Photon polarization in radiative B decays

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We study decay distributions in $B \to K\pi\pi\gamma$, combining contributions from several overlapping resonances in a $K\pi\pi$ mass range near 1400 MeV, $1^+ K_1(1400)$, $2^+ K_2^*(1430)$ and $1^- K^*(1410)$. A method is proposed for using these distributions to determine a photon polarization parameter in the effective radiative weak Hamiltonian. This parameter is measured through an up-down asymmetry of the photon direction relative to the $K\pi\pi$ decay plane in the $K_{\rm res}$ frame. We calculate a dominant up-down asymmetry of 0.33 ± 0.05 from the $K_1(1400)$ resonance, which can be measured with about 10^8 $B\bar{B}$ pairs, thus providing a new test for the standard model and a probe for some of its extensions.

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I. INTRODUCTION

Measurements of inclusive radiative B meson decays $B \to X_s \gamma$ [1] provide an important test for the standard model (SM), and set stringent bounds on physics beyond the SM [2]. In addition to the rather well predicted inclusive branching ratio, which was studied extensively both experimentally and theoretically [3], there is a unique feature of this process within the SM which drew only moderate theoretical attention and which has not yet been tested. Namely, the emitted photons are left handed in radiative B^- and $\bar{B}^0$ decays and are right handed in B^+ and B^0 decays. In the SM the photon in $b \to s\gamma$ is predominantly left handed, since the recoil s quark which couples to a W is left chiral. This prediction of approximately *maximal parity violation* holds in the SM to within a few percent, up to corrections of order m_s/m_b. It applies also in exclusive radiative decays when including long-distance effects [4]. While measurements of the inclusive radiative decay rate agree with SM calculations, no evidence exists for the helicity of the photons in inclusive and exclusive decays.

In several extensions of the SM the photon in $b \to s\gamma$ acquires an appreciable right-handed component due to chirality flip along a heavy fermion line in the electroweak loop process. Two well-known examples of such extensions are the left-right (LR) symmetric model and the unconstrained minimal supersymmetric standard model (MSSM). In the LR model chirality flip along the t quark line in the loop involves W_L-W_R mixing [5], while in the MSSM a chirality flip along the gluino line in the loop involves left-right squark mixing [6]. In both types of models it was found that, in certain allowed regions of the parameter space, the photons emitted in $b \to s\gamma$ can be largely right-handed polarized, without affecting the SM prediction for the inclusive radiative decay rate. This situation calls for an independent measurement of the photon helicity, which therefore becomes of immediate interest.

Several ways were suggested in the past five years to look for signals of physics beyond the SM through photon helicity effects in $B \to X_s \gamma$. To set the stage for the present proposal,

and in order to appreciate the immediate potential of applying this new idea at currently operating B factories, while other methods require higher luminosities or new experimental facilities, let us recall all previously proposed methods [7].

In the first suggested method [8] the photon helicity is probed through mixing-induced CP asymmetries. The time-dependent asymmetry of $B^0(t) \to X_{s(d)}^{CP}\gamma$, where $X_s^{CP} = K^{*0} \to K_S \pi^0$ or $X_d^{CP} = \rho^0 \to \pi^+\pi^-$, follows from interference between B^0 and $\bar{B}^0$ decay amplitudes into a common state of definite photon polarization. The asymmetry is proportional to the ratio of right-to-left polarization amplitudes A_R/A_L, for small values of this ratio (a few percent in the SM), and may reach a maximum value of order one in extensions of the SM. For a time-dependent measurement, one must measure the distance of the B decay point away from its production. It is hard to trace a K^{*0} decay back to its point of production, since in $K^{*0} \to K_S \pi^0$ the K_S decays after travelling some distance. This is not the case for $B^0 \to \rho^0\gamma$, where the ρ^0 decays promptly to $\pi^+\pi^-$, allowing thereby a time measurement. However, $B^0 \to \rho\gamma$ is suppressed by a factor $|V_{td}/V_{ts}|^2$ relative to $\mathcal{B}(B^0 \to K^*\gamma) \sim 4 \times 10^{-5}$ [9], and one expects its branching ratio to be only about 2×10^{-6}. CP asymmetries at a level of a few percent, as expected in the SM, require an order of 10^9 B mesons. A smaller number of order 10^8 B's, as envisaged in near future experiments at B factories, can provide constraints on possible large asymmetries, namely on large values of A_R/A_L, and would thereby improve bounds on certain parameters of the new physics models.

In a second scheme one studies angular distributions in $B \to \gamma(\to e^+e^-)K^*(\to K\pi)$, where the photon can be virtual [10] or real, converting in the beam pipe to an electron-positron pair [11]. The correlation between the e^+e^- and the $K^* \to K\pi$ planes is sensitive to the photon polarization. The distribution in the angle between the $K\pi$ and e^+e^- planes is isotropic for purely circular polarization, and the angular distribution is sensitive to interference between left and right polarization. Namely, the deviation from isotropy involves (again) a parameter A_R/A_L measuring the mixture of left and

right polarizations. One expects $\mathcal{B}(B \to K^* e^+ e^-) \sim (1-2) \times 10^{-6}$ [12]. Therefore, the number of B's required here to measure a photon polarization effect, in the SM or in the presence of new physics, is comparable to the corresponding number required for the previous method.

