Possible effects of noncommutative geometry on weak *CP* violation and unitarity triangles

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The possible effects of noncommutative geometry on weak CP violation and unitarity triangles are discussed by taking into account a simple version of the momentum-dependent quark mixing matrix in the noncommutative standard model. In particular, we calculate nine rephasing invariants of CP violation and illustrate the noncommutative CP-violating effect in a couple of charged D-meson decays. We also show how inner angles of the *deformed* unitarity triangles are related to CP-violating asymmetries in some typical B_d and B_s transitions into CP eigenstates. B-meson factories are expected to help probe or constrain noncommutative geometry at low energies in the near future.

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I. INTRODUCTION

One of the major goals of B-meson factories is to test the Kobayashi-Maskawa mechanism of CP violation in the standard model (SM) [1]. If this mechanism is correct, all CP-violating asymmetries in weak decays of quark flavors must be proportional to a universal and rephasing-invariant parameter $\mathcal{J}$ [2], defined through

$$\mathcal{J}^{ij}_{\alpha\beta} \equiv \mathrm{Im}(V_{\alpha i}V_{\beta j}V^{*}_{\alpha j}V^{*}_{\beta i}) = \mathcal{J}\sum_{\gamma,k}(\epsilon_{\alpha\beta\gamma}\epsilon_{ijk}), \tag{1.1}$$

where V denotes the Cabibbo-Kobayashi-Maskawa (CKM) matrix of quark flavor mixing, and its Greek and Latin subscripts run, respectively, over (u,c,t) and (d,s,b). A number of promising measurables of CP violation at B-meson factories are directly related to the unitarity triangle shown in Fig. 1(a), which describes the following orthogonal relation of V in the complex plane:

$$V^{*}_{ub}V_{ud}+V^{*}_{cb}V_{cd}+V^{*}_{tb}V_{td}=0. \tag{1.2}$$

The inner angles of this unitarity triangle are commonly defined as

$$\alpha \equiv \arg\left(-\frac{V^{*}_{tb}V_{td}}{V^{*}_{ub}V_{ud}}\right),$$

$$\beta \equiv \arg\left(-\frac{V^{*}_{cb}V_{cd}}{V^{*}_{tb}V_{td}}\right),$$

$$\gamma \equiv \arg\left(-\frac{V^{*}_{ub}V_{ud}}{V^{*}_{cb}V_{cd}}\right). \tag{1.3}$$

Of course, $\alpha+\beta+\gamma=\pi$ and $\mathcal{J}\propto\sin\alpha\propto\sin\beta\propto\sin\gamma$ hold. So far the CP-violating asymmetry in B^0_d vs $\overline{B}{}^0_d \to J/\psi K_S$ decays, which approximates to $\sin 2\beta$ to a high degree of accuracy in the SM, has been unambiguously measured at both KEK and SLAC [3]. Further experiments are expected to help determine all three angles of the unitarity triangle and test the consistency of the Kobayashi-Maskawa picture of CP violation.

Another major goal of B-meson factories is to detect possible new sources of CP violation beyond the SM. On the one hand, the Kobayashi-Maskawa mechanism of CP violation is unable to generate a sufficiently large matter-antimatter asymmetry of the universe observed today; and on the other hand, many extensions of the SM do allow the presence of new CP-violating phenomena [4]. Therefore it is

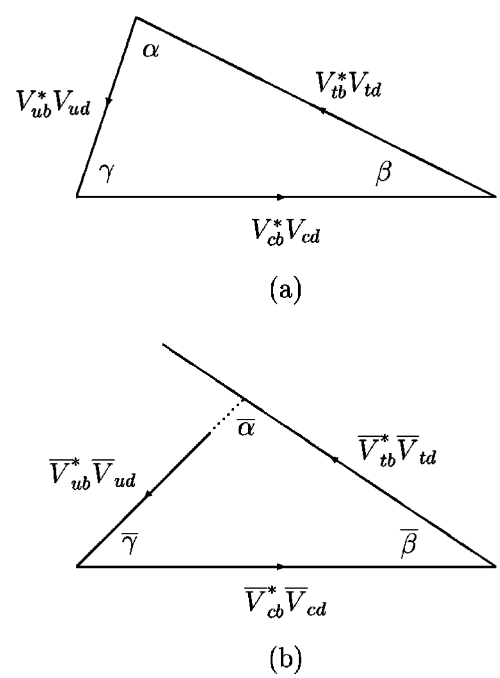

FIG. 1. The CKM unitarity triangle in the standard model (a) and its *deformed* counterpart in the noncommutative standard model (b).

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worthwhile to look for new sources of CP violation in various weak decays of quark (and lepton) flavors. A particularly interesting possibility is that new CP violation may stem from noncommutative geometry.

Noncommutative geometry plays a very important role in unraveling the properties of the Planck-scale physics. It has for a long time been suspected that the noncommutative spacetime might be a realistic picture of how spacetime behaves near the Planck scale [5]. Strong quantum fluctuations of gravity may make points fuzzy. In fact, the noncommutative geometry naturally enters the theory of open string in a background B field [6]. In particular, the noncommutative geometry makes the holography [7] [e.g., the AdS/conformal field theory (CFT) correspondence] of a higher-dimensional quantum system of gravity and lower-dimensional theory possible. It was also discovered that simple limits of M theory and superstring theory lead directly to the noncommutative gauge field theory [8,9]. The fluctuations of the D-brane are described by the noncommutative gauge field theory [10]. The noncommutative field theory has been intensively studied in the past two decades [11]. A standard model on noncommutative spacetime was even set up [12]. However, in recent years, the study of noncommutative geometry has been focused on the so-called Moyal plane, with the coordinates and their conjugate momenta satisfying the relations [13]

$$[x^{\mu} \overset{\star}{,} x^{\nu}] = i\,\theta^{\mu\nu},$$

$$[x^{\mu} \overset{\star}{,} p^{\nu}] = i\hbar\,\eta^{\mu\nu}, \tag{1.4}$$

where $\theta^{\mu\nu}$ is a constant antisymmetric matrix. Here the Moyal-Weyl star product can be defined by a formal power series:

$$(f\star g)(x) = e^{(i/2)\theta^{\mu\nu}(\partial/\partial x^{\mu})(\partial/\partial x^{\nu})} f(x) g(y)\big|_{x=y}. \tag{1.5}$$

