

$B \rightarrow D_s \pi$ and the tree amplitude in $B \rightarrow \pi^+ \pi^-$

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The recently observed decay $B^0 \rightarrow D_s^+ \pi^-$ is expected to proceed mainly by means of a tree amplitude in the factorization limit: $B^0 \rightarrow \pi^- (W^+)^*$, $(W^+)^* \rightarrow D_s^+$. Under this assumption, we predict the corresponding contribution of the tree amplitude to $B^0 \rightarrow \pi^+ \pi^-$. We indicate the needed improvements in data that will allow a useful estimate of this amplitude with errors comparable to those accompanying other methods. Since the factorization hypothesis for this process goes beyond that proved in most approaches, we also discuss independent tests of this hypothesis.

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The two-body hadronic decay process $B^0 \rightarrow \pi^+ \pi^-$ has been of great interest for a long time in the search for CP violation in B decays. Its branching ratio, smaller than one typically estimates on the basis of factorization and dominance of the tree-level amplitude T , may owe some suppression to destructive interference between T and the penguin amplitude P [1–3]. This interference could provide information on both the weak phase $\alpha = \phi_2$ and the relative strong phase of the tree and penguin amplitudes. Both quantities are helpful in testing the current picture of CP violation based on phases in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. However, to answer the question of tree-penguin interference in $B^0 \rightarrow \pi^+ \pi^-$ requires improved knowledge of $|T|$ and $|P|$. Since the tree amplitude is the dominant contribution to $B^0 \rightarrow \pi^+ \pi^-$, better knowledge of its magnitude is a key step toward such an improvement.

Within the factorization framework, if one simply takes form factor models and computes the tree level amplitude of $B^0 \rightarrow \pi^+ \pi^-$, a significant error will be obtained because of the large uncertainties in the form factor at large recoil, $F_0(q^2 \rightarrow 0)$, and in $|V_{ub}|$. Both of them have an error about 25%, resulting in an error of more than 35% on $|T|$.

In this Brief Report, we use the newly measured mode $B^0 \rightarrow D_s^+ \pi^-$ [4,5] to estimate $\Gamma_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$. The uncertainty can be reduced because in the ratio of $\Gamma_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)/\Gamma(B^0 \rightarrow D_s^+ \pi^-)$ the dominant error comes from the weak decay constant of D_s . Within the next two years, the CLEO-c program is expected to substantially improve the accuracy on various charm sector parameter measurements, including f_{D_s} . Therefore, we propose an alternative method to determine T for the $B^0 \rightarrow \pi^+ \pi^-$ decay. This method generally relies on a simple assumption about the pole structure of the relevant $B \rightarrow \pi$ form factor to relate these two processes at small and large recoil. The same

method can be applied to determining T_P for $B^0 \rightarrow \rho^+ \pi^-$, where the subscript P indicates that the spectator quark goes into a pseudoscalar meson in the final state.

The $B \rightarrow \pi$ weak transition matrix element is conventionally parametrized in the following way by two independent form factors:

$$\langle \pi(p) | \bar{u} \gamma_\mu b | B(p+q) \rangle = \left(2p + q - q \frac{m_B^2 - m_\pi^2}{q^2} \right)_\mu F_+(q^2) + q_\mu \frac{m_B^2 - m_\pi^2}{q^2} F_0(q^2). \quad (1)$$

Assuming factorization, the decay widths of $B^0 \rightarrow D_s^+ \pi^-$ and $B^0 \rightarrow \pi^+ \pi^-$ (tree) decay (as shown in Fig. 1) are given by

$$\Gamma(B^0 \rightarrow D_s^+ \pi^-) = \frac{G_F^2}{32\pi} |V_{ub}^* V_{cs}|^2 f_{D_s}^2 m_B \left(1 - \frac{m_\pi^2}{m_B^2} \right)^2 \times \lambda(m_B^2, m_{D_s}^2, m_\pi^2) a_1^2 |F_0(m_{D_s}^2)|^2, \quad (2)$$

$$\Gamma_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-) = \frac{G_F^2}{32\pi} |V_{ub}^* V_{ud}|^2 f_\pi^2 m_B \left(1 - \frac{m_\pi^2}{m_B^2} \right)^2 \times \lambda(m_B^2, m_\pi^2, m_\pi^2) a_1^2 |F_0(m_\pi^2)|^2, \quad (3)$$

