$\tau \rightarrow \mu\,\eta$ in supersymmetric models

Marc Sher

Nuclear and Particle Theory Group, College of William and Mary, Williamsburg, Virginia 23187

The existence of large ν_μ-ν_τ mixing suggests the likelihood of large smuon-stau mixing in supersymmetric models, leading to μ and τ number violation. In addition to interesting signatures in slepton and neutralino production and decay, this will lead to rare τ decays, such as $\tau \rightarrow \mu\gamma$. Recently, it has been pointed out that the $\tau \rightarrow 3\mu$ branching ratio could be substantial in the large $\tan\beta$ region of parameter space, due to an induced μ-τ-Higgs vertex. In this paper, another signature, $\tau \rightarrow \mu\eta$ is considered. In the large $\tan\beta$ region, it is shown that the branching ratio of $\tau \rightarrow \mu\eta$ is 8.4 times the branching ratio of $\tau \rightarrow 3\mu$, independent of any unknown parameters, and it will thus give the most stringent bound on Higgs-mediated lepton flavor violation, and may provide its first signature. In the other regions of parameter space, where $\tau \rightarrow \mu\gamma$ is the most prominent decay, the branching ratio for $\tau \rightarrow \mu\eta$ is always substantially lower.

The flavor physics of quarks and leptons is one of the most prominent mysteries in particle physics. The most surprising development in flavor physics in the past decade has been the observation [1] of very large mixing between muon and tau neutrinos. The mixing will, at some level, lead to mixing in the charged lepton sector, giving violations of muon and tau lepton number conservation.

In the standard model supplemented with right-handed neutrinos, such violation is generally small [2]. However, a much bigger effect will occur in supersymmetric models, by inducing mixing between the scalar muon and the scalar tau. In the most general supersymmetric standard model, even if neutrino mixing were not present, one would have large mixing between all scalar leptons and scalar quarks. This would lead to very large flavor (quark and lepton) changing neutral currents, which are not observed. It is thus generally assumed that the squark and slepton masses are equal at the unification scale, an assumption that occurs naturally in many models, such as supergravity or gauge-mediated supersymmetry breaking.

Even if the masses of the smuon and stau are equal at the unification scale, without mixing, the presence of nondiagonal neutrino mass terms (either from different Dirac mass terms or nondiagonal right-handed neutrino mass terms) will affect the masses through renormalization group running, and will generate mixing terms [3]. In this paper, we will consider only mixing between left-handed smuons and staus; in most models, such mixing is the largest. It should be kept in mind that solar neutrino oscillation experiments [4] also indicate large mixing between muon and electron neutrinos, and in models with inverted hierarchies, there could be substantial mixing between left-handed smuons and selectrons, although the very strong bounds on $\mu \rightarrow e\gamma$ will constrain these effects.

The mixing is characterized by the parameter $\delta_{23} \equiv M_{23}^2/\tilde{m}^2$, where M_{23} is the off-diagonal term in the slepton mass matrix and $\tilde{m}$ is the slepton mass scale. The value of δ_{23} is extremely model-dependent, of course, but in models in which the mixing arises entirely through renormalization group running [5–7], its value is typically between 0.1 and 1.

The most studied tau-number and muon-number violating process is $\tau \rightarrow \mu\gamma$ [12]. Many of these works consider various models for δ_{23}. Normalizing the rate to the current bound [8,9]

$$BR(\tau \rightarrow \mu\gamma) = 1.1 \times 10^{-6} \left(\frac{\delta_{23}}{1.4}\right)^2 \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2. \quad (1)$$

In the constrained minimal supersymmetric standard model (MSSM), where the parameter space is restricted to a manageable dimensionality, this process dominates in the low $m_{1/2}$ region [6].

In addition, one can consider tau-number and muon-number violation in production and decay of sleptons, neutralinos and charginos [6,10,11]. As an example, the process $\chi_2 \rightarrow \chi_1 \mu\tau$ where $\chi_{1,2}$ are neutralinos, can be searched for at the CERN Large Hadron Collider (LHC). In the constrained MSSM, this process dominates [6] in the $m_{1/2} > m_0$ region of parameter-space, and is thus complementary to $\tau \rightarrow \mu\gamma$.