There are two obstacles in the way of building a SM-like gauge field theory on the Moyal plane. The first one is the charge quantization in the noncommutative QED [14]. The charges of matter fields coupled to the $U_\star(1)$ gauge theory are fixed to only three possible values, ± 1 and 0, depending on the representation of particles. This is indeed a problem in view of the range of hypercharges in the $U(1)_Y$ part of the SM. The second one is due to extra $U_\star(1)$ gauge fields [15]. Under the infinitesimal gauge transformation $\hat{\delta}$, the vector gauge potential $\hat{V}_\mu$, the fundamental matter field $\hat{\Psi}$, and the Higgs field $\hat{\Phi}$ transform as

$$\hat{\delta}\hat{V}_\mu = \partial_\mu \hat{\Lambda} + i[\hat{\Lambda} \overset{\star}{,} \hat{V}_\mu],$$

$$\hat{\delta}\hat{\Psi} = i\hat{\Lambda}\star\hat{\Psi},$$

$$\hat{\delta}\hat{\Phi} = i\hat{\Lambda}\star\hat{\Phi} - i\hat{\Phi}\star\hat{\Lambda}'. \tag{1.6}$$

It should be noticed that the Moyal-Weyl product would destroy the closure condition of the $SU_\star(n)$. For example, two Lie algebra-valued consecutive transformations $\hat{\delta}_{\hat{\Lambda}}[=\Lambda_a(x)T^a]$ and $\hat{\delta}_{\hat{\Lambda}'}(=\Lambda'_a T^a)$ of the matter fields in the fundamental representation,

$$[\hat{\delta}_{\hat{\Lambda}} \overset{\star}{,} \hat{\delta}_{\hat{\Lambda}'}] = \frac{1}{2}\{\Lambda_a(x) \overset{\star}{,} \Lambda'_b(x)\}[T^a, T^b] + \frac{1}{2}[\Lambda_a(x) \overset{\star}{,} \Lambda'_b(x)]\{T^a, T^b\}, \tag{1.7}$$

are not equivalent to a Lie algebra-valued gauge transformation. The only group which admits a simple noncommutative extension is $U(N)$. However, there are extra $U_\star(1)$ factors in the $U_\star(N)$ gauge field theory compared to the extended SM on the noncommutative space. In order to construct an $SU_\star(3)\times SU_\star(2)\times U_\star(1)$ Yang-Mills theory [16], Wess and his collaborators [17–20] have extended the ordinary Lie algebra-valued gauge transformations to enveloping algebra-valued noncommutative gauge transformations,

$$\hat{\Lambda} = \Lambda_a^0(x) T^a + \Lambda_{ab}^1 :T^a T^b: + \Lambda_{abc}^2(x) :T^a T^b T^c: + \cdots, \tag{1.8}$$

where $:T^{a_1}T^{a_2}\cdots T^{a_m}:$ denotes a symmetric ordering under the exchange of the index a_i. This kind of extension of the gauge transformations and the Seiberg-Witten map [6] together solves the two main problems in building a noncommutative SM quite well.

The purpose of this paper is to examine possible effects of noncommutative geometry on weak CP violation and CKM unitarity triangles. In Sec. II, we elucidate a simple version of the momentum-dependent CKM matrix in the noncommutative SM, which consists of a new source of CP violation induced by nonvanishing $\theta^{\mu\nu}$. We calculate the rephasing invariants of CP violation in Sec. III, and find that the noncommutative CP-violating effects may be manifest in a couple of charged D-meson decays. In Sec. IV, we show how the CKM unitarity triangles in the SM get modified in the noncommutative SM. We also figure out the relations between inner angles of the *deformed* unitarity triangles and CP-violating asymmetries in some nonleptonic decays of B_d and B_s mesons. Section V is devoted to a brief summary of our main results.

II. MOMENTUM-DEPENDENT CKM MATRIX

The noncommutative SM [21,22] is an $SU_\star(3)\times SU_\star(2)\times U_\star(1)$ gauge field theory on the Moyal plane,

$$S_{\rm YM} = -\int d^4x \left[\frac{1}{2g'}{\rm Tr}_{u(1)}(\hat{F}_{\mu\nu}\star\hat{F}^{\mu\nu}) + \frac{1}{2g}{\rm Tr}_{su(2)}(\hat{F}_{\mu\nu}\star\hat{F}^{\mu\nu}) + \frac{1}{2g_S}{\rm Tr}_{su(3)}(\hat{F}_{\mu\nu}\star\hat{F}^{\mu\nu})\right]. \tag{2.1}$$

