Kaluza-Klein gravitino production with a single photon at e^+e^- colliders

Seungwon Baek, Seong Chan Park, and Jeonghyeon Song
Korea Institute for Advanced Study, 207-43 Cheongryangri-dong, Dongdaemun-gu, Seoul 130-012, Korea
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In a supersymmetric large extra dimension scenario, the production of Kaluza-Klein gravitinos accompanied by a photino at e^+e^- colliders is studied. We assume that a bulk supersymmetry is softly broken on our brane such that the low-energy theory resembles the MSSM. Low energy supersymmetry breaking is further assumed as in GMSB, leading to a sub-eV mass shift in each KK mode of the gravitino from the corresponding graviton KK mode. Since the photino decays within a detector due to the sufficiently large inclusive decay rate of $\tilde{\gamma} \to \gamma \tilde{G}$, the process $e^+e^- \to \tilde{\gamma}\tilde{G}$ yields single photon events with missing energy. Even if the total cross section can be substantial at $\sqrt{s}=500$ GeV, the KK graviton background of $e^+e^- \to \gamma G$ is kinematically advantageous and thus much larger. It is shown that the observable $\Delta\sigma_{LR} \equiv \sigma(e_L^- e_R^+) - \sigma(e_R^- e_L^+)$ can completely eliminate the KK graviton background but retain most of the KK gravitino signal, which provides a unique and robust method to probe the *supersymmetric* bulk.

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I. INTRODUCTION

The standard model (SM) has been thoroughly tested in various experiments even if the Higgs boson remains as the only missing ingredient. From the theoretical viewpoint, however, the SM has several unsatisfactory aspects such as the gauge hierarchy problem: The Higgs boson mass near the electroweak scale requires a fine-tuning to eliminate quadratically divergent radiative corrections. Low-energy supersymmetry is known to cancel the quadratic divergence by introducing a supersymmetric partner for each SM particle [1]. Supersymmetry protects the electroweak scale Higgs boson mass from the Planck scale, as the chiral (gauge) symmetry does for fermions (gauge bosons).

Recently another new route to the solution of the gauge hierarchy problem has been opened, based on advances in string theories, by introducing extra dimensions. Arkani-Hamed, Dimopoulos, and Dvali (ADD) proposed that the large volume of δ-dimensional extra dimensions can explain the observed huge Planck scale $M_{\rm Pl}$ [2]: The fundamental gravitational scale or string scale M_S is related to the Planck scale $M_{\rm Pl}$ and the size of an extra dimension R by $M_{\rm Pl}^2 = M_S^{\delta+2} R^\delta$; the hierarchy problem is resolved as M_S can be maintained around a TeV. Later Randall and Sundrum (RS) proposed another higher dimensional scenario based on two branes and a single extra dimension compactified in a slice of anti–de Sitter space [3]: The hierarchy problem is explained by a geometrical exponential factor. These extra dimensional models are very interesting and they can lead to distinct and rich phenomenological signatures in future colliders, characterized by low-energy gravity effects [4,5]. In the ADD case, for example, the multiplicity of gravitons below an energy scale E is proportional to $(ER)^\delta (= M_{\rm Pl}^2 E^\delta / M_S^{\delta+2})$, which is extremely large and compensates the small gravitational coupling.

An economical description of new physics to solve the gauge hierarchy problem would introduce either low-energy supersymmetry or extra dimensions. Nevertheless supersymmetric bulk is theoretically more plausible [6,7] since the most realistic framework of extra dimensional models, string or M theory [8], indeed possesses supersymmetry as a fundamental symmetry. Moreover, extra dimensions can play the role of supersymmetry breaking on a hidden brane [9] or in the bulk by Scherk-Schwarz compactification [10].

Obviously phenomenological signatures of supersymmetric bulk are crucially dependent on how many supersymmetries survive on our brane below the scale M_S. One interesting possibility is that a single supersymmetry is softly broken on our brane such that our low-energy effective theory yields supersymmetric spectra as in the minimal supersymmetric standard model (MSSM). One distinctive feature of this scenario is the presence of the gravitino, the superpartner of the graviton. Since this gravitino also propagates in the full dimensional space as it belongs to the same supermultiplet with the graviton, we have gravitino Kaluza-Klein modes on our brane. The soft and spontaneous breaking of a supersymmetry results in the mass shift between a graviton Kaluza-Klein (KK) mode and the corresponding gravitino KK mode, of order $\Lambda_{\rm SUSY}^2 / M_{\rm Pl}$. Here the four-dimensional Planck mass $M_{\rm Pl}$ scales the strength of gravitino coupling and $\Lambda_{\rm SUSY}$ is the supersymmetry breaking scale. If low-energy supersymmetry breaking is assumed, e.g., in the gauge mediated supersymmetry breaking (GMSB) scenario, the resulting mass shift is very light: For $\Lambda_{\rm SUSY} \sim 100$ TeV, $\Lambda_{\rm SUSY}^2 / M_{\rm Pl} \sim 1$ eV. Restricting ourselves to the ADD scenario, we have almost continuous spectrum of KK gravitinos with the zero mode mass at sub-eV scale. In Ref. [11], the four-dimensional effective theory in a supersymmetric ADD scenario has been derived, including the couplings of the bulk gravitino KK states to a fermion and its superpartner. At e^+e^- colliders, the virtual exchange of KK gravitinos can occur only in the selectron pair production which was shown to substantially enhance the total cross section and change the kinematic distributions.

Another distinctive signature of KK gravitinos is their production at high-energy colliders. A superlight gravitino, which becomes the stable lightest supersymmetric particle (LSP), escapes any detector, leading to missing energy

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events. Moreover the decay modes of a supersymmetric particle $\tilde{X}$ are now changed. Even if the $\tilde{X}$ is the lightest among supersymmetric partners of the SM particles, e.g., the photino, a new decay mode of $\tilde{\gamma}\to\gamma\tilde{G}$ is opened and dominant. As shall be discussed later, this decay rate is large enough for the photino to decay within a detector. Therefore, the process $e^+e^-\to\tilde{\gamma}\tilde{G}$ yields a typical signature of a single photon at large transverse momentum. And the summation over all possible extra-dimensional momenta yields a sizable production rate characterized by the M_S scale. This process has kinematic advantages over the selectron pair production in case the selectron is too heavy to be pair produced.

