \rightarrow l^\pm + \bar{\nu}_l$,

(2) $\pi^0 (\eta, \eta') \rightarrow \nu + \bar{\nu}$, and

(3) $\phi + \phi \rightarrow \phi + \nu + \bar{\nu}$,

and compare our results to those from previously studied ν-emission processes. In Sec. III, we calculate the photon emissivity from the pion electromagnetic radiative two-photon decay, $\pi^0 \rightarrow \tilde{\gamma} + \tilde{\gamma}$. This is followed by a discussion of photon-Goldstone boson interactions in the CFL phase. The cooling behavior of a neutron star is also governed by

the specific heat of the ambient matter. We therefore assess the contribution of the Goldstone modes in Sec. IV. Our principal findings are summarized in Sec. V. Utilizing our results for the emissivity and specific heat, we estimate the time scale over which a CFL phase core cools in the concluding Sec. VI. Here, we also comment on the expected role of neutrino interactions with Goldstone bosons in the evolution of a PNS, in which temperatures in the range of 20–40 MeV are encountered.

II. ELECTROWEAK INTERACTIONS OF GOLDSTONE MODES IN THE CFL PHASE

The low energy excitations of the CFL phase are two singlet modes associated with $U(1)_B$ and $U(1)_A$ symmetry and an octet of Goldstone modes associated with chiral symmetry breaking. The octet is described by a low energy theory that bears strong resemblance to chiral perturbation theory [22–24]. The Goldstone modes are parametrized by a 3×3 unitary matrix U which is a color singlet, transforming under $SU(3)_L \times SU(3)_R$ as $U \rightarrow g_L U g_R^\dagger$, where U is related to axial-like fluctuations of the left and right handed diquark fields:

$$L^{ai} \sim \epsilon^{abc} \epsilon^{ijk} \langle q_L^{bj} q_L^{ck} \rangle^*, \quad R^{ai} \sim \epsilon^{abc} \epsilon^{ijk} \langle q_R^{bj} q_R^{ck} \rangle^*,$$

and

$$U = LR^\dagger. \tag{1}$$

The fields L and R carry color (ijk) and flavor (abc), and transform under $g_f \subset SU(3)_f$ and $g_c \subset SU(3)_C$ (where f denotes left or right handed flavor) as

$$L \rightarrow g_L L g_C^\dagger \quad \text{and} \quad R \rightarrow g_R R g_C^\dagger, \tag{2}$$

respectively. The low energy effective theory is governed by the Lagrangian

$$\mathcal{L}_{eff} = \frac{f_\pi^2}{4} \text{Tr}[\partial_0 U \partial_0 U^\dagger - v_\pi^2 \partial_i U \partial_i U^\dagger] + \cdots, \tag{3}$$

where v_π is the Goldstone boson velocity and we have suppressed mass terms and higher derivative terms. The chiral field U can be related to the octet meson field by $U = \exp(i\pi^a \lambda^a / f_\pi)$, where the $SU(3)$ generators λ^a are normalized as $\text{tr}[\lambda^a \lambda^b] = 2\delta^{ab}$.

In the weak coupling limit, we have $v_\pi^2 = 1/3$. As a consequence of the breaking of Lorentz invariance in matter, there are two pion decay constants, f_T and f_S, which characterize the coupling of the pion to the temporal and the spatial components of the axial-vector current. The effective Lagrangian in Eq. (3) implies that $f_T = f_\pi$ and $f_S = v_\pi^2 f_\pi$. This result is consistent with axial-vector current conservation in the limit $m_\pi \rightarrow 0$. We observe that $f_T \omega^2 - f_S \mathbf{q}^2 = 0$ for an on-shell pion.

The extension to include electroweak interactions has been discussed in previous works [26,27]. Here, we follow [26] in introducing the electroweak gauge fields via minimal coupling:

$$D_\mu L = \partial_\mu L - ie A_\mu Q L - \frac{ig}{\sqrt{2}} [W_\mu^+ \sigma^+ + W_\mu^- \sigma^-] L$$

$$- \frac{ig}{2\cos\theta_W} Z_\mu^0 [\sigma^3 - 2Q\sin^2\theta_W] L, \quad (4)$$

$$D_\mu R = \partial_\mu R - ie A_\mu Q R + \frac{ig}{2\cos\theta_W} Z_\mu^0 [2Q\sin^2\theta_W] R.$$

Here, A is the electromagnetic field,[2] which couples with strength $Q = \mathrm{diag}(2/3, -1/3, -1/3)$ to the u, d, and s quarks, W and Z are the $SU(2)$ electroweak gauge fields,[3] g is the weak coupling constant, and θ_W is the Weinberg angle. The σ matrices are the Pauli matrices of $SU(2)$ embedded into flavor $SU(3)$:

$$\sigma^+ = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \sigma^- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and

$$\sigma^3 = \frac{1}{2} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (5)$$

In the charged current coupling, we have only taken into account the first two flavors [26]. We will return later to comment on charged current decays of kaons. We observe that the CFL pions, kaons, and etas have charged and neutral current interactions that are very similar to those of ordinary mesons. An important difference, however, is the fact that CFL mesons are very light, $m_{GB} \simeq 10$ MeV [24]; see [28,29] for a recent discussion. This implies that the pion and kaon are lighter than the muon and dominantly decay into electrons and neutrinos. Another difference, as compared to the zero density case, is the fact that the Goldstone boson dispersion relation is modified from its vacuum form:

$$E_p = \mu_{eff} + \sqrt{m_{GB}^2 + v_\pi^2 p^2}. \quad (6)$$

Here, μ_{eff} is an effective chemical potential that depends on the quark masses [30]. For pions, $\mu_{eff} \simeq 0$, but for kaons μ_{eff} is sizeable and may have consequences. The modified dispersion relation Eq. (6) leads to a number of unusual effects. One of them, the fact that a very fast pion becomes

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[2]The A field may be reexpressed in terms of the physical (massive) gluon and physical (massless) photon that arise as a consequence of photon-gluon mixing in the CFL phase.

[3]The weak gauge bosons W and Z would also mix with the gluons through the diquark condensate. This mixing leads to corrections of order $\alpha_s f_\pi^2 / M_Z^2$ in the decay rates from Goldstone bosons, and are numerically very much smaller than those from the leading order terms.

absolutely stable, was already discussed in [31]. Here, we will encounter a second one, the presence of the "helicity forbidden" decay $\pi^0 \to \nu\bar{\nu}$.

A. Neutrino emissivity from $\pi^\pm(p) \to l^\pm(p_1) + \bar{\nu}_l(p_2)$

Substituting Eq. (4) into the effective Lagrangian, we obtain the relevant interaction term for charged pion decay:

$$\mathcal{L}_{W\pi} = \frac{gf_T}{2} (W_0^- \partial^0 \pi^+ + W_0^+ \partial^0 \pi^-) - (f_T \to f_S, 0 \to i). \quad (7)$$

