Gauge origin of baryon number conservation and suppressed neutrino masses from five dimensions

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We consider a 5D SUSY $SU(3)_c\times SU(2)_L\times U(1)_Y\times \mathcal{U}(1)$ model compactified on an $S^{(1)}/Z_2$ orbifold. To cancel anomalies arising from the presence of $\mathcal{U}(1)$, we employ a Chern-Simons term and also chiral fields which could reside on the brane or in the bulk depending on the model. The presence of $\mathcal{U}(1)$ symmetry leads to baryon number conservation, gives rise to matter parity, and permits satisfactory neutrino masses and mixings even for a low fundamental scale. The brane Fayet-Iliopoulos D terms naturally break $\mathcal{U}(1)$, leaving $N=1$ SUSY unbroken in 4 dimensions.

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I. INTRODUCTION

The so far unsuccessful search for proton decay by the SuperKamiokande experiment [1] has yielded a lower bound of around 10^{33} yr on the lifetime, which proves especially challenging for supersymmetric models that allow the decay to proceed via the dimension five operators

$$\frac{\lambda}{M}qqql$$

and

$$\frac{\lambda'}{M}u^cu^cd^ce^c, \tag{1}$$

where $M\sim M_{\rm Pl}=2.4\times 10^{18}$ GeV denotes the reduced Planck mass. The dimensionless parameters λ,λ' must be $<10^{-8}$ or so, which demands some reasonable explanation. The suppression of such $d=5$ operators can be realized by either imposing discrete gauge [2], flavor [3], string-induced anomalous $U(1)$ [4] or R symmetries [5]. One must also suppress $d=5$ operators emerging through the exchange of additional states, such as the colored triplets appearing in grand unified theories (GUTs). Various mechanisms can be applied [4–7] to this end, making the nucleon sufficiently long lived [8].

The problem of B conservation becomes much more acute in extra dimensional theories with a low fundamental scale. The main phenomenological motivation for these kinds of models is the possibility of resolving the gauge hierarchy problem [9]. However, lowering the fundamental mass scale M_f down to a few TeV increases the $d=5$ operator induced nucleon decay amplitude by a factor of $M_{\rm Pl}/M_f\sim 10^{16}$ unless some additional mechanism for B conservation is applied. In Ref. [10], scenarios with gauged baryon number were considered and the matter sector was extended in order to cancel the anomalies. Reference [11] suggested scenarios in which quarks and leptons are localized on different 3-branes separated in the extra dimension(s). As a result, baryon number violating operators can be strongly suppressed. In Ref. [12], within the framework of a five dimensional (5D) $SU(5)$ orbifold GUT, certain $d=5$ operators were eliminated using special prescriptions of orbifold symmetry parities. It is also possible in such models to obtain GUT symmetry breaking and doublet-triplet splitting. However, Planck scale $d=5$ operators can still be problematic and additional care must be taken to suppress them [13].

In this paper, we present a new scenario in which baryon number arises as an accidental symmetry at the 4D level, which originates from 5D SUSY $SU(3)_c\times SU(2)_L\times U(1)_Y$, supplemented with a $\mathcal{U}(1)$ symmetry. After imposing a Z_2 projection, $\mathcal{U}(1)$ becomes anomalous on the fixed points. The 4D $\mathcal{U}(1)^3$ anomaly is canceled by a bulk Chern-Simons (CS) term [14–19]. The known quark, lepton and Higgs superfields carry nontrivial $\mathcal{U}(1)$ charges, whereas their $N=2$ mirrors carry opposite charges. The mixed anomalies are canceled through suitable assignments of $\mathcal{U}(1)$ charges for the quark-lepton superfields and by some additional chiral states. In the 5D bulk, we have a manifestly vectorlike theory. After imposing a $S^{(1)}/Z_2$ orbifold compactification, we obtain 4D $N=1$ SUSY $SU(3)_c\times SU(2)_L\times U(1)_Y$ supplemented with a $\mathcal{U}(1)$ gauge factor. The latter is crucial not only for suppressing B violating operators to the desired level, but also for obtaining appropriately suppressed neutrino masses and automatic matter parity. All this can be achieved for various values of the fundamental mass scale, with M_f as low as ~ 100 TeV. The $\mathcal{U}(1)$ symmetry can also be successfully employed as a flavor symmetry to explain the hierarchies among the charged fermion masses and their mixings.

II. 5D SUSY $SU(3)_C\times SU(2)_L\times U(1)_Y\times \mathcal{U}(1)$ ON AN $S^{(1)}/Z_2$ ORBIFOLD

It is a well known fact that after a Z_2 projection, bulk fermion fields can introduce an anomaly localized on both fixed points [14–20]. This anomaly can be written in the form $D_AJ^{a,A}(y)=Q^a(y)f(y)$, where $Q^a\equiv(g^2/32\pi^2)\epsilon^{\alpha\beta\gamma\delta}F^b_{\alpha\beta}F^c_{\gamma\delta}{\rm Tr}[T^a\{T^b,T^c\}]$. Provided $\int_0^{2\pi R}dy f(y)=0$, a bulk CS term can be added to cancel the anomalies from the fermions. As shown in Refs. [15–19],

$f(y)=\frac{1}{2}[\delta(y)+\delta(y-\pi R)]$, and the integral of f is nonzero, which means we cannot cancel the anomaly simply with a CS term. However, this can be remedied by adding additional fermion fields in such a way that the integral of f is zero. After this cancellation, the quantized theory will be free of local gauge anomalies [21,22]. Anomaly cancellation by adding a bulk CS term was considered in Refs. [14–20]. Here, we will exploit it for obtaining baryon number conservation in four dimensions [23,24]. In 5D, we will introduce a $\mathcal{U}(1)$ gauge symmetry which, prior to the addition of a CS term, is anomalous and suppresses dangerous baryon number violating operators to the desired level.