Recently, Babu and Kolda [5] pointed out that $\tau \rightarrow 3\mu$ was a promising signature in models with a large value of $\tan\beta$. Earlier [13], they had noticed that squark mixing would induce a flavor nondiagonal quark-quark-Higgs Yukawa coupling, and had examined the consequences for rare B decays. The same mechanism, however, will also induce a μ-τ-Higgs vertex, and thus directly to $\tau \rightarrow 3\mu$, through tree-level Higgs exchange (either the h, H, or A). The branching ratio, for a reasonable choice of mass parameters, is

$$\text{BR}(\tau \rightarrow 3\mu) = (1 \times 10^{-7}) \times \left(\frac{\tan\beta}{60}\right)^6 \times \left(\frac{100 \text{ GeV}}{m_A}\right)^4$$

where m_A is the pseudoscalar mass. This result is very insensitive to the SUSY spectrum, with the exception that it can increase by up to a factor of 4 for large μ. This branching ratio will be accessible at B factories in the near future.

In this paper, another signature of lepton-number violation is discussed: $\tau \rightarrow \mu\eta$. It will be shown that, in the large $\tan\beta$ region discussed in the previous paragraph, the branching

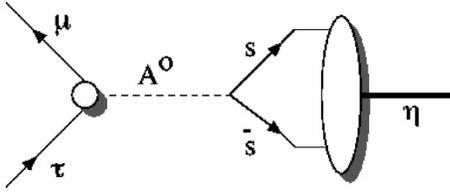FIG. 1. Diagram leading to $\tau \to \mu \eta$. The small circle is the lepton-flavor violating vertex of Babu and Kolda.

FIG. 2. Box diagram leading to $\tau \to \mu \eta$. A similar diagram will lead to $\tau \to \mu \pi$. The χ^i are neutralinos.

ratio is much higher than $\tau \to 3\mu$, and may be a much more sensitive test of Higgs-induced lepton flavor violation.

In the Babu-Kolda model [5], the lepton flavor violating interaction is given by

$$\mathcal{L}_{LFV} = (2G_F^2)^{1/4} \frac{m_\tau \kappa_{32}}{\cos^2 \beta} (\bar{\tau}_R \mu_L)[H^0 + iA^0] + \text{H.c.} \quad (2)$$

where H^0 and A^0 are the heavier scalar and the pseudoscalar, respectively, and we have chosen the generally preferred region of parameter-space in which $\sin(\alpha - \beta) \sim 1$. Here, κ_{32} is given in Ref. [5] and depends on loop integrals (but is relatively insensitive to SUSY parameters). For SUSY parameters $\mu = M_1 = m_2 = m_{\tilde{l}} = m_{\tilde{\nu}}^-$, $M_R = 10^{14}$ GeV and the off-diagonal Dirac neutrino coupling equal to the top quark Yukawa coupling [as expected in $SO(10)$ models], one has $\kappa_{32} = 4 \times 10^{-4}$.

With this interaction, one can have a τ convert into a μ and a virtual H^0 or A^0, which then converts into a $\mu^+ \mu^-$ pair. This gives the branching ratio mentioned above. However, one could equally well have an A^0 convert into a strange quark pair, which then becomes an η, as shown in Fig. 1, giving $\tau \to \mu \eta$. This will have both advantages and disadvantages. The two body phase space is a major advantage, and the extra color factor and slightly bigger coupling (since $m_s > m_\mu$) are also advantages. The disadvantage is in converting the strange quarks into an η. A general discussion of $\tau \to \mu \eta$ can be found in Ref. [14]. The relevant matrix element is

$$\langle 0|\bar{s}\gamma_5 s|\eta\rangle = -\sqrt{6}F_\eta^8 \frac{m_\eta^2}{m_u + m_d + 4m_s}. \quad (3)$$

Using this matrix element, it is straightforward to calculate the branching ratio. If one divides by the $\tau \to 3\mu$ branching ratio, the unknown parameters all cancel, and the result is (neglecting the muon mass)

$$\frac{\Gamma(\tau \to \mu \eta)}{\Gamma(\tau \to 3\mu)} = 54\pi^2 \left(\frac{F_\eta^8}{m_\mu}\right)^2 \left(\frac{m_\eta}{m_\tau}\right)^4 \left(1 - \frac{m_\eta^2}{m_\tau^2}\right)^2. \quad (4)$$

With $F_\eta^8 \sim 150$ MeV [15], this ratio is 8.4, giving a branching ratio of

$$\text{BR}(\tau \to \mu \eta) = (0.84 \times 10^{-6}) \times \left(\frac{\tan \beta}{60}\right)^6 \times \left(\frac{100 \text{ GeV}}{m_A}\right)^4.$$

