# $B\overline{B}$ mixing and $CP$ violation in $SU(2)_L \times SU(2)_R \times U(1)$ models

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(Received 18 June 2002; published 30 September 2002)

We reexamine the mass mixing and $CP$ violation in the $B\overline{B}$ system in general $SU(2)_L \times SU(2)_R \times U(1)$ models related to recent measurements without imposing manifest or pseudomanifest left-right symmetry. For certain parameter sets, the right-handed contributions can be sizable in $B\overline{B}$ mixing and $CP$ asymmetry in $B$ decays for a heavy $W'$ even with a mass about 3 TeV. On the other hand the lower bound on the mass of $W'$ can be taken down to approximately 300 GeV.

DOI: 10.1103/PhysRevD.66.055008 PACS number(s): 12.60.Cn, 11.30.Er, 14.70.Pw

## I. INTRODUCTION

The standard $SU(2)_L \times U(1)$ model (SM) has been very successful in describing the known weak interaction phenomena. But the consistency of the present experimental results with the general scheme of charged weak interactions and $CP$ violation in the SM is nontrivial so the model is challenged both experimentally and theoretically in its prediction of large $CP$ violation effects in the $B$ meson system [1]. As one of the simplest extensions of the standard model gauge group, and so a complement of the purely left-handed nature of the SM, the left-right theory with group $SU(2)_L \times SU(2)_R \times U(1)$ has been widely studied. In this model, even with two generations of quarks one could get $CP$ violation. With three generations of quarks, this model contains many parameters and many sources of $CP$ violation [2]. One of the main sources is the relative phase $\alpha$ between the two vacuum expectation values (VEVs) $k$ and $k'$ of the Higgs bidoublet $\Phi$. The other sources are the complex phases in the left- and right-handed quark mixing matrices $U^L$ and $U^R$, respectively. Here it would be convenient to regard $U^L$ as the usual Cabibbo-Kobayashi-Maskawa (CKM) matrix and shift all phases except one to $U^R$. Using the Wolfenstein parametrization [3], we can express the CKM matrix approximately as

$$U^L = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4), \tag{1}$$

where $\lambda$ ($\approx 0.22$) is a real expansion parameter, and $A$, $\rho$, and $\eta$ are also real quantities. From the above expression, the elements $U^L_{ub}$ and $U^L_{td}$ can be parametrized in terms of two phases $\gamma$ and $\beta$, respectively, which form a unitary triangle (Fig. 1) given by the orthogonality condition $\Sigma_{i=u,c,t} U_{id} U^*_{ib} = 0$. The recent significant measurements of $\beta_{expt}$ give [4]

$$\sin 2\beta_{expt} = \begin{cases} 0.59 \pm 0.14 \pm 0.05 & \text{(BABAR)}, \\ 0.99 \pm 0.14 \pm 0.06 & \text{(Belle)}. \end{cases} \tag{2}$$

If there are new physics effects involved, the experimental value $\beta_{expt}$ can be expressed through other parameters representing the new physics as well as the phase of $U^L_{td}$ in the SM.

In addition to the phases mentioned above, the masses ($M_{W_R}$) of the right-handed gauge bosons, the mixing angle $\xi$ between the left- and right-handed gauge bosons $W_L$ and $W_R$, and the right-handed gauge coupling constant $g_R$ play important roles in new physics effects as fundamental input parameters in the left-right model (LRM). The success of the SM in the low-energy phenomenology requires that the masses ($M_{W_R}$) of the right-handed gauge bosons are significantly larger than those ($M_{W_L}$) of left-handed gauge bosons. The first lower bound on $M_{W_R}$ came from a study of the low-energy charged current sector allowing $M_{W_R} \gtrsim 3 M_{W_L} \approx 240$ GeV [5]. Soon after, many theoretical limits were presented on $M_{W_R}$ and $\xi$ under various assumptions [6]. The recent experimental limits were obtained by DØ and the collider Detector at Fermilab (CDF) from direct searches for the decay channels of the extra gauge bosons $W'^{+} \to l^+_R \nu_R$. DØ found $M_{W'} > 720$ GeV for $m_{\nu_R} \ll M_{W'}$ or $M_{W'} > 650$ GeV for $m_{\nu_R} = M_{W'}/2$ [7]. CDF has the limit of $M_{W'} > 652$ GeV for $m_{\nu_R} \ll M_{W'}$ if $\nu_R$ is stable [8]. All of these limits were obtained assuming manifest ($U^R = U^L$) or pseudomanifest ($U^R = U^{L*} K$) left-right symmetry ($g_L = g_R$), where $K$ is a diagonal phase matrix [9]. In this paper, we will not impose discrete left-right symmetry which can cause trouble in explaining the cosmological baryon asymmetry and may lead to cosmological domain-wall problems [10]. However, we will also consider the possibility of the left-right symmetric case among other possibilities.

The main purpose of this paper is to investigate $CP$ violation in the $B^0\overline{B}^0$ system in the LRM related to the recent

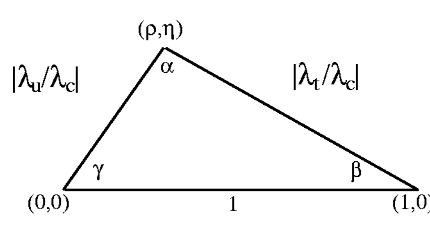


FIG. 1. Unitary triangle ($\lambda_i = U^*_{id} U_{ib}$).

 

experiments, since $B^0\overline{B^0}$ mixing has recently been advocated as a very sensitive probe for $CP$ violation and the presence of right-handed current. The SM contribution to $K^0\overline{K^0}$ mixing was previously computed for any internal quark mass by Inami and Lim [11]. The right-handed contribution in the LRM was investigated first by Beall, Bander, and Soni, assuming discrete left-right symmetry [12], and again by many authors [13] under various assumptions. But we notice that the contributions of the mixing angle $\xi$ to $B^0\overline{B^0}$ mixing and $CP$ asymmetry can be large due to the heaviness of the top quark mass and the possibility of enhancement in the right-handed quark mixing matrix in the general LRM. After reviewing the structure of the LRM in Sec. II, we will discuss $B^0\overline{B^0}$ mixing in Sec. III and $CP$ asymmetry in $B^0$ decay in Sec. IV in detail.