Consider then a 5D supersymmetric $SU(3)_c\times SU(2)_L\times U(1)_Y$ supplemented with a $\mathcal{U}(1)$ gauge symmetry. In 4D notation, the $N=2$ gauge superfield $\mathbf{V}_{N=2}=(V,\Phi)$ contains an $N=1$ gauge superfield V and a chiral superfield Φ, both of which are in the adjoint representation of the gauge group. The chiral supermultiplet $\mathbf{H}_{N=2}=(H,\overline{H})$ contains two $N=1$ chiral superfields H and $\overline{H}$ transforming as p and $\overline{p}$-plets respectively under the gauge group. H denotes all the "matter" and/or "scalar" superfields of the minimal supersymmetric standard model (MSSM), while $\overline{H}$ denotes their mirrors. In $N=1$ notation, the 5D action includes [15,25]:

$$S^{(5)}=\int d^5x(\mathcal{L}_V^{(5)}+\mathcal{L}_H^{(5)}), \tag{2}$$

where

$$\mathcal{L}_V^{(5)}=\frac{1}{4g^2}\int d^2\theta W^\alpha W_\alpha+\text{H.c.}+\frac{1}{g^2}\int d^4\theta((\sqrt{2}\partial_5 V+\Phi^+)e^{-V}(-\sqrt{2}\partial_5 V+\Phi)e^V+\partial_5 e^{-V}\partial_5 e^V), \tag{3}$$

$$\mathcal{L}_H^{(5)}=\int d^4\theta(H^+e^{-V}H+\overline{H}e^V\overline{H}^+)+\int d^2\theta\overline{H}\left(M_H+\partial_5-\frac{1}{\sqrt{2}}\Phi\right)H+\text{H.c.}, \tag{4}$$

and W_α are the supersymmetric field strengths. The action in Eq. (2) is invariant under the gauge transformations

$$e^V\rightarrow e^\Lambda e^V e^{\Lambda^+},\quad \Phi\rightarrow e^\Lambda(\Phi-\sqrt{2}\partial_5)e^{-\Lambda},$$
$$H\rightarrow e^\Lambda H,\quad \overline{H}\rightarrow\overline{H}e^{-\Lambda}. \tag{5}$$

In Eqs. (3)–(5)

$$\Phi=\frac{1}{\sqrt{2}}(\Sigma+iA_5)+\sqrt{2}\theta\psi+\theta\theta F, \tag{6}$$

where A_5 is the fifth component of a 5D gauge field and Σ is the real adjoint coming from 5D $N=1$ gauge supermultiplet.

We consider compactification on an $S^{(1)}/Z_2$ orbifold, with all fields having a definite Z_2 parity. States with positive and negative parities H_+, $\overline{H}_-$ can be expressed as

TABLE I. The $\mathcal{U}(1)$ charges and Z_2 parities of gauge, matter and scalar superfields.

$N=2$ supermultiplet	$\mathcal{U}(1)$ charge	Z_2 parity
All $\mathbf{V}_{N=2}=(V,\Phi)$	(0, 0)	$(+,-)$
$\mathbf{X}_{N=2}=(X,\overline{X})$	$(1,-1)$	$(+,-)$
$Q_{N=2}=(q,\overline{q})$	$(a,-a)$	$(+,-)$
$U^c_{N=2}=(u^c,\overline{u^c})$	$(-a+\alpha,a-\alpha)$	$(+,-)$
$D^c_{N=2}=(d^c,\overline{d^c})$	$(-a-n+\gamma,a+n-\gamma)$	$(+,-)$
$L_{N=2}=(l,\overline{l})$	$(b+\gamma,-b-\gamma)$	$(+,-)$
$E^c_{N=2}=(e^c,\overline{e^c})$	$(-b-n,b+n)$	$(+,-)$
$H^u_{N=2}=(h_u,\overline{h_u})$	$(-\alpha,\alpha)$	$(+,-)$
$H^d_{N=2}=(h_d,\overline{h_d})$	$(-\gamma,\gamma)$	$(+,-)$

$$H_+=\frac{\sqrt{2}}{\sqrt{\pi R}}\sum_{n=0}^{n=\infty}H^{(n)}(x)\eta^{(n)}\cos\left(\frac{ny}{R}\right),$$
$$\overline{H}_-=\frac{\sqrt{2}}{\sqrt{\pi R}}\sum_{n=1}^{n=\infty}\overline{H}^{(n)}(x)\sin\left(\frac{ny}{R}\right), \tag{7}$$

where $\eta^0=1/\sqrt{2}$ and $\eta^{(n)}=1$ for $n\neq 0$. As can be seen from Eq. (7), $\overline{H}_-$ does not have a zero mode. The fixed point $y=0$ is identified as the 3-brane corresponding to our 4D world.