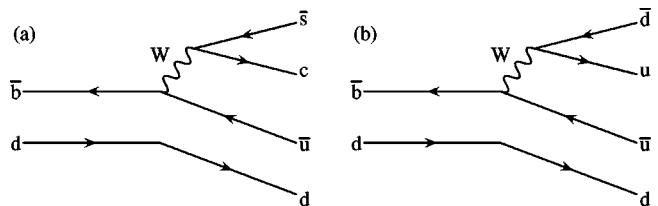


FIG. 1. Feynman diagrams for tree decays of a B^0 meson to $D_s^+ \pi^-$ and $\pi^+ \pi^-$.

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where $\lambda(a,b,c) \equiv \sqrt{a^2+b^2+c^2-2ab-2ac-2bc}$ and $a_1 \simeq 1$ is the Wilson coefficient. Note that only $F_0(q^2)$ contributes in these two decay modes. To illustrate our method, we will use the form factor model proposed in [6], where $F_0(q^2)$ has the following single pole structure:

$$F_0(q^2) = \frac{c_B(1-\alpha_B)}{1-q^2/(\beta_B m_{B^*}^2)}. \quad (4)$$

A lattice calculation by Abada *et al.* [7] gives $F_0(0) = c_B(1-\alpha_B) = 0.26 \pm 0.05 \pm 0.04$ and $\beta_B = 1.22 \pm 0.14^{+0.12}_{-0.03}$.

Let us define the ratio

$$\begin{aligned} \xi_B &\equiv \frac{\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)}{\mathcal{B}(B^0 \rightarrow D_s^+ \pi^-)} \\ &= \frac{\lambda(m_B^2, m_\pi^2, m_\pi^2)}{\lambda(m_B^2, m_{D_s}^2, m_\pi^2)} \left[\frac{|V_{ud}|}{|V_{cs}|} \frac{f_\pi F_0(m_\pi^2)}{f_{D_s} F_0(m_{D_s}^2)} \right]^2. \end{aligned} \quad (5)$$

In this ratio, the dependence upon $F_0(q^2=0)$ and $|V_{ub}|$ disappears and, therefore, some large sources of uncertainty are avoided. Neglecting the errors on meson masses, the pion decay constant, $f_\pi = 131$ MeV, and the CKM matrix elements (taking $|V_{ud}| = |V_{cs}|$ as suggested by unitarity), one sees that the major error in ξ_B comes from those of f_{D_s} and β_B . In comparison with the error from β_B , which is given by the lattice determination as mentioned earlier, a good portion of the uncertainty in the ratio ξ_B resides in the experimental determination of the D_s decay constant.

Current experimental determination of f_{D_s} uses the hadronic decay mode $D_s^+ \rightarrow \phi \pi^+$ as a ‘‘standard candle’’ and measures the ratio of $\mathcal{B}(D_s^+ \rightarrow \ell^+ \nu_\ell) / \mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$. Therefore, the systematic error is dominated by the knowledge of $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$, which has a 25% error [8]. Based on an experimental average of rates for $D_s \rightarrow \mu \nu$ and $D_s \rightarrow \tau \nu$ [9], we will use $f_{D_s} = (270 \pm 16) \sqrt{\mathcal{B}(D_s^+ \rightarrow \phi \pi^+) / 3.6\%}$ MeV for our numerical calculation, where the error is purely statistical. Here we single out the systematic error accompanying the $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$ mode. We will discuss its impact on the precision determination of $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$.

We first take the current value $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+) = (3.6 \pm 0.9)\%$ [8]. Since $f_{D_s}^2$ is proportional to the ratio $\mathcal{B}(D_s^+ \rightarrow \mu^+ (\tau^+) \nu) / \mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$, we predict

$$\xi_B = (0.216 \pm 0.027) \left[\frac{3.6\%}{\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)} \right], \quad (6)$$

where we have combined the statistical error from f_{D_s} and the error from β_B , leaving the systematic error of f_{D_s} in the square brackets. Although β_B has an error of $\sim 13\%$ by itself, it only results in a $\sim 3\%$ error in ξ_B . The statistical error of f_{D_s} , on the other hand, gives a dominant $\sim 12\%$ error.