Of great significance is to signal not only the extra dimensions, but also the supersymmetric extra dimensions, i.e., KK gravitinos. Unfortunately, single photon events with missing energy in this scenario have two more sources, the SM process of $e^+e^-\to\gamma\nu\bar{\nu}$ and the KK graviton production of $e^+e^-\to\gamma G$. With an appropriate cut to reduce the Z-pole contributions of the SM, the KK graviton production can be compatible with the SM background at the future e^+e^- collider. However the KK gravitino production rate is smaller than the KK graviton case by an order of magnitude. This is due to the kinematic suppression by the production of the massive photino while the dependence of the M_S and δ is the same for both the KK graviton and gravitino production. Total cross section alone cannot tell whether the bulk possesses supersymmetry or not. We shall show that the observable $\Delta\sigma_{LR}\equiv\sigma(e_L^- e_R^+)-\sigma(e_R^- e_L^+)$ completely eliminates the KK graviton effects, but retains most of the KK gravitino effects. This is because in a supersymmetric model the sign of the coupling of a left-handed electron with a photino (and a selectron) is opposite to that of a right-handed electron (a left-handed antielectron). The coupling with gravitino, which is gravitational, does not depend on the fermion chirality. Therefore, the scattering amplitudes of the t- and u-channel diagrams, where both couplings are involved, have opposite sign for e_L^- and e_R^- beams. As the t- and u-channel amplitudes are added to the s-channel one, mediated by a photon, the total scattering amplitudes are different for the left- and right-handed electron beam. For the KK graviton production accompanied by a single photon, all the involved interactions are chirality blind so that the corresponding $\Delta\sigma_{LR}$ vanishes.

Our paper is organized as follows. In Sec. II, we review the four-dimensional effective Lagrangian in a supersymmetric ADD scenario, and analytic expressions for photino decay rate into a photon and KK gravitinos and for the process $e^+e^-\to\tilde{\gamma}\tilde{G}$ are to be given. Section III is devoted to the phenomenological discussions of this scenario, including total cross section, kinematic distributions, a specific observable by using the polarization of electron beam, and so on. In Sec. IV we give our conclusions.

II. EFFECTIVE GRAVITINO LAGRANGIAN

In this paper, we assume that there are δ large and supersymmetric extra dimensions, and a single supersymmetry is softly broken on our brane such that our low-energy effective theory yields the MSSM spectra. The cases of more than three extra dimensions are to be considered since in the $\delta=2$ case astrophysical and cosmological constraints are too strong that the M_S is pushed up to about 100 TeV, disfavored as a solution of the gauge hierarchy problem [12]. And the MSSM superparticles are assumed to be confined on our brane. A new feature is another KK tower of the gravitino. The compactification of the gravitino field in supergravity theory leads to the four-dimensional effective action which is a sum of KK states of massive spin 3/2 gravitinos [11]. The free part of the effective Lagrangian gives the propagator of the $\vec{n}$-the KK mode of the gravitino with momentum k and mass m_n such as

$$\frac{i\mathcal{P}^{\vec{n}}_{\mu\nu}}{k^2-m_{\vec{n}}^2}. \tag{1}$$

Here $\mathcal{P}^{\vec{n}}_{\mu\nu}$ is

$$\begin{aligned}\mathcal{P}^{\vec{n}}_{\mu\nu}&\equiv\sum_{\lambda}\tilde{G}^{\vec{n}}_{\mu}(k,\lambda)\overline{\tilde{G}^{\vec{n}}_{\nu}}(k,\lambda)\\ &=(\not{k}+m_n)\left(\frac{k_\mu k_\nu}{m_n^2}-\eta_{\mu\nu}\right)\\ &\quad-\frac{1}{3}\left(\gamma^\mu+\frac{k^\mu}{m_n}\right)(\not{k}-m_n)\left(\gamma^\nu+\frac{k^\nu}{m_n}\right),\end{aligned} \tag{2}$$

where $\gamma^\mu\mathcal{P}^{\vec{n}}_{\mu\nu}=0$ and $k^\mu\mathcal{P}^{\vec{n}}_{\mu\nu}=0$.

The effective interaction Lagrangian for the KK gravitino is obtained from the general Noether technique, irrespective to the detailed supersymmetry breaking mechanism. The coupling of each KK mode of graviton and gravitino is determined by the Planck constant

$$M_{\rm Pl}^{-1}\equiv\kappa=\sqrt{8\pi G_{\rm Newton}}\approx\frac{1}{2.4\times10^{18}\ \text{GeV}}. \tag{3}$$

Minimally coupled to gravity, the interactions of a KK gravitino with a fermion and photon field to leading order in κ are [16]

$$\mathcal{L}_{f\tilde{f}\tilde{G}}=-\frac{\kappa}{\sqrt{2}}[\bar{\tilde{G}}_\mu\gamma^\nu\gamma^\mu\psi_L\partial_\nu\phi_L^*+\bar{\tilde{G}}_\mu\gamma^\nu\gamma^\mu\psi_R\partial_\nu\phi_R^*+\text{H.c.}], \tag{4}$$

$$\mathcal{L}_{\gamma\tilde{\gamma}\tilde{G}}=-\frac{\kappa}{4}\bar{\tilde{\gamma}}\gamma^\mu[\gamma^\rho,\gamma^\sigma]\tilde{G}_\mu\partial_\rho A_\sigma+\frac{\kappa}{4}\bar{\tilde{G}}_\mu[\gamma^\rho,\gamma^\sigma]\gamma^\mu\tilde{\gamma}\partial_\rho A_\sigma. \tag{5}$$

For later discussion, we present the interaction Lagrangian for the electron-selectron photino

$$\mathcal{L}_{f\tilde{f}\tilde{\gamma}}=-\sqrt{2}eQ_f[\bar{\tilde{\gamma}}_R\psi_L\phi_L^*+\bar{\psi}_L\tilde{\gamma}_R\phi_L-\bar{\tilde{\gamma}}_L\psi_R\phi_R^*-\bar{\psi}_R\tilde{\gamma}_L\phi_R]. \tag{6}$$

Since each KK mode of gravitons and gravitinos escapes a detector, experimentally applicable are inclusive rates with all the kinematically allowed KK modes summed up. Due to the very small mass splitting among KK modes, the summation can be approximated by a continuous integration over the KK mode mass m such as [4]

$$\sum_{\vec{n}} \to \int dm \frac{M_{\rm Pl}^2 m^{\delta-1}}{M_S^{2+\delta}} S_{\delta-1}, \tag{7}$$

where $S_{\delta-1}$ is the volume of the unit sphere in δ dimensions, given by $S_{\delta-1}=2\pi^{\delta/2}/\Gamma(\delta/2)$. The $M_{\rm Pl}^2$ in the numerator, implying the tremendous number of accessible KK modes, compensates the gravitational coupling. In effect, the Planck scale is lowered to the M_S of TeV scale.