## II. $SU(2)_L \times SU(2)_R \times U(1)$ MODELS

We briefly review here some of the main features of the LRM, which are needed to obtain our results. As the simplest extension of the SM, the gauge group of the LRM breaks down to that of the SM and it finally cascades down to $U(1)_{EM}$. The covariant derivative for the fermions $f_{L,R}$ with respect to the gauge group of the LRM appears as

$$D^\mu f_{L,R} = \partial^\mu f_{L,R} + i g_{L,R} W^{\mu a}_{L,R} T^a_{L,R} f_{L,R} + i g_1 B^\mu S f_{L,R}. \quad (3)$$

The electric charge which is the unbroken $U(1)$ generator is given by

$$Q = T^3_L + T^3_R + S. \quad (4)$$

The quarks and leptons transform under the gauge group of the LRM $(T_L, T_R, S)$ as

$$q'_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L \sim \left(\frac{1}{2}, 0, \frac{1}{6}\right), \quad q'_R = \begin{pmatrix} u' \\ d' \end{pmatrix}_R \sim \left(0, \frac{1}{2}, \frac{1}{6}\right),$$

$$l'_L = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_L \sim \left(\frac{1}{2}, 0, -\frac{1}{2}\right), \quad l'_R = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_R \sim \left(0, \frac{1}{2}, -\frac{1}{2}\right), \quad (5)$$

where the primes indicate that the fermions are gauge rather than mass eigenstates.

In order to generate masses for the fermions and implement the symmetry breaking, we need to include scalar fields into our theory. The simplest choice is to introduce one Higgs multiplet and two doublets:

$$\Phi = \begin{pmatrix} \phi^0_1 & \phi^+_1 \\ \phi^-_2 & \phi^0_2 \end{pmatrix} \sim \left(\frac{1}{2}, \frac{1}{2}^*, 0\right),$$

$$\chi_L = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix}_L \sim \left(\frac{1}{2}, 0, \frac{1}{2}\right), \quad \chi_R = \begin{pmatrix} \chi^+ \\ \chi^0 \end{pmatrix}_R \sim \left(0, \frac{1}{2}, \frac{1}{2}\right), \quad (6)$$

which acquire the vacuum expectation values

$$\langle \Phi \rangle = \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix}, \quad \langle \chi_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle \chi_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix}, \quad (7)$$

where $k$ and $k'$ are complex, and $v_L$ and $v_R$ are real. $\chi_R$ is needed to generate a large $M_{W_R}$ if $v_R \gg |k|, |k'|, v_L$. But $\chi_L$ is not essential unless we impose left-right symmetry. It is also possible to adopt other choices of Higgs field such as Higgs triplets instead [14]. The Lagrangian for the scalar field is

$$L_{scalar} = \mathrm{Tr}[(D^\mu \Phi)^\dagger D_\mu \Phi] + (D^\mu \chi_L)^\dagger D_\mu \chi_L + (D^\mu \chi_R)^\dagger D_\mu \chi_R - V(\Phi, \chi_L, \chi_R). \quad (8)$$

For the Higgs fields described above, the kinetic terms in the Lagrangian generate the charged $W$ boson matrix

$$M^2_{W^\pm} = \begin{pmatrix} g_L^2 (v_L^2 + K^2)/2 & -g_L g_R k^* k' \\ -g_L g_R k k'^* & g_R^2 (v_R^2 + K^2)/2 \end{pmatrix} \equiv \begin{pmatrix} M^2_{W_L} & M^2_{W_{LR}} e^{i\alpha} \\ M^2_{W_{LR}} e^{-i\alpha} & M^2_{W_R} \end{pmatrix}, \quad (9)$$

where $K^2 = |k|^2 + |k'|^2$ and $\alpha$ is the phase of $k^* k'$. After the mass matrix is diagonalized by a unitary transformation the eigenvalues can be expressed in terms of a mixing angle as

$$M^2_W = M^2_{W_L} \cos^2 \xi + M^2_{W_R} \sin^2 \xi + M^2_{W_{LR}} \sin 2\xi,$$

$$M^2_{W'} = M^2_{W_L} \sin^2 \xi + M^2_{W_R} \cos^2 \xi - M^2_{W_{LR}} \sin 2\xi. \quad (10)$$

Thus the mass eigenstates are written as

$$\begin{pmatrix} W^+ \\ W'^+ \end{pmatrix} = \begin{pmatrix} \cos \xi & e^{-i\alpha} \sin \xi \\ -\sin \xi & e^{-i\alpha} \cos \xi \end{pmatrix} \begin{pmatrix} W^+_L \\ W^+_R \end{pmatrix}, \quad (11)$$

where $\xi$ is a mixing angle defined by

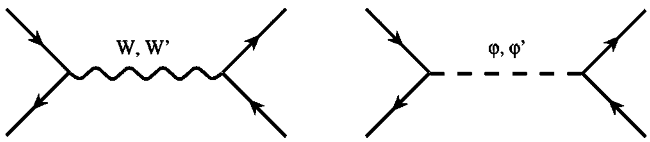


FIG. 2. Tree-level Feynman diagrams for the gauge boson $(W,W')$ and the Goldstone boson $(\varphi,\varphi')$ exchange.

$$\tan 2\xi = -\frac{2M_{W_{LR}}^2}{M_{W_R}^2 - M_{W_L}^2}. \tag{12}$$

For $v_R \gg |k|,|k'|,v_L$, the mass eigenvalues and the mixing angle reduce to

$$M_W^2 \approx \frac{1}{2}g_L^2(v_L^2+K^2), \quad M_{W'}^2 \approx \frac{1}{2}g_R^2 v_R^2, \quad \xi \approx \frac{2g_L|k^*k'|}{g_R v_R^2}. \tag{13}$$

Here, the Schwarz inequality requires that $\zeta \equiv M_W^2/M_{W'}^2 \geq \xi_g \equiv (g_L/g_R)\xi$. From the limits on deviations of muon decay parameters from the $V-A$ prediction, the lower bound on $M_{W'}$ can be obtained as follows [15]:

$$(g_R/g_L)^2\zeta < 0.033 \quad \text{or} \quad M_{W'} > (g_R/g_L)\times 440\ \text{GeV}. \tag{14}$$

We will use this number for our numerical analysis.

