Helicity conservation in inclusive nonleptonic decay $B\to VX$: Test of the long-distance final-state interaction

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The polarization measurement in inclusive B decay provides us with a simple test of how much the long-distance final-state interaction takes place as the energy of the observed meson varies in the final state. We give the expectation of perturbative QCD for the energy dependence of the helicity fractions in a semiquantitative form. Experiment will tell us for which decay processes the perturbative QCD calculation should be applicable.

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I. INTRODUCTION

It is of crucial importance to know how much long-distance final-state interaction (LDFSI) occurs in B decay. If LDFSI plays a significant role, we have no first-principle method to compute the decay amplitudes. Arguments have been presented in favor of short-distance (SD) dominance for the two-body decay in which a fast quark-antiquark pair moves almost collinearly in a colorless lump. Based on this color screening picture [1], a perturbative QCD computation has been carried out for two-body B decay [2,3]. Even if the SD dominance argument is valid in the infinite B mass limit, a quantitative question exists about the accuracy of the perturbative QCD calculation since the B meson mass is only 5.3 GeV in the real world. When the final mesons are highly excited states, the velocities of the mesons are less fast and the quarks inside them have larger transverse momenta. We expect that the SD dominance will be accordingly less accurate in such decays. In the large limit of the excited meson mass, the LDFSI should play a major role in determining the final state. We would like to verify experimentally the SD dominance in the two-body decay and see how the SD dominance disappears as the meson slows down in the inclusive decay.

One of the cleanest ways to test the breakdown of SD dominance or presence of LDFSI directly with experiment is to measure the helicity of a fast flying meson in the final state [4]. Since SD interactions do not flip helicities of light quarks (u,d,s), a fast light meson carries a memory of the quark helicities if no LDFSI enters. Because of the specific form of the weak interaction in the standard model, a fast light meson with spin must be polarized in the zero helicity state up to $O(1/M_B^2)$ in probability, when other hadrons fly away approximately in the opposite direction. One can determine the $h=0$ fraction of the meson by measuring the angular distribution of its decay products. In fact, this selection rule is so robust that it will be valid even if the right-handed W boson contributes to weak decays. It breaks down most likely by LDFSI, if at all.

Imagine that such a polarization measurement is made for the inclusive decay $B\to\rho X$ in which X is a highly excited meson state $(\overline{q}q)$ or a multiquark hadronic state. As the invariant mass m_X increases, it becomes more likely that LDFSI takes place between ρ and X. If so, we shall start seeing production of the ρ meson in the $h=\pm 1$ states. By measuring the ρ helicity as a function of m_X^2 or equivalently as a function of the ρ energy in the B rest frame, we can determine from experiment how much LDFSI enters the decay as ρ slows down or how much the color screening breaks down.

For the two-body decay, the polarization measurement is possible only when both final mesons have nonzero spins, for instance, $B\to 1^-1^-$. Meanwhile, most decay modes that are easily identifiable and high in branching fraction are $B\to 0^-0^-$ and 1^-0^-. Nonetheless, the polarization test will have a direct impact on these dominant decay modes of the B meson in the following way. In charmless B decay, the two-body decays $B\to\pi\pi$ and $\rho\pi$ are among the decay modes of primary interest from the viewpoint of CP violation. If our proposed test reveals that the $h=0$ state dominates in $B\to\rho\rho,\rho\omega$, and so forth, we shall feel more confident in computing the tree and penguin amplitudes of $B\to\pi\pi,\rho\pi$ in perturbative QCD. If on the contrary the $h=0$ dominance is substantially violated in $B\to\rho\rho,\rho\omega$, we should not trust the perturbative method of calculation for $B\to\pi\pi,\rho\pi$. In this case the only recourse would be to determine the $B\to\pi\pi$ amplitudes by experiment alone [5] without help of theoretical computation. And little could be done for $B\to\rho\pi$ with isospin invariance alone. The test proposed here is not for inventing a new method of calculation of decay amplitudes, but for learning from experiment for which decay modes we may perform the perturbative QCD calculation.

II. KINEMATICS OF $B\to VX$

We consider the inclusive B decay into a vector meson V of $J^P=1^-$:

$$B(\mathbf{P})\to V(\mathbf{q},h)+X(p_X)\to(\mathbf{k}_1)+b(\mathbf{k}_2)+X(p_X), \quad (1)$$

where a and b are spinless decay products of V ($m_b\neq m_b$ in general). Here we have $B\to\rho X$, K^*X, and ϕX in mind. The inclusive decay rate is written in the covariant form as

$$4(2\pi)^6 k_{10}k_{20}\frac{d\Gamma}{d^3k_1 d^3k_2}$$
$$=\sum_{ij}\int\frac{d^3\mathbf{q}}{4(2\pi)^3 q_0 P_0}\frac{g_{ab}^2}{2m_V\Gamma_V}(2\pi)^4\delta^4(k_1+k_2-q)$$
$$\times(\epsilon_i\cdot k_1-k_2)(\epsilon_j^*\cdot k_1-k_2)\epsilon_i^{\mu *}T_{\mu\nu}\epsilon_j^{\nu}, \quad (2)$$

where Γ_V is the decay width of V, g_{ab} is the decay coupling constant of V defined by $L_{int}=ig_{ab}(\phi_a^* \overleftrightarrow{\partial_\mu}\phi_b^*)V^\mu$, and the subscript of the polarization vector ϵ refers to three helicity states of V. The covariant tensor $T_{\mu\nu}$ is the inclusive structure function defined by