In 5D, we also introduce a SM singlet superfield $\mathbf{X}_{N=2}=(X,\overline{X})$ which carries a $\mathcal{U}(1)$ charge and is crucial for $\mathcal{U}(1)$ symmetry breaking in 4D. The field content of the 5D model is given by

$$Q_{N=2}=(q,\overline{q}),\quad U^c_{N=2}=(u^c,\overline{u^c}),\quad D^c_{N=2}=(d^c,\overline{d^c}),$$
$$L_{N=2}=(l,\overline{l}),\quad E^c_{N=2}=(e^c,\overline{e^c}), \tag{8}$$
$$H^u_{N=2}=(h_u,\overline{h_u}),\quad H^d_{N=2}=(h_d,\overline{h_d}),\quad \mathbf{X}_{N=2}=(X,\overline{X}). \tag{9}$$

The $\mathcal{U}(1)$ charges and Z_2 parities of the various components of the gauge $(\mathbf{V}_{N=2})$ and "matter"-"scalar" $(\mathbf{H}_{N=2})$ superfields are displayed in Table I. Note that a,b,α and γ are numbers to be specified later and n is a positive integer.

After projecting out states with negative Z_2 parity, we effectively have 4D $N=1$ MSSM supplemented with a $\mathcal{U}(1)$ gauge symmetry and a superfield X. In the next section we shall see that the $\mathcal{U}(1)^3$ anomaly from the fermions can be cancelled by a compensating contribution from a CS action involving the $\mathcal{U}(1)$ gauge field.

III. ANOMALY CANCELLATION

The 5D anomaly from bulk fermion fields on $S^{(1)}/Z_2$ is given by [14–20]

$$D_A J^{a,A}(y)=\frac{Q^a(y)}{2}[\delta(y)+\delta(y-\pi R)] \tag{10}$$

where D is the covariant derivative, A denotes the five spatial dimensions, a labels the T^a generator of the gauge group, and

$$Q^a \equiv \frac{g^2}{32\pi^2}\epsilon^{\alpha\beta\gamma\delta}F^b_{\alpha\beta}F^c_{\gamma\delta}\mathrm{Tr}[T^a\{T^b,T^c\}]. \tag{11}$$

By contrast, the anomalies due to brane fermion fields localized on the $y=0$ or the $y=\pi R$ brane are given by

$$D_A J^{a,A}(y)=Q^a(y)\,\delta(y), \quad D_A J^{a,A}(y)=Q^a(y)\,\delta(y-\pi R) \tag{12}$$

respectively. Note that the contributions to the anomaly from the bulk and brane fermion fields differ by a factor of 2. So, unless the (rational) $\mathcal{U}(1)$ charges of all the fermion fields satisfy $\mathrm{Tr}_{brane}[T_a\{T_b,T_c\}]=-\frac{1}{2}\mathrm{Tr}_{bulk}[T_a\{T_b,T_c\}]$ on both branes, we cannot cancel the anomalies induced by the bulk fermions simply by adding brane fermions. In general, if we insist on rational $\mathcal{U}(1)$ charges, such an assignment will not be possible. However, using a combination of additional fermion fields and a CS term in the action, we can cancel the local gauge anomalies everywhere.

The (nonsupersymmetric) CS action is given by

$$S_{CS}=\int_M \chi(y)\mathrm{Tr}\left[\mathbf{AF}^2-\frac{1}{4}\mathbf{A}^2\mathbf{F}-\frac{1}{4}\mathbf{FA}^2+\frac{1}{10}\mathbf{A}^4\right] \tag{13}$$

where $\mathbf{A}=A^a_\mu T^a dx^\mu$, $\mathbf{F}=\frac{1}{2}F^a_{\mu\nu}T^a dx^\mu\wedge dx^\nu$, and M is the spacetime manifold. This is a slightly modified form of the CS action because of the addition of a neutral field χ, which could either be a dynamical field whose VEV satisfies Eq. (20) or a nondynamical function. Since the Lagrangian must be even, χ has to have a negative Z_2 parity. So, unless χ is trivially zero everywhere, it has to have a y dependence.

Under an infinitesimal gauge transformation which transforms the fermion fields, ψ,

$$\psi \to \psi + i\omega\psi, \tag{14}$$

we can show that

$$\delta\mathbf{A}=i\omega\mathbf{A}-i\mathbf{A}\omega+\frac{1}{g}d\omega \tag{15}$$

$$\delta\mathbf{F}=i\omega\mathbf{F}-i\mathbf{F}\omega \tag{16}$$

$$\delta S_{\mathrm{CS}}=-\frac{1}{g}\int_M d\chi\,\mathrm{Tr}[\omega\mathbf{F}^2]. \tag{17}$$

From these equations, we have

$$\begin{aligned}\delta(S_{\mathrm{CS}}+S_{\mathrm{rest}})&=\mathrm{Tr}\left(\frac{\delta}{\delta\mathbf{A}}(S_{\mathrm{CS}}+S_{\mathrm{rest}})\cdot\delta\mathbf{A}\right)\\&\quad+\left(\delta\psi\frac{\delta}{\delta\psi}+\delta\phi\frac{\delta}{\delta\phi}+\cdots\right)(S_{\mathrm{CS}}+S_{\mathrm{rest}})\\&=\int d^4x\,dy\,\mathrm{Tr}\left(J^\mu\cdot\left(i[\omega,A_\mu]+\frac{1}{g}\partial_\mu\omega\right)\right)\\&=\int d^4x\,dy\,\mathrm{Tr}\left(iJ^\mu\cdot[\omega,A_\mu]-\frac{1}{g}\omega\nabla_\mu\cdot J^\mu\right),\end{aligned} \tag{18}$$