A. Decay rate of a photino

It has been known that the presence of a light gravitino alters the decay modes of supersymmetric particles as the gravitino becomes the LSP; the decay mode of $\tilde{X}\to X\tilde{G}$ becomes dominant [16,17]. Even though the coupling strength of the gravitino is Planck suppressed, the wave function of a light gravitino with mass $m_{3/2}$, momentum k^μ and helicity $\pm 1/2$ is an ordinary spin 1/2 wave function multiplied by the large factor $\sqrt{2/3}\,k^\mu/m_{3/2}$ [16]. The gravitino mass $m_{3/2}$, e.g., in the gauge mediation supersymmetry breaking (GMSB) where the supersymmetry breaking scale is generically low, is

$$m_{3/2}=\frac{\Lambda_{\rm SUSY}^2}{\sqrt{3}M_{\rm Pl}}\simeq 2.36\left(\frac{\Lambda_{\rm SUSY}}{100\ \ {\rm TeV}}\right)^2\ \ {\rm eV}. \tag{8}$$

Thus the $M_{\rm Pl}$ term in the $m_{3/2}$ cancels the gravitational coupling $M_{\rm Pl}$, so that the characteristic scale of the decay rate becomes the supersymmetry breaking scale $\Lambda_{\rm SUSY}$. The photino decay rate is known to be [17]

$$\Gamma(\tilde{\gamma}\to\gamma\tilde{G})=\frac{1}{48\pi}\frac{M_{\tilde{\gamma}}^5}{M_{\rm Pl}^2 m_{3/2}^2}=\frac{1}{16\pi}\frac{M_{\tilde{\gamma}}^5}{\Lambda_{\rm SUSY}^4}, \tag{9}$$

where $M_{\tilde{\gamma}}$ is the photino mass. For $\Lambda_{\rm SUSY}\lesssim 10^3$ TeV with $M_{\tilde{\gamma}}\sim 100$ GeV, the photino decays within a Collider Detector at Fermilab (CDF) type detector.

In a supersymmetric ADD scenario, a photino can decay into a photon and a KK mode of a gravitino, if kinematically allowed. The decay rate for the $\vec{n}$th KK gravitino is

$$\Gamma(\tilde{\gamma}\to\gamma\,\tilde{G}^{\vec{n}})=\frac{1}{48\pi}\frac{\kappa^2 M_{\tilde{\gamma}}^5}{m_n^2}\left(1-\frac{m_n^2}{M_{\tilde{\gamma}}^2}\right)^3\left(1+3\frac{m_n^2}{M_{\tilde{\gamma}}^2}\right). \tag{10}$$

The inclusive decay rate of a photino is obtained by the sum in Eq. (7):

$$\Gamma_{\rm tot}\equiv\sum_{\vec{n}}\ \Gamma(\tilde{\gamma}\to\gamma\tilde{G}^{\vec{n}})=\frac{f_\delta}{16\pi}\frac{M_{\tilde{\gamma}}^5}{M_S^4}\left(\frac{M_{\tilde{\gamma}}}{M_S}\right)^{\delta-2}, \tag{11}$$

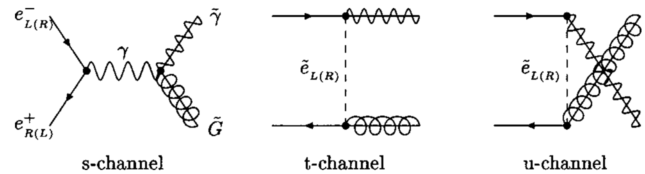

FIG. 1. Feynman diagrams for the process $e^+e^-\to\tilde{\gamma}\tilde{G}$.

where $f_\delta\equiv 64S_{\delta-1}/\{(\delta^2-4)(\delta+4)(\delta+6)\}$ of order one. Numerically $f_3\approx 2.55$, $f_4\approx 1.32$, $f_5\approx 0.81$, and $f_6\approx 0.52$. Here one should note that the $\Gamma_{\rm tot}$ does not depend on the exact value of $\Lambda_{\rm SUSY}$ which determines the zero mode mass of the KK gravitino, as long as the supersymmetry breaking ensures a superlight gravitino.

In general, the decay rate $\Gamma_{\rm tot}$ is quite large for M_S of order TeV even with the suppression of $(M_{\tilde{\gamma}}/M_S)^{\delta-2}$. For various numbers of extra dimensions, the magnitude of the inclusive photino decay rate is

$$\Gamma_{\rm tot}=\left(\frac{M_{\tilde{\gamma}}}{100\ {\rm GeV}}\right)^{\delta+3}\left(\frac{1\ {\rm TeV}}{M_S}\right)^{\delta+2}\times\begin{cases}50.8\ {\rm keV} & {\rm for}\quad \delta=3,\\ 2.62\ {\rm keV} & {\rm for}\quad \delta=4,\\ 0.16\ {\rm keV} & {\rm for}\quad \delta=5,\\ 0.01\ {\rm keV} & {\rm for}\quad \delta=6.\end{cases} \tag{12}$$

Then, the average distance traveled by a photino with energy E in the laboratory frame is

$$L=(E^2/M_{\tilde{\gamma}}^2-1)^{1/2}\left(\frac{100\ \ {\rm GeV}}{M_{\tilde{\gamma}}}\right)^{\delta+3}\left(\frac{M_S}{1\ \ {\rm TeV}}\right)^{\delta+2}\times\begin{cases}4.0\times10^{-10}\ \ {\rm cm} & {\rm for}\quad \delta=3,\\ 7.7\times10^{-9}\ \ {\rm cm} & {\rm for}\quad \delta=4,\\ 1.3\times10^{-7}\ \ {\rm cm} & {\rm for}\quad \delta=5,\\ 1.8\times10^{-6}\ \ {\rm cm} & {\rm for}\quad \delta=6.\end{cases} \tag{13}$$

Thus the photino decays within a detector, leaving a detectable photon signal. In the following, we investigate at e^+e^- collisions the production of a KK gravitino and a photino, which generates single photon events with missing energy.