The pion decay constant f_π has been estimated using various methods in the literature [24,25]. In Ref. [24], it was computed by matching a chiral effective theory with a microscopic theory of quasiparticles and holes near the Fermi surface. Explicitly,

$$f_\pi^2 = \frac{21 - 8\ln 2}{18} \frac{\mu^2}{2\pi^2}. \quad (8)$$

In the following, we will outline the calculation of the ν-emissivity for the π^- decay. The π^+ decay gives exactly the same result. The T-matrix element for the weak decay process is

$$\langle l^-(p_1)\bar{\nu}_l(p_2)|T|\pi^-(p)\rangle$$
$$= (2\pi)^4 \delta^4(p - p_1 - p_2) G_F \bar{u}_l(p_1)$$
$$\times (\gamma^0 p_0 f_T - \gamma \cdot \mathbf{p} f_S)(1 - \gamma_5) v_{\nu_l}(p_2), \quad (9)$$

where we have made the identification $G_F/\sqrt{2} = g^2/(8M_W^2)$. Using the equation of motion for the leptonic fields, the neutrino emissivity from this process is given by

$$\epsilon_\pi = \int \frac{d^3 p}{(2\pi)^3 2\omega_p} n_B(\omega_p) \int \frac{d^3 p_1}{(2\pi)^3 2\omega_{p_1}} \tilde{n}_F(p_1)$$

$$\times \int \frac{d^3 p_2}{(2\pi)^3 2\omega_{p_2}} (\omega_{p_2})(2\pi)^4 \delta^4(p - p_1 - p_2)$$

$$\times \sum_{spin} |M|^2, \quad (10)$$

$$\sum_{spin} |M|^2 = 8 G_F^2 (|\xi|^2 p_1 \cdot p_2 + |\chi|^2 \{\omega_p^2(\omega_{p_1}\omega_{p_2} + \mathbf{p}_1 \cdot \mathbf{p}_2)$$

$$+ m_e^2(p_1 \cdot p_2 - 2\omega_p\omega_{p_2})\}$$

$$+ 2|\xi||\chi| m_e(\omega_p\omega_{p_2} - p_1 \cdot p_2)), \quad (11)$$

where $\omega_p = \sqrt{m_{GB}^2 + v_\pi^2 p^2}$, $\xi = -im_e f_T$ and $\chi = -i(f_T - f_S)$. Here, m_e is the electron mass, $n_B(\omega_p)$ denotes the Bose occupation factor for the pion, and $\tilde{n}_F(p_1) = 1 - n_F(p_1)$ is the Pauli blocking factor for the outgoing electron. For the temperatures of relevance to long-term cooling,

Pauli blocking of the outgoing neutrino may be neglected. For bulk CFL matter at $T=0$, the electron chemical $\mu_e \rightarrow 0$ [30,32]. At nonzero temperature, there is a small electron chemical potential due to the fact that the energies of positively and negatively charged Goldstone modes are not the same. For $T<1$ MeV, we find $\mu_e \ll T$ and the effect on neutrino emission is negligible. In proto-neutron stars, CFL phases with charged meson condensates can also exist [33]. Since our focus here is on long-term cooling, we do not consider these possibilities. Finally, we expect that CFL matter inside a neutron star is in contact with a hadronic or quark phase that has a large electron chemical potential. As discussed in [34], this will lead to a thin charged surface layer which shields the CFL phase. In the following, we will assume that $\mu_e = 0$ in regions well separated from this layer.

The squared matrix element in Eq. (11) consists of three terms. We will evaluate the emissivities from each in turn. The first term involves $|\xi|^2$ and has the Lorentz invariant structure familiar from its vacuum counterpart. This is the only term that survives in the limit $f_T = f_S$. We also observe that the emissivity is proportional to the electron mass squared, as expected. The momentum dependence is of the form $p_1 \cdot p_2 = \omega_{p_1} \omega_{p_2}(1 - v_1 v_2 \cos \theta)$ with $v_1 = |\mathbf{p_1}|/\omega_{p_1}, v_2 = |\mathbf{p_2}|/\omega_{p_2}$, and $\theta = \angle(\mathbf{p_1}, \mathbf{p_2})$. The angular integral gives

$$\int d\Omega_p d\Omega_{p_1} d\Omega_{p_2}(1 - v_1 v_2 \cos \theta)(2\pi)^3 \delta^3(\mathbf{p} - \mathbf{p_1} - \mathbf{p_2})$$

$$= \frac{(2\pi)^5}{p p_1 p_2 \omega_{p_1} \omega_{p_2}}(2\omega_{p_1} \omega_{p_2} - (p^2 - p_1^2 - p_2^2))$$

$$\times \Theta(p + p_1 - p_2)\Theta(p + p_2 - p_1)$$

$$\times \Theta(p_1 + p_2 - p), \tag{12}$$

where p is now to be understood as $|\mathbf{p}|$ etc. The triangle inequalities imposed by momentum conservation restrict the range of the p_2 and p (or equivalently ω_p) integrals as follows:

$$\frac{\omega_p - p}{2} - \frac{m_e^2}{2(\omega_p + p)} \leq p_2 \leq \frac{\omega_p + p}{2} - \frac{m_e^2}{2(\omega_p - p)},$$

$$m_\pi \leq \omega_p \leq \sqrt{\frac{m_\pi^2 - m_e^2}{1 - v_\pi^2}}. \tag{13}$$

When these conditions are satisfied, the p_1 integration can be done trivially using the energy-delta function. The expression for the emissivity then becomes

$$\epsilon_\xi = \frac{A}{64\pi^3 v_\pi^2} \int_{m_\pi}^{\omega_p^{max}} d\omega_p n_B(\omega_p)$$

$$\times \left(\frac{m_\pi^2}{v_\pi^2} - m_e^2 + \frac{(v_\pi^2 - 1)}{v_\pi^2} \omega_p^2 \right)$$

$$\times \int_{p_2^{min}}^{p_2^{max}} dp_2 p_2 \frac{1}{1 + \exp(p_2 - \omega_p)/T}, \tag{14}$$

where $A = 8G_F^2 f_T^2 m_e^2$ and the limits $\omega_p^{max}, p_2^{min}$, and p_2^{max} are defined by Eq. (13). In obtaining Eq. (14), we have utilized the pion dispersion relation $\omega_p^2 = v_\pi^2 p^2 + m_\pi^2$ to change the variable of integration as $p dp/\omega_p = d\omega_p/v_\pi^2$. We have also used the identity $\tilde{n}_F(x) = n_F(-x)$. For long-term cooling, temperatures range from tens of keV to eV, much less than either the pion mass or electron mass (~ 10 MeV and 0.51 MeV, respectively). In the following, we will therefore calculate the emissivity in the low temperature limit when m_π/T and $m_e/T \gg 1$. Unless the pion mass becomes very small, $m_\pi \simeq m_e$, we can also neglect the electron mass in the integrand of Eq. (14). Within the range of the ω_p integration, we always have $p_2 \leq \omega_p$. In the low temperature limit, $m_\pi/T \gg 1$, we can replace the Pauli-blocking factor $n_F(-x)$ by unity. In this limit, we can also replace the Bose distribution function for the pion by the Boltzmann distribution. Introducing the scaled variables $x = \omega_p/T, y = p_2/T$, and $\psi = m_\pi/T$, Eq. (14) now reads

$$\epsilon_\xi = \frac{AT^5}{64\pi^3 v_\pi^2} \int_\psi^{\psi_{max}} dx\, e^{-x} \left(\frac{\psi^2}{v_\pi^2} - \left(\frac{1 - v_\pi^2}{v_\pi^2} \right) x^2 \right)$$

$$\times \int_{y_{min}}^{y_{max}} dy\, y, \tag{15}$$

where $\psi_{max} = \psi/\sqrt{1 - v_\pi^2}$. The y integration yields $x\sqrt{x^2 - \psi^2}/2v_\pi$ with the result that

$$\epsilon_\xi = \frac{AT^5}{64\pi^3 v_\pi^5} \int_\psi^{\psi_{max}} dx\, e^{-x} \frac{x\sqrt{x^2 - \psi^2}}{2}$$

$$\times (\psi^2 - (1 - v_\pi^2)x^2). \tag{16}$$

As ψ is parametrically large, most of the contribution to the integral comes near the lower limit. The upper limit can be extended to infinity with little impact on the numerical value of the integral. Using the substitution $x = \psi \cosh \theta$ (the limits on θ now run from 0 to ∞), the definition of the modified Bessel function of the second kind K_ν and their recursion relation, we finally obtain

$$\epsilon_\xi = \frac{A}{64\pi} m_\pi^2 \left[\frac{1}{v_\pi^3} \left(\frac{m_\pi T}{2\pi} \right)^{3/2} e^{-m_\pi/T} \right] = \frac{A}{64\pi} m_\pi^2 n_\pi, \tag{17}$$

where we have used the asymptotic form $K_\nu(z) = \sqrt{\pi/2z}e^{-z}$ valid for any ν when $z \gg 1$. The factor A is proportional to the interaction strength and the factor in the square bracket is the number density n_π of thermal nondegenerate pions. Note that the dispersion relation in Eq. (6) introduces an extra factor of $1/v_\pi^3$ in the number density of pions relative to the case with $v_\pi = 1$.