where ϕ represents the charged scalar fields and $\cdots$ represents the variation due to all the other charged fields. In the previous equation, we have split the action into three parts, $S=S_{\mathrm{CS}}+S_{\mathrm{gaugekinetic}}+S_{\mathrm{rest}}$ and used the definition $\mathbf{J}\equiv(\delta/\delta\mathbf{A})(S_{\mathrm{CS}}+S_{\mathrm{rest}})$ for the current. S_{rest} includes all the terms of the action except the CS and the gauge kinetic term and is a functional of $\mathbf{A}$ because the covariant derivative is used in the matter part of the action. But since S_{rest} is gauge invariant by assumption,

$$D_A J^{a,A}=\frac{d\chi}{dy}\epsilon^{\alpha\beta\gamma\delta}F^b_{\alpha\beta}F^c_{\gamma\delta}\mathrm{Tr}[T^a\{T^b,T^c\}]. \tag{19}$$

Since we only want anomaly cancellation on the fixed points, χ ought to have the following profile:

$$\chi=\begin{cases}\chi_0, & 0<y<\pi R,\\ -\chi_0, & \pi R<y<2\pi R.\end{cases} \tag{20}$$

With this form for χ, the 4D anomalous terms induced from variation of the CS action has opposite signs on both branes. Now, with the addition of brane fermions on the $y=0$ brane with the appropriate quantum numbers to contribute an anomaly of $-Q$, the anomaly on the $y=0$ brane is $-Q/2$ and the anomaly on the $y=\pi R$ brane is $Q/2$. But since the anomalies are now of opposite signs, they can be cancelled by the CS action with an appropriate value for χ_0. Another possibility is to add the brane fermions to the $y=\pi R$ brane instead. Now, the anomalies would be $Q/2$ and $-Q/2$ on the $y=0$ and the $y=\pi R$ branes respectively. This can also be cancelled by the CS action. A third possibility, of course, is to have the additional fermions in the bulk, obeying the same Z_2 projection as the other fields. The additional fermion fields would then have chiral zero modes and massive vector Kaluza-Klein modes from a 4D point of view. In this case, the anomalies cancel locally and no CS counterterm is needed. But in fact, however, it can be shown that in the limit as the absolute value of the 5D mass, $|M|$, of the additional fermion fields goes to infinity [26], the low energy effective theory would be that of a chiral brane field plus an effective CS action [18] with the appropriate value for χ to cancel the anomalies, reducing to the other two possibilities mentioned earlier.

As far as the mixed anomalies are concerned, for their cancellation we introduce some additional superfields. Namely, an $SU(3)_c$ triplet, F_1, and an $SU(3)_c$ antitriplet,

F_2, which are neutral under $SU(2)_L$ and $U(1)_Y$ [other possibilities seem to give rise to $\mathcal{U}(1)$ charge assignments in such a way that either suppression of proton decay does not hold, or the additional states obtain masses of order the electroweak scale]. These additional fields couple to each other on the brane through the interaction term $X^k F_1 F_2$, where $-k$ is the sum of the $\mathcal{U}(1)$ charges of both fields. Referring to Table I, we can see that the mixed $SU(3)_c^2-\mathcal{U}(1)$, $SU(2)_L^2-\mathcal{U}(1)$, $U(1)_Y^2-\mathcal{U}(1)$ and $U(1)_Y-\mathcal{U}(1)^2$ anomalies vanish if the following relations hold:

$$\gamma=n+k-\alpha,\quad b=\alpha-3a-\frac{2}{3}n-\frac{2}{3}k,$$

$$n=3k,\quad \alpha=\frac{1}{16}(60a+39k). \tag{21}$$

This leaves us with the $\mathcal{U}(1)^3$ and the $\mathcal{U}(1)-\mathrm{grav}^2$ anomalies. For cancellation of $\mathcal{U}(1)^3$ anomaly we invoke the bulk CS term. For $\mathcal{U}(1)-\mathrm{grav}^2$ anomaly cancellation we add additional $SU(3)\times SU(2)\times U(1)_Y$ singlet fields which are charged under $\mathcal{U}(1)$, such that $\mathrm{Tr}Q_{\mathcal{U}(1)}=0$. The latter condition also avoids divergences in the renormalization of the Fayet-Iliopoulos term (FI) [19].

IV. NEUTRINO MASSES

The 4D superpotential couplings which generate the charged fermion masses are given by

$$W_Y^{(4)}=qu^ch_u+\left(\frac{X}{M_f}\right)^n qd^ch_d+\left(\frac{X}{M_f}\right)^n le^ch_d, \tag{22}$$

where M_f denotes some fundamental mass scale. A nonzero VEV for the scalar component of X is guaranteed by a brane Fayet-Iliopoulos term for $V_{\mathcal{U}(1)}$, which is permitted by all 4D symmetries. One can also show that within the 5D orbifold framework, the brane FI term does not induce SUSY breaking. In Appendix A, we present a detailed analysis of these issues. We assume that $\langle X\rangle$ [$\mathcal{U}(1)$ breaking scale] is not too far below M_f, i.e.

$$\frac{\langle X\rangle}{M_f}\equiv\epsilon\simeq 0.2. \tag{23}$$

This value of ϵ is an important expansion parameter for understanding the charged fermion mass hierarchies and mixings [27]. Since $\tan\beta\simeq(m_t/m_b)\epsilon^n$, n has to take values between 0 and 3 to reproduce the observed masses. Here we consider two scenarios: (I) $M_f\simeq M_{\mathrm{Pl}}=2.4\times10^{18}$ GeV and (II) $M_f\sim 100$ TeV.