As well as the above charged gauge bosons, the charged would-be Goldstone bosons corresponding to the longitudinal components of the physical bosons take part in the charged current interactions. The coupling of the Goldstone fields to the fermions can be found from the detailed structure of the Higgs potential $V(\Phi,\chi_L,\chi_R)$ and the Yukawa couplings. However, one can directly determine the Goldstone couplings in terms of the gauge couplings without considering the Higgs potential, but using the Ward identities which ensure that the unphysical poles in the two diagrams shown in Fig. 2 should cancel each other [16]. The charged interaction Lagrangian is then given by

$$\begin{aligned} L_{CC} = -\frac{1}{\sqrt{2}}\overline{P}\gamma^\mu \Big\{ & [U^L g_L c_\xi L + U^R g_R s_\xi^+ R]W_\mu^+ + [-U^L g_L s_\xi L + U^R g_R c_\xi^+ R]W_\mu'^+ + [(U^L M_P g_L c_\xi - U^R M_N g_R s_\xi^+)L \\ & + (-U^L M_N g_L c_\xi + U^R M_P g_R s_\xi^+)R]\frac{\varphi_\mu^+}{M_W} + [-(U^L M_P g_L s_\xi + U^R M_N g_R c_\xi^+)L + (U^L M_N g_L s_\xi + U^R M_P g_R c_\xi^+)R]\frac{\varphi_\mu'^+}{M_{W'}}\Big\} N \\ & + \text{H.c.} + \ldots, \end{aligned} \tag{15}$$

where $c_\xi$ $(s_\xi) \equiv \cos\xi\,(\sin\xi)$, $s_\xi^\pm \equiv e^{\pm\alpha}\sin\xi$, $L,R \equiv (1\mp\gamma^5)/2$ denote left- and right-handed projection operators, $M_P = \text{diag}(m_u,m_c,m_t)$ and $M_N = \text{diag}(m_d,m_s,m_b)$ are the diagonalized quark mass matrices, $P(N)$ is the mass eigenstate corresponding to its eigenvalue $M_P$ $(M_N)$, and $U^L$ $(U^R)$ is the left- (right-) handed quark mixing matrix.

## III. $B^0\overline{B^0}$ MIXING

The effective Hamiltonian in the $B^0\overline{B^0}$ system is obtained by integrating out the internal loop in the box diagrams in Fig. 3 just as in the SM. We neglect external momenta and the $d$-quark mass, but the result is valid for general internal quark masses. One finds, using the Feynman–'t Hooft gauge, the charged gauge boson and Goldstone boson contributions to $B^0\overline{B^0}$ mixing in a straightforward manner:

$$H_{eff}^{B\overline{B}} = H_{eff}^{SM} + H_{eff}^{RR} + H_{eff}^{LR} \tag{16}$$

with

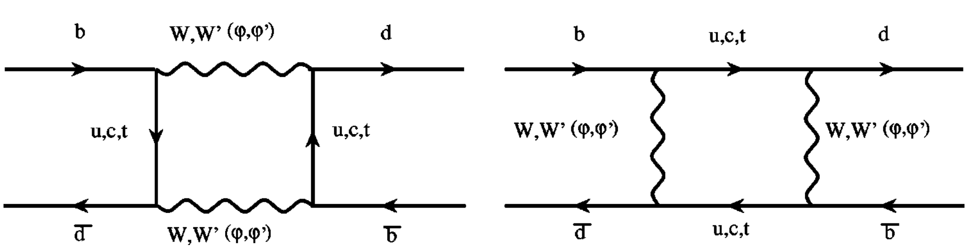


FIG. 3. Box diagrams for $B^0\overline{B^0}$ mixing with the gauge bosons $(W,W')$ and the Goldstone bosons $(\varphi,\varphi')$.

$$H_{eff}^{SM}=\frac{G_F^2M_W^2}{4\pi^2}\sum_{i,j=u,c,t}\lambda_i^{LL}\lambda_j^{LL}\left\{\left[\left(1+\frac{x_i^2x_j^2}{4}\right)f(x_i^2,x_j^2;1)-2x_i^2x_j^2g(x_i^2,x_j^2;1)\right](\overline{d}_L\gamma_\mu b_L)^2+x_b^2x_i^2x_j^2g(x_i^2,x_j^2;1)(\overline{d}_Lb_R)^2\right\}, \tag{17}$$

$$H_{eff}^{RR}=\frac{G_F^2M_W^2}{4\pi^2}\left(\frac{g_R}{g_L}\right)^4\sum_{i,j=u,c,t}\lambda_i^{RR}\lambda_j^{RR}\zeta f(x_i^2\zeta,x_j^2\zeta;1)(\overline{d}_R\gamma_\mu b_R)^2, \tag{18}$$

$$H_{eff}^{LR}=\frac{G_F^2M_W^2}{2\pi^2}\left(\frac{g_R}{g_L}\right)^2\sum_{i,j=u,c,t}\left\{\lambda_i^{LR}\lambda_j^{RL}x_ix_j\zeta[4g(x_i^2,x_j^2;\zeta)-f(x_i^2,x_j^2;\zeta)](\overline{d}_Lb_R)(\overline{d}_Rb_L)+\lambda_i^{LL}\lambda_j^{LR}x_jx_b\xi_g^+\left[x_i^2\left(g(x_i^2,x_j^2;1)\right.\right.\right.$$
$$\left.-\frac{1}{4}f(x_i^2,x_j^2;1)\right)(\overline{d}_L\gamma_\mu b_L)^2+(f(x_i^2,x_j^2;1)-x_i^2x_j^2g(x_i^2,x_j^2;1))(\overline{d}_Lb_R)^2\Big]+\lambda_i^{LL}\lambda_j^{RL}x_jx_b\xi_g^-\Big[x_i^2\Big(g(x_i^2,x_j^2;1)$$
$$\left.\left.\left.-\frac{1}{4}f(x_i^2,x_j^2;1)\right)(\overline{d}_L\gamma_\mu b_L)(\overline{d}_R\gamma_\mu b_R)+(f(x_i^2,x_j^2;1)-x_i^2x_j^2g(x_i^2,x_j^2;1))(\overline{d}_Lb_R)(\overline{d}_Rb_L)\right]\right\}, \tag{19}$$

where

$$\frac{G_F}{\sqrt{2}}\equiv\frac{g_L^2}{8M_W^2},\quad \xi_g^{\pm}\equiv e^{\pm\alpha}\xi_g,$$

$$\lambda_i^{AB}\equiv U_{id}^{A*}U_{ib}^{B},\quad x_i\equiv\frac{m_i}{M_W}(i=u,c,t), \tag{20}$$

and

$$f(x_i,x_j;\zeta)=\frac{\ln(1/\zeta)}{(1-\zeta)(1-x_i\zeta)(1-x_j\zeta)}+\left(\frac{x_i^2\ln x_i}{(x_i-x_j)(1-x_i)(1-x_i\zeta)}+(i\to j)\right),$$

$$g(x_i,x_j;\zeta)=\frac{\zeta\ln(1/\zeta)}{(1-\zeta)(1-x_i\zeta)(1-x_j\zeta)}+\left(\frac{x_i\ln x_i}{(x_i-x_j)(1-x_i)(1-x_i\zeta)}+(i\to j)\right). \tag{21}$$

Although the form of the charged interactions in Eqs. (17)–(19) is independent of our particular choice of scalar representation, the Ward identities require that the box diagrams contributing to $B^0\overline{B}^0$ mixing in the LRM are not gauge invariant [17]. In order to impose gauge invariance into our theory, we need to involve flavor-changing neutral Higgs bosons, but it is known that their contributions, even at the tree level as long as the mass of the flavor-changing Higgs boson is much heavier than $M_{W'}$,[1] are suppressed by approximately a factor of $\zeta$ compared to the above gauge boson contributions [18]. Therefore the above results, in the approximation of neglecting external momenta and the $d$-quark mass, provide the complete effective Hamiltonian contributing to $B^0\overline{B}^0$ mixing.