The other two contributions to the emissivity in Eq. (10) are evaluated along similar lines. The result for the mixed term ($|\xi||\chi|$) and the quadratic term ($|\chi|^2$) are

$$\epsilon_{\xi\chi} = \frac{B}{64\pi} m_\pi^2 n_\pi \quad \text{and} \quad \epsilon_\chi = \frac{C}{64\pi v_\pi^2} m_\pi^2 n_\pi, \tag{18}$$

respectively, with $B = 16 G_F^2 f_T (f_T - f_S) m_e^2$ and $C = 16 G_F^2 (f_T - f_S)^2 m_\pi T$. We observe that the coefficient C is independent of the electron mass. The term involving C arises purely as a consequence of Lorentz symmetry breaking in dense matter. We also note that all emissivities appear to diverge as $v_\pi \rightarrow 0$, which is the case near a phase transition. This is an artifact of the approximation $\psi_{max} \rightarrow \infty$ in Eq. (16). At fixed T, all rates are finite as $v_\pi \rightarrow 0$. Combining Eqs. (17) and (18), and using the perturbative result $v_\pi^2 = 1/3$, we obtain the neutrino emissivity from the electroweak decay of CFL pions as

$$\epsilon_\pi = \epsilon_\xi + \epsilon_{\xi\chi} + \epsilon_\chi$$

$$= \frac{1}{8\pi} (G_F^2 f_\pi^2 m_e^2) m_\pi^2 n_\pi \left(1 + 2 \left(\frac{\delta f_\pi}{f_\pi} \right) + \frac{2 m_\pi T}{v_\pi^2 m_e^2} \left(\frac{\delta f_\pi}{f_\pi} \right)^2 \right),$$

$$(19)$$

where $\delta f_\pi = f_T - f_S$. The rate from π^+ decay is exactly the same. This result is valid when temperatures fall below an MeV. Numerically, the emissivity may be expressed as

$$\epsilon_\pi = 1.18 \times 10^{28} \mu_{100}^2 m_{10}^2 (m_{10} T_9)^{3/2} e^{-116 m_{10}/T_9}$$

$$\times \left[1 + 1.14 \left(\frac{m_\pi}{m_e} \right) \left(\frac{T}{m_e} \right) \right] \; \text{erg cm}^{-3} \; \text{s}^{-1}, \quad (20)$$

where μ_{100} is the quark chemical potential in units of 100 MeV, m_{10} is the pion mass in units of 10 MeV, and $T_9 = T/10^9$ K.

It is interesting to compare the neutrino emissivity from pion decay with the emissivity from the quark direct Urca process in a gapped superfluid, as well as the quark pair breaking and formation (PBF) process. From [35,36], the emissivity from the quark direct Urca process is given by

$$\epsilon_{q\beta} \simeq 8.8 \times 10^{26} \alpha_s \left(\frac{n_B}{n_0} \right) Y_e^{1/3} T_9^6$$

$$\times e^{-\Delta/k_B T} \; \text{erg cm}^{-3} \; \text{s}^{-1}, \quad (21)$$

where $\alpha_s = g^2/4\pi$ is the strong coupling constant, n_B is the baryon density, $n_0 = 0.16$ fm^{-3} is the nuclear equilibrium density, and $Y_e = n_B/n_0$ is the electron concentration. For $n_B = 5 n_0$, $\alpha_s \simeq 1$, $Y_e \sim 10^{-6}$, $\Delta \simeq 100$ MeV, and $m_\pi \simeq 10$ MeV, we find that pion decay dominates the Urca emissivity for $T < 5 \times 10^{10}$ K. The emissivity from the quark PBF, which is effective in rapidly cooling the star during its early hundreds of years, is given by [21]

$$\epsilon_q^{\nu\bar{\nu}} \simeq 1.4 \times 10^{20} N_\nu T_9^7 F \, a_q \left(\frac{n_B}{n_0} \right)^{2/3} \; \text{erg cm}^{-3} \; \text{s}^{-1}, \quad (22)$$

where N_ν is the number of neutrino flavors, F is a temperature dependent function of order one that vanishes exponentially with the gap, and a_q is a flavor dependent numerical factor of order 0.1. In the vicinity of T_c, there is no expo-

nential suppression, but as $T \ll T_c$, the function F approaches $\exp(-2\Delta/k_B T)$ so that the emissivity from PBF becomes very small.

B. Neutrino emissivity from $\pi^0 (\eta, \eta') \rightarrow \nu + \bar{\nu}$

In the previous section, we noticed that the emissivity for $\pi^- \rightarrow e^- + \nu$ process in matter contains a term that is not proportional to m_e^2. This implies that the neutral current decays $\pi^0 \rightarrow \nu + \bar{\nu}$ and $\eta (\eta') \rightarrow \nu + \bar{\nu}$, which are forbidden by helicity selection rules in vacuum, can occur in matter. The reason these decays are forbidden in vacuum is that the two neutrinos have opposite chirality. Thus, the two neutrinos have total angular momentum one in the rest frame of the decaying meson. Hence, a scalar meson cannot decay into $\nu\bar{\nu}$. In a dense medium, it is still true that a scalar meson which is at rest with respect to the medium cannot decay into $\nu\bar{\nu}$ pairs. However, boost invariance is lost. This implies that the wave function of a scalar meson which is moving with respect to the medium can have higher angular momentum admixtures when viewed in its rest frame.

In the following, we calculate the emissivity due to the process $\pi^0 \rightarrow \nu + \bar{\nu}$. The interaction term relevant for this decay channel is given by

$$\mathcal{L}_{Z^0 \pi^0} = \frac{f_T g}{2 \cos \theta_W} (Z_0^0 \partial^0 \pi^0) - (f_T \rightarrow f_S, 0 \rightarrow i). \quad (23)$$

The corresponding interaction for the η decay has an extra factor of $1/\sqrt{3}$ ($1/\sqrt{6}$ for η' decay) from the normalization of the $SU(3)$ generators. For simplicity, we ignore π^0-η-η' mixing [24,29]. We will outline the emissivity calculation for the π^0 decay. The corresponding emissivity for the η is obtained by changing the overall factor and the mass. The neutrino pair emissivity from π^0 decay is given by

$$\epsilon_{\nu\bar{\nu}} = \int \frac{d^3 p}{(2\pi)^3 2 \omega_p} n_B(\omega_p) \int \frac{d^3 p_1}{(2\pi)^3 2 \omega_{p_1}} \int \frac{d^3 p_2}{(2\pi)^3 2 \omega_{p_2}}$$

$$\times (\omega_{p_1} + \omega_{p_2}) (2\pi)^4 \delta^4 (p - p_1 - p_2) \sum_{spin} |M|^2 \sum_{spin} |M|^2$$

$$= 4 N_\nu G_F^2 |\chi|^2 \omega_p^2 (\omega_{p_1} \omega_{p_2} + \mathbf{p}_1 \cdot \mathbf{p}_2), \quad (24)$$

where N_ν counts the number of neutrino flavors. The emissivity is proportional to $f_T - f_S$, so it is directly proportional to Lorentz symmetry breaking in matter. The integrals are performed in the limit $m_e \rightarrow 0$ and $m_\pi/T \gg 1$ as before, and the emissivity from this process is found to be

$$\epsilon_{\nu\bar{\nu}} = \frac{N_\nu}{4\pi v_\pi^2} (G_F^2 (\delta f_\pi)^2 m_\pi^2) T m_\pi n_\pi. \quad (25)$$