For case (I), the Planck scale $d=5$ operators $(lh_u)^2/M_{\mathrm{Pl}}$ (if permitted) induce neutrino masses that are much too low to explain the atmospheric neutrino anomaly via oscillations. To generate neutrino mass $\sim3\times10^{-2}$ eV, we have to introduce a right handed neutrino state. Introduce an MSSM singlet $N=2$ supermultiplet $\mathcal{N}_{N=2}=(\mathcal{N},\overline{\mathcal{N}})$ with $\mathcal{U}(1)$ charge $(Q_{\mathcal{N}},-Q_{\mathcal{N}})$ and Z_2 parity $(+,-)$. Then, only $\mathcal{N}$ will have a zero mode. The relevant 4D superpotential couplings responsible for neutrino masses are

$$W_\nu^{(4)}=\left(\frac{X}{M_{\mathrm{Pl}}}\right)^m l\mathcal{N}h_u+M_{\mathrm{Pl}}\left(\frac{X}{M_{\mathrm{Pl}}}\right)^p\mathcal{N}^2, \tag{24}$$

where m and p are non-negative integers. The light neutrino acquires mass of order of $h_u^2/(M_{\mathrm{Pl}}\epsilon^{p-2m})=(10^{-2}-1)$ eV for $\epsilon\simeq0.2$ and $p-2m=5$–8. This mass scale for the third generation neutrino suggest either hierarchical [27,28] or degenerate [29] masses for the neutrinos, if one wants to account for both the atmospheric and solar neutrino anomalies (see [30] and [31] respectively).

The couplings in Eqs. (24) and (21) and the prescriptions of Table I give

$$a=\frac{203}{36}k-\frac{2}{3}(p-2m)\quad\text{and}\quad\alpha=\frac{283}{12}k-\frac{5}{2}(p-2m). \tag{25}$$

(II) For a fundamental scale of $M_f\simeq100$ TeV, the situation is quite different in the neutrino sector. Here, we do not need to introduce right handed states. The suppression of (Majorana) neutrino masses can be guaranteed by $\mathcal{U}(1)$ symmetry. The relevant 4D coupling is

$$W_\nu^{(4)}=\left(\frac{X}{M_f}\right)^r\frac{(lh_u)^2}{M_f} \tag{26}$$

(where r is a positive integer), which gives $m_\nu\simeq h_u^2\epsilon^r/M_f\simeq(0.1$–$1)$ eV for $\epsilon\simeq0.2$ and $r=11$–13. The couplings in Eqs. (26) and (21) together with the prescriptions of Table I give

$$a=\frac{1}{252}(24r-53k)\quad\text{and}\quad\alpha=\frac{1}{168}(60r+277k). \tag{27}$$

The couplings in Eqs. (24) and (26) generate neutrino masses consistent with current atmospheric neutrino data [$m_\nu\sim(0.1$–$1)$ eV]. An appropriate scale for solar neutrinos can be obtained either by introducing heavy right handed neutrino states or using specific neutrino mass matrices. The latter can be generated if $\mathcal{U}(1)$ is applied as a flavor symmetry [27]. Indeed, this can ensure large, even maximal mixings between neutrinos [27], explaining both the solar and atmospheric neutrino data.

V. BARYON NUMBER CONSERVATION AND AUTOMATIC MATTER PARITY

It turns out that with suitable $\mathcal{U}(1)$ charge assignments, it is very easy to forbid all dangerous baryon number violating operators and obtain automatic matter parity. Table II lists some matter parity and baryon number violating operators and their $\mathcal{U}(1)$ charges for scenarios (I) and (II). To compute the $\mathcal{U}(1)$ charges of the couplings in the context of scenario (I), we use relations (21) and (25) and the prescriptions of Table I, while in the context of scenario (II), we use relations

TABLE II. $\mathcal{U}(1)$ charges of a few matter parity and baryon number violating operators for scenarios (I) and (II).

	Operator	Corresponding $\mathcal{U}(1)$ charge	
		Scenario (I)	Scenario (II)
(i)	$h_u l$	$-\frac{235}{6}k+\frac{9}{2}(p-2m)$	$\frac{53}{168}k-\frac{9}{14}r$
(ii)	$qd^c l$	$-\frac{175}{12}k+2(p-2m)$	$\frac{83}{28}k-\frac{2}{7}r$
(iii)	$e^c ll$	$-\frac{229}{6}k+\frac{9}{2}(p-2m)$	$\frac{221}{168}k-\frac{9}{14}r$
(iv)	$u^c d^c d^c$	$-\frac{77}{2}k+\frac{9}{2}(p-2m)$	$\frac{55}{56}k-\frac{9}{14}r$
(v)	$qqql$	$\frac{4}{3}k$	$\frac{4}{3}k$
(vi)	$u^c u^c d^c e^c$	$\frac{2}{3}k$	$\frac{2}{3}k$
(vii)	$qqqh_d$	$\frac{73}{2}k-\frac{9}{2}(p-2m)$	$-\frac{167}{56}k+\frac{9}{14}r$

(21) and (27). In scenario (I), as can be seen from Table II, the matter parity violating couplings (i)–(iii) are forbidden for $k=1$ (which gives $\tan\beta\sim$ unity) and $p-2m=5$–8 (to get the correct magnitude for the neutrino masses for $\epsilon \simeq 0.2$) because their effective $\mathcal{U}(1)$ charges are fractional. For $p-2m=5,7$, operator (iv) is allowed with suppressions ϵ^{16} and ϵ^{7} respectively which is not relevant phenomenologically. Baryon number violating $d=5$ operators (v) and (vi) have positive $\mathcal{U}(1)$ charges for any positive integer k and are therefore forbidden. The same applies to the $d=5$ operator (vii) which violates baryon number.