B. Cross section of $e^+e^-\to\tilde{\gamma}\tilde{G}$

For the process

$$e^-(p_1,\lambda_e)+e^+(p_2,\overline{\lambda_e})\to\tilde{\gamma}(k_1)+\tilde{G}^{\vec{n}}(k_2), \tag{14}$$

there are three Feynman diagrams mediated by the selectron and photon as depicted in Fig. 1. The Mandelstam variables are defined by $s=(p_1+p_2)^2$, $t=(p_1-k_1)^2$, and $u=(p_1-k_2)^2$. Then the helicity amplitudes apart from $i\kappa e$ factor, defined by $\mathcal{M}(\lambda_e,\overline{\lambda_e})\equiv i\kappa e\hat{\mathcal{M}}^{\lambda_e}$, are

$$\hat{\mathcal{M}}^{\mp}=\overline{v}_e(p_2)\gamma^\mu P_\mp u_e(p_1)\overline{\tilde{G}}_\nu(k_2)\Bigg[\pm\frac{1}{t-\tilde{m}^2_{e_\mp}}\times(p_1-k_1)^\nu\gamma_\mu P_\mp\mp\frac{1}{u-\tilde{m}^2_{e_\mp}}(p_1-k_2)^\nu\gamma_\mu P_\pm-\frac{1}{4s}[\not k_1+\not k_2,\gamma_\mu]\gamma^\nu\Bigg]v_{\tilde{\gamma}}(k_1), \tag{15}$$

where $P_\pm=(1\pm\gamma^5)/2$ and $\tilde{m}_{e_{-(+)}}=\tilde{m}_{e_{L(R)}}$.

The differential cross section is then

$$\frac{d^2\sigma}{dx_{\tilde{\gamma}}d\cos\theta_{\tilde{\gamma}}}(e^+e^-\to\tilde{\gamma}\tilde{G})=\frac{\alpha}{32}S_{\delta-1}\left(\frac{\sqrt{s}}{M_S}\right)^{\delta+2}\frac{1}{s}\left(1+\frac{M^2_{\tilde{\gamma}}}{s}-x_{\tilde{\gamma}}\right)^{\delta/2-1}\times\sqrt{\lambda_{\tilde{\gamma}}}f_{\tilde{G}}(x_{\tilde{\gamma}},\cos\theta), \tag{16}$$

where $x_{\tilde{\gamma}}\equiv 2E_{\tilde{\gamma}}/\sqrt{s}$ and $\lambda_{\tilde{\gamma}}\equiv\lambda(1,M^2_{\tilde{\gamma}}/s,1+M^2_{\tilde{\gamma}}/s-x_{\tilde{\gamma}})$. Here $\lambda(a,b,c)=a^2+b^2+c^2-2ab-2bc-2ac$ and

$$f_{\tilde{G}}(x_{\tilde{\gamma}},\cos\theta)\equiv\frac{|\hat{\mathcal{M}}^-|^2+|\hat{\mathcal{M}}^+|^2}{2s}. \tag{17}$$

The range of $x_{\tilde{\gamma}}$ is $[2M_{\tilde{\gamma}}/\sqrt{s},1+M^2_{\tilde{\gamma}}/s]$. The amplitudes squared are summarized in the Appendix. It is to be compared to the KK graviton production process

$$\frac{d^2\sigma}{dx_\gamma d\cos\theta}(e^+e^-\to\gamma G)=\frac{\alpha}{32}S_{\delta-1}\left(\frac{\sqrt{s}}{M_S}\right)^{\delta+2}\frac{1}{s}(1-x_\gamma)^{(\delta/2-1)}f_G(x_\gamma,\cos\theta), \tag{18}$$

where $x_\gamma\equiv 2E_\gamma/\sqrt{s}$ and for f_G we refer to Ref. [4].

Equations (16) and (18) show that both the differential cross sections have the same M_S dependence. The gravitino production accompanied by a massive photino is at a kinematically disadvantage, relative to the graviton production with a massless photon. The measurement of total cross section alone is not enough to probe supersymmetric bulk effects. Some kinematic distributions and other observables are needed.

We notice that there is one crucial characteristic for the gravitino production accompanied by a photino. As explicitly shown in Eq. (6), the coupling sign of a left-handed electron with a photino and a selectron is opposite to that of a right-handed electron: The holomorphy of the super potential requires that a fermion should belong to a (left-handed) chiral superfield; the right-handed electron is to be described by a left-handed antielectron, which possesses positive charge. The interaction with a gravitino, which is gravitational, does not distinguish the chirality of the involved fermion. Therefore, the scattering amplitudes of the t- and u-channel diagrams, which include one e-$\tilde{e}$-$\tilde{\gamma}$ and one e-$\tilde{e}$-$\tilde{G}$ coupling, have opposite sign for the left- and right-handed electron beam. In the s-channel diagram, the electron is coupled with the ordinary QED photon. Since two kinds of amplitudes (one changes the sign under the helicity flip of the electron beam, whereas the other does not) are added, we end up with chirality-sensitive total cross section. Note that without the s-channel diagram, the sign change in the amplitudes alone cannot yield any observable effect, as clearly shown in Eq. (A1). It is to be emphasized that this feature is generic in any supersymmetric model which ensures a light gravitino. In the ordinary MSSM, this point is hard to probe. For example, in the photino pair production, double vertices of e-$\tilde{e}$-$\tilde{\gamma}$ in the t- and u-channel Feynman diagrams eliminate the difference.

It is known that the availability of polarized electron and positron beams is highly expected at future linear collider [18]: The current LC performance goal is above 80% of electron polarization and 60% of positron polarization. We propose, therefore, that the effects of KK gravitinos can be most sensitively measured by

$$\Delta\sigma_{LR}\equiv\sigma(e^-_Le^+_R\to\gamma\not E_T)-\sigma(e^-_Re^+_L\to\gamma\not E_T). \tag{19}$$

For the graviton production with a photon, all the involved couplings are completely blind to the helicity of the electron beam; the $\Delta\sigma_{LR}$ vanishes. Moreover, in the SM, the main contribution from the Z pole to the $\Delta\sigma_{LR}$ is proportional to $[(g_V+g_A)^2-(g_V-g_A)^2]=4g_Vg_A$, where $g_V=-1/2+2\sin^2\theta_W$ and $g_A=-1/2$ [19]. The smallness of g_V suppresses the SM Z-pole background also.