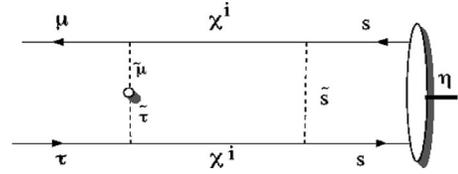One can get this result approximately without doing a calculation. Imagine that final state interactions are turned

off and that the strange quarks propagate as free particles. One expects the ratio of $\tau \to \mu \bar{s}s$ to $\tau \to 3\mu$ to have a factor of 3 for color and a factor of $(m_s/m_\mu)^2$ for the Yukawa coupling. The cross diagram in the muon case turns out to lower the rate by a factor of 3/2, so the overall ratio is $9m_s^2/2m_\mu^2 \sim 10$. Since the only two body decays would be $\mu \eta$ and $\mu \eta'$, and the latter is suppressed much more by phase space, the $\mu \eta$ rate should dominate this process.

Since the experimental bound [16] on the branching ratio for $\tau \to \mu \eta$ is 9.6×10^{-6}, and that [17] for $\tau \to 3\mu$ is 1.9×10^{-6}, it is clear that $\tau \to \mu \eta$ puts stronger constraints on the model. In order to reach the interesting region of parameter space ($\tan \beta \sim 60$ and $m_A \sim 100$ GeV), the bound on $\tau \to 3\mu$ would need to be improved by a factor of 20, whereas the bound on $\tau \to \mu \eta$ would need to be improved by a factor of 10.

Could these improvements be made? Both the $\tau \to 3\mu$ and $\tau \to \mu \eta$ bounds are based on the CLEO-II sample of 4.7 fb^{-1}, and the CLEO experiment has now accumulated a total of 5 times that luminosity (which would give a total of 24 million tau pairs). So in the absence of backgrounds, the current bounds could improve by a factor of 5. The efficiency for $\tau \to 3\mu$ is listed as 15%; the efficiency for $\tau \to \mu \eta$ is about 3%, when one includes the fact that they only search for the $\gamma\gamma$ channel for the η (thus the factor of 5 difference in the current bounds). Including the three-pion decay, or increasing the fiducial area for finding photons, could improve that efficiency substantially. Over the next years, BABAR and BELLE will reach 500 fb^{-1}, which could easily reach the needed sensitivity, depending on the point at which they become background-limited. Even if $\tan \beta$ is somewhat smaller, or m_A larger, the necessary sensitivity could possibly be reached at LHC, SuperKEKB or a tau-charm factory. Note that the $\tau \to 3\mu$ decay could still be dominant in the small region of parameter space in which $M_H \ll M_A$.

Are there any other processes that can give $\tau \to \mu \eta$? One can have the box diagram of Fig. 2, which will also yield τ decays into μ plus other mesons, including the π, ρ and ϕ. If we take the special case in which the neutralinos are pure photino, the rate for $\tau \to \mu \pi$ is given by

$$\frac{\Gamma(\tau \to \mu \pi)}{\Gamma(\tau \to \mu \gamma)} = \frac{32\pi\alpha F_\pi^2 m_{\tilde{l}}^8 (I_1^2 + I_2^2)}{81m_\tau^2 m_{\tilde{\gamma}}^8 M_3^2(x)} \quad (5)$$

where $x \equiv m_{\tilde{\gamma}}/m_{\tilde{l}}$, $m_{\tilde{\gamma}}$ is the photino mass, $m_{\tilde{l}}$ is the average slepton mass, $M_3(x)$ is given in Ref. [9], and the integrals are

$$(I_1, I_2) = \frac{1}{16\pi^2} \int_0^\infty dx \frac{\left(x, \frac{1}{4}x^2\right)}{(x-1)^2(x-a)(x-b)(x-c)}. \quad (6)$$

Here, a, b, c are the ratios of the squark, smuon and stau masses to the photino mass. In giving this expression, we have used the fact that $m_\pi^2/(m_u + m_d)$ is numerically close to m_τ. In the case of $\tau \to \mu \eta$, there will be a suppression of a factor of 4 from the s-quark charge, an increase of a factor of $(F_\eta^8/F_\pi)^2 \sim 3.0$ from the decay contents, and the coefficient of the I_1^2 term will be decreased by a factor of $(3m_\eta^4/8m_s^2)$ relative to m_τ^2, or a factor of about 3. This ratio has been evaluated for the entire SUSY parameter space, assuming sparticle masses in the range of $5-1000$ GeV [18], and is

always less than 10^{-2}. As a result, the $\tau \to \mu$ + meson arising from the box diagram is negligible.