Numerically, the emissivity (for one neutrino flavor) is

$$\epsilon_{\nu\bar{\nu}} = 4.52 \times 10^{28} \mu_{100}^2 m_{10}^2 (m_{10} T_9)^{5/2}$$

$$\times e^{-116 m_{10}/T_9} \ \text{erg cm}^{-3} \ \text{s}^{-1}. \quad (26)$$

C. Emissivities from kaon and massless Goldstone boson decays

In this section, we would like to provide a brief survey of other processes that contribute to the emissivity of CFL matter. Neutrino pair emission from plasmon decays in a stellar plasma, first proposed in [37] and subsequently investigated in detail in [38,39], can be the dominant energy loss mechanism for very hot and dense cores of red giant stars or white dwarfs. Blaschke *et al.* [16] studied the analogous plasmon decay of the massive gluon in the CFL phase. They found that the rate is suppressed by $\exp(-m_g/T)$, where $m_g \sim g\mu$ is the effective gluon mass. This implies that the plasmon rate is even more strongly suppressed than the quark Urca rate.

The physical photon $\widetilde{A}$ in the CFL phase is a linear combination of the ordinary photon (A) and gluon (G) fields

$$\widetilde{A}_\mu = A_\mu \cos\theta_{CFL} - G_\mu^8 \sin\theta_{CFL},$$

$$\tan\theta_{CFL} = 2e/\sqrt{3}g, \quad (27)$$

where e and g are the electromagnetic and strong coupling constants. The massive gluon $\widetilde{G}^8$ is the orthogonal combination. In an electron plasma, the photon is dressed by particle-hole excitations and acquires an effective mass. As a consequence, it can decay into $\nu\bar{\nu}$ pairs. In CFL matter at $T=0$, the photon is dressed by both particle-hole and particle-particle excitations, and the dielectric constant is larger than one, but it does not acquire a mass [40,41]. As a result, it cannot decay into $\nu\bar{\nu}$ pairs. At $T \neq 0$, the photons acquire a small mass, mainly because of thermal e^+e^- pairs, but the corresponding $\widetilde{\gamma} \to \nu + \bar{\nu}$ rate is very small.

In CFL matter, the usual spectrum of meson masses is partially inverted [24,30]. In particular, we expect that Goldstone bosons with positive strangeness, the K^0 and the K^+, are lighter than nonstrange Goldstone bosons such as pions and etas. The K^+ contributes to neutrino emission via the decay $K^+ \to e^+ + \bar{\nu}$. The emissivity for this process can be computed along the same lines as the emissivity from charged pion decay. However, there are two differences. First, kaon decay is suppressed by the Cabibbo angle $\sin^2(\theta_C)$, where $\theta_C \sim 15°$. The second difference is related to the fact that the kaon dispersion relation is modified by the effective chemical potential term; see Eq. (6). We find

$$\epsilon_{K^\pm} = \frac{A_K}{64\pi} E_{K^\pm}^2 n_{K^\pm} \left(1 + 2 \left(\frac{\delta f_\pi}{f_\pi} \pm \frac{\mu_{eff}}{E_{K^\pm}} \right) \right.$$

$$\left. + \frac{2 m_{K^\pm} T}{v_K^2 m_e^2} \left(\frac{\delta f_\pi}{f_\pi} \pm \frac{\mu_{eff}}{E_{K^\pm}} \right)^2 \right), \quad (28)$$

where $A_K = 8 \sin^2(\theta_C) G_F^2 f_T^2 m_e^2$ is the effective coupling, and $E_{K^\pm} = m_K \mp \mu_{eff}$ with $\mu_{eff} = m_s^2/(2\mu_q)$ is the energy of a kaon at rest. Note that in the limit $m_e \to 0$, $\delta f_\pi \to 0$, and

$v_K \to 1$, the emissivity remains finite because the terms containing μ_{eff} breaks Lorentz invariance. In this case,

$$\epsilon_{K^\pm} = \frac{\sin^2(\theta_C) G_F^2 f_T^2}{4\pi v_K^2} m_{K^\pm} n_{K^\pm} T \mu_{eff}^2. \quad (29)$$

For $T \sim 10^9$ K, this contribution to the emissivity would dominate over that of the first two terms in Eq. (28) that are proportional to m_e^2. We note that the K^+ emissivity is not equal to the K^- emissivity. This implies that, strictly speaking, it is not legitimate to ignore the effects of a nonzero electron chemical potential. We also note that, because of the smaller energy of a kaon compared to a pion, kaons will likely dominate the emissivity despite the Cabibbo suppression. Numerically, we find

$$\epsilon_K = 3.38 \times 10^{26} \mu_{100}^2 E_{10}^2 (m_{10} T_9)^{3/2} e^{-116 E_{10}/T_9}$$

$$\times \left[1 + 2 \left(\frac{\delta f_\pi}{f_\pi} \pm \frac{\mu_{eff}}{E_{K^\pm}} \right) \right.$$

$$\left. + \frac{2 m_{K^\pm} T}{v_K^2 m_e^2} \left(\frac{\delta f_\pi}{f_\pi} \pm \frac{\mu_{eff}}{E_{K^\pm}} \right)^2 \right] \ \text{erg cm}^{-3} \ \text{s}^{-1}, \quad (30)$$

where m_{10} and E_{10} are the kaon mass and energy in units of 10 MeV. In Sec. II B, we saw that in CFL matter the decay $\pi^0 \to \nu + \bar{\nu}$ is allowed if the π^0 has nonzero momentum. The analogous decay $K^0 \to \nu + \bar{\nu}$, however, remains strongly suppressed because it requires a second order weak $s \to d$ transition.

Since the CFL phase is characterized by unusually light Goldstone bosons, there is a possibility for Bose condensation to occur. Indeed, it has been argued that CFL matter at densities that can be achieved in neutron stars is likely to support a K^0 condensate [30,42,43]. Furthermore, because of the presence of trapped neutrinos, CFL matter in a proto-neutron star may have charged pion or kaon condensates [33]. In neutron matter, pion condensation (if it occurs) leads to a substantial increase of the neutrino emissivity through processes like $n + \pi^+ \to n + e^+ + \bar{\nu}$ [44,45]. In CFL matter, there are no ungapped fermions and the analogous process is suppressed by $\exp(-\Delta/(k_B T))$.

If charged pions or kaons become lighter, then the emissivity from the decay process $\pi^\pm \to e^\pm + \bar{\nu}$ will initially be enhanced. There is, however, no neutrino emission from the decay of a massless charged pion or kaon in a Bose condensed phase. The reason is that the decay of a massless boson with dispersion relation $\omega = (1/\sqrt{3})|\vec{k}|$ into pairs of leptons is kinematically forbidden. Indeed, we observe that the emissivity for the process $\pi^\pm \to e^\pm + \bar{\nu}$ is of the form $m_\pi^{7/2} \exp(-m_\pi/(k_B T))$ and vanishes as $m_\pi \to 0$.