$$T_{\mu\nu}(m_X^2)=4q_0P_0\sum_X (2\pi)^4\delta^4(q+p_X-P)\times\langle B(\mathbf{P})|H_{int}|V(\mathbf{q},j)X\rangle\langle V(\mathbf{q},i)X|H_{int}|B(\mathbf{P})\rangle, \tag{3}$$

where the states are normalized as $\langle\mathbf{p}|\mathbf{p}'\rangle=(2\pi)^3\delta(\mathbf{p}-\mathbf{p}')$ without $2E_{\mathbf{p}}$. The general tensor form of $T_{\mu\nu}$ is

$$T_{\mu\nu}=-g_{\mu\nu}A(m_X^2)+\frac{1}{M_B^2}P_\mu P_\nu B(m_X^2)+\frac{i}{M_Bm_V}\varepsilon_{\mu\nu\kappa\lambda}P^\kappa q^\lambda C(m_X^2), \tag{4}$$

where $m_X^2=(P-q)^2$ and the antisymmetric unit tensor is defined as $\varepsilon_{0123}=-1$. The scalar structure functions A–C are the absorptive parts of the analytic functions of the variable m_X^2 that are regular except on the segments of the real axis in the complex m_X^2 plane if V is treated as (approximately) stable. In particular, A–C are nonsingular ($\neq\infty$) in the physical region of the decay.

The helicity amplitudes H_h for $B\to V_hX$ in the B rest frame can be expressed in terms of A–C as

$$H_0=A+\frac{\mathbf{q}^2}{m_V^2}B,$$

$$H_{\pm1}=A\mp\frac{|\mathbf{q}|}{m_V}C. \tag{5}$$

In contracting $T_{\mu\nu}$ with ϵ, we must not make the approximation $\epsilon^\mu(\mathbf{q})\simeq q^\mu/m_V$ as we often do in the exclusive two-body decay $B\to V_1V_2$ where $g_{\mu\nu}\epsilon_1^\mu\cdot\epsilon_2^\nu\simeq(q_1\cdot q_2)/m^2$, because $g_{\mu\nu}\epsilon^{\mu *}\epsilon^\nu=-1$ while $g_{\mu\nu}q^\mu q^\nu/m_V^2=+1$ in the inclusive decay kinematics.

Carrying out the summation over the helicities in Eq. (2) with Eq. (4), we obtain the differential decay rate with respect to the direction of $\mathbf{k}_1$ and the energy of V. The result is

$$\left.\frac{d\Gamma(B\to VX\to abX)}{dq_0 d\cos\theta}\right|_{B \text{ at rest}}=\frac{g_{ab}^2|\mathbf{q}||\mathbf{k}_{cm}|^3}{32\pi^3m_V^2\Gamma_V}\left[A(m_X^2)+\frac{\mathbf{P}^2}{M_B^2}B(m_X^2)\cos^2\theta\right], \tag{6}$$

where q_0 is the energy of V in the rest frame of B, which is related to m_X by $m_X^2=M_B^2+m_V^2-2M_Bq_0$ so that $d\Gamma/dq_0=2M_Bd\Gamma/dm_X^2$, $\mathbf{k}_{cm}$ is the momentum of a in the rest frame of V, $\mathbf{P}$ is the momentum of B measured in the rest frame of V, and θ is the angle of $\mathbf{k}_{cm}$ measured from the direction of $\mathbf{P}$, namely, $(\mathbf{P}\cdot\mathbf{k}_{cm})=|\mathbf{P}||\mathbf{k}_{cm}|\cos\theta$.

We make two remarks on Eq. (6). Since the decay products a and b are spinless, the structure function of the $V\to ab$ decay, $g_{ab}^2(k_1-k_2)^\mu(k_1-k_2)^\nu$, is symmetric under $\mu\leftrightarrow\nu$ so that the function $C(m_X^2)$ does not enter the differential decay rate. This means according to Eq. (5) that we cannot separate the $h=-1$ decay from the $h=+1$ decay in this process. In order to distinguish between $h=\pm1$, we would have to choose a decay in which $J\neq0$ for a or b and measure the helicity of a or b through its decay. For instance, the triple product $\mathbf{q}\cdot(\mathbf{k}_1\times\mathbf{k}_1')$ in the sequence of decays $B\to a_2(\mathbf{q})X\to\pi(\mathbf{k}_1)\rho(\mathbf{k}_2)X\to\pi(\mathbf{k}_1)\pi(\mathbf{k}_1')\pi(\mathbf{k}_2')X$ contains such information.[1] The other comment is on the slow limit of V. In the limit of $\mathbf{q}\to0$ in Eq. (5), distinction among three different helicity states of V disappears for an obvious reason and all helicity functions H_h ($h=1,0,-1$) are given by $A(m_X^2)$ since $B(m_X^2)$ and $C(m_X^2)$ stay finite there:

$$H_1+H_{-1}\to2H_0,\quad H_1-H_{-1}\to0\ \text{ as }\ \mathbf{q}\to0. \tag{7}$$

In this limit only the $A(m_X^2)$ function survives in the differential decay rate of Eq. (6), as we expect, since $\mathbf{q}\to0$ means $\mathbf{P}\to0$.