The BaBar Collaboration [4,5] has recently measured the product

$$\begin{aligned} &\mathcal{B}(B^0 \rightarrow D_s^+ \pi^-) \times \mathcal{B}(D_s^+ \rightarrow \phi \pi^+) \\ &= (1.11 \pm 0.37 \pm 0.22) \times 10^{-6} \end{aligned} \quad (7)$$

based on a data sample of 56.4 fb^{-1} at $Y(4S)$ resonance, where the first error is statistical and the second is systematic. Using Eq. (6), we immediately have

$$\begin{aligned} &\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-) \\ &= 6.7(1 \pm 0.41) \times 10^{-6} \left[\frac{3.6\%}{\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)} \right]^2. \end{aligned} \quad (8)$$

Adding all the errors in quadrature, including that in $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$, we obtain

$$\begin{aligned} &\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-) = 6.7(1 \pm 0.64) \times 10^{-6}, \\ &|T| = 2.6(1 \pm 0.32) \times 10^{-3}. \end{aligned} \quad (9)$$

This is in good agreement with the values obtained in [10] and [11]. As stated before, direct calculation from Eq. (3) including the errors from $|V_{ub}|$ and $F_0(0)$ will have an uncertainty in the branching ratio at least as big as 70%, which would render the information useless.

As is obvious from the above analysis, the accuracy on the branching ratio of $D_s^+ \rightarrow \phi \pi^+$ plays a crucial role in the determination of $|T|$. It is thus of great importance to lower its error. The CLEO Collaboration proposes to explore the charm sector starting early 2003. CLEO-c [12] will be able to reach an accuracy of 1.9% on $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$ and in turn 1.7% on f_{D_s} . This will improve our determination of $|T|$ considerably. Moreover, if the data are enlarged from the current 56.4 fb^{-1} sample at BaBar to a combined BaBar and Belle sample of 300 fb^{-1} , one expects to be able to bring down the statistical error on $\mathcal{B}(B^0 \rightarrow D_s^+ \pi^-) \times \mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$ by a factor of ~ 2.3 . With such reduced errors on f_{D_s} and statistical error on the branching ratio product, our knowledge of $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-)$ can be improved to give

$$\begin{aligned} &\mathcal{B}_{\text{tree}}(B^0 \rightarrow \pi^+ \pi^-) = 6.7(1 \pm 0.25) \times 10^{-6}, \\ &|T| = 2.6(1 \pm 0.13) \times 10^{-3}. \end{aligned} \quad (10)$$

Now the error is dominated by the uncertainty in $\mathcal{B}(B^0 \rightarrow D_s^+ \pi^-) \times \mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$. Aside from reducing the statistical error as mentioned before, it is also possible to reduce the systematic error by, for example, improving the tagging techniques.

The anticipated error in Eq. (10) is not as good as that (about 5%) foreseen in Ref. [10] on the basis of forthcoming studies of $B \rightarrow \pi \ell \nu$. Instead, it provides a cross-check of the factorization hypothesis for the case in which the weak current produces a D_s . Present attempts to justify that hypothesis (see, e.g., [13]) do not expect it to be valid when the weak current produces such a heavy color-singlet meson. If we take the central values for the parameters appearing in Eq. (2), however, we obtain $\mathcal{B}(B^0 \rightarrow D_s^+ \pi^-) \simeq 2.9 \times 10^{-5} (|V_{ub}|/0.0036)^2$, consistent with the result presented in Refs. [4,5]. Therefore, current data do not indicate any

breakdown of factorization for D_s or D_s^* production by the weak current, but more conclusive tests are needed [14].