As far as scenario (II) is concerned, for $k=1$ and $r=11$–13 [which give the correct values for the neutrino mass (26) for $\epsilon\simeq 0.2$], all (i)–(vii) couplings carry noninteger $\mathcal{U}(1)$ charges and are therefore forbidden as a result. Thus, thanks to the $\mathcal{U}(1)$ symmetry, matter parity is present and baryon number conservation holds, even after taking account of dimension five operators.

In scenario (I), higher order baryon and lepton number violating operators are irrelevant from the phenomenological viewpoint since even if they are present, they are strongly suppressed by appropriate powers of M_{Pl}. Therefore, we can conclude that in scenario (I), with the help of $\mathcal{U}(1)$ symmetry and suitable choices for a and b, baryon number is essentially conserved.

In scenario (II), the situation can be different because of the low scale of $M_f \simeq 100$ TeV. Operators with $\Delta B=2$ can induce observable processes (such as $n-\bar{n}$ oscillations and deuteron two body decays $D\to K^*K$). $\Delta B=2$ operators of the form

$$\frac{1}{M_f^3}u^c d^c d^c u^c d^c d^c, \tag{28}$$

have a $\mathcal{U}(1)$ charge of $-\frac{9}{7}r+\frac{55}{28}k$ [see Eqs. (21) and (27) and Table I], which is fractional for $k=1$ and $r=11-13$ and therefore forbidden. Higher order operators with $\Delta B \geqslant 3$ are phenomenologically not relevant.

VI. CONCLUSIONS

Throughout our discussion so far, we have assumed flavor independent $\mathcal{U}(1)$ charges for chiral matter. However, automatic matter parity and baryon number conservation would hold even if $\mathcal{U}(1)$ is regarded as a flavor symmetry. This provides us with the possibility of explaining the hierarchies between the charged fermion masses and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements naturally. Also, one can construct various neutrino oscillation models in the spirit of Ref. [27], accommodating both the recent atmospheric and solar neutrino data.

In our considerations the breaking of $\mathcal{U}(1)$ symmetry was ensured by the Fayet-Iliopoulos term for $V_{\mathcal{U}(1)}$ vector superfield. An analogous term for $V_{U(1)_Y}$ must be avoided in order to avoid breaking either SUSY or the SM gauge group in an unacceptable way. Note that it will not be induced at the quantum level because for the MSSM field content we have $\mathrm{Tr}[Q_{U(1)_Y}]=0$. Let us also note that, since for both scenarios (I), (II) the scale of $\mathcal{U}(1)$ symmetry breaking lies well above the Z^0 boson mass, the mixed coupling $\int d^2\theta W_{\mathcal{U}(1)}W_{U(1)_Y}$ between the field strengths of $\mathcal{U}(1)$ and $U(1)_Y$ is not dangerous [10].

In conclusion, we considered a 5D orbifold construction of $SU(3)_c\times SU(2)_L\times U(1)_Y$ supplemented with an additional $\mathcal{U}(1)$ gauge factor. This $\mathcal{U}(1)$ symmetry allows us to solve various phenomenological puzzles of MSSM, such as baryon number conservation and the generation of the desired neutrino masses for the case where the fundamental scale is either $M_{Pl}=2.4\times 10^{18}$ GeV or relatively low (~ 100 TeV). It turns out that to cancel the mixed and pure anomalies arising from the presence of $\mathcal{U}(1)$, some additional (heavy) states and 5D Chern-Simons terms must be included. The $\mathcal{U}(1)$ symmetry can also play a role of flavor symmetry for understanding fermion masses and mixings.

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APPENDIX A: THE BRANE FI TERM AND THE VACUUM STRUCTURE OF THE FIELDS

In this appendix we will study the effects of a brane FI term. The latter gives rise not only to a nonzero VEV for the zero mode of X, but also nonzero VEVs for its KK states and $\Phi^{(k)}$. Here, V and Φ denote the states of the 5D $\mathcal{U}(1)$ gauge field.

The relevant terms for the gauge kinetic type couplings (3) are

$$\mathcal{L}_D=\frac{1}{g^2}D^2+\frac{1}{g^2}\left(-\frac{1}{\sqrt{2}}\partial_5 D(\Phi^*+\Phi)+F_\Phi^* F_\Phi\right), \tag{A1}$$

where in the right-hand side of Eq. (A1), the subscript Φ denotes the component of the superfield constructed from Φ (the same applies for X and $\bar{X}$).