Finally, let us express the differential decay rate in terms of H_h, noting that $|\mathbf{P}|/M_B=|\mathbf{q}|/m_V$ by the transformation between the B rest frame and the V rest frame. The result is

$$\left.\frac{d\Gamma(B\to VX\to abX)}{dq_0 d\cos\theta}\right|_{B \text{ at rest}}=\frac{g_{ab}^2|\mathbf{q}||\mathbf{k}_{cm}|^3}{32\pi^3m_V^2\Gamma_V}\left[H_0\cos^2\theta+\frac{1}{2}(H_1+H_{-1})\sin^2\theta\right]. \tag{8}$$

We are able to separate the longitudinal ($h=0$) and transverse ($h=\pm1$) polarization decay with the angular distribution of Eq. (8). Experiment will show us how the $h=0$ dominance goes away as m_X increases in the inclusive decay $B\to VX$. If the transverse polarization appears beyond the corrections to be discussed in the subsequent sections, it will be clear evidence for LDFSI.

III. LONGITUDINAL POLARIZATION DOMINANCE

For the weak interaction of the standard model, the zero-helicity function H_0 should dominate over all other H_λ for small m_X, if the strong interaction corrections are entirely of short distances except at hadron formation. We explain this rule for two-body decays [4], discuss the mass and orbital motion corrections to the rule, and extend it to the inclusive decay $B\to VX$. Our argument is based on the standard assumptions made in the perturbative calculation including the

[1] Such a measurement was actually proposed to determine the photon helicity in $B\to\gamma K_1\to\gamma K\pi\pi$ [6]. The strong phases due to the overlapping resonances are needed to detect the triple product.

light-cone formulation of mesons in $\bar{q}q$. The helicity selection rule should break down for sufficiently large values of m_X. The value at which the rule starts showing a significant departure from the $h=0$ dominance will provide us with a quantitative measure of the accuracy of the perturbative QCD calculation. We first discuss charmless decay and then move on to decays with charm.

A. Meson helicity and helicities of $\bar{q}q$

In nonleptonic B decay a pair of $\bar{q}q$ is produced by weak interaction nearly in parallel to form an energetic meson. In the case of a vector meson ($^3S_1$), we may approximate the $\bar{q}q$ pair as literally in parallel by ignoring the tiny $^3D_1$ component. For excited mesons such as $J^P=2^+(^3P_2)$, the transverse motion of q and $\bar{q}$ must be taken into account. This gives rise to an orbital angular momentum l between q and $\bar{q}$ as well as to the meson mass. This angular momentum is part of the meson spin. By simple kinematics, however, the state of $l_z=0$ dominates over all others when a meson moves fast. That is, to lowest order we may leave out the orbital motion of $\bar{q}q$ inside a meson even for an excited meson state with $l\neq 0$. Let us make this statement quantitative.

In the classical picture, the orbital angular momentum vector is squashed into the plane perpendicular to the meson momentum when a meson moves fast. To see it in quantum theory, let us expand the plane wave $e^{i\mathbf{p}\cdot\mathbf{r}}$ of a quark in spherical harmonics for $\mathbf{p}$ off the direction of the meson momentum $\mathbf{q}=|\mathbf{q}|\hat{z}$. Defining the directions of the vectors as

$$\begin{aligned}
\mathbf{r}&=r(\sin\vartheta\cos\varphi,\sin\vartheta\sin\varphi,\cos\vartheta),\\
\mathbf{p}&=|\mathbf{p}|(\sin\vartheta'\cos\varphi',\sin\vartheta'\sin\varphi',\cos\vartheta'),\\
\hat{\mathbf{r}}\cdot\hat{\mathbf{p}}&\equiv\cos\gamma.
\end{aligned}\tag{9}$$

We obtain by use of the well-known formulas the expansion of the plane wave in the form

$$\begin{aligned}
e^{i\mathbf{p}\cdot\mathbf{r}}&=\sum_l (2l+1)i^l j_l(|\mathbf{p}|r)P_l(\cos\gamma),\\
&=4\pi\sum_l i^l j_l(|\mathbf{p}|r)\sum_{m=-l}^{l} Y^*_{lm}(\vartheta',\varphi')Y_{lm}(\vartheta,\varphi).
\end{aligned}\tag{10}$$

Treating $\vartheta'\simeq|\mathbf{p}_T|/|\mathbf{p}|$ as small, we expand $Y^*_{lm}(\vartheta',\varphi')$ around $\vartheta'=0$. Then Eq. (10) turns into

$$\begin{aligned}
e^{i\mathbf{p}\cdot\mathbf{r}}&\simeq\sum_l \sqrt{4\pi(2l+1)}i^l j_l(|\mathbf{p}|r)\sum_{m=-l}^{l}\frac{(-1)^{m+|m|}}{2^{|m|}|m|!}\\
&\quad\times\sqrt{\frac{(l+|m|)!}{(l-|m|)!}}e^{-im\varphi'}\vartheta'^{|m|}Y_{lm}(\vartheta,\varphi).
\end{aligned}\tag{11}$$

In the sum over l_z (denoted by m above), the amplitudes of $l_z\neq 0$ are suppressed by $\vartheta'^{|l_z|}\simeq(\frac{1}{2}m_T/E)^{|l_z|}$ where m_T stands for the transverse meson mass ($\simeq\sqrt{\frac{2}{3}}\times$ meson mass). Repeat the argument for $\bar{q}$. Projecting the $\bar{q}q$ state with the quark distribution function of the meson, we find that the meson helicity consists entirely of the quark helicity $h_q+h_{\bar{q}}$ in the fast limit. The contribution of the $l_z\neq 0$ states generates a correction of $O[(\frac{1}{2}m_T/E)^{|l_z|}]$ in the amplitude for an excited meson and a multimeson state.

