Perturbative QCD analysis of $B \to \phi K^*$ decays

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We study the first observed charmless $B \to VV$ modes, the $B \to \phi K^*$ decays, in perturbative QCD formalism. The obtained branching ratios $B(B \to \phi K^*) \sim 15 \times 10^{-6}$ are larger than $\sim 9 \times 10^{-6}$ from QCD factorization. The comparison of the predicted magnitudes and phases of the different helicity amplitudes, and branching ratios with experimental data can test the power counting rules, the evaluation of annihilation contributions, and the mechanism of dynamical penguin enhancement in perturbative QCD, respectively.

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The branching ratios of the penguin-dominated $B \to K\pi$ decays, about 3–4 times larger than those of the tree-dominated $B \to \pi\pi$ decays, indicate that penguin contributions must be enhanced. This enhancement can be achieved either by large Wilson coefficients $C_{4,6}$ associated with the penguin operators in perturbative QCD (PQCD) [1–3], or by a large chiral symmetry breaking scale m_0 associated with the kaon in QCD factorization (QCDF) [4,5]. The latter mechanism, called chiral enhancement, corresponds to a characteristic scale of $O(m_b)$, at which we have $m_0(m_b) \sim 3$ GeV and the smaller Wilson coefficients $C_{4,6}(m_b)$. The former mechanism, called dynamical enhancement, corresponds to a characteristic scale of $O(\sqrt{\bar{\Lambda} m_b})$, $\bar{\Lambda} = M_B - m_b$ being the B meson and b quark mass difference, at which we have $m_0(\sqrt{\bar{\Lambda} m_b}) \sim 1.5$ GeV and the larger Wilson coefficients $C_{4,6}(\sqrt{\bar{\Lambda} m_b}) \sim 1.5 C_{4,6}(m_b)$. Recently, we have proposed the $B \to \phi K$ decays as the appropriate modes to clarify the above issue [6,7]. These modes are not chirally enhanced because ϕ is a vector meson, and they are insensitive to the variation of the unitarity angle ϕ_3 because they are pure penguin processes. If the data of the branching ratios $B(B \to \phi K)$ are settled down at values around 10×10^{-6} [7,8] instead of 4×10^{-6} [9,10], the dynamical enhancement of penguin contributions to charmless nonleptonic B meson decays will gain strong support.

Here we argue why the characteristic scale involved in two-body B meson decays must be of $O(\sqrt{\bar{\Lambda} M_B})$ in PQCD from two points of view. Consider a two-body nonleptonic decay, in which the two final-state light mesons move back-to-back with large momenta. The lowest-order diagram for its amplitude contains a hard gluon attaching the spectator quark. Intuitively, the spectator quark in the B meson, forming a soft cloud around the heavy b quark, carries momentum of order $\bar{\Lambda}$. The spectator quark on the light-meson side carries momentum of $O(M_B)$ in order to form the fast-moving light meson with the u quark produced in the b quark decay. Note that the end-point singularities from the small spectator momentum on the light-meson side do not exist in a self-consistent PQCD formalism, because of Sudakov suppression from k_T and threshold resummations [11,12]. Based on the above argument, the hard gluon is off-shell by order of $\bar{\Lambda} M_B$. This scale characterizes the corresponding quark-level hard amplitude, which involves the four-fermion decay vertex. Theoretically, the hard scale $\bar{\Lambda} M_B$ is essential for constructing a gauge invariant B meson wave function. This wave function, though being a nonlocal matrix element, is gauge invariant in the presence of the path-ordered Wilson line integral. A careful investigation [13,14] shows that the $O(\alpha_s^2)$ diagram with the second gluon attaching the hard gluon contributes to this line integral. That is, this diagram contains the soft divergence, which is factorized into the B meson wave function. This is possible only when the hard gluon is off shell by the intermediate scale $\bar{\Lambda} M_B$ rather than by M_B^2.

In this work we shall perform a PQCD analysis of the first observed charmless $B \to VV$ modes, the $B \to \phi K^*$ decays, which are similar to $B \to \phi K$, also appropriate for distinguishing the different penguin enhancing mechanism. Besides, the $B \to VV$ modes reveal dynamics of exclusive B meson decays more than the $B \to PP$ and VP modes through the measurement of the magnitudes and the phases of various helicity amplitudes. According to the power counting rules defined in [7], the longitudinal amplitude is leading, and the other two amplitudes are down by a power of M_ϕ / M_B or of M_{K^*}/M_B, M_ϕ and M_{K^*} being the ϕ and K^* meson masses, respectively. Since the $B \to \phi K^*$ decays are insensitive to the unitarity angle, the relative phases among the helicity amplitudes mainly arise from strong interaction. The annihilation contributions, which can be evaluated unambiguously in our

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approach, generate the strong phases. Therefore, comparing the predicted magnitudes and relative phases among the different helicity amplitudes, and the predicted branching ratios with experimental data, we test the power counting rules, the evaluation of annihilation contributions, and the mechanism of dynamical penguin enhancement in PQCD, respectively.

The idea of the PQCD factorization theorem for two-body nonleptonic B meson decays has been reviewed in [1,15,16], which is subject to corrections of $O(\alpha_s^2)$ and $O(\bar{\Lambda}/M_B)$. In this formalism decay amplitudes are expressed as the convolutions of the corresponding hard parts with universal meson distribution amplitudes [13,14], which are regarded as the nonperturbative inputs. Because of the Sudakov effects from k_T and threshold resummations, the end-point singularities do not exist as stated above. Therefore, PQCD involves inputs less than in QCDF, for which form factors, meson distribution amplitudes, and infrared cutoffs for regulating the end-point singularities are all independent parameters [4,5]. Strictly speaking, the infrared cutoffs, signifying important soft contributions to the nonfactorizable and annihilation amplitudes, imply that the factorization formulas in QCDF are not self-consistent.

We work in the frame with the B meson at rest, i.e., with the B meson momentum $P_1=(M_B/\sqrt{2})(1,1,\mathbf{0}_T)$ in the light-cone coordinates. Assume that the $\phi(K^*)$ meson moves in the plus (minus) z direction carrying the momentum $P_2(P_3)$ and the polarization vectors $\epsilon_2(\epsilon_3)$. The $B\to\phi K^*$ decay rates are written as

$$\Gamma=\frac{G_F^2 P_c}{16\pi M_B^2}\sum_{\sigma=L,T}\mathcal{M}^{(\sigma)\dagger}\mathcal{M}^{(\sigma)}, \tag{1}$$

where $P_c\equiv|P_{2z}|=|P_{3z}|$ is the momentum of either of the outgoing vector mesons, and the superscript σ denotes the helicity states of the two vector mesons with $L(T)$ standing for the longitudinal (transverse) component. The amplitude $\mathcal{M}^{(\sigma)}$ is decomposed into

$$\begin{aligned}\mathcal{M}^{(\sigma)}&=\epsilon_{2\mu}^*(\sigma)\epsilon_{3\nu}^*(\sigma)\left[a g^{\mu\nu}+\frac{b}{M_\phi M_{K^*}}P_1^\mu P_1^\nu\right.\\&\quad\left.+i\frac{c}{M_\phi M_{K^*}}\epsilon^{\mu\nu\alpha\beta}P_{2\alpha}P_{3\beta}\right],\\&\equiv M_B^2\mathcal{M}_L+M_B^2\mathcal{M}_N\epsilon_2^*(\sigma=T)\cdot\epsilon_3^*(\sigma=T)\\&\quad+i\mathcal{M}_T\epsilon^{\alpha\beta\gamma\rho}\epsilon_{2\alpha}^*(\sigma)\epsilon_{3\beta}^*(\sigma)P_{2\gamma}P_{3\rho},\end{aligned} \tag{2}$$

with the convention[1] $\epsilon^{0123}=1$ and the definitions

$$\begin{aligned}M_B^2\mathcal{M}_L&=a\,\epsilon_2^*(L)\cdot\epsilon_3^*(L)+\frac{b}{M_\phi M_{K^*}}\epsilon_2^*(L)\cdot P_1\epsilon_3^*(L)\cdot P_1,\\M_B^2\mathcal{M}_N&=a\,\epsilon_2^*(T)\cdot\epsilon_3^*(T),\\\mathcal{M}_T&=\frac{c}{M_\phi M_{K^*}}.\end{aligned} \tag{3}$$