The kinetic couplings (4) for X and $\overline{X}$ [with the $\mathcal{U}(1)$ charges Q_X and $-Q_X$ respectively],

$$\int d^4\theta(X^+e^{Q_XV}X+\overline{X}e^{-Q_XV}\overline{X}^+)+\int d^2\theta\overline{X}\left(\partial_5+\frac{Q_X}{\sqrt{2}}\Phi\right)X, \tag{A2}$$

are invariant under the gauge transformation

$$X\to e^{-Q_X\Lambda}X,\quad \overline{X}\to e^{Q_X\Lambda}\overline{X},$$
$$V\to V+\Lambda+\Lambda^+,\quad \Phi\to\Phi+\sqrt{2}\partial_5\Lambda. \tag{A3}$$

The relevant couplings coming from Eq. (A2) are

$$\mathcal{L}_X=F_X^*F_X+F_{\overline{X}}^*F_{\overline{X}}+F_{\overline{X}}\partial_5X+\overline{X}\partial_5F_X+\frac{Q_X}{2}DX^*X$$
$$-\frac{Q_X}{2}D\overline{X}^*\overline{X}+\frac{Q_X}{\sqrt{2}}(F_{\overline{X}}\Phi X+\overline{X}F_\Phi X+\overline{X}\Phi F_X). \tag{A4}$$

We also consider a 4D FI term on a fixed point of the form

$$\mathcal{L}_{\rm FI}^{(4)}=\xi\int d^4\theta V=\frac{1}{2}\xi D \tag{A5}$$

($\mathcal{L}_{\rm FI}^{(4)}$ is invariant under the 5D gauge transformation $V\to V+\Lambda+\Lambda^+$ since $\int d^4\theta\Lambda=\int d^4\theta\Lambda^+=0$). With the orbifold parities of Table I, we can expand X, $\overline{X}$, V and Φ as Eq. (7). Thus,

$$V=\sqrt{2}\sum_{n=0}^{\infty}V^{(n)}\eta^{(n)}\cos\frac{ny}{R},\quad \Phi=\sqrt{2}\sum_{n=1}^{\infty}\Phi^{(n)}\sin\frac{ny}{R}. \tag{A6}$$

Substituting in Eq. (A1) and integrating over the fifth coordinate y, we obtain

$$\mathcal{L}_D=\frac{1}{2g_4^2}\sum_{n=0}^{\infty}D^{(n)}D^{(n)}+\frac{1}{g_4^2}\sum_{n=1}^{\infty}\left(\frac{n}{R}D^{(n)}\Sigma^{(n)}+F_\Phi^{(n)*}F_\Phi^{(n)}\right), \tag{A7}$$

where

$$g_4\equiv\frac{g}{\sqrt{\pi R}}. \tag{A8}$$

The other terms in Eq. (A4) can be expanded to yield

$$\mathcal{L}_X^{(1)-(4)}=\sum_{n=0}^{\infty}F_X^{(n)*}F_X^{(n)}+\sum_{n=1}^{\infty}F_{\overline{X}}^{(n)*}F_{\overline{X}}^{(n)}-\sum_{n=1}^{\infty}\frac{n}{R}(F_{\overline{X}}^{(n)}X^{(n)}$$
$$+\overline{X}^{(n)}F_X^{(n)}), \tag{A9}$$

$$\mathcal{L}_X^{(5)}=\frac{Q_X}{2}\Bigg[D^{(0)}\sum_{n=0}^{\infty}X^{(n)*}X^{(n)}$$
$$+\frac{1}{\sqrt{2}}\sum_{n+p\neq0}D^{(n+p)}X^{(n)*}X^{(p)}\eta^{(n)}\eta^{(p)}$$
$$+\frac{1}{\sqrt{2}}\sum_{n\neq p}D^{(|n-p|)}X^{(n)*}X^{(p)}\eta^{(n)}\eta^{(p)}\Bigg] \tag{A10}$$

$$\mathcal{L}_X^{(6)}=-\frac{Q_X}{2}\Bigg[D^{(0)}\sum_{n=1}^{\infty}\overline{X}^{(n)*}\overline{X}^{(n)}$$
$$+\frac{1}{\sqrt{2}}\sum_{n\neq p}D^{(|n-p|)}\overline{X}^{(n)*}\overline{X}^{(p)}$$
$$-\frac{1}{\sqrt{2}}\sum_{n,p}D^{(n+p)}\overline{X}^{(n)*}\overline{X}^{(p)}\Bigg] \tag{A11}$$

$$\mathcal{L}_X^{(7)}=\frac{Q_X}{\sqrt{2}}\Bigg[\sum_{n=1}^{\infty}(X^{(0)}F_{\overline{X}}^{(n)}\Phi^{(n)}+X^{(0)}\overline{X}^{(n)}F_\Phi^{(n)}$$
$$+F_X^{(0)}\overline{X}^{(n)}\Phi^{(n)})+\frac{1}{\sqrt{2}}\sum_{n\neq p}(F_{\overline{X}}^{(p)}\Phi^{(n)}X^{(|n-p|)}$$
$$+\overline{X}^{(p)}F_\Phi^{(n)}X^{(|n-p|)}+\overline{X}^{(p)}\Phi^{(n)}F_X^{(|n-p|)})$$
$$-\frac{1}{\sqrt{2}}\sum_{n+p\neq0}(F_{\overline{X}}^{(p)}\Phi^{(n)}X^{(n+p)}+\overline{X}^{(p)}F_\Phi^{(n)}X^{(n+p)}$$
$$+\overline{X}^{(p)}\Phi^{(n)}F_X^{(n+p)})\Bigg]. \tag{A12}$$

The FI (A5) term is allowed on a brane

$$\int dy\,\delta(y)\mathcal{L}_{\rm FI}^{(4)}=\frac{\xi}{2}D^{(0)}+\frac{\xi}{\sqrt{2}}\sum_{n=1}^{\infty}D^{(n)}. \tag{A13}$$