At this stage, in order to analyze the obtained effective Hamiltonian quantitatively, we need to consider specific forms of the right-handed quark mixing matrices $U^R$. If the model has manifest or pseudomanifest left-right symmetry, the $W_R$ mass has a stringent bound $M_{W_R}\geqslant 1.6$ TeV [12], and the $W_R$ boson contributions to $B^0\overline{B}^0$ mixing and tree level $b$ decay are very small. But, in general, the form of $U^R$ is not necessarily restricted to manifest or pseudomanifest symmetric types, so the $W_R$ mass limit can be lowered to approximately 300 GeV by taking the following forms of $U^R$ [19]:

$$U_I^R=\begin{pmatrix} e^{i\omega} & \sim 0 & \sim 0\\ \sim 0 & c_Re^{i\alpha_1} & s_Re^{i\alpha_2}\\ \sim 0 & -s_Re^{i\alpha_3} & c_Re^{i\alpha_4}\end{pmatrix},$$

$$U_{II}^R=\begin{pmatrix} \sim 0 & e^{i\omega} & \sim 0\\ c_Re^{i\alpha_1} & \sim 0 & s_Re^{i\alpha_2}\\ -s_Re^{i\alpha_3} & \sim 0 & c_Re^{i\alpha_4}\end{pmatrix}, \tag{22}$$

where $c_R$ $(s_R)\equiv\cos\theta_R$ $(\sin\theta_R)(0°\leqslant\theta_R\leqslant 90°)$. Here the matrix elements indicated as $\sim 0$ may be $\lesssim 10^{-2}$ and unitarity requires $\alpha_1+\alpha_4=\alpha_2+\alpha_3$. From the $b\to c$ semileptonic decays of the $B$ mesons, we can get an approximate bound $\xi_g\sin\theta_R\lesssim 0.013$ by assuming $|U_{cb}^L|\approx 0.04$ [20].

The effective Hamiltonians obtained in Eqs. (17)–(19) are then further simplified using the Glashow-Iliopoulos-Maiani (GIM) cancellation $\Sigma_{i=u,c,t}\lambda_i=0$ and neglecting the $u$-quark mass:

$$H_{eff}^{SM}=\frac{G_F^2M_W^2}{4\pi^2}(\lambda_t^{LL})^2S(x_t^2)(\overline{d}_L\gamma_\mu b_L)^2, \tag{23}$$

[1]The tree-level flavor-changing neutral Higgs boson contributions with masses $M_H$ of order $M_{W'}$ in the manifest or pseudomanifest left-right symmetric model were discussed in Ref. [13].

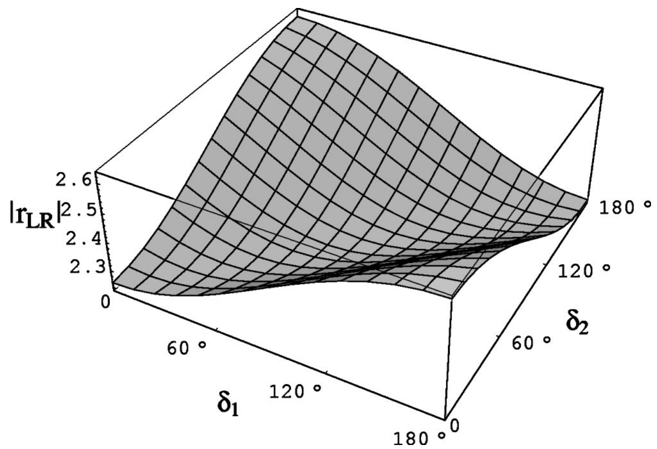


FIG. 4. Behavior of the ratio $|r_{LR}|$ as $\delta_{1,2}$ are varied.

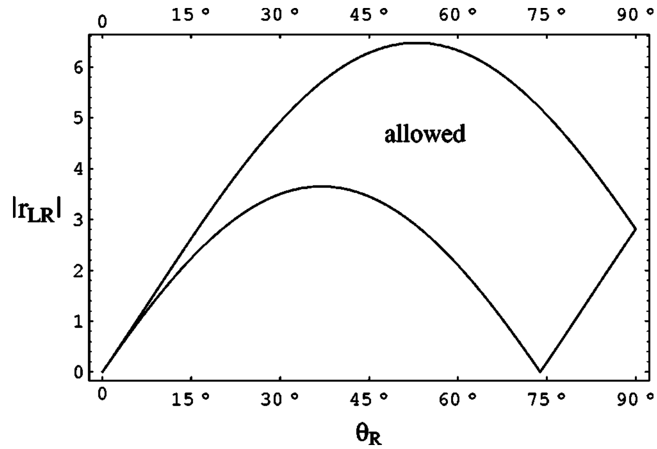


FIG. 5. Allowed region for $|r_{LR}|$ and $\theta_R$.

$$H_{eff}^{LR}=\frac{G_F^2 M_W^2}{2\pi^2}\left(\frac{g_R}{g_L}\right)^2\{[\lambda_c^{LR}\lambda_t^{RL}x_c x_t \zeta A_1(x_t^2,\zeta)$$
$$+\lambda_t^{LR}\lambda_t^{RL}x_t^2\zeta A_2(x_t^2,\zeta)](\overline{d}_L b_R)(\overline{d}_R b_L)$$
$$+\lambda_t^{LL}\lambda_t^{RL}x_b\xi_g^-[x_t^3 A_3(x_t^2)(\overline{d}_L\gamma_\mu b_L)(\overline{d}_R\gamma_\mu b_R)$$
$$+x_t A_4(x_t^2)(\overline{d}_L b_R)(\overline{d}_R b_L)]\}, \qquad (24)$$

where

$$S(x)=\frac{x(4-11x+x^2)}{4(1-x)^2}-\frac{3x^3\ln x}{2(1-x)^3},$$

$$A_1(x,\zeta)=\frac{(4-x)\ln x}{(1-x)(1-x\zeta)}+\frac{(1-4\zeta)\ln\zeta}{(1-\zeta)(1-x\zeta)},$$

$$A_2(x,\zeta)=\frac{4-x}{(1-x)(1-x\zeta)}+\frac{[4-2x+x^2(1-3\zeta)]\ln x}{(1-x)^2(1-x\zeta)^2}+\frac{(1-4\zeta)\ln\zeta}{(1-\zeta)(1-x\zeta)^2}, \qquad (25)$$

$$A_3(x)=\frac{7-x}{4(1-x)^2}+\frac{(2+x)\ln x}{2(1-x)^3},$$

$$A_4(x)=\frac{2x}{1-x}+\frac{x(1+x)\ln x}{(1-x)^2}.$$

Note that $S(x)$ is the usual Inami-Lim function, $A_1(x,\zeta)$ is obtained by taking the limit $x_c^2=0$, and $H_{eff}^{RR}$ is suppressed because it is proportional to $\zeta^2$. Also, in the case of $U_I^R$, one can see that there is no significant contribution of $H_{eff}^{LR}$ to $B^0\overline{B}^0$ mixing, so we will concentrate on the second type $U_{II}^R$ in this section.

