Interpreting experimental bounds on $D^0 - \overline{D^0}$ mixing in the presence of *CP* violation

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We analyze the most recent experimental data regarding $D^0 - \overline{D^0}$ mixing, allowing for *CP* violation. We focus on the dispersive part of the mixing amplitude, M_{12}^D , which is sensitive to new physics contributions. We obtain a constraint on the mixing amplitude: $|M_{12}^D| \le 6.2 \times 10^{-11}$ MeV at 95% C.L. This constraint is weaker by a factor of about three than the one which is obtained when no *CP* violation is assumed.

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I. INTRODUCTION

The ongoing searches for $D^0 \cdot \overline{D^0}$ mixing [1-8] have not yet detected a signal of such mixing. Thus, the experimental data place an upper bound on the mixing amplitude. The value of this upper bound, however, depends on the assumptions one makes when analyzing the experimental results. Specifically, the question of *CP* violation in $D^0 \cdot \overline{D^0}$ mixing has an important impact on the final answer. Most often, $D^0 \cdot \overline{D^0}$ mixing experiments are analyzed assuming no *CP* violation. While this assumption is valid for the standard model, it does not hold for many new physics models.¹ (See, for example, the supersymmetric models in [10,11].) Obviously, if the constraint on $D^0 \cdot \overline{D^0}$ mixing is to be used to test such new physics models, the experimental data should be interpreted in an appropriate framework [12].

The analysis consists of two steps which are potentially sensitive to *CP* violation. First, the expressions for the time dependent decay rates that are fitted to the data [Eqs. (13)–(16) below] have to allow for *CP* violation. Previous works have emphasized this aspect [12,13] and we update and expand their analysis. Second, the constraints on the quantity that can be predicted from theoretical models, the mixing amplitude $|M_{12}^D|$, have to be extracted from the data taking into account *CP* violation [Eq. (21) below]. This step is new here.

The organization of this work is as follows: In Sec. II we present our formalism. We review the most recent experimental data in Sec. III, and perform the analysis in Sec. IV. We conclude in Sec. V.

II. NOTATION AND FORMALISM

We follow mostly the formalism of Ref. [13]. The mass eigenstates are given by

$$|D_{1,2}\rangle = p|D^0\rangle \pm q|D^0\rangle. \tag{1}$$

The mass and the width differences are parametrized as follows:

$$x \equiv \frac{m_2 - m_1}{\Gamma}, \quad y \equiv \frac{\Gamma_2 - \Gamma_1}{2\Gamma}, \tag{2}$$

with the average mass and width defined as

$$m \equiv \frac{m_1 + m_2}{2}, \quad \Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2}.$$
 (3)

We define the D^0 and $\overline{D^0}$ decay amplitudes by

$$A_f \equiv \langle f | \mathcal{H}_d | D^0 \rangle, \quad \bar{A}_f \equiv \langle f | \mathcal{H}_d | D^0 \rangle, \tag{4}$$

and the complex observable λ_f as

$$\lambda_f \equiv \frac{q}{p} \frac{\bar{A}_f}{A_f}.$$
(5)

In almost all new physics models, the relevant decay processes are dominated by *W*-mediated tree level transitions. Hence, unlike $D^0 - \overline{D^0}$ mixing, where new physics can easily saturate the upper bound, the new physics contribution to the $\Delta c = 1$ processes can be safely neglected [13,14] (for an exception, however, see [15]). We therefore assume in our analysis

$$A_f = \overline{A}_{\overline{f}}.\tag{6}$$

Now we can parametrize the effects of indirect *CP* violation in the relevant decay processes: The doubly Cabibbosuppressed (DCS) $D^0 \rightarrow K^+ \pi^-$, the singly Cabibbosuppressed (SCS) $D^0 \rightarrow K^+ K^-$, the Cabibbo-favored (CF) $D^0 \rightarrow K^- \pi^+$, and the three conjugate processes. We denote²

$$|q/p|^2 = 1 + 2A_m, (7)$$

$$\lambda_{K^{+}\pi^{-}}^{-1} = \sqrt{R_{D}} (1 - A_{m}) e^{-i(\delta + \phi)}, \qquad (8)$$

$$\lambda_{K^-\pi^+} = \sqrt{R_D} (1 + A_m) e^{-i(\delta - \phi)}, \qquad (9)$$

$$\lambda_{K^+K^-} = -(1 + A_m)e^{i\phi}, \tag{10}$$

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¹In fact, since it has been recently suggested that the $D^0 - \overline{D^0}$ mixing amplitude may be large even in the standard model [9], *CP* violation may be the most valuable clue for new physics in this system.