Using Eqs. (A7)–(A13), the D and F terms for the zero modes are

$$D^{(0)}=-\frac{g_4^2}{2}\left(\xi+Q_X\sum_{n=0}^{\infty}X^{(n)*}X^{(n)}-Q_X\sum_{n=1}^{\infty}\overline{X}^{(n)*}\overline{X}^{(n)}\right), \tag{A14}$$

$$F_X^{(0)*}=-\frac{Q_X}{\sqrt{2}}\sum_{n=1}^{\infty}\overline{X}^{(n)}\Phi^{(n)}. \tag{A15}$$

The D and F terms of the corresponding Kaluza-Klein (KK) states are

$$D^{(k)}=-\frac{g_4^2}{2}\Bigg(\sqrt{2}\xi+\frac{2k}{g_4^2R}\Sigma^{(k)}+\frac{Q_X}{\sqrt{2}}\times\sum_{n+p=k}X^{(n)*}X^{(p)}\eta^{(n)}\eta^{(p)}+\frac{Q_X}{\sqrt{2}}\sum_{|n-p|=k}X^{(n)*}X^{(p)}\eta^{(n)}\eta^{(p)}-\frac{Q_X}{\sqrt{2}}\sum_{n+p=k}\overline{X}^{(n)*}\overline{X}^{(p)}-\frac{Q_X}{\sqrt{2}}\sum_{|n-p|=k}\overline{X}^{(n)*}\overline{X}^{(p)}\Bigg), \tag{A16}$$

$$F_{\overline{X}}^{(k)*}=\frac{k}{R}\overline{X}^{(k)}-\frac{Q_X}{2}\sum_{|n-p|=k}\overline{X}^{(p)}\Phi^{(n)}+\frac{Q_X}{2}\sum_{n+p=k}\overline{X}^{(p)}\Phi^{(n)}, \tag{A17}$$

$$F_X^{(k)*}=\frac{k}{R}X^{(k)}-\frac{Q_X}{\sqrt{2}}X^{(0)}\Phi^{(k)}-\frac{Q_X}{2}\sum_{n\neq k}\Phi^{(n)}X^{(|n-k|)}+\frac{Q_X}{2}\sum_{n+k\neq 0}\Phi^{(n)}X^{(n+k)}, \tag{A18}$$

$$F_\Phi^{(k)*}=-\frac{g_4^2}{2}Q_X\Bigg(\sqrt{2}X^{(0)}\overline{X}^{(k)}+\sum_{p\neq k}\overline{X}^{(p)}X^{(|p-k|)}-\sum_{p+k\neq 0}\overline{X}^{(p)}X^{(p+k)}\Bigg). \tag{A19}$$

It is easy to see that there is a solution with zero D and F terms and nonzero vacuum expectation values (VEVs) for the $X^{(k)}$ and $\Phi^{(k)}$ states. Assuming $\langle\overline{X}^{(k)}\rangle=0$ for all k, from Eqs. (A15), (A17) and (A19), we see that

$$F_{\overline{X}}^{(0)}=F_{\overline{X}}^{(k)}=F_\Phi^{(k)}=0. \tag{A20}$$

If we require all the other D and F terms to vanish, from Eqs. (A14), (A16) and (A18), we obtain

$$\xi+Q_X\sum_{n=0}^{\infty}X^{(n)*}X^{(n)}=0, \tag{A21}$$

$$\sqrt{2}\xi+\frac{2k}{g_4^2R}\Sigma^{(k)}+\frac{Q_X}{\sqrt{2}}\sum_{n+p=k}X^{(n)*}X^{(p)}\eta^{(n)}\eta^{(p)}+\frac{Q_X}{\sqrt{2}}\sum_{|n-p|=k}X^{(n)*}X^{(p)}\eta^{(n)}\eta^{(p)}=0,\quad k\neq 0, \tag{A22}$$

$$\frac{k}{R}X^{(k)}-\frac{Q_X}{\sqrt{2}}X^{(0)}\Phi^{(k)}-\frac{Q_X}{2}\sum_{n\neq k}\Phi^{(n)}X^{(|n-k|)}+\frac{Q_X}{2}\sum_{n+k\neq 0}\Phi^{(n)}X^{(n+k)}=0,\quad k\neq 0. \tag{A23}$$

If one assumes that the VEVs of all the $\Phi^{(k)}$ states vanish, then, from Eq. (A23), we deduce $\langle X^{(k)}\rangle=0$ (for $k\neq 0$) and so, we cannot satisfy Eq. (A22). Thus, we can conclude that, in order to satisfy Eqs. (A21)–(A23) simultaneously, the states $\Phi^{(k)}$ must have nonzero VEVs. In order to satisfy Eq. (A21), we need opposite signs for ξ and Q_X. Without any loss of generality, one can assume $\xi<0$ and $Q_X>0$. If we restrict Eqs. (A21)–(A23) to the first k KK modes of Φ, the first k' modes of X and the zero mode $X^{(0)}$, we are left with $k+k'+1$ nontrivial equations. Therefore, the number of equations and variables coincides and there will always be a solution where all the D and F terms vanish. In particular, the $X^{(0)}$ state has a nonzero VEV.

We have shown that within the framework of 5D $S^{(1)}/Z_2$ orbifold models, the brane FI term for the $\mathcal{U}(1)$ gauge superfield ensures a nonzero VEV for the X field and SUSY remains unbroken. It turns out that the VEV of the scalar component of X is crucial for the generation of sufficiently suppressed neutrino masses and to explain hierarchies between fermion masses and mixings if $\mathcal{U}(1)$ is applied as a flavor symmetry.

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