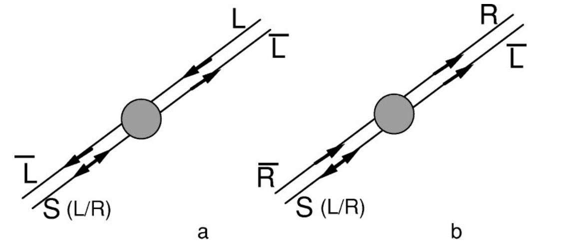

FIG. 1. The quark helicities in the two-body $B(\bar{b}q)$ decay. L ($\bar{L}$) and R ($\bar{R}$) denote left and right chiral quarks (antiquarks), respectively. The spectator quark q_{spec} is denoted by S. The arrows represent the dominant spin directions. (a) The case of $\bar{b}\rightarrow\bar{q}_L q_L\bar{q}_L$. (b) The case of $\bar{b}\rightarrow\bar{q}_L q_R\bar{q}_R$.

B. Helicity selection rule; charmless decay

The fundamental weak interaction is dressed or improved into the effective decay operators by the renormalization group down to the scale m_b. In the standard model, the chiral structures of the decay operators relevant to the charmless decay are $(\bar{b}_L q_L)(\bar{q}_L q_L)+\text{H.c.}$ and $(\bar{b}_L q_L)(\bar{q}_R q_R)+\text{H.c.}$, where q stands for the light quark. The short-distance interaction below the scale m_b does not generate any new chiral structure. It can add $\bar{q}_L q_L+\bar{q}_R q_R$ through quark pair emission by a hard gluon. The chirality of the spectator quark is indefinite so that it can be in either helicity $+\frac{1}{2}$ or $-\frac{1}{2}$ when it forms a meson.

Let us start with the two-body charmless decay $B\rightarrow VM$ ($J\geqslant 1$ for M too). When one of V and M is formed with $\bar{q}_L q_L$ or with $\bar{q}_R q_R$, this meson is in the $h=0$ state. The angular momentum conservation along the decay momenta in the B rest frame requires that the helicity of the other meson must also be zero (Fig. 1a). Therefore H_0 dominates in this case. Alternatively, with $(\bar{b}_L q_L)(\bar{q}_R q_R)$, if $\bar{q}_L q_R(h=+1)$ is combined to form one meson, the other meson must be made of the spectator q_{spec} and $\bar{q}_R(h=-\frac{1}{2})$. Then the net helicity of the second meson can be only 0 or -1, which does not match the helicity $h=+1$ of the first meson (Fig. 1b). Therefore the only two-meson state compatible with the helicities and the overall angular momentum conservation is $V_{h=0}M_{h=0}$. This argument is valid only in the limit that the massless q and $\bar{q}$ move strictly in parallel and there is no relative motion between them inside the meson.

C. Mass corrections

The relative motion of $\bar{q}q$ generates a correction to this helicity selection rule. Since the motion of light quarks makes up the entire mass of a nonflavored meson, this correction should be $O(|\mathbf{p}_T|/E)=O(\frac{1}{2}m/E)$ in amplitude, where m is the meson mass and $E\approx\frac{1}{2}M_B$ for two-body decays. When either mass of V and M is large, the correction is large and the accuracy of the rule is reduced accordingly. Let us examine this correction.

In the case of the $B(\bar{b}q)$ meson decaying through the interaction $(\overline{b_L q_L})(\overline{q_L q_L})$, the quarks in the final state are $\overline{q_L} q_L \overline{q_L} q_{spec}$ where q_{spec} stands for the spectator. The $h=+1$ state of the meson $(\overline{q_L q_L})$ can arise from a small opposite helicity component of a single q_L while $h=+1$ is allowed for the other meson $(\overline{q_L q_{spec}})$ because of the indefinite helicity of q_{spec}. On the other hand, formation of the $h=-1$ meson state requires small opposite helicity components of two $\overline{q_L}$'s, one in V and one in M (Fig. 1a). Consequently, H_1 arises as a first-order correction while H_{-1} can arise only as a second-order correction. The same conclusion follows when B decays through $(\overline{b_L q_L})(\overline{q_R q_R})$ (Fig. 1b). If we define the longitudinal and transverse fractions of helicity decay rates by

$$\Gamma_L = \frac{H_0}{H_1 + H_0 + H_{-1}}, \quad \Gamma_T = 1 - \Gamma_L, \tag{12}$$

the mass corrections are expressed as $\Gamma_T = O(m^2/M_B^2)$ and $\Gamma_L = 1 - O(m^2/M_B^2)$ in the case of the two-body decay $B(\bar{b}q) \to VM$ [4].[2] Here m is the mass of the meson which does not receive the spectator quark or its descendant. The reason is obvious from the preceding argument: It is the meson formed by the energetic $\bar{q}q$ originating from the effective decay interaction that primarily determines the helicity state, since the helicity of the other side that receives the spectator has a twofold uncertainty due to the indefinite spectator helicity. The helicity of the meson carrying the spectator is constrained by the overall angular momentum conservation. In the case of the $\bar{B}(b\bar{q})$ meson, the mass corrections to H_1 and H_{-1} are interchanged in the same argument.

We should recall that there is also the $l_z \neq 0$ correction of $O(m^2/M_B^2)$ in probability in the case that a meson has $l \neq 0$. This correction contributes to H_1 and H_{-1} in the same order, namely ϑ^2. In terms of $\Gamma_T \propto H_1 + H_2$ the correction takes the same form for excited mesons.