III. NUMERICAL RESULTS

The cross section of the process $e^+e^-\to\tilde{\gamma}\tilde{G}$ obviously depends sensitively on the mass spectrum of the involved supersymmetric particles, a photino, and the left- and right-handed selectron. Since the contribution of the $\tilde{m}_{e_R}$ to the total cross section is very small, two mass scales ($M_{\tilde{\gamma}}$ and $\tilde{m}_{e_L}$) effectively determine the production rate. One can obtain the mass spectrum of superparticles by specifying a concrete supersymmetry breaking model, such as the GMSB model which guarantees a light gravitino. Instead we rather consider the experimental mass bounds when the decay mode of $\tilde{X}\to X\tilde{G}$ is open: At the LEP the negative results of the $\gamma\not E_T$ event search from $e^+e^-\to\tilde{G}\tilde{\chi}^0_1(\tilde{\chi}^0_1\to\gamma\tilde{G})$ lead to $M_{\tilde{\gamma}}\gtrsim 82.5$ GeV, and those of the $\gamma\gamma\not E_T$ from $e^+e^-\to\tilde{\chi}^0_1\tilde{\chi}^0_1(\tilde{\chi}^0_1\to\gamma\tilde{G})$ to $M_{\tilde{\gamma}}\gtrsim 86.5$ GeV [13]. Similarly, the CERN e^+e^- collider (LEP) bound with a light gravitino is $\tilde{m}_e\gtrsim 77$ GeV. In the following numerical analysis, we adopt the lower mass bounds of a photino and a left-handed selectron as $M_{\tilde{\gamma}}\geqslant 90$ GeV and $\tilde{m}_{e_L}\geqslant 80$ GeV. The $\tilde{m}_{e_R}$ is set to be 200 GeV, which affects little the total cross section. Figure 2 presents the total cross section as a function of $M_{\tilde{\gamma}}$ and $\tilde{m}_{e_L}$

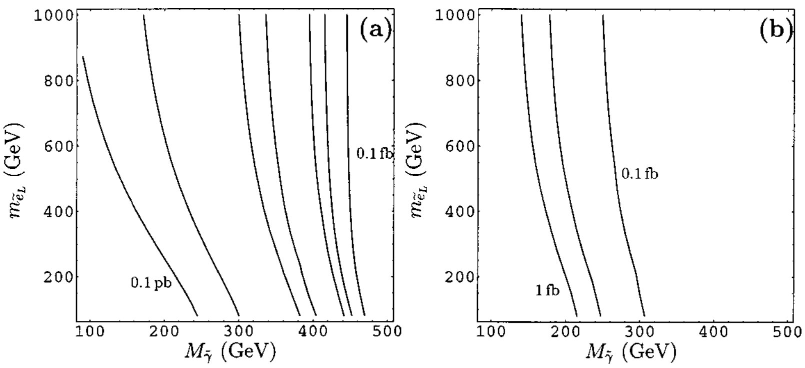

FIG. 2. At $\sqrt{s}=500$ GeV with $M_S=1$ TeV, the total cross section of $e^+e^-\to\tilde{\gamma}\tilde{G}$ as a function of $M_{\tilde{\gamma}}$ and $\tilde{m}_{e\_}$. (a) is for $\delta=3$, with the contours from the left denoting $\sigma_{\rm tot}=100$, 50, 10, 5, 1, 0.5, 0.1 fb. (b) is for $\delta=6$ with $\sigma_{\rm tot}=1$, 0.5, 0.1 fb.

for $\delta=3$ (a) and $\delta=6$ (b). We set $\sqrt{s}=500$ GeV and $M_S=1$ TeV with the kinematic cut of $|\cos\theta_{\tilde{\gamma}}|<0.95$. For the case of $M_{\tilde{\gamma}}=90$ GeV and $\tilde{m}_{e_L}=80$ GeV where the total cross section reaches its maximum $\sigma_{\rm tot}=321.4$ fb for $\delta=3$ and $\sigma_{\rm tot}=8.8$ fb for $\delta=6$. With the design luminosity of 500 (100) fb^{-1}/yr of the DESY TeV Energy Superconducting linear Accelerator (TESLA) [Japan Linear Collider (JLC) and Next Linear Collider (NLC)] [14,15], even the $\delta=6$ case with $M_{\tilde{\gamma}}\lesssim 300$ GeV can produce substantial events. Being conservative, we present the parameter space of $(M_{\tilde{\gamma}},\tilde{m}_{e_L})$ for $\sigma_{\rm tot}>0.1$ fb. It can been seen that in the $\tilde{e}_L$ decoupling range with $\tilde{m}_{e_L}\gtrsim 500$ GeV, the s-channel diagram alone can produce sizable cross section. As expected from the presence of light KK gravitinos, this single photino production mode can probe the photino mass much higher than $\sqrt{s}/2$, the kinematic maximum for the photino pair production: For $\delta=3$, the photino with $M_{\tilde{\gamma}}\lesssim 460$ GeV can be sufficiently produced; for $\delta=6$, that with $M_{\tilde{\gamma}}\lesssim 260$ GeV. In the following, we set $M_{\tilde{\gamma}}=90$ GeV and $\tilde{m}_{e_L}=80$ GeV.

In Fig. 3, we compare the polarized cross sections of the single photon production at e^+e^- collisions, with the neutrino pair in the SM $(e^+e^-\to\gamma\nu\bar{\nu})$ denoted by $\sigma^{\pm}_{\rm SM}$, with the KK gravitons $(e^+e^-\to\gamma G)$ by $\sigma^{\pm}(G)$, and with the KK gravitinos $(e^+e^-\to\tilde{\gamma}\tilde{G})$ by $\sigma^{\pm}(\tilde{G})$. Here superscript $\pm$ denotes the chirality of the electron beam. To eliminate the SM Z-pole contribution as much as possible, we employ the following kinematic cuts:

$$20\ \text{GeV}<E_{\gamma(\tilde{\gamma})}<\frac{s-M_Z^2}{2\sqrt{s}}-20\ \text{GeV and } |\cos\theta_{\gamma(\tilde{\gamma})}|<0.95. \tag{20}$$

Since the $\sigma_{\rm SM}$ with the above cuts are mainly through the t- and u-channel diagrams mediated by the W boson, the $\sigma^{+}_{\rm SM}$ is much smaller than the $\sigma^{-}_{\rm SM}$. For the KK graviton production, the blindness of the interactions of the graviton and the photon to the fermion chirality guarantees the equality of $\sigma^-(G)$ and $\sigma^+(G)$. For the KK gravitino production, there are several interesting points. First its cross section is only a few tens of percents of that for the KK graviton production. This is due to the kinematic suppression by the massive photino. Second, the behavior of the cross section with respect to $\sqrt{s}$ is the same as the KK graviton case, which increases due to the use of four-dimensional *effective* Lagrangian. Finally the opposite sign of the photino coupling with the left- and right-handed electron leads to the domination of the $\sigma^-(\tilde{G})$ over the $\sigma^+(\tilde{G})$. In Fig. 4, we present the ratio of $\Delta\sigma_{LR}(\tilde{\gamma}\tilde{G})$ to $\Delta\sigma_{LR}(SM)$. As discussed before, the $\Delta\sigma_{LR}$ vanishes for the KK graviton production. Therefore, any deviation of the $\Delta\sigma_{LR}$ from the SM background hints the presence of *supersymmetric* extra dimensions. And this deviation increases with the beam energy.