The dispersive part of the $B^0\overline{B}^0$ mixing matrix element can then be written as

$$M_{12}=M_{12}^{SM}+M_{12}^{LR}=M_{12}^{SM}\left\{1+\left(\frac{g_R}{g_L}\right)^2 r_{LR}\right\}, \qquad (26)$$

where

$$\left(\frac{g_R}{g_L}\right)^2 r_{LR}\equiv\frac{M_{12}^{LR}}{M_{12}^{SM}}=\frac{\langle\overline{B^0}|H_{eff}^{LR}|B^0\rangle}{\langle\overline{B^0}|H_{eff}^{SM}|B^0\rangle}. \qquad (27)$$

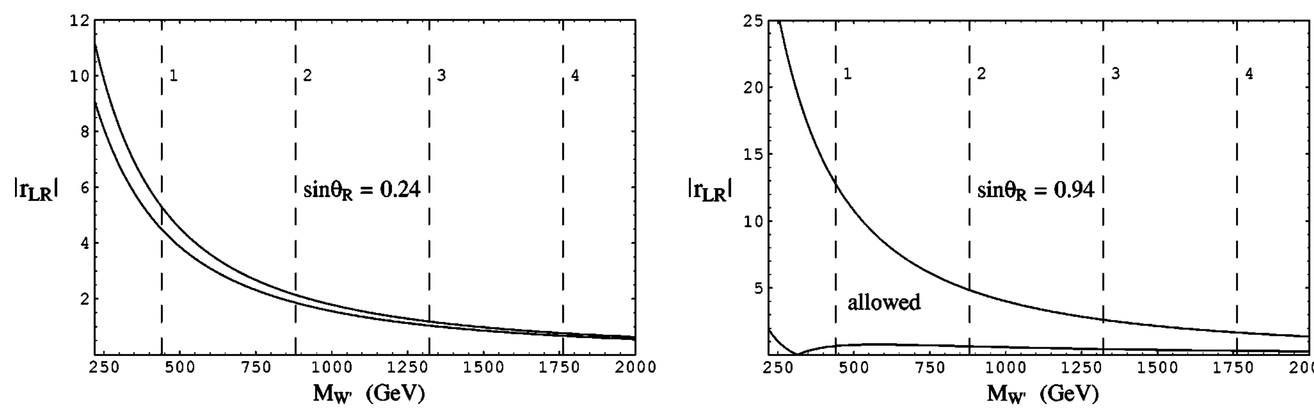


FIG. 6. Allowed regions for $|r_{LR}|$ and $M_{W'}$ for $g_R/g_L \geqslant 0.5$. The dashed lines correspond to the lower bounds on $M_{W'}$ in Eq. (14) for the ratio $g_R/g_L=1,2,3,$ and 4, respectively.

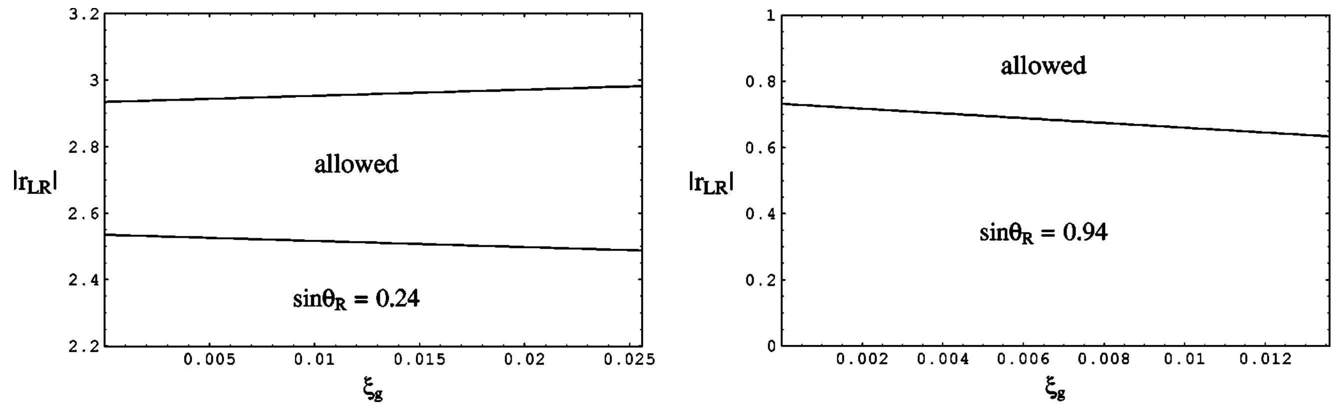


FIG. 7. Allowed regions for $|r_{LR}|$ and $\xi_g$ for $M_{W'}=700$ GeV.

For specific phenomenological estimates one needs the hadronic matrix elements of the operators in Eqs. (23), (24) in order to evaluate the mixing matrix element. We use the following parametrization:

$$\langle \overline{B^0}|(\overline{d_L}\gamma_\mu b_L)^2|B^0\rangle = \frac{1}{3}B_1 f_B^2 m_B,$$

$$\langle \overline{B^0}|(\overline{d_L}\gamma_\mu b_L)(\overline{d_R}\gamma_\mu b_R)|B^0\rangle = -\frac{5}{12}B_2 f_B^2 m_B, \qquad (28)$$

$$\langle \overline{B^0}|(\overline{d_L}b_R)(\overline{d_R}b_L)|B^0\rangle = \frac{7}{24}B_3 f_B^2 m_B,$$

where

$$\langle 0|\overline{d_\beta}\gamma^\mu b_\alpha|B^0\rangle = -\langle \overline{B^0}|\overline{d_\beta}\gamma^\mu b_\alpha|0\rangle = -\frac{i f_B p_B^\mu}{\sqrt{2m_B}}\frac{\delta_{\alpha\beta}}{3}, \qquad (29)$$

and where $f_B$ is the $B$ meson decay constant and $B_i(i=1,2,3)$ are the bag factors. In the vacuum-insertion method [21], $B_i=1$ in the limit $m_b \simeq m_B$. We will use $f_B B_i^{1/2}=(210\pm40)$ MeV for our numerical estimates [22]. Using the standard values of the quark masses and $|U_{cd}^L|\approx 0.222$, one can express $r_{LR}$ in terms of the mixing angle and phases in Eq. (22) as