We define the helicity amplitudes

$$\begin{aligned}A_0&=-\xi M_B^2\mathcal{M}_L,\\A_\parallel&=\xi\sqrt{2}M_B^2\mathcal{M}_N,\\A_\perp&=\xi M_\phi M_{K^*}\sqrt{2(r^2-1)}\mathcal{M}_T,\end{aligned} \tag{4}$$

with the normalization factor $\xi=\sqrt{G_F^2P_c/(16\pi M_B^2\Gamma)}$ and the ratio $r=P_2\cdot P_3/(M_\phi M_{K^*})$. These helicity amplitudes satisfy the relation

$$|A_0|^2+|A_\parallel|^2+|A_\perp|^2=1, \tag{5}$$

following the helicity summation in Eq. (1). We also introduce another equivalent set of helicity amplitudes,

$$\begin{aligned}H_0&=M_B^2\mathcal{M}_L,\\H_\pm&=M_B^2\mathcal{M}_N\mp M_\phi M_{K^*}\sqrt{r^2-1}\mathcal{M}_T,\end{aligned} \tag{6}$$

with the helicity summation,

$$\sum_\sigma \mathcal{M}^{(\sigma)\dagger}\mathcal{M}^{(\sigma)}=|H_0|^2+|H_+|^2+|H_-|^2. \tag{7}$$

The $B\to\phi K^*$ decays involve the emission and annihilation topologies, both of which are classified into factorizable diagrams, where hard gluons attach the valence quarks in the same meson, and nonfactorizable diagrams, where hard gluons attach the valence quarks in different mesons. The amplitudes are written as

$$\mathcal{M}_H=f_\phi V_t^*F_{He}^{(s)}+V_t^*\mathcal{M}_{He}^{(s)}+f_BV_t^*F_{Ha}^{(d)}+V_t^*\mathcal{M}_{Ha}^{(d)}, \tag{8}$$

$$\begin{aligned}\mathcal{M}_H&=f_\phi V_t^*F_{He}^{(s)}+V_t^*\mathcal{M}_{He}^{(s)}+f_BV_t^*F_{Ha}^{(u)}+V_t^*\mathcal{M}_{Ha}^{(u)}\\&\quad-f_BV_u^*F_{Ha}-V_u^*\mathcal{M}_{Ha},\end{aligned} \tag{9}$$

for the $B_d^0\to\phi K^{*0}$ and $B^+\to\phi K^{*+}$ modes, respectively, where the subscript $H=L,N,T$ denotes the different helicity amplitudes, $e(a)$ denotes the emission (annihilation) topology, and $V_q=V_{qs}^*V_{qb}$ are the products of the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements. The hard parts for the factorizable amplitudes F and for the nonfactorizable amplitudes $\mathcal{M}$ are derived by contracting the following structures to the lowest-order one-gluon-exchange diagrams:

$$\frac{1}{\sqrt{2N_c}}(\not{P}_1+M_B)\gamma_5\Phi(x,b), \tag{10}$$

$$\frac{1}{\sqrt{2N_c}}[M_\phi\not{\epsilon}_2(L)\Phi_\phi(x)+\not{\epsilon}_2(L)\not{P}_2\Phi_\phi^t(x)+M_\phi I\Phi_\phi^s(x)], \tag{11}$$

[1]This convention corresponds to $\mathrm{tr}(\gamma_5\not{a}\not{b}\not{c}\not{d})=-4i\epsilon^{\alpha\beta\gamma\rho}a_\alpha b_\beta c_\gamma d_\rho$.

$$\frac{1}{\sqrt{2N_c}}\left[M_\phi \not\epsilon_2(T)\Phi_\phi^v(x)+\not\epsilon_2(T)\not P_2\Phi_\phi^T(x)\right.$$
$$\left.+\frac{M_\phi}{P_2\cdot n_-}i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_2^\nu(T)P_2^\rho n_-^\sigma\Phi_\phi^a(x)\right], \tag{12}$$

$$\frac{1}{\sqrt{2N_c}}[M_{K^*}\not\epsilon_3(L)\Phi_{K^*}(x)+\not\epsilon_3(L)\not P_3\Phi_{K^*}^t(x)$$
$$+M_{K^*}I\Phi_{K^*}^s(x)], \tag{13}$$

$$\frac{1}{\sqrt{2N_c}}\left[M_{K^*}\not\epsilon_3(T)\Phi_{K^*}^v(x)+\not\epsilon_3(T)\not P_3\Phi_{K^*}^T(x)\right.$$
$$\left.+\frac{M_{K^*}}{P_3\cdot n_+}i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon_3^\nu(T)P_3^\rho n_+^\sigma\Phi_{K^*}^a(x)\right], \tag{14}$$

where $n_+=(1,0,\mathbf{0}_T)$ and $n_-=(0,1,\mathbf{0}_T)$ are dimensionless vectors on the light cone. Equations (11) and (12) are associated with the longitudinally and transversely polarized ϕ mesons, respectively. The structures associated with the K^* meson are similar as shown above.

To extract the contributions to the helicity amplitude $\mathcal{M}_L$, the following parametrization for the longitudinal polarization vectors is useful:

$$\epsilon_2(L)=\frac{P_2}{M_\phi}-\frac{M_\phi}{P_2\cdot n_-}n_-,$$
$$\epsilon_3(L)=\frac{P_3}{M_{K^*}}-\frac{M_{K^*}}{P_3\cdot n_+}n_+, \tag{15}$$

which satisfy the normalization $\epsilon_2^2(L)=\epsilon_3^2(L)=-1$ and the orthogonality $\epsilon_2(L)\cdot P_2=\epsilon_3(L)\cdot P_3=0$ for the on-shell conditions $P_2^2=M_\phi^2$ and $P_3^2=M_{K^*}^2$. We first keep the full dependence on the light meson masses M_ϕ and M_{K^*} in the momenta P_2 and P_3. After deriving the factorization formulas, which are well defined in the limit $M_\phi, M_{K^*}\to 0$, we drop the terms proportional to $r_\phi^2, r_{K^*}^2\sim 0.04$, with the ratios $r_\phi=M_\phi/M_B$ and $r_{K^*}=M_{K^*}/M_B$. Under this approximation, the expressions of the ϕ and K^* meson momenta are then as simple as

$$P_2=\frac{M_B}{\sqrt{2}}(1,0,\mathbf{0}_T),\qquad P_3=\frac{M_B}{\sqrt{2}}(0,1,\mathbf{0}_T). \tag{16}$$

For the extraction of the helicity amplitudes $\mathcal{M}_N$ and $\mathcal{M}_T$, Eq. (16) and the transverse polarization vectors,

$$\epsilon_2(T)=(0,0,\mathbf{1}_T),\qquad \epsilon_3(T)=(0,0,\mathbf{1}_T), \tag{17}$$

can be adopted directly. The explicit factorization formulas are collected in the Appendix.