The gauge field strength $\hat{F}_{\mu\nu}$ is given by

$$\hat{F}_{\mu\nu}=\partial_\mu \hat{V}_\nu-\partial_\nu \hat{V}_\mu - i[\hat{V}_\mu \overset{\star}{,} \hat{V}_\nu], \tag{2.2}$$

where $\hat{V}_\mu$ is the vector potential of the $SU_\star(3)\times SU_\star(2)\times U_\star(1)$ gauge field, which is related to the ordinary potential

$$V_\mu = g' A_\mu(x) Y + g\sum_{a=1}^{3} B_{\mu a}(x) T_L^a + g_S \sum_{a=1}^{8} G_{\mu a}(x) T_S^a, \tag{2.3}$$

by the Seiberg-Witten map (to the first order of $\theta^{\mu\nu}$)

$$\hat{V}_\mu = V_\mu + \frac{1}{4}\theta^{\lambda\rho}\{V_\rho, \partial_\lambda V_\mu\} + \frac{1}{4}\theta^{\lambda\rho}\{F_{\lambda\mu}, V_\nu\} + O(\theta^2). \tag{2.4}$$

Here $F^{\mu\nu}=\partial^\mu V^\nu - \partial^\nu V^\mu - i[V^\mu, V^\nu]$ is the ordinary field strength, and Y, T_L^a, and T_S^a are the generators of $U(1)_Y$, $SU(2)_L$, and $SU(3)_C$, respectively.

The parameter $\hat{\Lambda}$ of the gauge transformations on the noncommutative space is determined by the ordinary gauge parameter Λ via the Seiberg-Witten map,

$$\hat{\Lambda} = \Lambda + \frac{1}{4}\theta^{\mu\nu}\{V_\nu, \partial_\mu \Lambda\} + O(\theta^2), \tag{2.5}$$

where the ordinary gauge parameter Λ is of the form

$$\Lambda = g' \tau(x) Y + g\sum_{a=1}^{3}\tau_a^L(x) T_L + g_S \sum_{a=1}^{8}\tau_a^S(x) T_S^a. \tag{2.6}$$

The Seiberg-Witten maps for the Higgs field $\hat{\Phi}$ and the fermion field $\hat{\Psi}$ are given as

$$\hat{\Phi} = \Phi + \frac{1}{2}\theta^{\mu\nu} V_\nu \left[\partial_\mu \Phi - \frac{i}{2}(V_\mu \Phi - \Phi V'_\mu)\right] + \frac{1}{2}\theta^{\mu\nu}\left[\partial_\mu \Phi - \frac{i}{2}(V_\mu \Phi - \Phi V'_\mu)\right] V'_\nu + O(\theta^2),$$

$$\hat{\Psi} = \Psi + \frac{1}{2}\theta^{\mu\nu} V_\nu \partial_\mu \Psi + \frac{i}{8}\theta^{\mu\nu}[V_\mu, V_\nu]\Psi + O(\theta^2). \tag{2.7}$$

At this stage, we can say that a SM-like gauge field theory on the noncommutative spacetime is set up consistently. Many interesting properties of noncommutative spacetime can be investigated directly within the framework of the noncommutative SM [22,23].

In the noncommutative SM, the W-quark-quark $SU(2)_L$ vertex in the flavor basis can be written as

$$\mathcal{L}_{Wqq} = \overline{(u' \quad c' \quad t')_L}\; J_{\rm cc} \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix}_L + \text{H.c.}, \tag{2.8}$$

where the superscript "prime" denotes the flavor or interaction eigenstates of quarks, and

$$J_{\rm cc} = \frac{\sqrt{2}}{2} g \gamma^\mu W_\mu^+ - ig\frac{\sqrt{2}}{4}\left(\frac{1}{2}\theta^{\mu\nu}\gamma^\alpha + \theta^{\nu\alpha}\gamma^\mu\right) \times (\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)\partial_\alpha \tag{2.9}$$

represents the charged current. Note that the charged-current interactions with more than one $W^\pm$ and (or) Z bosons as well as those with gluons [21] are not included in Eqs. (2.8) and (2.9), since they are not closely associated with our subsequent discussions about weak CP violation and unitarity triangles. To diagonalize the Yukawa interactions of quarks with the Higgs boson, one should make proper unitary rotations on the up- and down-type quark fields. In the basis where the Yukawa coupling matrices are diagonal, the W-quark-quark $SU(2)_L$ vertex in Eq. (2.8) becomes

$$\mathcal{L}_{Wqq} = \overline{(u \quad c \quad t)_L}\; U J_{\rm cc} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + \text{H.c.} \tag{2.10}$$

Within the SM (i.e., $\theta^{\mu\nu}=0$), U turns out to be the CKM matrix V after a spontaneous breakdown of the $SU(2)_L$ symmetry.

Making use of the antisymmetric property of $\theta^{\mu\nu}$ and taking into account the $SU(2)_L$ symmetry, we have the following relations for the W-u-d vertex:

$$\begin{aligned}\int d^4x[\overline{u(p)}\theta^{\nu\alpha}\gamma^\mu \partial_\mu W_\nu^+ \partial_\alpha d(q)] &= -\int d^4x[\overline{u(p)}\theta^{\nu\alpha}\gamma^\mu(p_\mu - q_\mu) q_\alpha W_\nu^+ d(q)] \\ &= 0,\end{aligned} \tag{2.11}$$

and

$$\begin{aligned}\int d^4x\left[\overline{u(p)}\frac{1}{2}\theta^{\mu\nu}\gamma^\alpha(\partial_\mu W_\nu^+ - \partial_\nu W_\mu^+)\partial_\alpha d(q)\right] &= -\int d^4x[\overline{u(p)}\theta^{\mu\nu}(p_\mu - q_\mu)\gamma^\alpha q_\alpha W_\nu^+ d(q)] \\ &= 0.\end{aligned} \tag{2.12}$$