The process $\pi^+ \pi^- \to \nu\bar{\nu}$, which involves two Goldstone bosons (GB) in the initial state, also contributes to the neutrino emission. If meson condensation does not occur, this process is suppressed by the factor $\exp(-m_{GB}/T)$ compared to the emissivity from single meson decays. This is because, for $T \ll m_{GB}$, thermal occupation factors severely diminish

the probability of processes involving two or more massive Goldstone bosons in the initial state. In the presence of meson condensation, in which one species forms the background condensate, the mass of the charge conjugated species is increased relative to that in the condensate. This once again leads to an exponential suppression compared to the single meson decays unless $T \gg m_{GB}$.

Instead, we have to consider neutrino bremsstrahlung in Goldstone boson scattering, or Goldstone boson scattering followed by the decay of a Goldstone boson into $\nu\bar{\nu}$ or $e^{\pm}\bar{\nu}$. In the case of massive Goldstone bosons, these processes are suppressed by an additional power of $\exp(-m_{GB}/(k_B T))$ with respect to the direct Goldstone boson decay process. For massless Goldstone bosons, however, neutrino emission is dominated by Goldstone boson scattering.

In CFL matter, there is always at least one exactly massless Goldstone boson which is associated with the breaking of the $U(1)_B$ of baryon number. The $U(1)_B$ Goldstone boson contributes to the neutrino emissivity through the process $\phi + \phi \rightarrow \phi + \nu + \bar{\nu}$. The coupling of ϕ to the Z boson is given by $(g/[12\cos(\theta_W)])Z_\mu f \partial^\mu \phi$ where f is the $U(1)_B$ decay constant [24]. The $\pi\pi$ and KK scattering amplitudes are fixed by the leading order $O(p^2)$ chiral Lagrangian. The $\phi\phi$ scattering amplitude, on the other hand, vanishes at leading order in the low energy expansion. The $\phi\phi$ scattering amplitude is on the order of $A \sim p^4/(f^2 \mu^2)$ [46]. The emissivity will then scale as

$$\epsilon_{GB} \sim \frac{G_F^2}{f^2 \mu^4} T^{15}. \tag{31}$$

Including numerical factors associated with the scattering amplitude and the phase space integral,

$$\epsilon_{GB} \simeq 10^{-11} \times T_9^{15} \mu_{100}^{-6} \ \mathrm{erg\,cm^{-3}\,s^{-1}}. \tag{32}$$

This estimate is adequate for $T \ll \Delta$. For the range of temperatures considered in this work, this process is not likely to play an important role. Since there is no exponential suppression, it will, however, dominate neutrino emission at very late times.

III. PHOTON EMISSIVITY AND PROPAGATION IN THE CFL PHASE

So far, we have identified sources for neutrino emission from CFL quark matter. Another interesting issue is the photon emissivity from this phase. A first step towards computing the emissivity of photons from the CFL phase was taken in [41], where the contributions from pair-correlated quarks and $q\bar{q}$ annihilations were found to be comparable in magnitude to the rates from a hot hadron gas, at temperatures of several tens of MeVs. At keV temperatures, however, the rates are vanishingly small due to the fact that quark gaps are of order 100 MeV.

Another source of photons in CFL matter is the anomalous $\pi^0 \rightarrow \tilde{\gamma} + \tilde{\gamma}$ decay, which proceeds via the non-Abelian

axial anomaly. The photon emissivity due to this process is given by

$$\begin{aligned}
\epsilon_{\tilde{\gamma}} = &\frac{1}{2} \int \frac{d^3q\, n(\omega_q)}{2\omega_q(2\pi)^3} \omega_q \int \frac{d^3p(1+n(\omega_p))}{2\omega_p(2\pi)^3} \\
&\times \int \frac{d^3k(1+n(\omega_k))}{2\omega_k(2\pi)^3}(2\pi)^4 \delta^4(q-p-k) \\
&\times \sum_{spin} |M(\pi^0 \rightarrow 2\tilde{\gamma})|^2 \sum_{spin} |M(\pi^0 \rightarrow 2\tilde{\gamma})|^2 \\
= &2A_{\tilde{\gamma}}^2 (p \cdot k)^2.
\end{aligned} \tag{33}$$

The factor 1/2 accounts for the two identical bosons in the final state. The distribution of photons in the final state is Bose enhanced. The constant $A_{\tilde{\gamma}}$ is related to the coefficient of the electromagnetic anomaly for the isospin axial vector current. Anomalous electromagnetic processes in the CFL phase can be studied in analogy with the vacuum case [26,47] through the replacement $e \rightarrow \tilde{e} = e \cos \theta$.[4] We have

$$\partial_\mu j_5^{\mu 3} = -\frac{\tilde{e}^2}{32\pi^2} \epsilon^{\alpha\beta\mu\nu} \tilde{F}_{\alpha\beta} \tilde{F}_{\mu\nu}, \tag{34}$$

which leads to the identification $A_{\tilde{\gamma}} = \tilde{e}^2/4\pi^2 f_\pi$ [48]. Unlike in vacuum, the matrix element $(p \cdot k)^2$ now depends on the pion momentum:

$$(p \cdot k)^2 = \frac{1}{4v_\pi^4}(m_\pi^4 + (1-v_\pi^2)^2 \omega_q^4 - 2m_\pi^2(1-v_\pi^2)\omega_q^2). \tag{35}$$

In vacuum, this expression reduces to $m_\pi^4/4$, which is also the case for a pion with zero momentum in the rest frame of dense matter. Note that the matrix element vanishes if either of the photons become soft. This helps to tame potential divergences from the Bose enhancement factors. The emissivity calculation proceeds similarly to that of the electroweak decay, where we had ignored the electron mass in the kinematics. Rescaling energies by the temperature as $x = \omega_q/T, y = \omega_p/T$, and $\psi = m_\pi/T$, we obtain

$$\begin{aligned}
\epsilon_{\tilde{\gamma}} = &\frac{A_{\tilde{\gamma}}^2 T^7}{2^7 \pi^3 v_\pi^2} \int_\psi^{\sqrt{3/2}\psi} dx \frac{x}{e^x - 1} \\
&\times \left(\frac{\psi^2}{v_\pi^2} - \left(\frac{1-v_\pi^2}{v_\pi^2} \right) x^2 \right)^2 \\
&\times \int_{y_{min}}^{y_{max}} dy \frac{1}{1 + e^{-x} - e^{-y} - e^{y-x}}.
\end{aligned} \tag{36}$$

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[4]Note that the sum of the left and right-handed chiral currents considered in Ref. [47] also leads to Eq. (34).

For the restricted range of y and x above, the exponential factors in the y integral can be dropped, which is equivalent to neglecting Bose enhancement factors. Any possible divergence from this factor is tamed by vanishing matrix elements and phase space. The y integral is then easily performed yielding

$$\epsilon_{\tilde{\gamma}} = \frac{A_{\tilde{\gamma}}^2 T^7}{2^7 \pi^3 v_\pi^3} \int_\psi^{\sqrt{3/2}\psi} dx \, x \, e^{-x} \sqrt{x^2 - \psi^2}$$
$$\times \left(\frac{\psi^2}{v_\pi^2} - \left(\frac{1 - v_\pi^2}{v_\pi^2} \right) x^2 \right)^2. \tag{37}$$

With the substitution $x = \psi \cosh \theta$, the above integral can be expressed as a combination of modified Bessel functions $K_\nu(\psi)$, in the limiting case of $m_\pi/T \gg 1$. As their argument is very large, we can utilize the asymptotic forms of K_ν to obtain the result

$$\epsilon_{\tilde{\gamma}} = \frac{A_{\tilde{\gamma}}^2 m_\pi^4}{64\pi} \frac{1}{v_\pi^3} \left(\frac{m_\pi T}{2\pi} \right)^{3/2} e^{-m_\pi/T} = \frac{A_{\tilde{\gamma}}^2 m_\pi^4}{64\pi} n_\pi. \tag{38}$$

For purposes of numerical estimation, we note that for the typical central densities of $(5-10)n_0$, where n_0 is the nuclear saturation density, the value of the strong coupling g is still much larger than e, so that θ_{CFL} is small. The emissivity can then be expressed as

$$\epsilon_{\tilde{\gamma}} = 5.1 \times 10^{40} m_{10}^4 (m_{10} T_9)^{3/2} \mu_{100}^{-2}$$
$$\times e^{-116 m_{10}/T_9} \text{ erg cm}^{-3} \text{ s}^{-1}. \tag{39}$$

Since thermal pions have an extremely small number density at keV temperatures, the photon emissivity is negligible despite the large cross section for this process.