The power counting rules in PQCD [7] tell that the factorizable amplitude F_{Le} (corresponding to the $B\to K^*$ transition form factor) is leading, and the other factorizable amplitudes are at least down by a power of r_ϕ or r_{K^*}. The nonfactorizable amplitudes $\mathcal{M}$ are suppressed by a power of $\bar\Lambda/M_B$. Hence, the formalism presented in this work is complete at $O(M_{\phi,K^*}/M_B)$, and subject to corrections of $O(\bar\Lambda/M_B)$. Equation (4) then implies that the helicity amplitude A_0 is leading in the heavy-quark limit, and $A_\parallel$ and $A_\perp$ are next to leading. The factorizable annihilation amplitudes F_{Ha}, being suppressed only by $M_{\phi,K^*}/M_B$ and almost imaginary, are the major source of the strong phases in PQCD. Since the $B\to\phi K^*$ decays are the pure penguin processes with a weak dependence on the unitarity angle ϕ_3, these strong phases determine the relative phases among the helicity amplitudes $A_0, A_\parallel$ and $A_\perp$.

For the B meson wave function, we employ the model [1],

$$\Phi_B(x,b)=N_Bx^2(1-x)^2\times\exp\left[-\frac{1}{2}\left(\frac{xM_B}{\omega_B}\right)^2-\frac{\omega_B^2b^2}{2}\right], \tag{18}$$

where the shape parameter $\omega_B=0.4$ GeV has been adopted in all our previous analyses of exclusive B meson decays. The normalization constant $N_B=91.784$ GeV is related to the decay constant $f_B=190$ MeV (in the convention $f_\pi=130$ MeV). It is known that there are two B meson wave functions Φ_B and $\bar\Phi_B$, which are related to the three-parton B meson wave functions through a set of equations of motion [17–20]. Because of the unknown three-parton wave functions, the equations of motion in fact do not impose any constraint on the functional form of Φ_B and $\bar\Phi_B$. Our simple choice of the model wave functions corresponds to Φ_B in Eq. (18) and $\bar\Phi_B=0$. This choice is legitimate, since the contribution from $\bar\Phi_B$ is suppressed by a power of $\bar\Lambda/M_B$ [11], and negligible within the accuracy of the current formalism.

The ϕ and K^* meson distribution amplitudes up to twist 3 are given by [21]

$$\Phi_\phi(x)=\frac{3f_\phi}{\sqrt{2N_c}}x(1-x), \tag{19}$$

$$\Phi_\phi^t(x)=\frac{f_\phi^T}{2\sqrt{2N_c}}\left\{3(1-2x)^2+1.68C_4^{1/2}(1-2x)\right.$$
$$\left.+0.69\left[1+(1-2x)\ln\frac{x}{1-x}\right]\right\}, \tag{20}$$

$$\Phi_\phi^s(x)=\frac{f_\phi^T}{4\sqrt{2N_c}}\left[3(1-2x)(4.5-11.2x+11.2x^2)\right.$$
$$\left.+1.38\ln\frac{x}{1-x}\right], \tag{21}$$

$$\Phi_\phi^T(x)=\frac{3f_\phi^T}{\sqrt{2N_c}}x(1-x)[1+0.2C_2^{3/2}(1-2x)], \tag{22}$$

$$\Phi_\phi^v(x)=\frac{f_\phi}{2\sqrt{2N_c}}\left\{\frac{3}{4}[1+(1-2x)^2]+0.24\times[3(1-2x)^2-1]+0.96C_4^{1/2}(1-2x)\right\}, \quad (23)$$

$$\Phi_\phi^a(x)=\frac{3f_\phi}{4\sqrt{2N_c}}(1-2x)[1+0.93(10x^2-10x+1)], \quad (24)$$

$$\Phi_{K^*}(x)=\frac{3f_{K^*}}{\sqrt{2N_c}}x(1-x)[1+0.57(1-2x)+0.07C_2^{3/2}(1-2x)], \quad (25)$$

$$\Phi_{K^*}^t(x)=\frac{f_{K^*}^T}{2\sqrt{2N_c}}\{0.3(1-2x)[3(1-2x)^2+10(1-2x)-1]+1.68C_4^{1/2}(1-2x)+0.06(1-2x)^2[5(1-2x)^2-3]+0.36\{1-2(1-2x)[1+\ln(1-x)]\}\}, \quad (26)$$

$$\Phi_{K^*}^s(x)=\frac{f_{K^*}^T}{2\sqrt{2N_c}}\{3(1-2x)[1+0.2(1-2x)+0.6(10x^2-10x+1)]-0.12x(1-x)+0.36[1-6x-2\ln(1-x)]\}, \quad (27)$$

$$\Phi_{K^*}^T(x)=\frac{3f_{K^*}^T}{\sqrt{2N_c}}x(1-x)[1+0.6(1-2x)+0.04C_2^{3/2}\times(1-2x)], \quad (28)$$

$$\Phi_{K^*}^v(x)=\frac{f_{K^*}}{2\sqrt{2N_c}}\left\{\frac{3}{4}[1+(1-2x)^2+0.44(1-2x)^3]+0.4C_2^{1/2}(1-2x)+0.88C_4^{1/2}(1-2x)+0.48[2x+\ln(1-x)]\right\}, \quad (29)$$

$$\Phi_{K^*}^a(x)=\frac{f_{K^*}}{4\sqrt{2N_c}}\{3(1-2x)[1+0.19(1-2x)+0.81(10x^2-10x+1)]-1.14x(1-x)+0.48[1-6x-2\ln(1-x)]\}, \quad (30)$$

with the Gegenbauer polynomials,

$$C_2^{1/2}(\xi)=\frac{1}{2}(3\xi^2-1),$$

$$C_4^{1/2}(\xi)=\frac{1}{8}(35\xi^4-30\xi^2+3),$$

$$C_2^{3/2}(\xi)=\frac{3}{2}(5\xi^2-1). \quad (31)$$

TABLE I. Helicity amplitudes and relative phases.

Mode	$BR(10^{-6})$	$\lvert A_0\rvert^2$	$\lvert A_\parallel\rvert^2$	$\lvert A_\perp\rvert^2$	$\phi_\parallel$(rad)	$\phi_\perp$(rad)
ϕK^{*0}	14.86	0.750	0.135	0.115	2.55	2.54
ϕK^{*+}	15.96	0.748	0.133	0.111	2.55	2.54

We employ $G_F=1.16639\times 10^{-5}$ GeV^{-2}, the Wolfenstein parameters $\lambda=0.2196$, $A=0.819$, and $R_b=0.38$, the unitarity angle $\phi_3=90°$, the masses $M_B=5.28$ GeV, $M_\phi=1.02$ GeV and $M_{K^*}=0.89$ GeV, the decay constants $f_\phi=237$ MeV, $f_\phi^T=220$ MeV, $f_{K^*}=200$ MeV, and $f_{K^*}^T=160$ MeV, and the $B_d^0(B^+)$ meson lifetime $\tau_{B^0}=1.55$ ps($\tau_{B^+}=1.65$ ps) [22]. We have confirmed that the above distribution amplitudes and decay constants lead to the $B\to K^*$ transition form factors [23] in agreement with those from light-cone QCD sum rules [24]. We have also confirmed that the averaged values of the running hard scales t defined by Eqs. (A20) and (A21) in the Appendix are indeed about $\sqrt{\bar\Lambda M_B}\sim 1.6$ GeV. Note that the $B\to\phi K^*$ branching ratios are insensitive to the variation of ϕ_3. The results for the helicity amplitudes A_0, $A_\parallel$ and $A_\perp$, including their relative phases $\phi_\parallel\equiv\mathrm{Arg}(A_\parallel/A_0)$ and $\phi_\perp\equiv\mathrm{Arg}(A_\perp/A_0)$, are displayed in Table I. The contributions to the $B\to\phi K^*$ branching ratios mainly arise from the longitudinal polarizations A_0 because of the relation $|A_0|^2\gg|A_\parallel|^2\sim|A_\perp|^2$, which is expected from the power counting rules. It is easy to observe that the ratios $|H_-/H_0|^2$ and $|H_+/H_0|^2$ obtained in PQCD are close to those in QCDF [25]. The annihilation contributions are the major source of the strong phases, and the nonfactorizable contributions are the minor one. The values of $\phi_\parallel$ and $\phi_\perp$ in the rows (I)–(III) of Table II indicate that the phases from the former are about 4–5 times those from the latter (but opposite in sign). Without these sources, we have $\phi_\parallel=\phi_\perp=\pi$. Note that the relative phases among the different helicity amplitudes cannot be predicted unambiguously

TABLE II. Helicity amplitudes and relative phases: (I) without annihilation and nonfactorizable contributions, (II) without annihilation contributions, and (III) without nonfactorizable contributions.