$$r_{LR}\approx l\left\{18.1l\left(\frac{1-\zeta-(3.49-14.0\zeta)\ln(1/\zeta)}{1-5.68\zeta}\right)\zeta s_R^2 e^{i\delta_1}-739\left(\frac{1-5.04\zeta-(0.483-1.93\zeta)\ln(1/\zeta)}{1-10.4\zeta+31.3\zeta^2}\right)\zeta s_R c_R e^{i\delta_2}-7.68\xi_g s_R e^{i\delta_3}\right\}, \qquad (30)$$

where $l=0.009/|U_{td}^L|$, $\delta_1=-2\beta+\alpha_2-\alpha_3$, $\delta_2=-\beta-\alpha_3+\alpha_4$, $\delta_3=-\beta-\alpha_3$, and the mixing phase $\alpha$ was absorbed in $\alpha_i$ by redefining $\alpha_i+\alpha\rightarrow\alpha_i$.

Now we investigate numerically the behavior of the ratio $|r_{LR}|$, which is the deviation of $M_{12}$ from the SM value, under variation of $M_{W'}$, $\xi_g$, $\theta_R$, and the phases in $U^R$, assuming $l=1$. Although we use the average value of $|U_{td}|$, which might be different from the actual value of $|U_{td}^L|$, it should not affect the order of magnitude in our estimates. First, in order to see the dependence of $|r_{LR}|$ on the phases, we fix $M_{W'}=800$ GeV,$\xi_g=0.005$, $\theta_R=15°$, and set $\delta_3=\pi$ because its effect is relatively much smaller than that of $\delta_1$ and $\delta_2$. The plot is shown in Fig. 4. From Eq. (30) and Fig. 4, one can see that $|r_{LR}|$ becomes maximal when $\delta_{1,3}=\pi$ and $\delta_2=0$, and minimal when $\delta_{1,2,3}=\pi$ if $\theta_R\lesssim 70°$ (or $\delta_{1,2}=\pi$ and $\delta_3=0$ if $\theta_R\gtrsim 70°$). This behavior also holds for other values of $M_{W'}$ and $\xi_g$. Since $|r_{LR}|$ is a continuously varying function of the phases, we can probe the allowed region for $|r_{LR}|$ with respect to the parameters $M_{W'}$, $\xi_g$, and $\theta_R$. Next, we fix $M_{W'}=800$ GeV, $\xi_g=0.005$, and evaluate $|r_{LR}|$ by varying $\theta_R$. Note that $|r_{LR}|$ can approach zero at a nonzero $\theta_R$ near 73° as shown in Fig. 5. Otherwise, it is larger than 1, which means that generally it is possible to have $|M_{12}^{LR}|\gg|M_{12}^{SM}|$. In Fig. 6, we consider the behavior of $|r_{LR}|$ for $g_R/g_L\geqslant 0.5$, $\xi_g=0.0004$, and $\theta_R=14°,70°$ as $M_{W'}$ is varied. The behavior of $|r_{LR}|$ exhibits a substantial dependence on $M_{W'}$, and $|r_{LR}|$ can be larger than 1 even for $M_{W'}\sim 2$ TeV. Moreover, it can be seen that $|r_{LR}|$ falls near $M_{W'}\sim 300$ GeV at certain angles and phases in the mixing matrices. This reflects the possibility of relatively light masses of $W'$ compared to the previously known bound. We will return to this point in Sec. IV. The dependence of $|r_{LR}|$ on $\xi_g$ satisfying $\xi_g\sin\theta_R\lesssim 0.013$ at fixed $M_{W'}=700$ GeV and $\theta_R=14°,70°$ is shown in Fig. 7. As one can see, $|r_{LR}|$ can be enhanced up to 10% of the SM contribution for the given inputs. Although its effect is smaller than that of the other parameters, it is not negligible and can be dominant in $|r_{LR}|$ if the first two $\zeta$ dependent terms in Eq. (30) cancel each other.

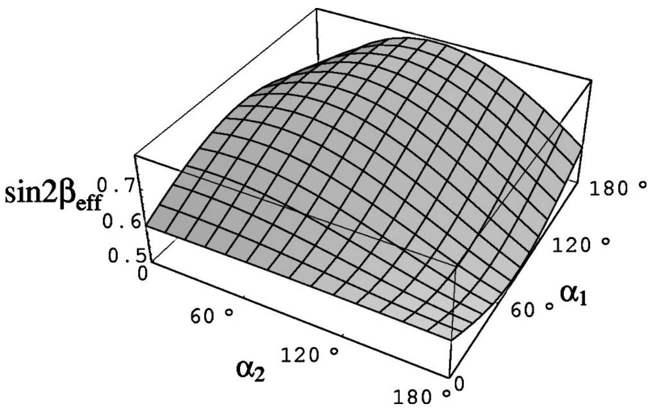


FIG. 8. Behavior of $\sin 2\beta_{eff}$ as $\alpha_{1,2}$ are varied.

As we mentioned previously, the average value of $|U_{td}|$ might be different from the actual value of $|U^L_{td}|$, and there is also ambiguity from errors in $f_B B_i^{1/2}$. Therefore the mass mixing $\Delta M_B^{SM}$ can be either much larger or smaller than $\Delta M_B^{expt}$. However, if we assume that $0.5 \lesssim |\Delta M_B^{SM}/\Delta M_B^{expt}| \lesssim 2$, we can get specific bounds on the mass $M_{W'}$ and the angle $\theta_R$ using the experimental value $\Delta M_B^{expt} \simeq 0.472\times 10^{12}\,\mathrm{s}^{-1}$. We will estimate the lowest possible bound on $M_{W'}$ with respect to $\theta_R$ in their parameter space with numerical consideration of $\sin 2\beta$ in the next section.