It is amusing to estimate the mean free path of photons due to the inverse process $\tilde{\gamma} + \tilde{\gamma} \to \pi^0$. For the purpose of illustration only, we consider a thermal distribution of photons. The thermally averaged mean free path may be written as

$$\langle \lambda \rangle = v_{\tilde{\gamma}} \langle \tau \rangle, \tag{40}$$

where the photon velocity is [40]

$$v_{\tilde{\gamma}} = c/\sqrt{1 + \tilde{\kappa}}, \quad \tilde{\kappa} = \frac{\tilde{e}^2}{18\pi^2} \frac{\mu^2}{\Delta^2} \ll 1. \tag{41}$$

Setting $v_{\tilde{\gamma}} \approx c$ and utilizing the thermal average of the inverse rate

$$\frac{1}{\langle \tau \rangle} = \frac{1}{n_{\tilde{\gamma}}} \int \frac{d^3 p \, n(\omega_p)}{(2\pi)^3} \frac{1}{\tau(\omega_p)}, \tag{42}$$

where $1/\tau(\omega_p)$ is the typical inverse lifetime of a photon interacting with other thermal photons to form a π^0 excitation, we find

$$\langle \lambda \rangle = \frac{64 \pi c}{A_{\tilde{\gamma}}^2 m_\pi^3} \frac{n_{\tilde{\gamma}}}{n_\pi}. \tag{43}$$

This result clearly displays the dependences on the vacuum decay rates and on the baryon density (through f_π and m_π in $A_{\tilde{\gamma}}$) and temperature (through $n_{\tilde{\gamma}}$ and n_π). Inserting the expression for the number density of thermal photons, $n_{\tilde{\gamma}} = 2\xi(3)T^3/\pi^2$, into Eq. (42), we obtain

$$\langle \lambda \rangle = 1.89 \times 10^{-7} \mu_{100}^2 T_9^{3/2} m_{10}^{-9/2} e^{116(m_{10}/T_9)} \text{ cm}. \tag{44}$$

For the temperatures of interest, this number is exponentially large. Thus, photons in the CFL phase are extremely unlikely to recombine into a π^0.

Unlike photon-photon interactions, the processes that contribute dominantly to the mean free path are Compton scatterings off charged mesons in the CFL phase. Since photons stem from the decay of thermal pions, most of the photons would have energies well below m_{GB}, since $T \ll m_{GB} < \Delta$. Thus, the Compton cross section may be taken to be adequately represented by the Thomson scattering limit:

$$\sigma_{\tilde{\gamma}\pi} \sim \sigma_T^e \left(\frac{m_e}{m_{GB}} \right)^2 \sim \frac{1}{400} \sigma_T^e,$$

$$\sigma_T^e = \frac{8}{3} \left(\frac{e^2}{m_e c^2} \right)^2 = 66.5 \text{ fm}^2. \tag{45}$$

The mean free path from scattering off charged pions can be estimated as

$$l \simeq \frac{1}{\sigma_{\tilde{\gamma}\pi} n_\pi} \sim 2 \times 10^{40} \text{ km}. \tag{46}$$

Since the density of scatterers, charged Goldstone bosons, is exponentially small, the mean free path is very large. Hence, at low temperatures, the CFL phase can be regarded as being transparent to photons [12] even in the presence of Goldstone bosons.

The mean free path of a photon is essentially the size of the CFL quark core of the neutron star, since photons equilibrate rapidly by interacting with the surrounding ordinary matter. For normal nuclear matter at a density $n_B = 3n_0$, with $n_0 = 0.16$ fm^{-3}, a similar estimate as above yields

$$l' \simeq \frac{1}{\sigma_T^B n_B} \sim 10^{-8} \text{ cm}. \tag{47}$$

We conclude that both photon emissivities and opacities in the CFL phase are negligible at the temperatures relevant for the long-term cooling of a neutron star.

IV. SPECIFIC HEAT OF CFL MATTER

In order to determine the cooling history of a neutron star, we also need the specific heat of CFL matter. The specific heat of ordinary quark matter is dominated by the quark contribution

$$c_q = \sum_{f=u,d,s} \pi^2 n_f \left(\frac{k_B T}{\mu_f} \right)$$

$$= 2.44 \times 10^{20} T_9 \left(\frac{n_B}{n_0} \right)^{2/3} \text{ erg cm}^{-3} \text{ K}^{-1}, \quad (48)$$

where for simplicity all quarks have been taken as massless and thus μ_f is the same for all quark flavors.

The gluon contribution is

$$c_g = N_g \frac{4\pi^2}{15} T^3 = 3N_g \times 10^{13} T_9^3 \text{ erg cm}^{-3} \text{ K}^{-1}, \quad (49)$$

with $N_g = N_c^2 - 1 = 8$. The photon contribution is identical to the gluon contribution with $N_\gamma = 1$.

In ordinary quark matter, there is a nonzero electron chemical potential. As a result, electrons provide a significant contribution to the specific heat:

$$c_e = \pi^2 n_e \left(\frac{k_B T}{\mu_e} \right)$$

$$= 0.56 \times 10^{20} T_9 \left(\frac{Y_e n_B}{n_0} \right)^{2/3} \text{ erg cm}^{-3} \text{ K}^{-1}. \quad (50)$$

In CFL quark matter, the contributions from quarks are suppressed by $\exp(-\Delta/(k_B T))$, where Δ is the gap in the appropriate channel. In addition, $Y_e \ll 1$ in the CFL phase. Note, however, that at $T_9 = 1, c_e > c_g + c_\gamma$ till $Y_e = 10^{-9}$.

The specific heat of CFL matter receives significant contributions from the Goldstone modes. The contribution of massive thermal bosons (that obey the dispersion relation $\omega^2 = v_\pi^2 k^2 + m^2$) in the CFL phase is modified by the factor $1/v_\pi^3$ relative to the case with $v_\pi = 1$. For example, for pions in the nondegenerate limit, we have

$$c_{GB}(m \neq 0) = \frac{3}{2} n_\pi \left(1 + \frac{5}{2} \frac{T}{m} + \cdots \right)$$

$$= 7.12 \times 10^{15} (m_{10} T_9)^{3/2}$$

$$\times e^{-116 m_{10}/T_9} \text{ erg cm}^{-3} \text{ K}^{-1}. \quad (51)$$

The contribution of a Goldstone mode with dispersion relation $\omega_{GB} = v_\pi k$ is also modified by a factor $1/v_\pi^3$ over that of a massless mode with $\omega = k$. With $v_\pi = 1/\sqrt{3}$, we find

$$c_{GB} = \frac{6\sqrt{3}\pi^2}{15} T^3 = 7.8 \times 10^{13} T_9^3 \text{ erg cm}^{-3} \text{ K}^{-1}. \quad (52)$$

At temperatures on the order of several MeV, there are 10 almost massless Goldstone modes. When the temperature drops below 1 MeV, the specific heat of CFL matter is dominated by the exactly massless $U(1)_B$ Goldstone boson. If CFL matter is kaon condensed, then there are two massless Goldstone bosons.