Figure 5 presents the differential cross section of the KK gravitino production with respect to the photino energy fraction $x_{\tilde{\gamma}}(\equiv 2E_{\tilde{\gamma}}/\sqrt{s})$ for various δ. In the $\delta<4$ case, a rapid increase occurs as the $x_{\tilde{\gamma}}$ reaches its maximum; energetic photinos are more likely produced. This behavior can be un-

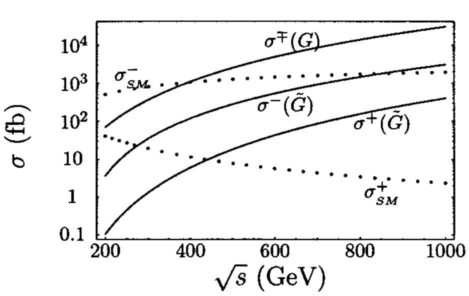

FIG. 3. The polarized cross sections for the $e^+e^-\to\gamma\nu\bar{\nu}$ in the SM denoted by $\sigma_{\rm SM}$, $e^+e^-\to\gamma G^{\rm KK}$ by $\sigma(G)$, and $e^+e^-\to\tilde{\gamma}\tilde{G}$ by $\sigma(\tilde{G})$ when $\delta=3$. The SM $\gamma Z\to\gamma\nu\bar{\nu}$ background is reduced by a kinematic cut.

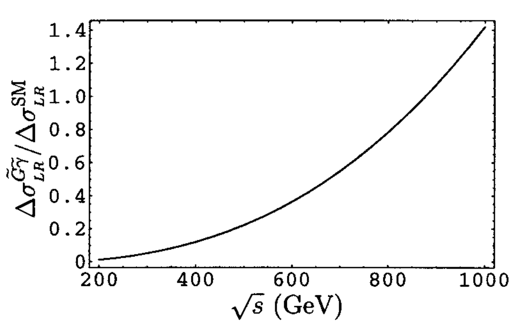

FIG. 4. The ratio of the $\Delta\sigma_{LR}(e^+e^-\to\tilde{\gamma}\tilde{G})$ to the $\Delta\sigma_{LR}$(SM) as a function of $\sqrt{s}$.

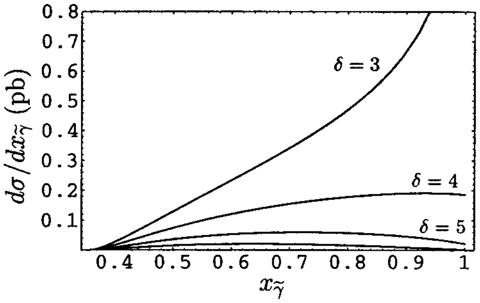

FIG. 5. The differential cross section of the KK gravitino production with respect to the photino energy fraction $x_{\tilde{\gamma}}(\equiv 2E_{\tilde{\gamma}}/\sqrt{s})$.

derstood from Eqs. (16), (17), and (A1). Near the maximum of $x_{\tilde{\gamma}}$, light KK gravitinos are produced, where the differential cross section behaves as

$$\lim_{m^2\to 0} \frac{d\sigma}{dx_{\tilde{\gamma}}} \propto \lim_{m^2\to 0} \frac{(m^2)^{\delta/2-1}}{m^2}, \tag{21}$$

which the m^2 in the denominator comes from the amplitude squared in Eq. (A1). The different behavior of the $\delta<4$ case is explained. The measurement of this differential cross section can tell whether the number of extra dimensions is 3 or more. In the $\delta=3$ case, the scattering angle of the photino can be well approximated by that of the photon decayed from the energetic photino. Figure 6 exhibits the angular distribution shapes for the $\delta=3$ case, by plotting $(1/\sigma)d\sigma/dz_\gamma$ with $z_\gamma\equiv\cos\theta_\gamma$. The normalization by the total cross section reveals the generic shape of the angular distribution. For the SM and the KK graviton production, the shapes are very similar: Most of the photons are produced toward the beam line. The KK gravitino production shows different behavior: The angular distribution shape is rather flat. In Tables I and II, we summarize the sensitivity to the M_S at 95% C.L. in two cases, when only the KK gravitons are produced and when the KK gravitinos are also produced. Table I is for $\sqrt{s}=183$ GeV with the luminosity of 55.3 pb^{-1}, and Table II for $\sqrt{s}=500$ GeV with the luminosity of 100 fb^{-1}. We have applied the kinematic cuts in Eq. (20). With the KK gravitinos, the increased cross section generally raises the sensitivity bound on the M_S. Unfortunately, the resulting change is practically negligible.

IV. CONCLUSIONS

Originally, extra dimensional models have been introduced to solve the gauge hierarchy problem without resort to supersymmetry. However if the ultimate theory is string theory, we live in higher dimensional spacetime which has supersymmetry as a fundamental symmetry and branes tend to break supersymmetry. An interesting scenario is that there are large and supersymmetric extra dimensions and at least one supersymmetry survives on our brane below the scale M_S so that the low-energy effective theory on our brane resembles the MSSM.

TABLE I. The M_S bound in GeV from the $\sigma_{\rm tot}$ with the kinematic cuts in Eq. (20) at $\sqrt{s}=183$ GeV and the luminosity of 55.3 pb^{-1} at 95% C.L.

	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$
G^{KK}	764.5	621.5	525.6	457.5
$G^{KK}+\tilde{G}^{KK}$	782.4	625.2	526.5	457.6

The gravity supermultiplet resides in the bulk, which includes the graviton and its super-partner, the gravitino. On our brane, we have Kaluza-Klein towers of the graviton and gravitino. If supersymmetry is not broken, KK modes of the graviton would have the same mass spectrum as those of the gravitino; the zero mode of gravitino remains massless. As the supersymmetry is broken by an expectation value of order $\Lambda_{\rm SUSY}$, each gravitino KK mode acquires additional mass of $\Lambda_{\rm SUSY}^2/M_{\rm Pl}$. Under the assumption of low-energy $\Lambda_{\rm SUSY}$, this mass shift is sub-eV scale. In practice, KK gravitinos exist with almost continuous mass spectrum from zero.

In this scenario, we have studied the KK gravitino production at e^+e^- collisions. With R-parity conservation, the KK gravitino is produced with a supersymmetric particle, e.g., the photino. Since light KK gravitinos become the lightest supersymmetric particle (LSP), the photino decays into a photon and a KK gravitino (missing energy). It has been shown that the inclusive decay rate of $\tilde{\gamma}\to\gamma\tilde{G}^{\vec{n}}$ is large enough for the photino to decay within a detector. Therefore, the process $e^+e^-\to\tilde{\gamma}\tilde{G}$ yields a typical signature of a single photon with missing energy. In the phenomenological allowed parameter space of $(M_{\tilde{\gamma}},\tilde{m}_{e_L})$, we have shown that the total cross section can be substantial: At $\sqrt{s}=500$ GeV, $\sigma_{\rm tot}>0.1$ fb for $M_{\tilde{\gamma}}\lesssim 460$ GeV in the $\delta=3$ case and for $M_{\tilde{\gamma}}\lesssim 260$ GeV in the $\delta=6$ case. The dependence of $\tilde{m}_{e_L}$ is rather weak; even in the range of $\tilde{m}_{e_L}\gtrsim 500$ GeV, we have sizable cross section.