Mode	$BR(10^{-6})$	$\lvert A_0\rvert^2$	$\lvert A_\parallel\rvert^2$	$\lvert A_\perp\rvert^2$	$\phi_\parallel$(rad)	$\phi_\perp$(rad)
ϕK^{*0}(I)	14.48	0.923	0.040	0.035	π	π
(II)	13.25	0.860	0.072	0.063	3.30	3.33
(III)	16.80	0.833	0.089	0.078	2.37	2.34
ϕK^{*+}(I)	15.45	0.923	0.040	0.035	π	π
(II)	14.17	0.860	0.072	0.063	3.30	3.33
(III)	17.98	0.830	0.094	0.075	2.37	2.34

in QCDF due to the arbitrary complex cutoffs for the evaluation of the nonfactorizable and annihilation contributions.

We examine the theoretical uncertainty from the variation of the hard scales t, which are defined as the invariant masses of the internal particles and are required to be higher than the factorization scales $1/b$, b being the transverse extents of the mesons. This examination estimates higher-order corrections to the hard amplitudes, which are the most important theoretical uncertainty for penguin-dominated B meson decays. The light meson distribution amplitudes have been determined in QCD sum rules. The possible 30% variation of the coefficients of the Gegenbauer polynomials in these distribution amplitudes lead only to little changes of our predictions. We consider the hard scales t located between 0.75–1.25 times the invariant masses of the internal particles. The predictions for the $B\to\phi K$ branching ratios from the above range are consistent with the data with uncertainty [7]. We then obtain the $B\to\phi K^*$ branching ratios,

$$B(B_d^0\to\phi K^{*0})=(14.86^{+4.88}_{-3.36})\times 10^{-6},$$

$$B(B^\pm\to\phi K^{*\pm})=(15.96^{+5.24}_{-3.61})\times 10^{-6}. \tag{32}$$

The relative phases $\phi_\parallel$ and $\phi_\perp$, and the magnitudes $|A_0|^2$, $|A_\parallel|^2$ and $|A_\perp|^2$ of the helicity amplitudes are quite stable under the variation of the hard scales t. They change within 0.05 rad and within 0.01, respectively. There is another minor source of theoretical uncertainty from the light meson decay constants $f_\phi^{(T)}$ and $f_{K^*}^{(T)}$. If they reduce by 5%, the predicted branching ratios will decrease by 10%. The CP asymmetries of the $B\to\phi K^*$ modes are, as of $B\to\phi K$, vanishingly small (less than 2%).

The above branching ratios are larger than those from QCDF [25],

$$B(B_d^0\to\phi K^{*0})=8.71\times 10^{-6},$$

$$B(B^\pm\to\phi K^{*\pm})=9.30\times 10^{-6}, \tag{33}$$

due to the dynamical enhancement of penguin contributions. We emphasize that the annihilation amplitudes, though not negligible, are not responsible for the large branching ratios in PQCD, since they are mainly imaginary. This is understood by comparing the branching ratios in Table I and in row (II) of Table II. The nonfactorizable contributions are not shown either by the branching ratios in Table I or in the row (III) of Table II. However, the annihilation contributions, parametrized as being real, are important in QCDF in order to explain the large $B\to\phi K$ branching ratios. With the almost real annihilation contributions, the $B\to\phi K$ branching ratios obtained in QCDF can increase from 4×10^{-6} to 7×10^{-6} [9]. The values quoted in Eq. (33) do not include the annihilation contributions. The current experimental data of $B(B^0\to\phi K^{*0})$,

$$\text{CLEO [26]:}\quad (11.5^{+4.5+1.8}_{-3.7-1.7})\times 10^{-6},$$

$$\text{BELLE [27]:}\quad (15^{+8}_{-6}\pm 3)\times 10^{-6},$$

$$\text{BABAR [28]:}\quad (8.6^{+2.8}_{-2.4}\pm 1.1)\times 10^{-6}, \tag{34}$$

and those of $B(B^\pm\to\phi K^{*\pm})$,

$$\text{CLEO [26]:}\quad (10.6^{+6.4+1.8}_{-4.9-1.6})\times 10^{-6},$$

$$\text{BELLE [27]:}\quad <36\times 10^{-6},$$

$$\text{BABAR [28]:}\quad (9.7^{+4.2}_{-3.4}\pm 1.7)\times 10^{-6}, \tag{35}$$

are not yet precise enough to distinguish the two different approaches.

In this paper we have studied the first observed $B\to VV$ modes, the $B\to\phi K^*$ decays, using the PQCD formalism. It has been stressed that two-body heavy meson decays are characterized by a scale of $O(\bar{\Lambda}M_B)$ in PQCD, for which penguin contributions are dynamically enhanced. This enhancement makes penguin-dominated decay modes acquire branching ratios larger than those in QCDF, even when the final-state particles are vector mesons. We have proposed the $B\to\phi K^{(*)}$ decays as the ideal modes to test the significance of this mechanism. If their branching ratios are as large as $10\times 10^{-6}(15\times 10^{-6})$ (independent of the unitarity angle ϕ_3), dynamical enhancement will be convincing. We have also emphasized that the relative importance and the relative strong phases among the different helicity amplitudes in the $B\to VV$ modes can be predicted unambiguously in PQCD, which are determined by the power counting rules and by the annihilation contributions, respectively. These predictions are insensitive to the variation of the hard scales. Therefore, the comparison of the results presented here with future experimental data will provide a stringent confrontation of the PQCD approach.

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APPENDIX: FACTORIZATION FORMULAS

In this appendix we present the explicit expressions of the factorizable and nonfactorizable amplitudes in Eq. (9). The effective Hamiltonian for the flavor-changing $b\to s$ transition is given by

$$H_{\rm eff}=\frac{G_F}{\sqrt{2}}\sum_{q=u,c}V_q\left[C_1(\mu)O_1^{(q)}(\mu)+C_2(\mu)O_2^{(q)}(\mu)+\sum_{i=3}^{10}C_i(\mu)O_i(\mu)\right], \tag{A1}$$

with the CKM matrix elements $V_q=V_{qs}^*V_{qb}$ and the operators

$$O_1^{(q)}=(\bar{s}_iq_j)_{V-A}(\bar{q}_jb_i)_{V-A},$$
$$O_2^{(q)}=(\bar{s}_iq_i)_{V-A}(\bar{q}_jb_j)_{V-A},$$
$$O_3=(\bar{s}_ib_i)_{V-A}\sum_q(\bar{q}_jq_j)_{V-A},$$
$$O_4=(\bar{s}_ib_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V-A},$$
$$O_5=(\bar{s}_ib_i)_{V-A}\sum_q(\bar{q}_jq_j)_{V+A},$$
$$O_6=(\bar{s}_ib_j)_{V-A}\sum_q(\bar{q}_jq_i)_{V+A},$$
$$O_7=\frac{3}{2}(\bar{s}_ib_i)_{V-A}\sum_q e_q(\bar{q}_jq_j)_{V+A},$$
$$O_8=\frac{3}{2}(\bar{s}_ib_j)_{V-A}\sum_q e_q(\bar{q}_jq_i)_{V+A},$$
$$O_9=\frac{3}{2}(\bar{s}_ib_i)_{V-A}\sum_q e_q(\bar{q}_jq_j)_{V-A},$$
$$O_{10}=\frac{3}{2}(\bar{s}_ib_j)_{V-A}\sum_q e_q(\bar{q}_jq_i)_{V-A}, \tag{A2}$$

i and j being the color indices. Using the unitarity condition, the CKM matrix elements for the penguin operators O_3–O_{10} can also be expressed as $V_u+V_c=-V_t$. The unitarity angle ϕ_3 is defined via

$$V_{ub}=|V_{ub}|\exp(-i\phi_3). \tag{A3}$$

Here we adopt the Wolfenstein parametrization for the CKM matrix up to $O(\lambda^3)$,