It is easy to see here that the $h=0$ dominance holds even if the right-handed current enters the weak interaction. Only $H_1 > H_{-1}$ or $H_{-1} > H_1$ in the mass correction depends on $V-A$ or $V+A$. In order to violate the $h=0$ dominance, we would need such an exotic weak interaction as $b \to q_L \overline{q_R} q_L$. If the $h=0$ dominance breaks down, therefore, the most likely source is LDFSI.

[2]Such a mass correction can be seen in the $U(6) \times U(6)$ model calculation of the charmless decay $B \to 1^- 1^-$ by Ali *et al.* [7]. Cheng *et al.* recently referred to this correction in their improved factorization calculation of $B \to J/\psi K^*$ [8]. Many other model calculations in the past based on the factorization, however, do not follow this pattern of mass corrections since vector and axial-vector form factors were introduced without chiral constraints.

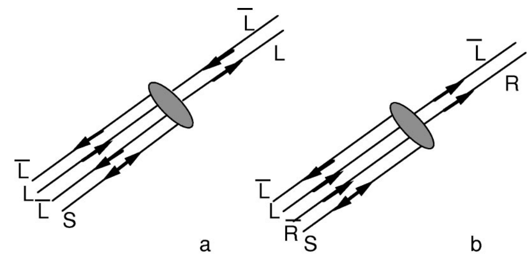

FIG. 2. The helicities in the inclusive $B(\bar{b}q)$ decay where an additional hard pair of $\bar{q}q$ is produced and leads to the final state $\bar{q}q\bar{q}q\bar{q}q_{spec}$.

D. Inclusive charmless decay

The argument in the preceding section can be immediately extended to the inclusive decay $B \to VX$ in the case that X is described as excited $q\bar{q}$ states. When a $q\bar{q}$ pair is created almost collinearly by a hard gluon and turns X into a $q\bar{q}q\bar{q}$ state, the added pair $\overline{q_L q_L}$ or $\overline{q_R q_R}$ has net helicity zero and does not contribute to the helicity of X (Fig. 2a). In this case the previous argument of the $h=0$ dominance is unaffected. It can happen alternatively that the hard q and $\bar{q}$ are emitted back to back. Imagine, for instance, that q_R enters V and $\overline{q_R}$ goes into X so that $V \sim \overline{q_L} q_R$ and $X \sim \overline{q_R} q_L \overline{q_L} q_{spec}$ (Fig. 2b). Then the net helicities are $h=+1$ for V and $h=0,-1$ for X, so the additional hard pair of $\bar{q}q$ cannot realize $V_{h=\pm 1} X_{h=\pm 1}$. We can easily see that the helicities of V and X do not match for $h=\pm 1$ even when V receives q_{spec}. The only helicity final state compatible with the overall angular momentum conservation is still $V_{h=0} X_{h=0}$ in the collinear limit. Therefore, the preceding argument for the two-body decay $B \to VM$ is carried over to the inclusive decay $B \to VX$.

However, the collinear quark limit becomes a poor approximation as m_X increases in the inclusive decay. The transverse quark momenta $\mathbf{p}_T$ in X become large with respect to $\mathbf{p}_X$ so that the corrections grow with m_X. The mass correction depends on whether the spectator q_{spec} enters V or X. For the same reason as in the two-body decay, the final helicity state is determined primarily by the meson (V) or the group of mesons (X) that does not receive q_{spec} of indefinite helicity. Making an appropriate substitution in the mass corrections for the two-body decay, we obtain, for $m_X \gg m_V$,

$$(1-\Gamma_L)_{mass} \approx \frac{m_V^2 M_B^2}{(M_B^2 - m_X^2)^2} \quad (q_{spec}\,\text{in}\,X), \tag{13}$$

$$(1-\Gamma_L)_{mass} \approx \frac{m_X^2 M_B^2}{(M_B^2 + m_X^2)^2} \quad (q_{spec}\,\text{in}\,V).$$

The right-hand sides indicate the orders of magnitude. It is difficult even within perturbative QCD to compute their coefficients with good accuracy since they depend on the quark distributions inside mesons and other details. The coefficients are highly dependent on individual decay modes. Nonetheless, the rise of Γ_T with m_X^2, particularly in the case that X is produced without the spectator, is an important

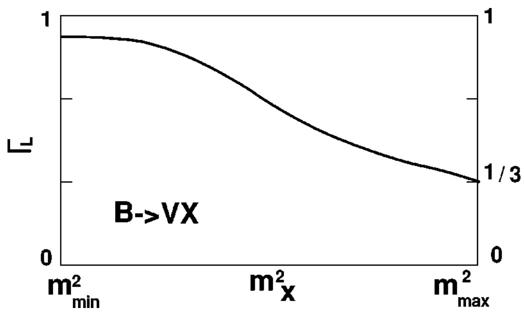

FIG. 3. The qualitative behavior of Γ_L against m_X^2. While $\Gamma_L = 1/3$ at $m_X = m_{max}$ is a kinematical constraint, the behavior of Γ_L near the small end of m_X is only the expectation of perturbative QCD.

trend. It simply means that the ''small opposite helicity component'' of $O(m_X/E_X)$ ceases to be small when m_X becomes large.