²Note that A_m in our definition is twice smaller than the A_m used by CLEO [2].

where the ϕ and δ are the weak phase and the strong phase, respectively, and

$$R_{D} = \left| \frac{A_{K^{+}\pi^{-}}}{\bar{A}_{K^{+}\pi^{-}}} \right|^{2} = \left| \frac{\bar{A}_{K^{-}\pi^{+}}}{A_{K^{-}\pi^{+}}} \right|^{2}.$$
 (11)

Next we define

$$x' \equiv x \cos \delta + y \sin \delta,$$

$$y' \equiv y \cos \delta - x \sin \delta.$$
 (12)

The rates of the DCS, SCS and CF decays are expanded for short times $t \leq 1/\Gamma$ as

$$\Gamma[D^{0}(t) \to K^{+} \pi^{-}] = e^{-\Gamma t} |A_{K^{-} \pi^{+}}|^{2} \times \left[R_{D} + \sqrt{R_{D}} (1 + A_{m}) \right] \times (y' \cos \phi - x' \sin \phi) \Gamma t + \frac{1 + 2A_{m}}{4} (y^{2} + x^{2}) (\Gamma t)^{2} , \qquad (13)$$

$$\Gamma[\overline{D^{0}}(t) \rightarrow K^{-} \pi^{+}] = e^{-\Gamma t} |A_{K^{-} \pi^{+}}|^{2} \times \left[R_{D} + \sqrt{R_{D}} (1 - A_{m}) \times (y' \cos \phi + x' \sin \phi) \Gamma t + \frac{1 - 2A_{m}}{4} (y^{2} + x^{2}) (\Gamma t)^{2} \right], \quad (14)$$

$$\Gamma[D^{0}(t) \rightarrow K^{+}K^{-}] = e^{-\Gamma t} |A_{K^{+}K^{-}}|^{2} \times [1 - (1 + A_{m})(y' \cos \phi - x' \sin \phi)\Gamma t], \qquad (15)$$

$$\Gamma[D^{0}(t) \to K^{-} \pi^{+}] = \Gamma[\overline{D^{0}(t)} \to K^{+} \pi^{-}]$$

= $e^{-\Gamma t} |A_{K^{-} \pi^{+}}|^{2}.$ (16)

Several experiments measure the parameter y_{CP} , defined by

$$y_{CP} = \frac{\tau(D^0 \to K^- \pi^+)}{\tau(D^0 \to K^+ K^-)} - 1, \qquad (17)$$

with τ being the measured lifetime fitted to a pure exponential decay rate for the specific modes [1,13]. If *CP* is a good symmetry in the relevant processes, this definition of y_{CP} corresponds to

$$y_{CP} \equiv \frac{\Gamma(CP \text{ even}) - \Gamma(CP \text{ odd})}{\Gamma(CP \text{ even}) + \Gamma(CP \text{ odd})},$$
(18)

since then the K^+K^- state is an even *CP* state and the $K^-\pi^+$ state is an equal mixture of *CP* even and *CP* odd states. By fitting the decay rates in Eqs. (13) and (15) to exponents, and expanding for small A_m we get [13]

TABLE I. Measurements of y_{CP} .