The above method can be similarly applied to the determination of the tree amplitude T_P in the $B^0 \rightarrow \rho^+ \pi^-$ decay using the experimental branching ratio of $B^0 \rightarrow D_s^+ \pi^-$. Using the same notation introduced before,

$$\Gamma_{\text{tree}}(B^0 \rightarrow \rho^+ \pi^-) = \frac{G_F^2}{32\pi} |V_{ub}^* V_{ud}|^2 f_\rho^2 \left(\frac{\lambda(m_B^2, m_\rho^2, m_\pi^2)}{m_B} \right)^3 \times a_1^2 |F_+(m_\rho^2)|^2, \quad (11)$$

where $f_\rho = 208$ MeV (see, for example, Ref. [15]). We consider an analogous ratio

$$\xi_B' \equiv \frac{\mathcal{B}_{\text{tree}}(B^0 \rightarrow \rho^+ \pi^-)}{\mathcal{B}(B^0 \rightarrow D_s^+ \pi^-)} = \frac{\lambda(m_B^2, m_\rho^2, m_\pi^2)^3}{m_B^4 \lambda(m_B^2, m_{D_s}^2, m_\pi^2)} \left(1 - \frac{m_\pi^2}{m_B^2} \right)^{-2} \times \left[\frac{|V_{ud}|}{|V_{cs}|} \frac{f_\rho F_+(m_\rho^2)}{f_{D_s} F_0(m_{D_s}^2)} \right]^2. \quad (12)$$

This ratio generally involves additional model dependence because of $F_+(q^2)$. Reference [7] suggests the following parametrization:

$$F_+(q^2) = \frac{c_B(1 - \alpha_B)}{(1 - q^2/m_{B^*}^2)(1 - \alpha_B q^2/m_{B^*}^2)}, \quad (13)$$

where α_B has a value of $0.40 \pm 0.15 \pm 0.09$. Again, $F_+(0)$ cancels with $F_0(0)$ in the ratio in Eq. (12) and we find

$$\xi_B' = (0.541 \pm 0.018 \pm 0.004) \left[1 \pm \left(\frac{\Delta \mathcal{B}(D_s \rightarrow \ell \nu)}{\mathcal{B}(D_s \rightarrow \ell \nu)} \right) \right] \times \left[\frac{3.6\%}{\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)} \right], \quad (14)$$

where the first error comes from β_B , the second error comes from α_B , and we have taken the central value of f_{D_s} mentioned previously. Considering the same physics reach at CLEO-c and BaBar discussed in the previous section, we obtain

$$\mathcal{B}_{\text{tree}}(B^0 \rightarrow \rho^+ \pi^-) = 16.7(1 \pm 0.25) \times 10^{-6}, \\ |T_P| = 4.1(1 \pm 0.13) \times 10^{-3}, \quad (15)$$

where the latter number agrees well with the estimate given in Ref. [15]. One may also contemplate estimating the above quantities using the ratio $\mathcal{B}_{\text{tree}}(B^0 \rightarrow \rho^+ \pi^-)/\mathcal{B}(B^0 \rightarrow D_s^{*+} \pi^-)$. This has the advantage that the dependence on β_B disappears because both of the VP modes involve only the form factor F_+ . Although it is not observed yet, the branching ratio of $B^0 \rightarrow D_s^{*+} \pi^-$ is estimated to be of the same order as that of $B^0 \rightarrow D_s^+ \pi^-$, except that it will have a bigger error due to the γ detection efficiency in D_s^{*+} decay. Currently, the BaBar group observes a 2.2σ hint of the decay and sets an upper limit $\mathcal{B}(B^0 \rightarrow D_s^{*+} \pi^-) < 4.3 \times 10^{-5}$ at 90% confidence level [5]. Nevertheless, a measurement of the former mode will still serve as a useful check.

We have shown in this Brief Report that within large experimental uncertainties the present measurement of the branching ratio for $B^0 \rightarrow D_s^+ \pi^-$ is compatible with the factorization hypothesis for production of the heavy meson D_s^+ by the weak current. Improvements in data [particularly in the knowledge of $\mathcal{B}(D_s^+ \rightarrow \phi \pi^+)$] are pinpointed which will permit a more conclusive test of this hypothesis. It is also shown how observation of the decay $B^0 \rightarrow D_s^{*+} \pi^-$ can provide a value of the tree amplitude in $B^0 \rightarrow \rho^+ \pi^-$ which can be compared with that obtained through other means (see, e.g., Ref. [15]) to further test factorization in this unexpected domain of its validity.

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