Unfortunately, the background processes (the SM reaction of $e^+e^-\to\gamma\nu\bar{\nu}$ and the KK graviton production of $e^+e^-\to\gamma G$) have much larger cross sections. With the M_S of TeV, the KK graviton production becomes compatible with the

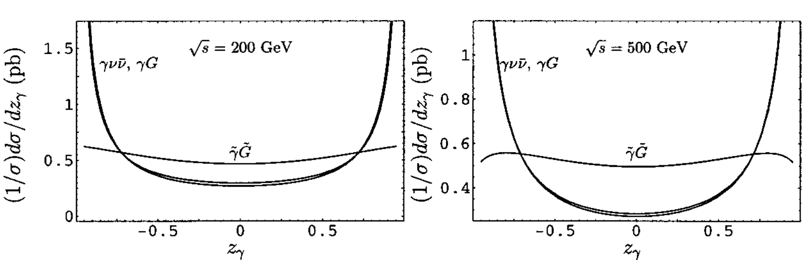

FIG. 6. $(1/\sigma)d\sigma/dz_\gamma$, the angular distributions of the cross sections for the SM, the KK graviton, and KK gravitino production.

TABLE II. The same M_S bound in GeV at $\sqrt{s}=500$ GeV and the luminosity of 100 fb^{-1}.

	$\delta=3$	$\delta=4$	$\delta=5$	$\delta=6$
G^{KK}	3250	2505	2037	1719
$G^{KK}+\tilde{G}^{KK}$	3398	2559	2061	1732

SM background around $\sqrt{s}=500$ GeV. However the production of a massive photino kinematically suppresses the KK gravitino production rate compared to the KK graviton case by an order of magnitude, since the M_S dependence is the same. To single out the effect of KK gravitinos, the total cross section is not enough.

We have noticed that the observable $\Delta\sigma_{LR}\equiv\sigma(e_L^- e_R^+) - \sigma(e_R^- e_L^+)$ can completely eliminate the KK graviton background. This is because both the gravitational and QED interactions, which are involved in the KK graviton production, do not distinguish the electron beam chirality; $\Delta\sigma_{LR}(\gamma G)$ vanishes. For the KK gravitino production accompanied by a photino, the electron chirality becomes important since the interaction of $e_L^- - \tilde{e}_L - \tilde{\gamma}$ has opposite sign to that of $e_R^- - \tilde{e}_R - \tilde{\gamma}$, such that $\sigma(e_L^- e_R^+) \gg \sigma(e_L^- e_R^+)$. The ratio of $\Delta\sigma_{LR}(SM)$ to $\Delta\sigma_{LR}(\tilde{\gamma}\tilde{G})$ is demonstrated to increase with the beam energy, implying that the observable $\Delta\sigma_{LR}$ is unique and robust to probe the *supersymmetric* bulk.

We also found that the differential cross section with respect to the photino energy fraction $x_{\tilde{\gamma}}$ can tell whether the number of extra dimensions is three or more: In the $\delta=3$ case, the $d\sigma/dx_{\tilde{\gamma}}$ increases rapidly as $x_{\tilde{\gamma}}$ approaches its maximum; energetic photinos are more likely produced. And the angular distribution shapes, e.g., for the $\delta=3$ case, are presented: For the KK gravitino it is more or less flat, while for the SM and the KK graviton they rapidly increase toward the beam line. The sensitivity bound of the M_S at 95% C.L. does not practically change by taking into account of KK gravitino effects due to the kinematic suppression of the KK gravitino production cross section.

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APPENDIX: THE SQUARED AMPLITUDES OF $E^+E^-\to\tilde{\gamma}\tilde{G}$

For the process $e^+e^-\to\tilde{\gamma}\tilde{G}^{\vec{n}}$, the amplitudes squared in terms of the Mandelstam variables defined in the text are

$$|\hat{\mathcal{M}}_s^{\mp}|^2 = -\frac{2}{3s}\Bigg(\frac{(t+u)(t^2+u^2)}{m_n^2}+2(st+su-2tu) + 2m_n\{m_n(m_n^2-M_{\tilde{\gamma}}^2-s)+4M_{\tilde{\gamma}}s\}\Bigg), \quad \text{(A1)}$$

$$|\hat{\mathcal{M}}_t^{\mp}|^2 = \frac{2}{3m_n^2}\frac{(M_{\tilde{\gamma}}^2-t)(m_n^2-t)^3}{(\tilde{m}_{e_\mp}^2-t)^2},$$

$$|\hat{\mathcal{M}}_u^{\mp}|^2 = \frac{2}{3m_n^2}\frac{(M_{\tilde{\gamma}}^2-u)(m_n^2-u)^3}{(\tilde{m}_{e_\mp}^2-u)^2},$$

$$2\,\mathrm{Re}\hat{\mathcal{M}}_s^{\mp\dagger}\hat{\mathcal{M}}_t^{\mp} = \pm\frac{4}{3m_n^2}\frac{1}{t-\tilde{m}_{e_\mp}^2}[t(m_n^2-t)^2 + m_nM_{\tilde{\gamma}}\{m_n^2(s-2t)+2t(s+t) + 2M_{\tilde{\gamma}}^2(m_n^2-t)\}],$$

$$2\mathrm{Re}\hat{\mathcal{M}}_s^{\mp\dagger}\hat{\mathcal{M}}_u^{\mp} = \pm\frac{4}{3m_n^2}\frac{1}{u-\tilde{m}_{e_\mp}^2}[u(m_n^2-u)^2 + m_nM_{\tilde{\gamma}}\{m_n^2(s-2u)+2u(s+u) + 2M_{\tilde{\gamma}}^2(m_n^2-u)\}],$$

$$2\mathrm{Re}\hat{\mathcal{M}}_t^{\mp\dagger}\hat{\mathcal{M}}_u^{\mp} = -\frac{4M_{\tilde{\gamma}}s}{3m_n(t-\tilde{m}_{e_\mp}^2)(u-\tilde{m}_{e_\mp}^2)} \times(2M_{\tilde{\gamma}}^2m_n^2+m_n^2s-2tu).$$