## IV. CP ASYMMETRY IN $B^0$ DECAY

The $CP$ angle $\beta$ in the CKM matrix can be measured in $B\to J/\psi K_S$ decays. In $B$ decays into a final $CP$ eigenstate $J/\psi K_S$, $\beta$ is related to the parametrization invariant quantity $\lambda$ as follows [1]:

$$\sin 2\beta_{eff} = \mathrm{Im}\lambda(B^0\to J/\psi K_S), \tag{31}$$

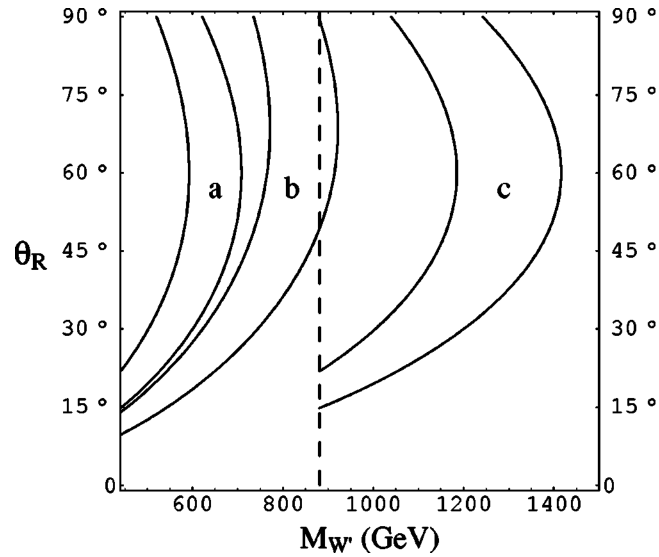


FIG. 9. Contour plots corresponding to $\sin 2\beta_{eff}=0.99$ for $\sin 2\beta=0.60$: (a) $\xi_g=\zeta/2$ and $g_R=g_L$, (b) $\xi_g=\zeta$ and $g_R=g_L$, and (c) $\xi_g=\zeta/2$ and $g_R=2g_L$. The dashed line corresponds to the lower bound on $M_{W'}$ for $g_R=2g_L$.

where

$$\lambda \equiv -\left(\frac{q}{p}\right)_B \frac{A(\overline{B^0}\to J/\psi K_S)}{A(B^0\to J/\psi K_S)}, \qquad \left(\frac{q}{p}\right)_B \simeq \frac{M_{12}^*}{|M_{12}|}. \tag{32}$$

The minus sign in the above expression comes from the fact that $J/\psi K_S$ is $CP$ odd. As mentioned earlier, $\beta_{eff}=\beta$ in the SM.

In the LRM, the two types of $U^R$ give us two distinct results. In the case of $U_I^R$, the $W'$ contribution to the mixing parameter $(q/p)_B$ is negligible so that $(q/p)_B \simeq (q/p)_{SM} = e^{-2i\beta}$. Then the $CP$ angle $\beta_{eff}$ can be expressed as

$$\begin{aligned}\sin 2\beta^I_{eff} &\simeq -\mathrm{Im}\left( e^{-2i\beta}\frac{U^{L*}_{cs}U^L_{cb}+(g_R/g_L)^2(-2U^{L*}_{cs}U^R_{cb}\xi_g^+ + U^{R*}_{cs}U^R_{cb}\zeta)}{U^L_{cs}U^{L*}_{cb}+(g_R/g_L)^2(-2U^L_{cs}U^{R*}_{cb}\xi_g^- + U^R_{cs}U^{R*}_{cb}\zeta)}\right)\\ &\simeq -\mathrm{Im}\left( e^{-2i\beta}\frac{1+25(g_R/g_L)^2(-2s_R\xi_g e^{i\alpha_2}+c_R s_R\zeta e^{i(\alpha_2-\alpha_1)})}{1+25(g_R/g_L)^2(-2s_R\xi_g e^{-i\alpha_2}+c_R s_R\zeta e^{-i(\alpha_2-\alpha_1)})}\right),\end{aligned} \tag{33}$$

where the mixing angle $\alpha$ is absorbed in $\alpha_i$ again, and we ignored the $K\overline{K}$ mixing and assumed that

$$\langle J/\psi K_s|\overline{c_L}\gamma_\mu s_L \overline{b_L}\gamma^\mu c_L|B^0\rangle \simeq \langle J/\psi K_s|\overline{c_R}\gamma_\mu s_R \overline{b_R}\gamma^\mu c_R|B^0\rangle \simeq -\frac{1}{2}\langle J/\psi K_s|\overline{c_L}\gamma_\mu s_L \overline{b_R}\gamma^\mu c_R|B^0\rangle. \tag{34}$$

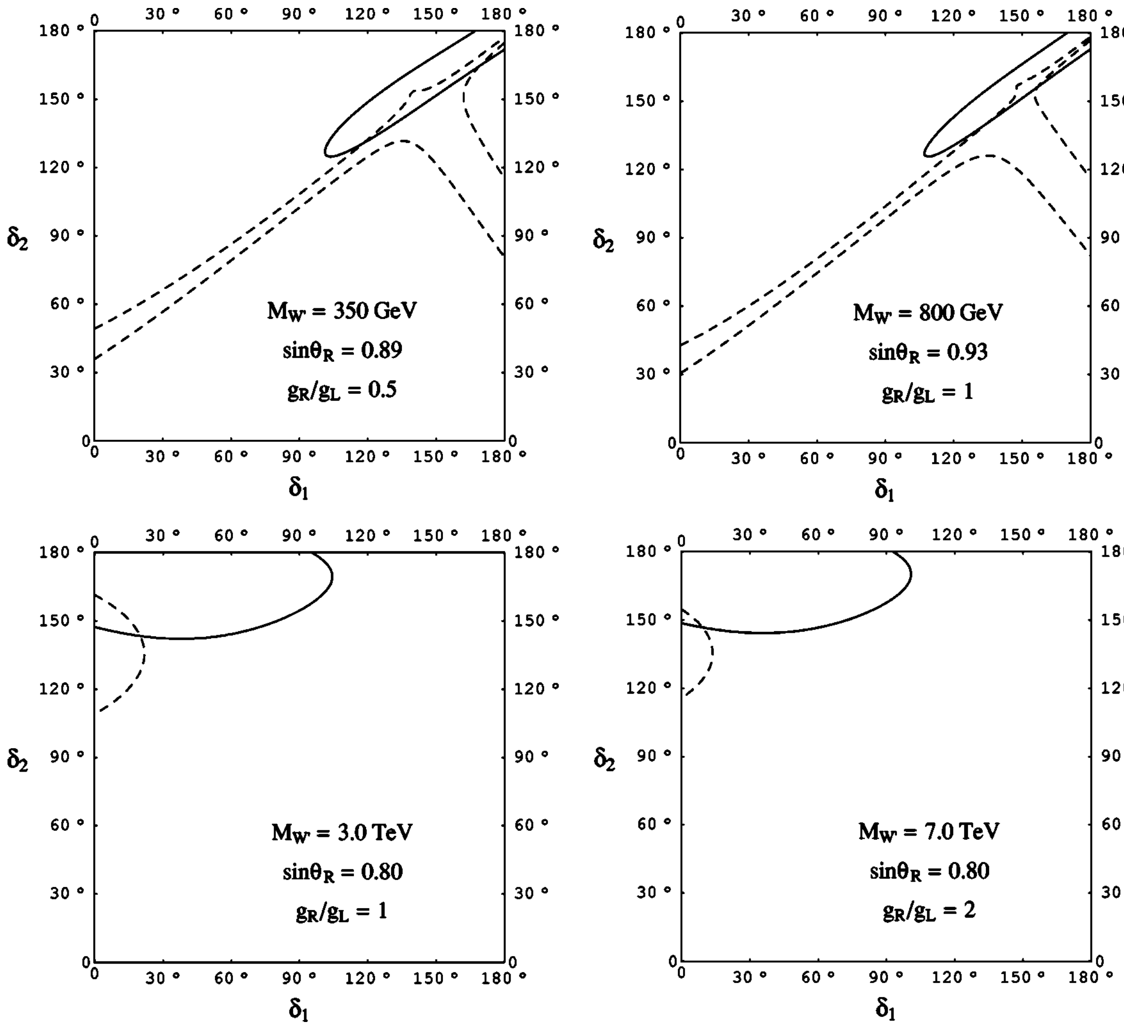


FIG. 10. Contour plots of $\Delta M_B$ (solid line) for $|U_{td}^L|=0.009$ and those corresponding to $\sin 2\beta_{eff}=0.99$ (dashed line) for $\sin 2\beta=0.60$.