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}=\begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{A4}$$

with the parameters [29],

$$\lambda=0.2196\pm0.0023,$$
$$A=0.819\pm0.035,$$
$$R_b\equiv\sqrt{\rho^2+\eta^2}=0.41\pm0.07. \tag{A5}$$

The factorizable amplitudes $F_{He}^{(q)}$ and $F_{Ha}^{(q)}=F_{Ha4}^{(q)}+F_{Ha6}^{(q)}$ are written as

$$F_{Le}^{(q)}=8\pi C_FM_B^2\int_0^1dx_1dx_3\int_0^\infty b_1db_1b_3db_3\Phi_B(x_1,b_1)\{[(1+x_3)\Phi_{K*}(x_3)+r_{K*}(1-2x_3)(\Phi_{K*}^t(x_3)+\Phi_{K*}^s(x_3))]E_e^{(q)}(t_e^{(1)})h_e(x_1,x_3,b_1,b_3)+2r_{K*}\Phi_{K*}^s(x_3)E_e^{(q)}(t_e^{(2)})h_e(x_3,x_1,b_3,b_1)\}, \tag{A6}$$

$$F_{Ne}^{(q)}=8\pi C_FM_B^2\int_0^1dx_1dx_3\int_0^\infty b_1db_1b_3db_3\Phi_B(x_1,b_1)r_\phi\{[\Phi_{K*}^T(x_3)+2r_{K*}\Phi_{K*}^v(x_3)+r_{K*}x_3(\Phi_{K*}^v(x_3)-\Phi_{K*}^a(x_3))]E_e^{(q)}(t_e^{(1)})h_e(x_1,x_3,b_1,b_3)+r_{K*}[\Phi_{K*}^v(x_3)+\Phi_{K*}^a(x_3)]E_e^{(q)}(t_e^{(2)})h_e(x_3,x_1,b_3,b_1)\}, \tag{A7}$$

$$F_{Te}^{(q)}=16\pi C_FM_B^2\int_0^1dx_1dx_3\int_0^\infty b_1db_1b_3db_3\Phi_B(x_1,b_1)r_\phi\{[\Phi_{K*}^T(x_3)+2r_{K*}\Phi_{K*}^a(x_3)-r_{K*}x_3(\Phi_{K*}^v(x_3)-\Phi_{K*}^a(x_3))]E_e^{(q)}(t_e^{(1)})h_e(x_1,x_3,b_1,b_3)+r_{K*}[\Phi_{K*}^v(x_3)+\Phi_{K*}^a(x_3)]E_e^{(q)}(t_e^{(2)})h_e(x_3,x_1,b_3,b_1)\}, \tag{A8}$$

$$F_{La4}^{(q)}=8\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{[-(1-x_3)\Phi_\phi(x_2)\Phi_{K*}(x_3)+2r_\phi r_{K*}\Phi_\phi^s(x_2)(x_3\Phi_{K*}^t(x_3)+(2-x_3)\Phi_{K*}^s(x_3))]E_{a4}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)+[x_2\Phi_\phi(x_2)\Phi_{K*}(x_3)+2r_\phi r_{K*}\Phi_{K*}^s(x_3)((1-x_2)\times\Phi_\phi^t(x_2)-(1+x_2)\Phi_\phi^s(x_2))]E_{a4}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\},\tag{A9}$$

$$F_{Na4}^{(q)}=-8\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3r_\phi r_{K*}\{[(2-x_3)(\Phi_\phi^v(x_2)\Phi_{K*}^v(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^a(x_3))+x_3(\Phi_\phi^v(x_2)\Phi_{K*}^a(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^v(x_3))]E_{a4}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)-[(1+x_2)(\Phi_\phi^v(x_2)\Phi_{K*}^v(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^a(x_3))-(1-x_2)(\Phi_\phi^v(x_2)\Phi_{K*}^a(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^v(x_3))]E_{a4}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\},\tag{A10}$$

$$F_{Ta4}^{(q)}=-16\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3r_\phi r_{K*}\{[x_3(\Phi_\phi^v(x_2)\Phi_{K*}^v(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^a(x_3))+(2-x_3)\times(\Phi_\phi^v(x_2)\Phi_{K*}^a(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^v(x_3))]E_{a4}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)+[(1-x_2)(\Phi_\phi^v(x_2)\Phi_{K*}^v(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^a(x_3))-(1+x_2)(\Phi_\phi^v(x_2)\Phi_{K*}^a(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^v(x_3))]E_{a4}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\},\tag{A11}$$

$$F_{La6}^{(q)}=16\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{[r_{K*}(1-x_3)\Phi_\phi(x_2)(\Phi_{K*}^t(x_3)+\Phi_{K*}^s(x_3))-2r_\phi\Phi_\phi^s(x_2)\Phi_{K*}(x_3)]\times E_{a6}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)+[r_\phi x_2(\Phi_\phi^t(x_2)-\Phi_\phi^s(x_2))\Phi_{K*}(x_3)+2r_{K*}\Phi_\phi(x_2)\Phi_{K*}^s(x_3)]E_{a6}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\},\tag{A12}$$

$$F_{Na6}^{(q)}=16\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{r_\phi(\Phi_\phi^v(x_2)+\Phi_\phi^a(x_2))\Phi_{K*}^T(x_3)E_{a6}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)+r_{K*}\Phi_\phi^T(x_2)(\Phi_{K*}^v(x_3)-\Phi_\phi^a(x_3))E_{a6}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\},\tag{A13}$$

$$F_{Ta6}^{(q)}=32\pi C_F M_B^2\int_0^1 dx_2dx_3\int_0^\infty b_2db_2b_3db_3\{r_\phi(\Phi_\phi^v(x_2)+\Phi_\phi^a(x_2))\Phi_{K*}^T(x_3)E_{a6}^{(q)}(t_a^{(1)})h_a(x_2,1-x_3,b_2,b_3)+r_{K*}\Phi_\phi^T(x_2)(\Phi_{K*}^v(x_3)-\Phi_{K*}^a(x_3))E_{a6}^{(q)}(t_a^{(2)})h_a(1-x_3,x_2,b_3,b_2)\}.\tag{A14}$$

The expression of the factorizable amplitudes F_{Ha} from the tree operators O_1 and O_2 are the same as $F_{Ha4}^{(q)}$ but with the evolution factor $E_{a4}^{(q)}$ replaced by $E_{a1}^{(q)}$.