Therefore, we can generally rewrite the W-quark-quark $SU(2)$ vertex in the form

$$\mathcal{L}_{Wqq} = \frac{\sqrt{2}}{2} g \; \overline{(u \quad c \quad t)_L} \; \bar{U} \gamma^{\mu} W^{+}_{\mu} \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L + \text{H.c.}, \tag{2.13}$$

where we have used the notation

$$\bar{U}_{\alpha k}(p,q) = U_{\alpha k}\left(1 - \frac{i}{2} p^{\mu}_{\alpha} \theta_{\mu\nu} q^{\nu}_{k}\right), \tag{2.14}$$

with α and k running, respectively, over (u,c,t) and (d,s,b). The momentum-dependent matrix $\bar{U}$ is not guaranteed to be unitary, and its new phases (induced by nonzero $\theta^{\mu\nu}$) may lead to new CP-violating effects in weak interactions.

Indeed, the aforementioned property of $\bar{U}$ has been observed by Hinchliffe and Kersting in Ref. [24]. They point out that the signal for noncommutative geometry at low energies can simply be a momentum-dependent CKM matrix $\bar{V}$, which is defined in analogy with $\bar{U}$ as follows:

$$\bar{V} = V - \frac{i}{2} \begin{pmatrix} V_{ud} x_{ud} & V_{us} x_{us} & V_{ub} x_{ub} \\ V_{cd} x_{cd} & V_{cs} x_{cs} & V_{cb} x_{cb} \\ V_{td} x_{td} & V_{ts} x_{ts} & V_{tb} x_{tb} \end{pmatrix}, \tag{2.15}$$

where $x_{\alpha k} \equiv p^{\mu}_{\alpha} \theta_{\mu\nu} q^{\nu}_{k}$ for $\alpha = u,c,t$ and $k = d,s,b$. This *effective* flavor mixing matrix arises from an approximation of the exact noncommutative SM in the leading order of $\theta^{\mu\nu}$. Subsequently, we explore some phenomenological implications of $\bar{V}$ on weak CP violation and unitarity triangles.

III. REPHASING INVARIANTS OF *CP* VIOLATION

The momentum-dependent CKM matrix $\bar{V}$ is not unitary in general, as one can see from Eq. (2.15). Note that the following normalization relations hold up to $O(x^2_{\alpha i})$:

$$\sum_{\alpha} |\bar{V}_{\alpha i}|^2 = \sum_{\alpha} |V_{\alpha i}|^2 = 1,$$

$$\sum_{i} |\bar{V}_{\alpha i}|^2 = \sum_{i} |V_{\alpha i}|^2 = 1. \tag{3.1}$$

On the other hand, we obtain ($i \neq j$ and $\alpha \neq \beta$)

$$\sum_{\alpha} (\bar{V}^*_{\alpha i} \bar{V}_{\alpha j}) = i \sum_{\alpha} \left[(V^*_{\alpha i} V_{\alpha j}) \frac{x_{\alpha i} - x_{\alpha j}}{2} \right],$$

$$\sum_{i} (\bar{V}^*_{\alpha i} \bar{V}_{\beta i}) = i \sum_{i} \left[(V^*_{\alpha i} V_{\beta i}) \frac{x_{\alpha i} - x_{\beta i}}{2} \right], \tag{3.2}$$

which do not vanish unless $(x_{\alpha i} - x_{\alpha j}) = \text{const}$ and $(x_{\alpha i} - x_{\beta i}) = \text{const}$.

The observables of CP violation in the noncommutative SM must depend upon the imaginary parts of nine rephasing invariants $(\bar{V}_{\alpha i} \bar{V}_{\beta j} \bar{V}^*_{\alpha j} \bar{V}^*_{\beta i})$. Up to $O(x_{\alpha i})$, we have

$$\bar{\mathcal{J}}^{ij}_{\alpha\beta} \equiv \text{Im}(\bar{V}_{\alpha i} \bar{V}_{\beta j} \bar{V}^*_{\alpha j} \bar{V}^*_{\beta i}) = \mathcal{J} \sum_{\gamma,k} (\epsilon_{\alpha\beta\gamma} \epsilon_{ijk}) + \mathcal{R}^{ij}_{\alpha\beta} \xi^{ij}_{\alpha\beta}, \tag{3.3}$$

where

$$\mathcal{R}^{ij}_{\alpha\beta} \equiv \text{Re}(V_{\alpha i} V_{\beta j} V^*_{\alpha j} V^*_{\beta i}),$$

$$\xi^{ij}_{\alpha\beta} \equiv \frac{1}{2}(x_{\alpha j} + x_{\beta i} - x_{\alpha i} - x_{\beta j}), \tag{3.4}$$

and the subscripts (α,β,γ) and (i,j,k) run, respectively, over (u,c,t) and (d,s,b). If $\bar{V}$ were unitary (i.e., $\xi^{ij}_{\alpha\beta} = 0$), the term associated with $\mathcal{R}^{ij}_{\alpha\beta}$ would vanish and the equality $\bar{\mathcal{J}}^{ij}_{\alpha\beta} = \mathcal{J}^{ij}_{\alpha\beta}$ would hold. Otherwise, both the magnitude and the sign of $\bar{\mathcal{J}}^{ij}_{\alpha\beta}$, rely on the momentum-dependent parameter $\xi^{ij}_{\alpha\beta}$ which signifies the effect of noncommutative geometry. To get an order-of-magnitude feeling about the SM and noncommutative SM contributions to $\bar{\mathcal{J}}^{ij}_{\alpha\beta}$, we adopt the Wolfenstein parametrization [25] for the CKM matrix V and then obtain