The orbital motion inside X is not restricted to $l=0$. Therefore l_z of X can make up for violation of the overall angular momentum conservation when V is formed with $\overline{q}_L q_R$ $(h=+1)$ or $\overline{q}_R q_L$ $(h=-1)$. In terms of the helicity fraction, the l_z correction to X generates the leading correction that grows rapidly with m_X:

$$(1-\Gamma_L)_{l_z} \approx \frac{m_X^2 m_B^2}{(M_B^2+m_X^2)^2}. \tag{14}$$

When $\Gamma_T = 1-\Gamma_L$ becomes a substantial fraction of unity, LDFSI is clearly important.[3] As m_X approaches the kinematical upper limit corresponding to $\mathbf{q}=0$, Γ_L should reach 1/3 according to the limiting behavior of Eq. (7):

$$\Gamma_L \to 1/3 \quad \text{as } m_X \to m_{max}. \tag{15}$$

Future experiments on the inclusive decay will determine Γ_L as a function of m_X interpolating between $1-O(m_V^2/M_B^2)$ and $\frac{1}{3}$, as sketched qualitatively in Fig. 3. We should keep in mind that the corrections presented here are the expectation based on perturbative QCD. It is only a theoretical prediction which should be tested by experiment. While the helicity test of the charmless decay is of primary interest, no experimental data exist on $\Gamma_{T,L}$ for any charmless decay mode at present.

One problem exists in performing an inclusive measurement of the charmless decay $B \to VX$. One has to make sure that X does not contain charm or hidden charm. Since the charmless decays are the *rare* decays, the region above the charm threshold for m_X is overwhelmed by the background, which is much higher in branching. In practice, the charmless inclusive decay will be analyzed only in the region separated from the charm background by kinematics, that is,

$$m_X < m_D. \tag{16}$$

Above m_D, the dominant process is $B \to VX_c^-$ where X_c^- contains an anticharmed meson. Fortunately, Eq. (16) is the mass range where many interesting results will be extracted from the charmless decay. For the decays into X_c^-, the helicity selection rule holds in a manner almost identical to that in charmless decay. We shall see that the Fig. 3 applies to $B \to VX_c^-$ as well. Therefore, separate tests of the rule will be possible with $B \to VX_c^-$ in the range above $m_X = m_D$.

As for V, reconstruction of ρ from $\pi\pi$ may encounter an excessive combinatorial background. If this happens, ϕ will be a clean alternative for V in the environment of the BaBar and Belle experiments.[4] As a last resort, we can work on fully reconstructed B events with reduced statistics.

IV. DECAY INTO CHARMED *X* OR CHARMED *V*

We extend the argument for the charmless decay to the charmed meson production decay $B \to VX_c^-$ and $B \to V_c^- X$. We ignore here the small contribution from the penguin-type processes for this class of decay. When V is formed without involving the spectator, V carries $h=0$ of $\overline{q}_L q_L$ up to the small mass correction given by the first line of Eq. (13). The $h=0$ dominance remains true even when an extra $\overline{q}q$ pair is produced: Imagine, for instance, that $\overline{q}_R$ and q_R are produced secondarily by a hard gluon and enter both V and X. Then $V = \overline{q}_L q_R$ and $X = \overline{c}_L q_L \overline{q}_R q_{spec}$ can satisfy the overall angular momentum conservation only with the help of $l_z = +1$ or the opposite component of q_L or $\overline{q}_R$. In the case of $V = \overline{q}_R q_L$ and $X = \overline{c}_L q_R \overline{q}_L q_{spec}$, both $l_z = -1$ and the opposite helicity of $\overline{c}_L$ are needed.[5] In the two-body decay where X_c^- is $\overline{D}^*$ $(l=0)$ and q_{spec} enters D^*, therefore, the correction to the $h=0$ rule is dominated by the mass correction to V,

$$1-\Gamma_L \approx \frac{m_V^2 M_B^2}{(M_B^2 - m_X^2)^2}. \tag{17}$$

This correction will apply to $B^0/\overline{B^0} \to \rho^\pm D^{*\mp}$ since the quark distribution function disfavors formation of $\rho^\pm$ with the spectator. Because of the large branching, experiments have already measured the helicity fractions with good accuracy for the two-body decay $B^0/\overline{B^0} \to \rho^\pm D^{*\mp}$ many years ago. The experimental result was in agreement with the $h=0$ dominance [9]:

$$\Gamma_L = 0.93 \pm 0.05 \pm 0.05. \tag{18}$$

The deviation from unity of Γ_L is consistent with the mass correction (≈ 0.03) that we expect from Eq. (17). Even when $X_{c/c}^-$ is a higher state of $l \neq 0$, the correction to the $h=0$ rule

[3]It is possible that X consists of a widely separated pair of mesons interacting only through SDFSI. In this case, the final state is a three-jet state and the decay may be a SD process calculable by perturbative QCD for $m_B \to \infty$. However, such a contribution is suppressed by $O(\alpha_s/\pi)$ and not expected to be a significant portion of the inclusive decay. One should be able to check by actually examining the final states whether this is the case or not.

[4]The author owes thanks to R. N. Cahn for this remark.

[5]The opposite helicity content of c_L is larger, $(m_c^2 + \mathbf{p}_T^2)^{1/2}/E_c$ instead of $|\mathbf{p}_T|/E_c$.

is determined by $\rho^{\pm}$ and grows rather slowly with m_X according to Eq. (17) since q_{spec} enters $X^-_{\bar{c}/c}$ in the dominant process of $B^0/\bar{B}^0 \to \rho^{\pm} X^-_{\bar{c}/c}$.