The factors $E(t)$ contain the evolution from the W boson mass to the hard scales t in the Wilson coefficients $a(t)$, and from t to the factorization scale $1/b$ in the Sudakov factors $S(t)$:

$$E_e^{(q)}(t)=\alpha_s(t)a_e^{(q)}(t)S_B(t)S_{K*}(t),$$

$$E_{ai}^{(q)}(t)=\alpha_s(t)a_i^{(q)}(t)S_\phi(t)S_{K*}(t).\tag{A15}$$

The Wilson coefficients a in the above formulas are given by

$$a_1^{(q)}=C_2+\frac{C_1}{N_c},$$

$$a_3^{(q)}=\left(C_3+\frac{C_4}{N_c}\right)+\frac{3}{2}e_q\left(C_9+\frac{C_{10}}{N_c}\right),$$

$$a_4^{(q)}=\left(C_4+\frac{C_3}{N_c}\right)+\frac{3}{2}e_q\left(C_{10}+\frac{C_9}{N_c}\right),$$

$$a_5^{(q)}=\left(C_5+\frac{C_6}{N_c}\right)+\frac{3}{2}e_q\left(C_7+\frac{C_8}{N_c}\right),$$

$$a_6^{(q)}=\left(C_6+\frac{C_5}{N_c}\right)+\frac{3}{2}e_q\left(C_8+\frac{C_7}{N_c}\right),$$

$$a_e^{(q)}=a_3^{(q)}+a_4^{(q)}+a_5^{(q)}.$$

k_T resummation of large logarithmic corrections to the B, ϕ and K^* meson distribution amplitudes lead to the exponentials S_B, S_ϕ and S_{K*}, respectively,

$$S_B(t)=\exp\left[-s(x_1P_1^+,b_1)-2\int_{1/b_1}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}^2))\right],$$

$$S_\phi(t)=\exp\left[-s(x_2P_2^+,b_2)-s((1-x_2)P_2^+,b_2)\right.$$
$$\left.-2\int_{1/b_2}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}^2))\right],$$

$$S_{K^*}(t)=\exp\left[-s(x_3P_3^-,b_3)-s((1-x_3)P_3^-,b_3)\right.$$
$$\left.-2\int_{1/b_3}^{t}\frac{d\bar{\mu}}{\bar{\mu}}\gamma(\alpha_s(\bar{\mu}^2))\right], \tag{A16}$$

with the quark anomalous dimension $\gamma=-\alpha_s/\pi$. The variables b_1, b_2, and b_3 conjugate to the parton transverse momenta k_{1T}, k_{2T}, and k_{3T}, and represent the transverse extents of the B, ϕ, and K^* mesons, respectively. The expression for the exponent s is referred to in [30,31]. The above Sudakov exponentials decrease fast in the large b region [11,12], such that the $B\to\phi K^*$ hard amplitudes remain sufficiently perturbative in the end-point region.

The hard functions h's are

$$h_e(x_1,x_3,b_1,b_3)$$
$$=K_0(\sqrt{x_1x_3}M_Bb_1)S_t(x_3)[\theta(b_1-b_3)K_0$$
$$\times(\sqrt{x_3}M_Bb_1)I_0(\sqrt{x_3}M_Bb_3)+\theta(b_3-b_1)$$
$$\times K_0(\sqrt{x_3}M_Bb_3)I_0(\sqrt{x_3}M_Bb_1)], \tag{A17}$$

$$h_a(x_2,x_3,b_2,b_3)$$
$$=\left(\frac{i\pi}{2}\right)^2H_0^{(1)}(\sqrt{x_2x_3}M_Bb_2)S_t(x_3)[\theta(b_2-b_3)$$
$$\times H_0^{(1)}(\sqrt{x_3}M_Bb_2)J_0(\sqrt{x_3}M_Bb_3)+\theta(b_3-b_2)$$
$$\times H_0^{(1)}(\sqrt{x_3}M_Bb_3)J_0(\sqrt{x_3}M_Bb_2)]. \tag{A18}$$

We have proposed the parametrization for the evolution function $S_t(x)$ from threshold resummation [11,32],

$$S_t(x)=\frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c, \tag{A19}$$

where the parameter c is chosen as $c=0.4$ for the $B\to\phi K^*$ decays. This factor modifies the end-point behavior of the meson distribution amplitudes, making them vanish faster at $x\to 0$. Threshold resummation for nonfactorizable diagrams is weaker and negligible. K_0, I_0, H_0 and J_0 are the Bessel functions.

The hard scales t are chosen as the maxima of the virtualities of the internal particles involved in the hard amplitudes, including $1/b_i$:

$$t_e^{(1)}=\max(\sqrt{x_3}M_B,1/b_1,1/b_3),$$
$$t_e^{(2)}=\max(\sqrt{x_1}M_B,1/b_1,1/b_3), \tag{A20}$$
$$t_a^{(1)}=\max(\sqrt{1-x_3}M_B,1/b_2,1/b_3),$$
$$t_a^{(2)}=\max(\sqrt{x_2}M_B,1/b_2,1/b_3). \tag{A21}$$

When the PQCD formalism is extended to $O(\alpha_s^2)$, the hard scales can be determined more precisely and the scale independence of our predictions will be improved. Before this calculation is carried out, we consider the variation of, for example, t_e in the following range:

$$\max(0.75\sqrt{x_3}M_B,1/b_1,1/b_3)$$
$$<t_e^{(1)}<\max(1.25\sqrt{x_3}M_B,1/b_1,1/b_3),$$
$$\max(0.75\sqrt{x_1}M_B,1/b_1,1/b_3)<$$
$$t_e^{(2)}<\max(1.25\sqrt{x_1}M_B,1/b_1,1/b_3), \tag{A22}$$

in order to estimate the $O(\alpha_s^2)$ corrections. The range for t_a is chosen in a similar way.

The nonfactorizable amplitudes $\mathcal{M}_{He}^{(q)}=\mathcal{M}_{He3}^{(q)}+\mathcal{M}_{He4}^{(q)}+\mathcal{M}_{He5}^{(q)}+\mathcal{M}_{He6}^{(q)}$ and $\mathcal{M}_{Ha}^{(q)}=\mathcal{M}_{Ha3}^{(q)}+\mathcal{M}_{Ha5}^{(q)}$, depending on kinematic variables of all the three mesons [33], are written as

$$\mathcal{M}_{Le3}^{(q)}=16\pi C_FM_B^2\sqrt{2N_c}\int_0^1d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)\{\Phi_\phi(x_2)[-(x_2+x_3)\Phi_{K^*}(x_3)+r_{K^*}x_3(\Phi_{K^*}^t(x_3)$$
$$+\Phi_{K^*}^s(x_3))]E_{e3}^{(q)\prime}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+\Phi_\phi(x_2)[(1-x_2)\Phi_{K^*}(x_3)+r_{K^*}x_3(\Phi_{K^*}^t(x_3)$$
$$-\Phi_{K^*}^s(x_3))]E_{e3}^{(q)\prime}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A23}$$

$$\mathcal{M}_{Ne3}^{(q)}=16\pi C_FM_B^2\sqrt{2N_c}\int_0^1d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_\phi\{[x_2(\Phi_\phi^v(x_2)+\Phi_\phi^a(x_2))\Phi_{K^*}^T(x_3)-2r_{K^*}(x_2+x_3)$$
$$\times(\Phi_\phi^v(x_2)\Phi_{K^*}^v(x_3)+\Phi_\phi^a(x_2)\Phi_{K^*}^a(x_3))]E_{e3}^{(q)\prime}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+(1-x_2)(\Phi_\phi^v(x_2)$$
$$+\Phi_\phi^a(x_2))\Phi_{K^*}^T(x_3)E_{e3}^{(q)\prime}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A24}$$

$$\mathcal{M}^{(q)}_{Te3}=32\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_\phi\{[x_2(\Phi^v_\phi(x_2)+\Phi^a_\phi(x_2))\Phi^T_{K^*}(x_3)-2r_{K^*}(x_2+x_3)\times(\Phi^v_\phi(x_2)\Phi^a_{K^*}(x_3)+\Phi^a_\phi(x_2)\Phi^v_{K^*}(x_3))]E^{(q)\prime}_{e3}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+(1-x_2)(\Phi^v_\phi(x_2)+\Phi^a_\phi(x_2))\Phi^T_{K^*}(x_3)E^{(q)\prime}_{e3}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A25}$$