$$\mathcal{J} \approx A^2 \lambda^6 \eta, \tag{3.5}$$

and

$$\begin{aligned}
\mathcal{R}^{ds}_{uc} &\approx -\lambda^2, \\
\mathcal{R}^{ds}_{ut} &\approx -A^2\lambda^6(1-\rho), \\
\mathcal{R}^{ds}_{ct} &\approx A^2\lambda^6(1-\rho), \\
\mathcal{R}^{db}_{uc} &\approx -A^2\lambda^6\rho, \\
\mathcal{R}^{db}_{ut} &\approx A^2\lambda^6[\rho(1-\rho)-\eta^2], \\
\mathcal{R}^{db}_{ct} &\approx -A^2\lambda^6(1-\rho), \\
\mathcal{R}^{sb}_{uc} &\approx A^2\lambda^6\rho, \\
\mathcal{R}^{sb}_{ut} &\approx -A^2\lambda^6\rho, \\
\mathcal{R}^{sb}_{ct} &\approx -A^2\lambda^4,
\end{aligned} \tag{3.6}$$

where $A \approx 0.81$, $\lambda \approx 0.22$, $\rho \approx 0.15$, and $\eta \approx 0.34$ extracted from a global fit of current experimental data in the framework of the SM [26]. The values of λ and A, which are extracted, respectively, from the semileptonic K_{e3} and $B \rightarrow \overline{D}^{(*)} l^+ \nu_l$ decays without any loop-induced pollution, remain unchanged even in the presence of noncommutative geometry. In contrast, ρ and η are sensitive to possible new physics induced by loop (box and penguin) effects, which may reside in B^0_d-$\bar{B}^0_d$ mixing, B^0_s-$\bar{B}^0_s$ mixing, and CP violation in K^0-$\overline{K}^0$ mixing. Hence the results of ρ and η are expected to deviate somehow from their SM values in a new analysis of current experimental data, when the noncommutative SM takes the place of the SM. It is unnecessary to know an accurate range of ρ or η, however, for our purpose to illustrate the effects of noncommutative geometry on weak CP violation and unitarity triangles. One can see that $\mathcal{J} \ll |\mathcal{R}^{sb}_{ct}| \ll |\mathcal{R}^{ds}_{uc}|$ holds, while the other seven $\mathcal{R}^{ij}_{\alpha\beta}$ have comparable sizes as $\mathcal{J}$. Note in particular that

$$\overline{\mathcal{J}}^{ds}_{uc} \approx A^2\lambda^6\eta - \lambda^2 \xi^{ds}_{uc},$$

$$\overline{\mathcal{J}}^{sb}_{ct} \approx A^2\lambda^6\eta - A^2\lambda^4 \xi^{sb}_{ct}. \tag{3.7}$$

Thus the noncommutative CP-violating effect may be comparable with or dominant over the SM one, if ξ^{ds}_{uc} is of O (λ^4) or larger in $\overline{\mathcal{J}}^{ds}_{uc}$, and if ξ^{sb}_{ct} is of $O(\lambda^2)$ or larger in $\overline{\mathcal{J}}^{sb}_{ct}$.

To see how the rephasing invariants $\overline{\mathcal{J}}^{ij}_{\alpha\beta}$ are related to CP-violating asymmetries in specific weak decays, let us take $D^{\pm}_s \to K^{\pm} K_S$ for example. Direct CP violation arises from the interference between the Cabibbo-allowed channel and the doubly Cabibbo-suppressed channel of $D^{\pm}_s$ decays into the final states $K^{\pm}K_S$, where K^0-$\overline{K}^0$ mixing leads to an additional CP-violating effect of magnitude $2\,\mathrm{Re}\epsilon_K \approx 3.3 \times 10^{-3}$ [27]. The latter dominates over the former in the SM, because two interferring amplitudes of D^+_s or D^-_s transitions have a small relative weak phase $\arg[(V_{cd}V^*_{ud})/(V_{cs}V^*_{us})] \approx A^2\lambda^4\eta \sim 5\times 10^{-4}$ and a small relative size $|V_{cd}V^*_{us}|/|V_{cs}V^*_{ud}| \approx \lambda^2 \sim 5 \times 10^{-2}$ [28]:

$$A(D^+_s \to K^+K_S) \propto (V_{cs}V^*_{ud}) q^*_K + (V_{cd}V^*_{us}) p^*_K R_s \;\; e^{i\delta_s},$$

$$A(D^-_s \to K^-K_S) \propto (V^*_{cs}V_{ud}) p^*_K + (V^*_{cd}V_{us}) q^*_K R_s e^{i\delta_s}, \tag{3.8}$$

where p_K and q_K are the K^0-$\overline{K}^0$ mixing parameters,[1] δ_s denotes the relative strong phase difference between two interferring decay amplitudes, and $R_s \approx 1 + a_2/a_1 \approx -1.2$ in the factorization approximation for relevant hadronic matrix elements [$a_1 \approx 1.1$ and $a_2 \approx -0.5$ being the effective Wilson coefficients at the $\mathcal{O}(m_c)$ scale [30]]. When noncommutative geometry is taken into consideration, the relative weak phase between two interferring decay amplitudes of a D^+_s or D^-_s meson becomes associated with $\mathrm{Im}[(\overline{V}_{cd}\overline{V}^*_{ud})/(\overline{V}_{cs}\overline{V}^*_{us})]$. In this case, we obtain the momentum-dependent CP-violating asymmetry between the partial rates of $D^-_s \to K^-K_S$ and $D^+_s \to K^+K_S$ decays as follows:

$$\mathcal{A}_s \equiv \frac{|A(D^-_s \to K^-K_S)|^2 - |A(D^+_s \to K^+K_S)|^2}{|A(D^-_s \to K^-K_S)|^2 + |A(D^+_s \to K^+K_S)|^2}$$

$$\approx 2\,\mathrm{Re}\,\epsilon_K - 2\overline{\mathcal{J}}^{ds}_{uc} R_s \sin\delta_s. \tag{3.9}$$