The correction is a little different for the so-called color-disfavored decays. Take $B^0(\bar{b}d) \to \rho^0 \bar{D}^{*0}$ as an example: The ρ^0 meson must be formed with the spectator when the decay occurs through the dominant operator for this decay. The final helicity is constrained by D^{*-} and the correction is $1-\Gamma_L \approx m_X^2 M_B^2/(M_B^2+m_X^2)^2$. Therefore we expect that the correction is larger in $B^0 \to \rho^0 \bar{D}^{*0}$ than in $B^0 \to \rho^+ D^{*-}$:

$$\Gamma_T(B^0 \to \rho^0 \bar{D}^0) > \Gamma_T(B^0 \to \rho^+ D^{*-}). \tag{19}$$

The recent measurement [15] of the factorization-disfavored two-body decays $B^0 \to \bar{D}^{(*0)} X^0$ $(X^0 = \pi^0, \omega, \eta)$ seems to show that the branching fractions for these decays are larger than their lowest-order perturbative QCD calculations [2]. The helicity analysis of $B^0 \to \rho^0 X_c^0$ and $K^{*0} X_c^0$ will help us toward better understanding of how much LDFSI is involved here.

Let us move to the other inclusive measurement where a charmed meson is identified instead of a light meson: $B \to \bar{D}^* X$. There is an experimental advantage in reconstructing $\bar{D}^*$ through its soft decay into $\bar{D}\pi$. The $\bar{D}^*$ meson can be formed with or without the spectator. With the spectator $(\bar{D}^* = \bar{c}_L q_{spec})$, the accuracy of the $h=0$ dominance is controlled by the helicity of X, which is determined by $\bar{q}_L q_L$, $\bar{q}_L q_L \bar{q}_L q_L$, $\bar{q}_L q_L \bar{q}_R q_R$, $\ldots$. The correction is given by the second line of Eq. (13) and grows rapidly with m_X. On the other hand, when X receives the spectator, $X = \bar{q}_L q_{spec}$, $\bar{q}_L q_{spec} \bar{q} q$, $\ldots$ can be in either $h = +1$ or 0 with a 50/50 chance. Then it is $\bar{D}^* = \bar{c}_L q_L$ that determines the final helicity. The dominant helicity is again $h=0$ and the correction is given by the first line of Eq. (13), but the magnitude is large because of the larger opposite helicity content in $\bar{c}_L$.

Finally we comment on the decays $B \to V X_{c\bar{c}}^-$ and $V_{c\bar{c}}^- X$. A pair of $\bar{c}c$ is produced by weak interaction and forms one of charmonia or turns into $\bar{D}^{(*)} D^{(*)}$. V is most likely formed with the spectator since little phase space is left for production of a fast pair of $\bar{q}q$. In this case, the helicity content is determined by $\bar{c}_L c_L$. Since c_L and $\bar{c}_L$ are heavy and slow, the opposite helicity content of $O(\frac{1}{2} m_{c\bar{c}}^- / E_{c\bar{c}}^-)$ does not give an accurate estimate. Nonetheless, let us stretch for the moment the mass correction formula for Γ_T such that the coefficient in front be adjusted to give the kinematical constraint $\Gamma_T = \frac{2}{3}$ at the maximum value of m_X. Then the prediction on Γ_T would be

$$\Gamma_T \simeq \frac{8}{3} \times \frac{m_{c\bar{c}}^2 M_B^2}{(M_B^2 + m_{c\bar{c}}^2)^2}, \tag{20}$$

where $m_{c\bar{c}}^-$ is the invariant mass of all hadrons but V. $X_{c\bar{c}}^-$ is most likely one of charmonia. Detailed measurements were made for the helicity content of $B \to J/\psi K^*$. For this decay mode, Eq. (20) gives a "correction" of $\Gamma_L \simeq 0.49$. The latest result of the helicity analysis by BaBar [10] can be expressed as

$$\Gamma_L = 0.597 \pm 0.028 \pm 0.024, \tag{21}$$

which is not far from 0.49. However, the agreement is probably fortuitous since the Lorentz factor γ of J/ψ is only 1.12 in this decay.

In the decay $B \to J/\psi K^*$, the K^* meson moves with $\gamma \simeq 2$. If we make the approximation of K^* being fast, $K^*(\bar{s}_L q_{spec})$ can be only in helicity $+1$ or 0, not in -1. Therefore $H_{-1} \simeq 0$ is predicted for $B \to J/\psi K^*$ if one assumes perturbative QCD for K^*. The transversity angular analysis [10] allows two solutions, $H_1 \gg H_{-1}$ and $H_1 \ll H_{-1}$, but cannot resolve the twofold ambiguity. At present, experiment still does not exclude the possibility that perturbative QCD is applicable to the light meson side (K^*) of the decay.[6]

The Belle Collaboration recently measured the branching fraction for the factorization-suppressed decay $B \to \chi_0 K$ [15] at a level comparable with the factorization-favored decays $B \to \eta_c K$, $J/\psi K$, and $\chi_1 K$. It shows that the simple factorization clearly fails in the decay $B \to$ charmonium.

The decay $B \to \bar{D}^* D^*$ is being analyzed at the B factories. The branching fraction was reported for $D^{*+} D^{*-}$ [16]. After accumulation of more events, helicity analysis will become feasible. Comparison of this decay with $B \to J/\psi K^*$ may provide additional useful information about the dynamics in $b \to c\bar{c}q$.