Experiment	Value
FOCUS [1]	$(3.42 \pm 1.39 \pm 0.74)\%$
E791 [4]	$(0.8 \pm 2.9 \pm 1.0)\%$
CLEO [5]	$(-1.2\pm2.5\pm1.4)\%$
BELLE [6]	$(-0.5 \pm 1.0^{+0.7}_{-0.8})\%$
BABAR [8]	$(1.4 \pm 1.0^{+0.6}_{-0.7})\%$

$$y_{CP} = y \cos \phi - A_m x \sin \phi. \tag{19}$$

We are interested in the dispersive part of the mixing amplitude, M_{12}^D : Short distance contributions from new physics can affect M_{12}^D in a significant way. In terms of measurable quantities, $|M_{12}^D|$ is given by [16]

$$|M_{12}^D|^2 = \frac{4(\Delta m)^2 + A_m^2 (\Delta \Gamma)^2}{16(1 - A_m^2)},$$
(20)

or, using Eq. (2),

$$|M_{12}^D|^2 = \Gamma^2 \frac{x^2 + A_m^2 y^2}{4(1 - A_m^2)}.$$
 (21)

III. EXPERIMENTAL DATA ON $D^0 - \overline{D^0}$ MIXING

The neutral D system is studied by various experiments. First, the CLEO experiment [2] measures the rates (13), (14):

$$R_D = (0.48 \pm 0.13)\%,$$

$$y' \cos \phi = (-2.5^{+1.4}_{-1.6})\%,$$

$$x' = (0.0 \pm 1.5)\%,$$

$$2 A_m = 0.23^{+0.63}_{-0.80},$$

$$\sin \phi = 0.00 \pm 0.60.$$
 (22)

The FOCUS experiment [3] provides a measurement of the ratio between the branching ratio of the DCS and CF decays. This measurement is consistent with CLEO data at the level of $\sim 0.8\sigma$. However, as no direct measurement of the parameters is done, no stronger bounds on the parameters result.

The value of y_{CP} is measured by the various experiments. Table I presents the various results. The world weighted average of y_{CP} is, hence,

$$y_{CP} = (1.0 \pm 0.7)\%$$
. (23)

IV. INTERPRETATION OF THE EXPERIMENTAL DATA

Our aim is to constrain the $D^0 - \overline{D^0}$ mixing amplitude M_{12}^D . First we combine Eqs. (12) and (19) to get

$$y_{CP} + A_m \sin \phi(x' \cos \delta + y' \sin \delta)$$

= y' \cos \phi \cos \delta - x' \cos \phi \sin \delta. (24)

The measured values of Eqs. (22) and (23) can be used to constrain $\cos \delta$. Assuming first³ $A_m = 0$ and also $|\sin \phi| \approx 0$ we find

$$(1.0\pm0.7)\% = (-2.5^{+1.4}_{-1.6})\%\cos\delta - (0.0\pm1.5)\%\sin\delta,$$

(25)

which implies a certain distribution for $\cos \delta$. Due to the sign difference between y_{CP} and y' and due to the relative smallness of x' it is expected that this distribution of $\cos \delta$ will be biased to negative values. By a full analysis, considering the measured values of A_m and $\sin \phi$ we can characterize the bias by stating the total confidence level value:

$$\cos \delta \lesssim 0.7$$
 (95% C.L.) (26)

(and $\cos \delta \leq 0.0$ at 68% C.L.).

Since we now have a distribution for x', y' and $\cos \delta$, we may invert Eq. (12) to solve for x and y:

$$x = x' \cos \delta - y' \sin \delta,$$

$$y = y' \cos \delta + x' \sin \delta.$$
 (27)

We note that the signs of x and y in Eq. (27) are not measured by current experimental results. Since the measured value for x' is distributed around zero the sign for y is determined by the sign of y' which, in turn, depends on the sign of $\cos \phi$. This sign is not provided by any measurement (all we know is that $|\cos \phi| \approx 1$). Similarly, the sign of x is determined by the signs of both y' and sin δ , which are not measured.