The existence of large $\nu_\mu - \nu_\tau$ mixing implies mixing between the left-handed smuon and stau. While one could look for this mixing directly in neutralino/slepton interactions, one can also look at τ decays. The decay $\tau \to \mu \gamma$ is one signature, however Babu and Kolda have noted the mixing will also lead, especially in the large $\tan \beta$ region, to $\tau \to 3\mu$. In this paper, it has been pointed out that $\tau \to \mu \eta$ will also occur in this Higgs-mediated model, with a branching ratio 8.4 times bigger, and is thus more sensitive. In other models, where $\tau \to \mu \gamma$ is the main signature, the $\tau \to \mu \eta$ rate is substantially smaller.

[1] SuperKamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **85**, 3999 (2000).

[2] If the Majorana neutrinos are very heavy, there can be muon and tau number nonconservation in Higgs boson decays; A. Pilaftsis, Phys. Lett. B **285**, 68 (1992), as well as in rare electronic tau decays; A. Ilakovac, B.A. Kniehl, and A. Pilaftsis, Phys. Rev. D **52**, 3993 (1995).

[3] F. Borzumati and A. Masiero, Phys. Rev. Lett. **57**, 961 (1986).

[4] SuperKamiokande Collaboration, S. Fukuda *et al.*, Phys. Rev. Lett. **86**, 5656 (2001); SNO Collaboration, Q.R. Ahmad *et al.*, *ibid.* **87**, 071301 (2001); **89**, 011301 (2002).

[5] K.S. Babu and C. Kolda, hep-ph/0206310.

[6] D.F. Carvalho, J. Ellis, M.E. Gomez, S. Lola, and J.C. Romao, hep-ph/0206148.

[7] I. Hinchliffe and F.E. Paige, Phys. Rev. D **63**, 115006 (2001).

[8] J.L. Feng, Y. Nir, and Y. Shadmi, Phys. Rev. D **61**, 113005 (2000).

[9] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996).

[10] J. Kalinowski, hep-ph/0207051.

[11] J. Hisano, M.M. Nojiri, Y. Shimizu, and M. Tanaka, Phys. Rev. D **60**, 055008 (1999).

[12] J. Hisano, T. Moroi, M. Yamaguchi, and T. Yanagida, Phys. Lett. B **357**, 579 (1995); J. Hisano, T. Moroi, K. Tobe, and M. Yamaguchi, Phys. Rev. D **53**, 2442 (1996); J. Hisano and D.

Nomura, *ibid.* **59**, 116005 (1999); J.R. Ellis, M.E. Gomez, G.K. Leontaris, S. Lola, and D.V. Nanopoulos, Eur. Phys. J. C **14**, 319 (2000); W. Buchmuller, D. Delepine, and L.T. Handoko, Nucl. Phys. **B576**, 445 (2000); J.L. Feng, Y. Nir, and Y. Shadmi, Phys. Rev. D **61**, 113005 (2000); J. Sato and K. Tobe, *ibid.* **63**, 116010 (2001); S. Lavignac, I. Masina, and C.A. Savoy, Phys. Lett. B **520**, 269 (2001); J.A. Casas and A. Ibarra, Nucl. Phys. **B618**, 171 (2000); A. Rossi, hep-ph/0207006.

[13] K.S. Babu and C. Kolda, Phys. Rev. Lett. **84**, 228 (2000).

[14] D. Black, T. Han, H.-J. He, and M. Sher, Phys. Rev. D (to be published), hep-ph/0206056.

[15] T. Feldmann, Int. J. Mod. Phys. A **15**, 159 (2000).

[16] CLEO Collaboration, G. Bonvicini *et al.*, Phys. Rev. Lett. **79**, 1221 (1997).

[17] CLEO Collaboration, D.W. Bliss *et al.*, Phys. Rev. D **57**, 5903 (1998).

[18] In D. Choudhury, H. Dreiner, P. Richardson, and S. Sarkar, Phys. Rev. D **61**, 095009 (2000), it is noted that the KARMEN time anomaly could be explained by a 33.9 MeV neutralino. In this model, such a light neutralino would lead to a tree-level decay of $\tau \to \mu \chi^0 \chi^0$, with a large branching ratio. Although we have not done a detailed analysis of the bound in this case, it is probably ruled out by universality, since it would appear as a universality-violating leptonic τ decay. We thus assume the neutralino is heavier than the τ.