V. HIGHER SPIN $(J \geq 2)$

A helicity test can be performed for higher-spin inclusive processes $B \to MX \to abX$ with $J \geq 2$ for M. For $J=0$ for a and b, the differential decay rate in the B rest frame takes the form

$$\frac{d\Gamma}{dq_0 d\cos\theta} \propto |\mathbf{q}| \sum_{\lambda=-J}^{J} H_\lambda(m_X^2) |d^J_{\lambda,0}(\theta)|^2, \tag{22}$$

where λ is the helicity of M. The momentum $\mathbf{q}$ and the angle θ are defined in the same way as in Eq. (6). In the case of $J \neq 0$ for a and/or b, an additional λ dependence enters through the decay $M \to a+b$. The dominant helicity structure function is H_0 and then $H_{\pm 1}$ for both B and $\bar{B}$ decays, since the l_z correction to M contributes to H_1 and H_{-1} in the same order. If perturbative QCD is valid, the function H_λ with $|\lambda| \geq 2$ cannot arise without the l_z correction. H_h with

[6]Following earlier experimental papers [11], the BaBar analysis [10] quotes only one solution, $\phi_\parallel - \phi_\perp \simeq \pi$, which would lead to $H_1 \ll H_{-1}$ in the ordinary sign convention chosen in Ref. [12]. It might look as if the BaBar result were in direct conflict with the prediction of perturbative QCD for K^*. In fact, the other solution $\phi_\parallel - \phi_\perp \simeq 0$ leading to $H_1 \gg H_{-1}$ is also allowed by this experiment, although not explicitly quoted as such [13]. Therefore no conclusion can be drawn from this experiment as to which is larger between H_1 and H_{-1} in $B \to J/\psi K^*$. The same comment applies to the latest Belle analysis [14].

$|h|\geqslant 2$ beyond the l_z correction will be clear evidence for LDFSI. As m_X tends to its maximum value, Γ_L should approach $1/(2J+1)$. In the decay $B\to f_2X\to\pi\pi X$, for instance, the angular dependence $|d^2_{\pm 20}|^2=\frac{3}{8}(1-\cos^2\theta)^2$ appears as f_2 slows down. The appearance of $(1-\cos^2\theta)^2$ indicates that the orbital angular momentum of $\bar{q}q$ inside f_2 becomes important in the B rest frame. One might think of attributing the appearance of $|h|\geqslant 2$ to possible breakdown of the $\bar{q}q$ description of f_2. But it is unlikely in the face of the static quark model: the $\bar{q}q$ description of low-lying mesons works well both in the infinite momentum limit and in the static limit, albeit the physical nature of quarks is different between the two limits. As $\mathbf{q}\to 0$, all l_z states of f_2 are equally produced and Γ_L should approach 1/5.

VI. COMPARISON WITH OTHER TESTS

Various tests have so far been proposed concerning the validity of the factorization. The most straightforward is to compute as many decay amplitudes as possible with theoretical resources at hand. In some simple cases we are fortunate to have only a single dominant decay process in the factorization limit. An example is $B^0\to D^-\pi^+$. Otherwise the decay amplitude for a given process is the sum of the competing contributions of more than one decay process. Once short-distance QCD corrections are included, the quark operators producing mesons are nonlocal. Then we need to know not only the decay constants, namely, the wave functions at the origin, but also the entire light-cone quark distribution functions in order to obtain a single decay amplitude. Furthermore, the relevant energy scale of the QCD coupling $\alpha_s(E)$ can take different values depending on how and where it appears. Therefore the final number for the total decay amplitude is sensitive to small theoretical uncertainties of each contribution particularly when different terms enter with different signs. These added uncertainties make the comparison of theory with experiment less decisive. For this reason we give up here attempting a numerical estimate of the coefficients of the corrections to the $h=0$ dominance rule even for the simplest two-body decay $B\to 1^-1^-$.

A while ago Ligeti *et al.* [17] proposed a test of factorization in the decay $B\to\overline{D}^{(*)}X$. They proposed to compare the m_X distribution of this inclusive decay with the $m_{l\nu}$ distribution of the semileptonic decay $B\to\overline{D}^{(*)}\overline{l}\nu$. It appears to be a clean test. In order for this test to work, however, X must be produced from a single weak current just as $\overline{l}\nu$ is. Therefore, it applies to $B^0\to D^{(*)-}X^+$ (and the conjugate) through the dominant decay operator, but not to $B^+\to\overline{D}^{(*)0}X^+$ (and the conjugate) since X^+ can pick either the current quark u or the spectator u in the B^+ decay. Only the neutral B decay is possibly related to the semileptonic decay. The most important difference from our test is that the comparison with the semileptonic decay tests only the validity of the factorization before the perturbative QCD improvement. The SDFSI surely plays a significant role in the final state and breaks down the similarity between the nonleptonic and semileptonic decays. An alternative to this test was proposed for two-body decays and the importance of spin was mentioned [18], but it is not free of uncertainties and complications in the theoretical computation. In contrast, the inclusive helicity measurement tests not just the lowest-order factorization but its perturbative QCD corrections to all orders independent of theoretical details. It will provide us with important information as to how much long-distance QCD interactions enter a given process and allow us to use it for related processes. A negative side of the helicity test is, of course, the common drawback of LDFSI that, after LDFSI is found, we cannot compute phases or magnitudes of decay amplitudes from first principles. However, just measuring CP violations beyond the B^0-$\overline{B}^0$ mixing effect will be important even if we cannot easily relate them to fundamental parameters of theory. Only when LDFSI is significant do we have a chance to detect direct CP violation from particle-antiparticle asymmetry. The helicity test will hopefully tell us which decay modes we should go after in search of direct CP violations.

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