$$\mathcal{M}^{(q)}_{Le5}=16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_\phi\{[-x_2(\Phi^t_\phi(x_2)-\Phi^s_\phi(x_2))\Phi_{K^*}(x_3)+r_{K^*}x_2(\Phi^t_\phi(x_2)-\Phi^s_\phi(x_2))(\Phi^t_{K^*}(x_3)-\Phi^s_{K^*}(x_3))+r_{K^*}x_3(\Phi^t_\phi(x_2)+\Phi^s_\phi(x_2))(\Phi^t_{K^*}(x_3)+\Phi^s_{K^*}(x_3))]\times E^{(q)\prime}_{e5}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+[-(1-x_2)(\Phi^t_\phi(x_2)+\Phi^s_\phi(x_2))\Phi_{K^*}(x_3)+r_{K^*}(1-x_2)(\Phi^t_\phi(x_2)+\Phi^s_\phi(x_2))(\Phi^t_{K^*}(x_3)-\Phi^s_{K^*}(x_3))+r_{K^*}x_3(\Phi^t_\phi(x_2)-\Phi^s_\phi(x_2))(\Phi^t_{K^*}(x_3)+\Phi^s_{K^*}(x_3))]E^{(q)\prime}_{e5}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A26}$$

$$\mathcal{M}^{(q)}_{Ne5}=-16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_{K^*}x_3\Phi^T_\phi(x_2)(\Phi^v_{K^*}(x_3)-\Phi^a_{K^*}(x_3))\times\{E^{(q)\prime}_{e5}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+E^{(q)\prime}_{e5}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A27}$$

$$\mathcal{M}^{(q)}_{Te5}=2\mathcal{M}^{(q)}_{Ne5}, \tag{A28}$$

$$\mathcal{M}^{(q)}_{Le6}=-16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)\Phi_\phi(x_2)\{[x_2\Phi_{K^*}(x_3)+r_{K^*}x_3(\Phi^t_{K^*}(x_3)-\Phi^s_{K^*}(x_3))]E^{(q)\prime}_{e5}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+[-(1-x_2+x_3)\Phi_{K^*}(x_3)+r_{K^*}x_3(\Phi^t_{K^*}(x_3)+\Phi^s_{K^*}(x_3))]E^{(q)\prime}_{e5}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A29}$$

$$\mathcal{M}^{(q)}_{Ne6}=-16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_\phi\{x_2(\Phi^v_\phi(x_2)-\Phi^a_\phi(x_2))\Phi^T_{K^*}(x_3)E^{(q)\prime}_{e5}\times(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+[(1-x_2)(\Phi^v_\phi(x_2)-\Phi^a_\phi(x_2))\Phi^T_{K^*}(x_3)-2r_{K^*}(1-x_2+x_3)(\Phi^v_\phi(x_2)\Phi^v_{K^*}(x_3)-\Phi^a_\phi(x_2)\Phi^a_{K^*}(x_3))]E^{(q)\prime}_{e5}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A30}$$

$$\mathcal{M}^{(q)}_{Te6}=-32\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_\phi\{x_2(\Phi^v_\phi(x_2)-\Phi^a_\phi(x_2))\Phi^T_{K^*}(x_3)\times E^{(q)\prime}_{e5}(t_d^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+[(1-x_2)(\Phi^v_\phi(x_2)-\Phi^a_\phi(x_2))\Phi^T_{K^*}(x_3)-2r_{K^*}(1-x_2+x_3)\times(\Phi^v_\phi(x_2)\Phi^a_{K^*}(x_3)-\Phi^a_\phi(x_2)\Phi^v_{K^*}(x_3))]E^{(q)\prime}_{e5}(t_d^{(2)})h_d^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A31}$$

$$\mathcal{M}^{(q)}_{La3}=16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)[\{(1-x_3)\Phi_\phi(x_2)\Phi_{K^*}(x_3)+r_\phi r_{K^*}[(1+x_2-x_3)\times(\Phi^t_\phi(x_2)\Phi^t_{K^*}(x_3)-\Phi^s_\phi(x_2)\Phi^s_{K^*}(x_3))-(1-x_2-x_3)(\Phi^t_\phi(x_2)\Phi^s_{K^*}(x_3)-\Phi^s_\phi(x_2)\Phi^t_{K^*}(x_3))]\}\times E^{(q)\prime}_{a3}(t_f^{(1)})h_f^{(1)}(x_1,x_2,x_3,b_1,b_2)-[x_2\Phi_\phi(x_2)\Phi_{K^*}(x_3)-2r_\phi r_{K^*}(\Phi^t_\phi(x_2)\Phi^t_{K^*}(x_3)+\Phi^s_\phi(x_2)\Phi^s_{K^*}(x_3))+r_\phi r_{K^*}(1+x_2-x_3)(\Phi^t_\phi(x_2)\Phi^t_{K^*}(x_3)-\Phi^s_\phi(x_2)\Phi^s_{K^*}(x_3))+r_\phi r_{K^*}(1-x_2-x_3)(\Phi^t_\phi(x_2)\Phi^s_{K^*}(x_3)-\Phi^s_\phi(x_2)\Phi^t_{K^*}(x_3))\}E^{(q)\prime}_{a3}(t_d^{(2)})h_f^{(2)}(x_1,x_2,x_3,b_1,b_2)] \tag{A32}$$

$$\mathcal{M}_{Na3}^{(q)}=-32\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_\phi r_{K*}[\Phi_\phi^v(x_2)\Phi_{K*}^v(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^a(x_3)]$$
$$\times E_{a3}^{(q)\prime}(t_f^{(2)})h_f^{(2)}(x_1,x_2,x_3,b_1,b_2), \tag{A33}$$

$$\mathcal{M}_{Ta3}^{(q)}=-64\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)r_\phi r_{K*}[\Phi_\phi^v(x_2)\Phi_{K*}^a(x_3)+\Phi_\phi^a(x_2)\Phi_{K*}^v(x_3)]$$
$$\times E_{a3}^{(q)\prime}(t_f^{(2)})h_f^{(2)}(x_1,x_2,x_3,b_1,b_2), \tag{A34}$$

$$\mathcal{M}_{La5}^{(q)}=16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)\{[r_{K*}(1-x_3)\Phi_\phi(x_2)(\Phi_{K*}^t(x_3)-\Phi_{K*}^s(x_3))$$
$$-r_\phi x_2(\Phi_\phi^t(x_2)+\Phi_\phi^s(x_2))\Phi_{K*}(x_3)]E_{a5}^{(q)\prime}(t_f^{(1)})h_f^{(1)}(x_1,x_2,x_3,b_1,b_2)+[-r_\phi(2-x_2)(\Phi_\phi^t(x_2)$$
$$+\Phi_\phi^s(x_2))\Phi_{K*}(x_3)+r_{K*}(1+x_3)\Phi_\phi(x_2)(\Phi_{K*}^t(x_3)-\Phi_{K*}^s(x_3))]E_{a5}^{(q)\prime}(t_f^{(2)})h_f^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A35}$$

$$\mathcal{M}_{Na5}^{(q)}=16\pi C_F M_B^2\sqrt{2N_c}\int_0^1 d[x]\int_0^\infty b_1db_1b_2db_2\Phi_B(x_1,b_1)\{[r_\phi x_2(\Phi_\phi^v(x_2)+\Phi_\phi^a(x_2))\Phi_{K*}^T(x_3)-r_{K*}$$
$$\times(1-x_3)\Phi_\phi^T(x_2)(\Phi_{K*}^v(x_3)-\Phi_{K*}^a(x_3))]E_{a5}^{(q)\prime}(t_f^{(1)})h_d^{(1)}(x_1,x_2,x_3,b_1,b_2)+[r_\phi(2-x_2)(\Phi_\phi^v(x_2)$$
$$+\Phi_\phi^a(x_2))\Phi_{K*}^T(x_3)-r_{K*}(1+x_3)\Phi_\phi^T(x_2)(\Phi_{K*}^v(x_3)-\Phi_{K*}^a(x_3))]E_{a5}^{(q)\prime}(t_f^{(2)})h_f^{(2)}(x_1,x_2,x_3,b_1,b_2)\}, \tag{A36}$$

$$\mathcal{M}_{Ta5}^{(q)}=2\mathcal{M}_{Na5}^{(q)}. \tag{A37}$$

The expressions of the nonfactorizable amplitudes $\mathcal{M}_{Ha}$ and $\mathcal{M}_{He4}$ are the same as $\mathcal{M}_{Ha3}^{(q)}$ and $\mathcal{M}_{He3}^{(q)}$ but with the evolution factors $E_{a3}^{(q)\prime}$ and $E_{e3}^{(q)\prime}$ replaced by $E_{a1}^{(q)\prime}$ and $E_{e4}^{(q)\prime}$, respectively.