If $\delta_s \sim \mathcal{O}(1)$ and $\xi^{ds}_{uc} \sim \mathcal{O}(\lambda^2)$ or $\overline{\mathcal{J}}^{ds}_{uc} \sim \mathcal{O}(\lambda^4)$ held, two different contributions to $\mathcal{A}_s$ would be comparable in magnitude. Therefore, a significant deviation of $\mathcal{A}_s$ from $2\,\mathrm{Re}\,\epsilon_K$, if experimentally observed, would signal the presence of new physics, which is likely to be noncommutative geometry.

IV. UNITARITY TRIANGLES IN *B*-MESON DECAYS

In the complex plane, the vector $\overline{V}^*_{\alpha i}\overline{V}_{\beta i}$ can be obtained from rotating the vector $V^*_{\alpha i}V_{\beta i}$ anticlockwise to a small angle $(x_{\alpha i} - x_{\beta i})/2$. It is therefore expected that $\overline{V}^*_{ub}\overline{V}_{ud}$, $\overline{V}^*_{cb}\overline{V}_{cd}$, and $\overline{V}^*_{tb}\overline{V}_{td}$ do not form a close triangle, as shown in Fig. 1(b). Nevertheless, one may define three angles by using these three vectors:

$$\overline{\alpha} \equiv \arg\left(-\frac{\overline{V}^*_{tb}\overline{V}_{td}}{\overline{V}^*_{ub}\overline{V}_{ud}}\right),$$

$$\overline{\beta} \equiv \arg\left(-\frac{\overline{V}^*_{cb}\overline{V}_{cd}}{\overline{V}^*_{tb}\overline{V}_{td}}\right),$$

$$\overline{\gamma} \equiv \arg\left(-\frac{\overline{V}^*_{ub}\overline{V}_{ud}}{\overline{V}^*_{cb}\overline{V}_{cd}}\right). \tag{4.1}$$

Comparing between Eqs. (1.3) and (4.1), we find

$$\overline{\alpha} = \alpha + \xi^{db}_{tu},$$

$$\overline{\beta} = \beta + \xi^{db}_{ct},$$

$$\overline{\gamma} = \gamma + \xi^{db}_{uc}. \tag{4.2}$$

By definition in Eq. (3.4), $\xi^{db}_{tu} + \xi^{db}_{ct} + \xi^{db}_{uc} = 0$ holds. It turns out that

$$\overline{\alpha} + \overline{\beta} + \overline{\gamma} = \alpha + \beta + \gamma = \pi \tag{4.3}$$

holds too. In Ref. [24], the momentum-dependent features of $\overline{\alpha}$, $\overline{\beta}$, and $\overline{\gamma}$ are illustrated in the assumption of $\eta = 0$ or $\mathcal{J} = 0$ (i.e., CP violation from the SM is switched off).

Besides α, β, and γ, CP violation in weak B-meson decays is also associated with the following three angles of the CKM unitarity triangles in the SM [29]:

$$\gamma' \equiv \arg\left(-\frac{V^*_{ub}V_{tb}}{V^*_{us}V_{ts}}\right),$$

$$\delta \equiv \arg\left(-\frac{V^*_{tb}V_{ts}}{V^*_{cb}V_{cs}}\right),$$

[1] Since CP violation in the kaon system is tiny, we expect that the weak phase of K^0-$\overline{K}^0$ mixing is nearly the same as that of K^0 vs $\overline{K}^0$ decays, which amounts to $(V_{us}V^*_{ud})/(V^*_{us}V_{ud})$ at the tree level [29]. It is therefore plausible to take $q_K/p_K = [(V_{us}V^*_{ud})(1-\epsilon_K)]/[(V^*_{us}V_{ud})(1+\epsilon_K)]$ as an effective description of the weak phase and the associated CP violation in K^0-$\overline{K}^0$ mixing.

TABLE I. Typical B_d and B_s decays and associated CP-violating asymmetries in the noncommutative standard model.

Class	Sub-process	Decay mode	CP asymmetry
$1d$	$\overline{b}\to\overline{c}\overline{c}\overline{s}$	$B_d^0\to J/\psi K_S$	$+\sin 2(\overline{\beta}+\overline{\omega})$
$2d$	$\overline{b}\to\overline{c}\overline{c}\overline{d}$	$B_d^0\to D^+D^-$	$-\sin 2\overline{\beta}$
$3d$	$\overline{b}\to\overline{u}\overline{u}\overline{d}$	$B_d^0\to\pi^+\pi^-$	$+\sin 2\overline{\alpha}$
$4d$	$\overline{b}\to\overline{s}\overline{s}\overline{s}$	$B_d^0\to\phi K_S$	$-\sin 2(\overline{\alpha}+\overline{\gamma'})$
$1s$	$\overline{b}\to\overline{c}\overline{c}\overline{s}$	$B_s^0\to D_s^+D_s^-$	$+\sin 2\overline{\delta}$
$2s$	$\overline{b}\to\overline{c}\overline{c}\overline{d}$	$B_s^0\to J/\psi K_S$	$-\sin 2(\overline{\gamma}-\overline{\gamma'})$
$3s$	$\overline{b}\to\overline{u}\overline{u}\overline{d}$	$B_s^0\to\rho K_S$	$+\sin 2\overline{\gamma'}$
$4s$	$\overline{b}\to\overline{s}\overline{s}\overline{s}$	$B_s^0\to\eta'\eta'$	0

$$\omega\equiv\arg\left(-\frac{V_{us}^*V_{ud}}{V_{cs}^*V_{cd}}\right). \tag{4.4}$$