[1] H.E. Haber and G.L. Kane, Phys. Rep. **117**, 75 (1985); H.P. Nilles, *ibid.* **110**, 1 (1984).
[2] N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **429**, 263 (1998); Phys. Rev. D **59**, 086004 (1999); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos, and G. Dvali, Phys. Lett. B **436**, 257 (1998).
[3] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
[4] G.F. Giudice, R. Rattazzi, and J.D. Wells, Nucl. Phys. **B544**, 3 (1999).
[5] T. Han, J.D. Lykken, and R. Zhang, Phys. Rev. D **59**, 105006 (1999); E.A. Mirabelli, M. Perelstein, and M.E. Peskin, Phys. Rev. Lett. **82**, 2236 (1999); J.L. Hewett, *ibid.* **82**, 4765 (1999); K.Y. Lee, H.S. Song, and J. Song, Phys. Lett. B **464**, 82 (1999); K.Y. Lee, S.C. Park, H.S. Song, J. Song, and C. Yu, Phys. Rev. D **61**, 074005 (2000); H. Davoudiasl, J.L. Hewett, and T.G. Rizzo, Phys. Rev. Lett. **84**, 2080 (2000); Phys. Rev. D **63**, 075004 (2001).
[6] D. Atwood, C.P. Burgess, E. Filotas, F. Leblond, D. London, and I. Maksymyk, Phys. Rev. D **63**, 025007 (2001); D. Marti and A. Pomarol, *ibid.* **64**, 105025 (2001); D.E. Kaplan and T.M. Tait, J. High Energy Phys. **06**, 020 (2000); D.E. Kaplan,

G.D. Kribs, and M. Schmaltz, Phys. Rev. D **62**, 035010 (2000); A. Delgado, A. Pomarol, and M. Quiros, *ibid.* **60**, 095008 (1999); T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000).

[7] R. Altendorfer, J. Bagger, and D. Nemeschansky, Phys. Rev. D **63**, 125025 (2001); J. Bagger, D. Nemeschansky, and R.J. Zhang, J. High Energy Phys. **08**, 057 (2001); T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000); **B602**, 3 (2001).

[8] For reviews see, for example: J.H. Schwarz, Nucl. Phys. B (Proc. Suppl.) **55B**, 1 (1997); M.J. Duff, Int. J. Mod. Phys. A **11**, 5623 (1996); J. Polchinski, hep-th/9611050; P.K. Townsend, hep-th/9612121; C. Bachas, hep-th/9806199; C.V. Johnson, hep-th/9812196.

[9] N. Arkani-Hamed, L.J. Hall, Y. Nomura, D. Smith, and N. Weiner, Nucl. Phys. **B605**, 81 (2001); E.A. Mirabelli and M.E. Peskin, Phys. Rev. D **58**, 065002 (1998); N. Arkani-Hamed, T. Gregoire, and J. Wacker, J. High Energy Phys. **03**, 055 (2002); L. Randall and R. Sundrum, Nucl. Phys. **B557**, 79 (1999); Z. Chacko, M.A. Luty, A.E. Nelson, and E. Ponton, J. High Energy Phys. **01**, 003 (2000); A.E. Nelson and N.J. Weiner, Phys. Rev. Lett. **88**, 231802 (2002).

[10] J. Scherk and J.H. Schwarz, Phys. Lett. **82B**, 60 (1979); Nucl. Phys. **B153**, 61 (1979); S. Ferrara, C. Kounnas, M. Porrati, and F. Zwirner, *ibid.* **B318**, 75 (1989); I. Antoniadis, Phys. Lett. B **246**, 377 (1990); C. Kounnas and B. Rostand, Nucl. Phys. **B341**, 641 (1990); E. Dudas and C. Grojean, *ibid.* **B507**, 553 (1997); I. Antoniadis, S. Dimopoulos, A. Pomarol, and M. Quirós, *ibid.* **B544**, 503 (1999); I. Antoniadis, E. Dudas, and A. Sagnotti, Phys. Lett. B **464**, 38 (1999); I. Antoniadis, K. Benakli, and A. Laugier, hep-th/0111209; R. Barbieri, L.J. Hall, and Y. Nomura, Phys. Rev. D **63**, 105007 (2001).

[11] J.L. Hewett and D. Sadri, hep-ph/0204063.

[12] S. Cullen and M. Perelstein, Phys. Rev. Lett. **83**, 268 (1999); V.D. Barger, T. Han, C. Kao, and R.J. Zhang, Phys. Lett. B **461**, 34 (1999); C. Hanhart, D.R. Phillips, S. Reddy, and M.J. Savage, Nucl. Phys. **B595**, 335 (2001); L.J. Hall and D.R. Smith, Phys. Rev. D **60**, 085008 (1999); S. Hannestad and G.G. Raffelt, Phys. Rev. Lett. **88**, 071301 (2002).

[13] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[14] ECFA/DESY LC Physics Working Group Collaboration, J.A. Aguilar-Saavedra *et al.*, hep-ph/0106315.

[15] American Linear Collider Working Group Collaboration, T. Abe *et al.*, in Proceedings of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001), edited by R. Davidson and C. Quigg, SLAC-R-570 Resource book for Snowmass 2001, Snowmass, Colorado.

[16] A. Mendez and F.X. Orteu, Nucl. Phys. **B256**, 181 (1985); P. Fayet, Phys. Lett. B **175**, 471 (1986); Phys. Lett. **84B**, 421 (1979).

[17] G.F. Giudice and R. Rattazzi, Phys. Rep. **322**, 419 (1999); S. Ambrosanio, G.L. Kane, G.D. Kribs, S.P. Martin, and S. Mrenna, Phys. Rev. D **54**, 5395 (1996); D.R. Stump, M. Wiest, and C.P. Yuan, *ibid.* **54**, 1936 (1996).

[18] ECFA/DESY LC Physics Working Group Collaboration, J.A. Aguilar-Saavedra *et al.*, hep-ph/0106315; American Linear Collider Working Group Collaboration, T. Abe *et al.*, in Proceedings of the APS/DPF/DPB Summer Study on the Future of Particle Physics (Snowmass 2001), edited by R. Davidson and C. Quigg, hep-ex/0106058.

[19] F.A. Berends, G.J. Burgers, C. Mana, M. Martinez, and W.L. van Neerven, Nucl. Phys. **B301**, 583 (1988); A.D. Dolgov, L.B. Okun, and V.I. Zakharov, *ibid.* **B41**, 197 (1972); E. Ma and J. Okada, Phys. Rev. Lett. **41**, 287 (1978); **41**, 1759(E) (1978); K.J. Gaemers, R. Gastmans, and F.M. Renard, Phys. Rev. D **19**, 1605 (1979).