The evolution factors are given by

$$E_{ei}^{(q)\prime}(t)=\alpha_s(t)a_i^{(q)\prime}(t)S(t)|_{b_3=b_1},$$
$$E_{ai}^{(q)\prime}(t)=\alpha_s(t)a_i^{(q)\prime}(t)S(t)|_{b_3=b_2}, \tag{A38}$$

with the Sudakov factor $S=S_BS_\phi S_{K*}$. The Wilson coefficients a appearing in the above formulas are

$$a_1'=\frac{C_1}{N_c},$$
$$a_3^{(q)\prime}=\frac{1}{N_c}\left(C_3+\frac{3}{2}e_qC_9\right),$$
$$a_4^{(q)\prime}=\frac{1}{N_c}\left(C_4+\frac{3}{2}e_qC_{10}\right),$$
$$a_5^{(q)\prime}=\frac{1}{N_c}\left(C_5+\frac{3}{2}e_qC_7\right),$$
$$a_6^{(q)\prime}=\frac{1}{N_c}\left(C_6+\frac{3}{2}e_qC_8\right).$$

The hard functions $h^{(j)}, j=1$ and 2, are written as

$$h_d^{(j)}=[\theta(b_1-b_2)K_0(DM_Bb_1)I_0(DM_Bb_2)$$
$$+\theta(b_2-b_1)K_0(DM_Bb_2)I_0(DM_Bb_1)]$$
$$\times K_0(D_jM_Bb_2)\qquad \text{for}\ D_j^2\geqslant 0,$$
$$\times\frac{i\pi}{2}H_0^{(1)}(\sqrt{|D_j^2|}M_Bb_2)\quad \text{for}\ D_j^2\leqslant 0, \tag{A39}$$

$$h_f^{(j)}=\frac{i\pi}{2}[\theta(b_1-b_2)H_0^{(1)}(FM_Bb_1)J_0(FM_Bb_2)$$
$$+\theta(b_2-b_1)H_0^{(1)}(FM_Bb_2)J_0(FM_Bb_1)]$$
$$\times K_0(F_jM_Bb_1)\qquad \text{for}\ F_j^2\geqslant 0,$$
$$\times\frac{i\pi}{2}H_0^{(1)}(\sqrt{|F_j^2|}M_Bb_1)\qquad \text{for}\ F_j^2\leqslant 0, \tag{A40}$$

with the variables

$$D^2=x_1x_3,$$
$$D_1^2=(x_1-x_2)x_3,$$
$$D_2^2=-(1-x_1-x_2)x_3, \tag{A41}$$
$$F^2=x_2(1-x_3),$$

$$F_1^2=(x_1-x_2)(1-x_3),$$

$$F_2^2=x_1+x_2+(1-x_1-x_2)(1-x_3). \tag{A42}$$

The hard scales $t^{(j)}$ are chosen as

$$t_d^{(1)}=\max(DM_B,\sqrt{|D_1^2|}M_B,1/b_1,1/b_2),$$

$$t_d^{(2)}=\max(DM_B,\sqrt{|D_2^2|}M_B,1/b_1,1/b_2),$$

$$t_f^{(1)}=\max(FM_B,\sqrt{|F_1^2|}M_B,1/b_1,1/b_2),$$

$$t_f^{(2)}=\max(FM_B,\sqrt{|F_2^2|}M_B,1/b_1,1/b_2). \tag{A43}$$

[1] Y.Y. Keum, H.-n. Li, and A.I. Sanda, Phys. Lett. B **504**, 6 (2001); Phys. Rev. D **63**, 054008 (2001).
[2] Y.Y. Keum and H.-n. Li, Phys. Rev. D **63**, 074006 (2001).
[3] C.D. Lü, K. Ukai, and M.Z. Yang, Phys. Rev. D **63**, 074009 (2001).
[4] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999); Nucl. Phys. **B591**, 313 (2000).
[5] M. Beneke, G. Buchalla, M. Neubert, and C.T. Sachrajda, Nucl. Phys. **B606**, 245 (2001).
[6] H.-n. Li, talk presented at the 4th International Workshop on B Physics and CP violation, Ise-Shima, Japan, 2001, hep-ph/0103305; Y. Y. Keum, H.-n. Li, and A. I. Sanda, talk presented at the 9th international Symposium on Heavy Flavor Physics, Caltech, Pasadena, 2001, hep-ph/0201103.
[7] C.H. Chen, Y.Y. Keum, and H.-n. Li, Phys. Rev. D **64**, 112002 (2001).
[8] S. Mishima, Phys. Lett. B **521**, 252 (2001).
[9] H.Y. Cheng and K.C. Yang, Phys. Rev. D **64**, 074004 (2001).
[10] X.G. He, J.P. Ma, and C.Y. Wu, Phys. Rev. D **63**, 094004 (2001).
[11] T. Kurimoto, H.-n. Li, and A.I. Sanda, Phys. Rev. D **65**, 014007 (2002).
[12] Z.T. Wei and M.Z. Yang, hep-ph/0202018.
[13] H.-n. Li, Phys. Rev. D **64**, 014019 (2001).
[14] M. Nagashima and H.-n. Li, hep-ph/0202127.
[15] C.H. Chang and H.-n. Li, Phys. Rev. D **55**, 5577 (1997).
[16] T.W. Yeh and H.-n. Li, Phys. Rev. D **56**, 1615 (1997).
[17] A.G. Grozin and M. Neubert, Phys. Rev. D **55**, 272 (1997).
[18] M. Beneke and T. Feldmann, Nucl. Phys. **B592**, 3 (2000).
[19] S. Descotes and C.T. Sachrajda, Nucl. Phys. **B625**, 239 (2002).
[20] H. Kawamura, J. Kodaira, C.F. Qiao, and K. Tanaka, Phys. Lett. B **523**, 111 (2001); hep-ph/0112174.
[21] P. Ball, V.M. Braun, Y. Koike, and K. Tanaka, Nucl. Phys. **B529**, 323 (1998).
[22] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[23] C.H. Chen and C.Q. Geng, Phys. Rev. D **63**, 114025 (2001); Nucl. Phys. **B636**, 338 (2002).
[24] A. Ali, P. Ball, L.T. Handoko, and G. Hiller, Phys. Rev. D **61**, 074024 (2001); A. Ali and A.Ya. Parkhomenko, Eur. Phys. J. C **23**, 89 (2002).
[25] H.Y. Cheng and K.C. Yang, Phys. Lett. B **511**, 40 (2001).
[26] CLEO Collaboration, R.A. Briere *et al.*, Phys. Rev. Lett. **86**, 3718 (2001).
[27] BELLE Collaboration, A. Bozek, Proceedings for the 4th International Workshop on B Physics and CP Violation, Ise-Shima, Japan, 2001, p. 81.
[28] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **87**, 151801 (2001).
[29] Particle Data Group, C. Caso *et al.*, Eur. Phys. J. C **3**, 1 (1998).
[30] J. Botts and G. Sterman, Nucl. Phys. **B325**, 62 (1989).
[31] H.-n. Li and G. Sterman, Nucl. Phys. **B381**, 129 (1992).
[32] H.-n. Li, hep-ph/0102013.
[33] C.Y. Wu, T.W. Yeh, and H.-n. Li, Phys. Rev. D **55**, 237 (1997).