V. PRINCIPAL FINDINGS

We have studied neutrino emission from the decay and scattering processes of Goldstone modes in high density quark matter in the superconducting color-flavor locked (CFL) phase. Such a phase might occur in the core of a neutron star whose mass is near its maximum allowed value.

The neutrino emissivities from the new processes that we have identified scale as

$$\pi^\pm \rightarrow l^\pm + \bar{\nu}_l: \quad \epsilon_\pi \sim (G_F^2 f_\pi^2 m_e^2) m_\pi^2 n_\pi \left(1 + 2 \left(\frac{\delta f_\pi}{f_\pi} \right) + \frac{2m_\pi T}{v_\pi^2 m_e^2} \left(\frac{\delta f_\pi}{f_\pi} \right)^2 \right)$$

$$K^\pm \rightarrow l^\pm + \bar{\nu}_l: \quad \epsilon_{K^\pm} \sim (\sin^2\theta_C G_F^2 f_\pi^2 m_e^2) E_{K^\pm}^2 n_{K^\pm} \left(1 + 2 \left(\frac{\delta f_\pi}{f_\pi} \pm \frac{\mu_{eff}}{E_{K^\pm}} \right) + \frac{2m_{K^\pm} T}{v_K^2 m_e^2} \left(\frac{\delta f_\pi}{f_\pi} \pm \frac{\mu_{eff}}{E_{K^\pm}} \right)^2 \right)$$

$$\pi^0(\eta, \eta') \rightarrow \nu + \bar{\nu}: \quad \epsilon_{\nu\nu} \sim (G_F^2 m_\pi^2 \delta f_\pi^2) T m_\pi n_\pi$$

$$\phi + \phi \rightarrow \phi + \nu + \bar{\nu}: \quad \epsilon_{GB} \sim \frac{G_F^2}{f^2 \mu^4} T^{15}, \quad (53)$$

where $\delta f_\pi = f_T - f_S$ with $f_S = f_\pi/3$, and n_π and n_K are the number densities of pions and kaons, respectively. Since the masses of these Goldstone bosons are expected to be of order 10 MeV and $T \lesssim 0.1$ MeV, these Goldstone bosons are nondegenerate and obey Boltzmann statistics. Hence, their number densities scale as $(mT)^{3/2} \exp(-m/T)$. This is in contrast to quarks, which are degenerate since $\mu_B/T \gg 1$. However, emissivities from gapped quarks are suppressed by a factor $\exp(-\Delta/T)$, with the gap Δ being of order 100 MeV in the CFL phase. The hierarchy of physical scales, $\mu_B > \Delta > m_{GB} > T$ (for the most part, we have taken $\mu_e = 0$ in the CFL phase) thus assures that the neutrino emissivities from massive Goldstone modes dominate over those from gapped quarks.

The unique feature of the $\pi^0 \rightarrow \nu + \bar{\nu}$ process is that it requires Lorentz symmetry breaking, which manifests itself in the difference of the spatial and temporal pion decay constants, f_S and f_T. This emissivity is thus proportional to

δf_π^2. In vacuum, helicity conservation forbids such a decay, but in matter where boost invariance is lost, this process is allowed for all nonzero pion momenta.

The emissivities from the massive Goldstone modes are expressible in terms of simple products interaction strength × relevant energies × the number density of thermal bosons, mainly because bosons are nondegenerate. Such simplicity is lost when these modes exist in partially degenerate or degenerate conditions because of Bose enhancement effects. It must be stressed that the emissivities from the massive Goldstone modes, even though they dominate those from gapped quarks, remain rather small, chiefly because of their very small number densities at the temperatures of relevance to long-term cooling. For example, at $T_9 = 1, n_\pi \sim 3.5 \times 10^{-58}$ fm^{-3}.

Neutrino emission from the scattering of massless Goldstone modes (numbering one in the CFL phase and two in the kaon condensed phase) is not exponentially penalized. However, this process appears only at $\mathcal{O}(p^4)$ in a chiral effective Lagrangian, which leads to a numerically small emissivity. Nevertheless, it is likely to be the principal source of neutrino emission from the CFL phase.

We have also calculated the photon emissivity from the CFL phase. The anomalous decay process $\pi^0 \to \tilde{\gamma} + \tilde{\gamma}$ yields an intuitive expression for the emissivity, $\epsilon_{\tilde{\gamma}} = (A_\gamma^2 m_\pi^4 / 64\pi) n_\pi$, when $m_\pi \gg T$. This emissivity may be readily understood as the product of the energy per unit time carried by the photons times the number density of thermal pions from which they are emitted. Numerically, the emissivity is exponentially small, not because the cross-section for the anomalous process is small, but because, at $T_9 = 1$, the thermal pion density is exceedingly small. Our estimation of the photon mean free path, due to Compton scattering in the Thomson limit, shows that the CFL phase is transparent to photons even in the presence of Goldstone bosons. However, photons are thermalized quickly in the high density normal baryonic matter that surrounds the CFL phase.

Since electrons are strictly absent in bulk CFL matter, its specific heat per unit volume c_V is dominated by the massless $U(1)$ mode associated with superfluidity (two such modes exist for the kaon condensed phase). This bosonic excitation (ϕ) is relatively easier to excite than all other massive Goldstone modes, which are exponentially suppressed by thermal occupation factors. Thermal gradients arising in the CFL phase are rapidly reduced due to the large thermal conductivity, which is a consequence of small cross sections for $\phi\phi$ scattering as well as their low number density. As expected, the transport properties of the CFL phase are controlled essentially by its superfluid nature.

VI. IMPLICATIONS FOR THE COOLING OF COMPACT STARS

Our results for the neutrino emissivity in the CFL phase lead naturally to the issue of their efficiency in cooling the *interior* of the star. The thermal evolution is determined by the equations of radiative transport and energy balance which may be cast to read as [49]

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 K \frac{\partial T}{\partial r}\right) = c_V \frac{\partial T}{\partial t} = -(\epsilon + H), \quad (54)$$

where T is the temperature, r is the radial coordinate (general relativistic corrections have been suppressed for simplicity), K is the thermal conductivity, and c_V is the total specific heat per unit volume. The total emissivity due to photons and neutrinos is denoted by ϵ and possible internal heating sources are contained in H. We can estimate the cooling time scales Δt of the CFL phase in isolation with the expression

$$\Delta t = -\int_{T_i}^{T_f} dT \frac{c_V}{\epsilon}. \quad (55)$$

In a strictly electron-free CFL phase, the largest contributions to the specific heat comes from those of gluons, photons, and the massless Goldstone modes, since those of the massive Goldstone modes are severely suppressed. For the same reason, the emissivity is dominated by the scattering of massless Goldstone modes. Using $c_V \simeq 3.5 \times 10^{14} T_9$ erg cm^{-3} K^{-1} and $\epsilon \simeq 10^{-11} \times T_9^{15}$ erg cm^{-3} s^{-1} from Secs. II and III, we find that $\Delta t \sim 10^{26} T_9^{-11} \mu_{100}^6$ y. This time scale is extremely long

Note that, were the massive Goldstone bosons to operate in isolation, the cooling time would be vastly different. Using

$$c_V \simeq 7.2 \times 10^{15} (m_{10} T_9)^{3/2}$$
$$\times \exp(-116 m_{10}/T_9) \text{ erg cm}^{-3} \text{ K}^{-1}$$

and

$$\epsilon \simeq 1.2 \times T_9^{28} \mu_{100}^2 m_{10}^2 (m_{10} T_9)^{3/2}$$
$$\times \exp(-116 m_{10}/T_9) \text{ erg cm}^{-3} \text{ s}^{-1},$$

we obtain

$$\Delta t = 6 \times 10^{-4} \times (T_9(i) - T_9(f)) \mu_{100}^{-2} m_{10}^{-2} \text{ s}.$$

In this case, c_v / ϵ becomes independent of T, which leads to a cooling rate that is linear in T, and also exceedingly rapid.