The resulting distributions for x and y are therefore in the form of two superimposed distributions for the two possible sign choices (denoted by the \pm sign). We obtain

$$x \approx (\pm 2.8 \pm 2.5)\%,$$

 $y \approx (\pm 0.9 \pm 3.6)\%.$ (28)

We note that these values are different from those quoted in [17] where it is assumed that $\delta = \phi = 0$. When we consider the obtained distribution of $\cos \delta$, the value of x is calculated not only from x', which is rather small, but also from y' which is larger. The result is a weaker constraint on x (and hence on Δm) by a factor of about 2.2. The opposite happens regarding the bound on y (and $\Delta \Gamma$), which becomes stronger due to the contributions from the small x'. For comparison, Table II shows the 95% C.L. ranges for x and y in the two cases: One which assumes $\cos \delta = 1$ and $\cos \phi = 1$, and one which takes the values mentioned.

We evaluate now the $D^0 \cdot \overline{D^0}$ mixing amplitude. Taking the average decay width [17]

TABLE II. Comparison between mass and width difference parameters at 95% C.L. with different assumptions on mixing parameters.

Assuming $\cos \delta = 1$, $\cos \phi = 1$	No assumption
$ x \le 2.9\%$ - 5.8% $\le y \le 1.0\%$	$\begin{aligned} x &\leq 6.4\% \\ y &\leq 4.9\% \end{aligned}$

$$\Gamma_D = (1.595 \pm 0.011) \times 10^{-9}$$
 MeV, (29)

and using Eq. (21), we obtain a distribution for M_{12} which is maximal near zero:

$$|M_{12}^D| \le 6.2 \times 10^{-11} \text{ MeV}$$
 (95% C.L.) (30)

(and $|M_{12}^D| \le 3.3 \times 10^{-11}$ MeV at 68% C.L.).

It is interesting to compare this value to the ones obtained by using some simplifying assumptions. First, assuming no *CP* violation in mixing, we set $A_m = 0$ but allow for $\delta, \phi \neq 0$. We get

$$|M_{12}^D| \le 5.4 \times 10^{-11} \text{ MeV} (95\% \text{ C.L.}).$$
 (31)

Second, we set $A_m = \phi = 0$ and allow $\delta \neq 0$. We get

$$|M_{12}^D| \le 4.0 \times 10^{-11} \text{ MeV} (95\% \text{ C.L.}).$$
 (32)

Third, we set $\delta = 0$, but allow $A_m, \phi \neq 0$. We get

$$|M_{12}^D| \le 3.9 \times 10^{-11} \text{ MeV} (95\% \text{ C.L.}).$$
 (33)

Last, we set $A_m = \phi = \delta = 0$ and get⁴

$$|M_{12}^D| \le 2.3 \times 10^{-11} \text{ MeV}$$
 (95% C.L.). (34)

This is the value which appears in [17]. Thus, allowing *CP* violation, the resulting constraint is about 2.7 times weaker (i.e. larger) than the one which is obtained with the maximal set of assumptions.

V. CONCLUSIONS

We interpret the most recent data from the experimental searches for $D^0 - \overline{D^0}$ mixing. Allowing *CP* violation in mixing, we obtain the upper bound

$$|M_{12}^D| \le 6.2 \times 10^{-11} \text{ MeV}$$
 (95% C.L.), (35)

which is 2.7 times weaker than the naive calculation.

The actual upper bound for the $D^0 - \overline{D^0}$ mixing amplitude depends, therefore, on the model in question. Assuming that CP is conserved in $D^0 - \overline{D^0}$ mixing, as is the case in the standard model, the bound is the one in Eq. (32). [If, in

³A similar procedure was followed in Ref. [13].

⁴Actually, it is enough to assume $A_m = \delta = 0$ since, in this case, the value of ϕ affects only y, which does not contribute to M_{12}^D .

addition, one is willing to assume that SU(3)-flavor symmetry holds in D decays, the bound is given by Eq. (34).] For a more general model, with new sources of CP violation, Eq. (35) gives the present bound. Taking into account the correct bound is most significant in models which predict $D^0 \cdot \overline{D^0}$ mixing of magnitude comparable to current experimental sensitivity. The weaker bound then implies that such models

are still viable in a larger part of parameter space [18] compared to analyses that consider only the CP conserving bound [19–21].

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