As one can easily see in Eq. (33), $\sin 2\beta_{eff} = \sin 2\beta$ if $\alpha_{1,2}=0$ or $\pi$.

For illustration of the possible effect of the new interaction on the effective value of $\sin 2\beta$:$\sin 2\beta_{eff}$, we assume that the SM contribution produces $\sin 2\beta=0.60$, and show the region of parameters where the effective value is shifted to $\sin 2\beta_{eff}\sim 1$. We first plot $\sin 2\beta_{eff}$ in Fig. 8 for the typical values $M_{W'}=800$ GeV, $\xi_g=0.005$, $\theta_R=15°$, and $g_R=g_L$ as $\alpha_{1,2}$ are varied. In the figure, $\sin 2\beta_{eff}$ has a maximum variation from $\sin 2\beta$ near $\alpha_2\approx\pi/2$ and $\alpha_1=\pi$, and this behavior holds for other values of $M_{W'}$, $\xi_g$, and $\theta_R$. Next, we plot the contour corresponding to $\sin 2\beta_{eff}=0.99$ satisfying $\xi_g \sin\theta_R \lesssim 0.013$ in the parameter space of $M_{W'}$ and $\theta_R$ for $\alpha_2=\pi/2$, $\alpha_1=\pi$, $\xi_g=\zeta/2,\zeta$, and $g_R/g_L=1,2$ in Fig. 9.[2] As one can see, the upper bound of $M_{W'}$ goes down with decreasing $\xi_g$ and $g_R/g_L$. Therefore, under the given assumption, $g_R \ll g_L$ is disfavored, and so is $M_{W'}\gg 1$ TeV unless $g_R \gg g_L$.

[2]We did the same analysis for $g_R/g_L=0.5$ but there was no allowed region.

In the case of $U_{II}^R$, one has $U_{cs}^R\sim 0$ so that the $\zeta(M_{W'})$ dependent term in Eq. (33) is very small. However the ratio $(q/p)_B$ depends on $M_{W'}$. Thus the $W'$ contribution enters in a somewhat different way:

$$\sin 2\beta_{eff}^{II}\simeq -\mathrm{Im}\left( e^{-2i\beta}\frac{[1+(g_R/g_L)^2 r_{LR}^*]}{|1+(g_R/g_L)^2 r_{LR}|}\times\frac{[1-50(g_R/g_L)^2 s_R\xi_g e^{i\alpha_2}]}{[1-50(g_R/g_L)^2 s_R\xi_g e^{-i\alpha_2}]}\right). \quad (35)$$

Unlike in the previous case, we need to consider here the mass mixing of $B^0\overline{B}^0$ in order to analyze $\sin 2\beta_{eff}$ numerically. Assuming that $|U_{td}^L|=0.009$ and $\sin 2\beta=0.60$, we plot the contours corresponding to $\sin 2\beta_{eff}=0.99$ and $\Delta M_B^{LR}=\Delta M_B^{expt}$ in the parameter space of $\delta_{1,2}$ for $\delta_3=\pi$, $\xi_g=\zeta/4$, and $g_R/g_L=0.5,1,2$ by varying $\theta_R$ and $M_{W'}$ in Fig. 10. Because of the nontriviality of the dependence of $\sin 2\beta_{eff}$ on $\delta_i$, we repeated this analysis until the two contours overlapped by varying $M_{W'}$ from 350 GeV to 8 TeV, and found that the overlap appeared where 350 GeV$\lesssim M_{W'}$

$\lesssim 1.3$ TeV if $g_R/g_L = 0.5$, 440 GeV$\lesssim M_{W'} \lesssim 3.1$ TeV if $g_R/g_L = 1$, and 880 GeV$\lesssim M_{W'} \lesssim 7.1$ TeV if $g_R/g_L = 2$. Even though the existence of a heavy $W'$ with the mass $M_{W'} > 7$ TeV may be allowed by the numerical analysis of $\Delta M_B^{LR}$, it is excluded by that of $\sin 2\beta_{eff}$ under the given assumption. For different values of $\xi_g$, we also have similar results.

## V. CONCLUSION

In the LRM, if one does not impose manifest or pseudomanifest left-right symmetry, the $W'$ contributions to $B^0\overline{B}^0$ mixing and $CP$ asymmetry in $B^0$ decays are highly dependent upon the phases in the mass mixing matrix $U^{L,R}$. For certain phases, the contribution of $W'$ with a heavy mass of about a few TeV to $B^0\overline{B}^0$ mixing can be sizable. On the other hand, there is also the possibility of the existence of $W'$ with a light mass of about a few hundred GeV, whose contribution can be either very large or small, and so the contribution of the mixing angle $\xi$ is not negligible. Since the existence of a light $W'$ requires a small $g_R$, $g_R \lesssim g_L$, one can see from Eq. (26) and Fig. 6 that its contribution is limited. Therefore even assuming that $\Delta M_B^{LR} \lesssim \Delta M_B^{SM}$, we find that there is the possibility of a light $W'$ with a mass $M_{W'} \sim 300$ GeV.

This possibility also arises from a numerical analysis of the $CP$ asymmetry in $B^0$ decay. Since $U_{td}^L$ is not known with sufficient accuracy, estimates of the pure right-handed current contributions to $\Delta M_B$ and $\sin 2\beta$ are somewhat uncertain. But, for certain values of the parameter sets, one can see from Fig. 9 and Fig. 10 that the $CP$ asymmetry parameter $\sin 2\beta$ can be as large as almost 1, and the mass of $W'$ can be as small as about 350 GeV. Therefore, the existence of the light $W'$ can be tested once future experiments confirm the values of $\sin 2\beta$ and $|U_{td}^L|$.

## ACKNOWLEDGMENTS

The author would like to thank M. B. Voloshin for helpful comments.

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