It is easy to check that the relation $\delta+\omega=\gamma-\gamma'$ holds. The counterparts of γ', δ, and ω in the noncommutative SM are defined as

$$\overline{\gamma'}\equiv\arg\left(-\frac{\overline{V}_{ub}^*\overline{V}_{tb}}{\overline{V}_{us}^*\overline{V}_{ts}}\right),$$

$$\overline{\delta}\equiv\arg\left(-\frac{\overline{V}_{tb}^*\overline{V}_{ts}}{\overline{V}_{cb}^*\overline{V}_{cs}}\right),$$

$$\overline{\omega}\equiv\arg\left(-\frac{\overline{V}_{us}^*\overline{V}_{ud}}{\overline{V}_{cs}^*\overline{V}_{cd}}\right). \tag{4.5}$$

Of course, the similar relation $\overline{\delta}+\overline{\omega}=\overline{\gamma}-\overline{\gamma'}$ holds. Comparing between Eqs. (4.4) and (4.5), we obtain

$$\overline{\gamma'}=\gamma'+\xi_{ut}^{sb},$$

$$\overline{\delta}=\delta+\xi_{tc}^{sb},$$

$$\overline{\omega}=\omega+\xi_{uc}^{ds}. \tag{4.6}$$

One can see that ω or $\overline{\omega}$ is actually the weak phase associated with $D_s^\pm\to K^\pm K_S$ decays discussed above. As $|\delta|\approx\lambda^2\eta\sim2\times10^{-2}$ and $|\omega|\approx A^2\lambda^4\eta\sim5\times10^{-4}$ in the SM, the noncommutative effect may be comparable with δ in $\overline{\delta}$ and dominant over ω in $\overline{\omega}$. In particular, the latter could be a sensitive window to probe or constrain noncommutative geometry at low energies.

The weak angles $\overline{\alpha}$, $\overline{\beta}$, $\overline{\gamma}$, $\overline{\gamma'}$, $\overline{\delta}$, and $\overline{\omega}$ can be determined from direct and indirect CP-violating asymmetries in a variety of weak B decays. Here let us consider neutral B_d and B_s decays into CP eigenstates. In the neglect of penguin-induced pollution, indirect CP violation in such decay modes may arise from the interplay of direct B_q^0 and $\overline{B}_q^0$ decays (for $q=d$ or s) and B_q^0-$\overline{B}_q^0$ mixing [31]. If the final state consists of a K_S or K_L meson, then K^0-$\overline{K}^0$ mixing should also be taken into account. In the box-diagram approximation of the SM, the weak phase of B_q^0-$\overline{B}_q^0$ mixing is associated with the CKM factor $(V_{tb}^*V_{tq})/(V_{tb}V_{tq}^*)$. On the other hand, the weak phase of K^0-$\overline{K}^0$ mixing can simply be taken as $(V_{us}V_{ud}^*)/(V_{us}^*V_{ud})$, since CP violation is tiny in the kaon system [29]. When noncommutative geometry is concerned, all $V_{\alpha i}$ should be replaced by $\overline{V}_{\alpha i}$.

To illustrate how the inner angles of *deformed* unitarity triangles are related to the CP-violating asymmetries in neutral B-meson decay modes, we take B_d^0 vs $\overline{B}_d^0\to J/\psi K_S$ and B_s^0 vs $\overline{B}_s^0\to J/\psi K_S$ transitions for example. Their indirect CP-violating asymmetries Δ_d and Δ_s are given, respectively, as

$$\Delta_d=-\mathrm{Im}\left(\frac{\overline{V}_{tb}^*\overline{V}_{td}}{\overline{V}_{tb}\overline{V}_{td}^*}\cdot\frac{\overline{V}_{cb}\overline{V}_{cs}^*}{\overline{V}_{cb}^*\overline{V}_{cs}}\cdot\frac{\overline{V}_{us}\overline{V}_{ud}^*}{\overline{V}_{us}^*\overline{V}_{ud}}\right)$$
$$=+\sin2(\overline{\beta}+\overline{\omega}),$$

$$\Delta_s=-\mathrm{Im}\left(\frac{\overline{V}_{tb}^*\overline{V}_{ts}}{\overline{V}_{tb}\overline{V}_{ts}^*}\cdot\frac{\overline{V}_{cb}\overline{V}_{cd}^*}{\overline{V}_{cb}^*\overline{V}_{cd}}\cdot\frac{\overline{V}_{us}^*\overline{V}_{ud}}{\overline{V}_{us}\overline{V}_{ud}^*}\right)$$
$$=-\sin2(\overline{\gamma}-\overline{\gamma'}). \tag{4.7}$$

Here we have taken into account the fact that $J/\psi K_S$ is a CP-odd state. Possible deviations of such momentum-dependent observables from the SM predictions are worth searching for at B-meson factories.

In Table I, we list a number of typical decay channels of B_d and B_s mesons and their CP-violating asymmetries, including two examples given above. One can see that the weak angles $\overline{\alpha}$, $\overline{\beta}$, $\overline{\gamma}$, $\overline{\gamma'}$, $\overline{\delta}$, and $\overline{\omega}$ are (in principle) measurable. The self-consistent relations such as $\overline{\alpha}+\overline{\beta}+\overline{\gamma}=\pi$ and $\overline{\gamma}-\overline{\gamma'}=\overline{\delta}+\overline{\omega}$ could be tested, if the relevant angles were able to be determined at the same momentum scale.