The CFL phase is characterized by a very large thermal conductivity due to the smallness of both the Goldstone boson number densities and cross sections in CFL matter. The temperature gradient in the CFL core will thus be very small, and the CFL core will be nearly isothermal, with its temperature pinned to that of the surrounding hadronic matter [this is easily verified by seeking separable solutions to Eq. (54)]. In the realistic case in which a neutron star has a small CFL core, the thermal content and the emissivity of the CFL core will be negligible compared to those of the surrounding

hadronic matter.[5] The picture that then emerges is that such a star will cool nearly identically to a star without a CFL core. The presence of a CFL phase in dense matter will apparently leave no observable trace in the long-term cooling history of the star.

The interface between electron-rich nuclear matter and electron-deficient CFL quark matter was analyzed in Ref. [34] with the conclusion that a thin charged layer (~ 10 fm) separates the two bulk phases if the surface energy σ_s is large (dimensional estimates place $\sigma_s \sim 400$ MeV, which is too costly for a mixed phase). In this case, electrons can leak in to the CFL phase. Owing to the partially inverted mass spectrum of Goldstone bosons in the CFL phase (in particular, $m_K < m_\pi$), electrons can decay into kaons through electroweak interactions near the CFL side of the interface, resulting in neutrino emission and the formation of a K^- condensate. This causes the proton concentration on the nuclear side of the interface to rise substantially in order to maintain charge neutrality. Consequently, there is the possibility that, within this very thin layer, the direct Urca process $n \rightarrow p + e^- + \bar{\nu}$ would occur as long as the proton fraction is of order 11–14 % for which momentum conservation between the participating fermions becomes possible [50]. Inasmuch as this thin interface is embedded within the star, the occurence of a such a rapid cooling process would be masked by cooling from the much thicker exterior layers in which similar processes are also possible. From an observational standpoint, such cooling would resemble cooling from normal stars.

In Ref. [34], the phase structure of baryon and CFL quark matter was also studied as a function of the magnitude of σ_s. For a sufficiently small σ_s, a homogeneous mixed phase with domains of CFL quark matter and nuclear matter was found to be preferred over a wide range of density. In this case, neutrino emission would be dominated by the nuclear bubbles which can cool via the rapid processes characteristic of the nuclear phase (see footnote 3).

Recently, the cooling of a self-bound quark star with a bare surface has been studied in Ref. [52] by considering

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[5]In baryonic matter, the simplest possible ν emitting processes are the direct Urca processes $f_1 + \ell \rightarrow f_2 + \nu_\ell, f_2 \rightarrow f_1 + \ell + \bar{\nu}_\ell$, where f_1 and f_2 are baryons and ℓ is either an electron or a muon. These processes can occur whenever momentum conservation is satisfied among f_1, f_2 and ℓ. If the unsuppressed direct Urca process for *any* component occurs, a neutron star will rapidly cool because of large energy losses due to neutrino emission: the star's interior temperature T will drop below 10^9 K in minutes and reach 10^7 K in about a hundred years. This is the so-called rapid cooling paradigm [50,51]. If no direct Urca processes are allowed, or they are all suppressed due to baryon superfluidity (with gaps of order 1 MeV), cooling instead proceeds through the significantly less rapid modified Urca process in which an additional fermion enables momentum conservation. This situation could occur if no hyperons are present, or the nuclear symmetry energy has a weak density dependence [50,51]. Comparatively less rapid, but faster than that due to modified Urca cooling occurs in the presence of Bose condensates [4].

both the CFL and 2SC phases. If the entire star is in the CFL phase, which permits no electrons, cooling occurs chiefly through processes that involve the Goldstone modes. As noted above, the cooling times for this case are too long to be observable. If the CFL phase is surrounded by the 2SC phase at lower densities, cooling is dominated by neutrino emission from direct Urca processes involving unpaired quarks in the 2SC phase (which allows electrons to exist) and by the annihilation of $e^+ e^-$ pairs produced by an intense electrical field which binds electrons to the surface of quark matter. This is essentially the Schwinger mechanism in the presence of a Fermi sea of electrons, and can occur only at finite temperature [53]. Beginning at a temperature of $\sim 10^{11}$ K (~ 10 MeV) down to $\sim 10^8$ K, the temperature versus age curve for the case with the 2SC phase in which gaps are of order 100 MeV differs little from that of the normal phase, chiefly because pairing reduces neutrino emission and the specific heat by similar amounts. Substantial differences from the cooling of the normal phase occur, however, if quarks not participating in 2SC pairing also pair through residual interactions, but with gaps of order 1 MeV. In this case, the hard x-ray spectrum with a mean energy of $\sim 10^2$ keV is proffered as an observational signature of bare quark stars.

In this work, we have focused on the long-term cooling epoch when most neutrinos have left the star and temperatures are in the range of hundreds of keV to tens of eV. In a proto-neutron star, neutrinos are trapped and temperatures range from 20–40 MeV. The CFL phase is likely to appear during the PNS evolution. During this stage, $m_{GB}/T \lesssim 1$ and hence the Goldstone modes are in the partially degenerate or degenerate regime. As $m_{GB}/T \ll \Delta/T$, both neutral and charged current neutrino interactions with the Goldstone bosons offer new sources of opacity. Some examples of possible processes are $\nu(\bar{\nu}) + \pi^\pm \rightarrow \nu(\bar{\nu}) + \pi^\pm$, $\nu + \pi^- \rightarrow e^- + \pi^0$, $\bar{\nu} + \pi^+ \rightarrow e^+ + \pi^0$ and $\nu(\bar{\nu}) + \pi^0 \rightarrow e^{\mp} + \pi^\pm$. Note that, with fast neutrinos, the processes $\nu \rightarrow \pi^0 + \nu$ and $\bar{\nu} \rightarrow \pi^+ + e^-$ also become allowed and contribute to the opacity. The resultant neutrino opacities are expected to be significantly larger than those from scattering and absorption on gapped quarks, since temperatures are high enough to admit large pion densities. For example, in the range $20 < T/\text{MeV} < 40$, the number densities of a given species of a Goldstone mode whose mass lies in the range $0 < m_{GB}/\text{MeV} < 10$ are $n_{GB} \sim 10^{-3} - 10^{-4}$ fm^{-3}, substantially more than those encountered in the long-term cooling epoch.

Note added. (a) Concurrently and independently of our work, Reddy, Sadzikowsky, and Tachibana [54] have performed a detailed study of ν-opacities and emissivities in the CFL phase at tens of MeV temperatures encountered in the evolution of a protoneutron star. Their quantitative results underscore the important role of Goldstone boson excitations in the CFL phase and provide the groundwork for calculations of observable neutrino luminosities to be performed. (b) Shovkovy and Ellis [55] have performed a calculation of the thermal conductivity in the CFL phase at temperatures of relevance to long-term cooling. They find that the dominant contribution to the conductivity comes from photons and

Goldstone bosons. This result is a necessary ingredient in quantitative calculations of a compact star's long-term cooling. (c) Page and Usov [52] have calculated the thermal evolution and light curves of young bare (self-bound) quark stars considering both the 2SC and CFL phases. They find that the energy gap of superconducting quark matter may be estimated from the light curves if it is in the range ~ 0.5 MeV to a few MeV.

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