Note that it is possible to distinguish noncommutative geometry from some other sources of new physics in indirect CP-violating asymmetries of B_d and B_s decays. Taking a variety of supersymmetric standard models, for example, we

TABLE II. Typical B_d and B_s decays and associated CP-violating asymmetries in the noncommutative standard model and in the presence of supersymmetric $\Delta B=2$ effects.

Class	Sub-process	Decay mode	CP asymmetry
1d	$\bar{b}\to\bar{c}c\bar{s}$	$B^0_d\to J/\psi K_S$	$+\sin 2(\bar{\beta}+\bar{\omega}+\theta_d)$
2d	$\bar{b}\to\bar{c}c\bar{d}$	$B^0_d\to D^+D^-$	$-\sin 2(\bar{\beta}+\theta_d)$
3d	$\bar{b}\to\bar{u}u\bar{d}$	$B^0_d\to \pi^+\pi^-$	$+\sin 2(\bar{\alpha}-\theta_d)$
4d	$\bar{b}\to\bar{s}s\bar{s}$	$B^0_d\to \phi K_S$	$-\sin 2(\bar{\alpha}+\bar{\gamma}'-\theta_d)$
1s	$\bar{b}\to\bar{c}c\bar{s}$	$B^0_s\to D^+_s D^-_s$	$+\sin 2(\bar{\delta}-\theta_s)$
2s	$\bar{b}\to\bar{c}c\bar{d}$	$B^0_s\to J/\psi K_S$	$-\sin 2(\bar{\gamma}-\bar{\gamma}'+\theta_s)$
3s	$\bar{b}\to\bar{u}u\bar{d}$	$B^0_s\to \rho K_S$	$+\sin 2(\bar{\gamma}'+\theta_s)$
4s	$\bar{b}\to\bar{s}s\bar{s}$	$B^0_s\to \eta'\eta'$	$-\sin 2\theta_s$

find that those CP-violating asymmetries listed in Table I may get corrections from gauginos, Higgsinos, and squarks through box diagrams which produce nonstandard $\Delta B=2$ effects. This kind of new physics can be parametrized in terms of two phases [29],

$$\theta_d \equiv \frac{1}{2}\arg\left(\frac{\langle B^0_d|\mathcal{H}^{\rm full}_{\rm eff}|\bar{B}^0_d\rangle}{\langle B^0_d|\mathcal{H}^{\rm SM}_{\rm eff}|\bar{B}^0_d\rangle}\right),$$

$$\theta_s \equiv \frac{1}{2}\arg\left(\frac{\langle B^0_s|\mathcal{H}^{\rm full}_{\rm eff}|\bar{B}^0_s\rangle}{\langle B^0_s|\mathcal{H}^{\rm SM}_{\rm eff}|\bar{B}^0_s\rangle}\right), \tag{4.8}$$

where $\mathcal{H}^{\rm full}_{\rm eff}$ is the effective Hamiltonian consisting of both standard and supersymmetric contributions, and $\mathcal{H}^{\rm SM}_{\rm eff}$ consists only of the contribution from the SM box diagrams. In the presence of θ_d and θ_s, the CP-violating asymmetries given in Table I get modified. We list the new results for those asymmetries in Table II. Comparing between Tables I and II, one can see that noncommutative and supersymmetric effects are actually distinguishable, if some of those CP-violating asymmetries are measured at B-meson factories. In particular, the CP asymmetry in B^0_s vs $\bar{B}^0_s\to\eta'\eta'$ decay modes is nonvanishing only if there is new physics contributing to B^0_s-$\bar{B}^0_s$ mixing.

In our discussions, the penguin-induced effects have been neglected. This approximation is expected to be reasonable for those B^0_d and B^0_s transitions occurring through the subprocesses $\bar{b}\to\bar{c}c\bar{d}$ and $\bar{b}\to\bar{c}c\bar{s}$, but it might be problematic for those charmless decay modes whose tree-level amplitudes are strongly CKM-suppressed. In the latter case, significant new noncommutative CP-violating effects may appear via the QCD penguins as a result of CP-odd phase factors in the relevant quark-gluon vertices [21]. The entanglement of different types of noncommutative CP violation in weak decays of quark flavors should be carefully analyzed. Such an analysis, which must involve much complexity and subtlety of the noncommutative SM, is beyond the scope of this paper.

V. SUMMARY

We have examined the possible effects of noncommutative geometry on weak CP violation and unitarity triangles based on a simple version of the momentum-dependent CKM matrix in the noncommutative SM. Among nine rephasing invariants of CP violation, we find that two of them are sensitive to the noncommutative corrections. In particular, the noncommutative CP-violating effect could be comparable with or dominant over the SM one in $D^\pm_s\to K^\pm K_S$ decays. We have also illustrated how the CKM unitarity triangles get deformed in the noncommutative SM. Simple relations are established between inner angles of the *deformed* unitarity triangles and CP-violating asymmetries in some typical decays of B_d and B_s mesons into CP eigenstates, such as $B_d\to J/\psi K_S$ and $B_s\to D^+_s D^-_s$. We anticipate that B-meson factories may help probe or constrain noncommutative geometry at low energies in the near future.

Finally, we remark that further progress in the noncommutative gauge field theory will allow us to study the phenomenology of noncommutative geometry on a more solid ground.

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