In a third method using Λ_b decays [13], $\Lambda_b \to \Lambda \gamma \to p \pi \gamma$, one measures directly the photon polarization. The forward-backward asymmetry of the proton with respect to the Λ_b in the Λ rest frame is proportional to the photon polarization. Using polarized Λ_b's [14], one can also measure the forward-backward asymmetry of the Λ momentum with respect to the Λ_b boost axis. This asymmetry is proportional to the product of the Λ_b and photon polarizations. This last scheme can only be applied in extremely high luminosity $e^+ e^-$ Z factories. We note that the two methods based on Λ_b decays measure directly the photon polarization, whereas the other two types of measurements using B decays are sensitive to interference between amplitudes involving photons with left- and right-handed polarization. All methods can probe deviations from approximately pure left handedness as predicted in the SM.

In the present paper we wish to elaborate further on a method proposed very recently in a short letter [15] (see also [16]), which measures directly a fundamental parameter in the effective radiative weak Hamiltonian describing the photon polarization. This method, based on radiative B decays to excited kaons, makes use of angular correlations among the three-body decay products of the excited kaons. It was shown [15] that in decays $B^+ \to [K_1^+(1400) \to K^0 \pi^+ \pi^0] \gamma$ and $B^0 \to [K_1^0(1400) \to K^+ \pi^- \pi^0] \gamma$ the up-down asymmetry of the photon momentum with respect to the $K \pi \pi$ decay plane in the K_{res} frame measures the photon polarization with a rather high efficiency. For approximately complete polarization, as expected in the SM, the asymmetry integrated over the entire Dalitz plot was calculated to be 0.34. Here we study this asymmetry in some more detail than in [15]. In particular, we calculate carefully theoretical uncertainties due to an admixture of S and D waves in $K_1 \to K^* \pi$ and due to a possible small decay rate into ρK. Assuming the radiative branching ratio into $K_1(1400)$ to be around but somewhat below 10^{-5} [17], such an asymmetry can be measured at currently operating B factories.

The $K \pi \pi$ invariant mass region around 1400 MeV contains several kaon resonances with different quantum numbers. We give further details for calculations of angular distributions corresponding to these resonance states. Whereas the axial-vector state $K_1(1400)$ introduces a large up-down asymmetry, the other two states, a tensor $K_2^*(1430)$ and a vector $K_1^*(1400)$, lead to a much smaller asymmetry and to no asymmetry, respectively. Separation or projection of these individual resonance contributions is therefore crucial, in order to achieve a high efficiency in measuring the photon polarization parameter in the effective radiative weak Hamiltonian. It is shown that this parameter measures the photon polarization in decays to all individual K resonances. Interference between the different resonances, which was disregarded in [15], would introduce uncertainties in the measurement of the polarization parameter. Here we present details

of a method, by which interference effects between overlapping resonances can be resolved, thereby providing a way of measuring the photon polarization parameter with only minimal model dependence.

In addition to the above-mentioned final states which involve a neutral pion, we also consider extensions of the method to two other resonance decay channels involving only charged pions, $K^+ \pi^+ \pi^-$ and $K^0 \pi^+ \pi^-$, not considered in [15]. We explain the sources of theoretical uncertainties in studying polarization effects in these decays.

The basic idea of the method is introduced in Sec. II, where it is shown in general that certain observables in $B \to (K_{\text{res}} \to K \pi \pi) \gamma$ decays are sensitive to the photon polarization. Several relevant final states are listed to which this measurement can be applied. These final states involve a kaon and two pions of specific charges. Section III considers, for resonance states of specific quantum numbers, the general structure of the weak decay amplitude of $B \to (K_{\text{res}} \to K \pi \pi) \gamma$ in terms of the photon polarization. It is shown that the polarizations corresponding to individual resonance states are all identical to a polarization parameter defined by Wilson coefficients in the effective weak Hamiltonian. Strong decays of several resonance states with different quantum numbers are studied in Sec. IV. Angular decay distributions of $B \to K \pi \pi \gamma$, sensitive to the photon polarization, are calculated in Sec. V. A method is presented for determining the photon polarization parameter from decay distributions, in spite of involving contributions from several overlapping kaon resonances. Section VI presents numerical results for the expected up-down asymmetry parameters. Section VII contains a discussion of the experimental feasibility of the method, followed by conclusions in Sec. VIII.

II. WHY AND WHICH THREE-BODY DECAYS OF K_{res}?

The first measured radiative B decays [9] were exclusive decays into the first excited kaon resonance state, $B \to K^*(892) \gamma$, with branching ratios of about 4×10^{-5}. Radiative decays into a higher excited resonance state, $K_2^*(1430)$, were observed more recently, both by the CLEO and Belle Collaborations:

$$\mathcal{B}(B \to K_2^*(1430) \gamma)$$

$$= (1.66^{+0.59}_{-0.53} \pm 0.13) \times 10^{-5} \quad (\text{CLEO [18]}),$$

$$= (1.50^{+0.58+0.11}_{-0.53-0.13}) \times 10^{-5} \quad (\text{Belle [19]}). \quad (1)$$

In these experiments the $K^*(892)$ and $K_2^*(1430)$ resonance states were identified through their $K \pi$ decay channels. In both cases the corresponding $K \pi$ decay branching ratios are large, $(49.9 \pm 1.2)\%$ in the case of K_2^* [20].

In the case of K_2^*, decay branching ratios into $K^*(892) \pi$ and ρK are also sizable, $(24.7 \pm 1.5)\%$ and $(8.7 \pm 0.8)\%$, respectively [20]. These modes lead to $K \pi \pi$ final states. As we will argue below, in order to probe the helicity of the K_2^* (or any other resonance), and thereby determine the photon polarization, one must study the resonance decays into final states involving at least three particles. First measurements

of B decays into a photon and three-body hadronic final states, involving a $K\pi\pi$ invariant mass in the kaon resonance region, were reported recently by the Belle Collaboration [19]. As will be explained below, this experiment can be used to measure the photon polarization.

Let us explain first the necessary conditions for a measurement of the photon polarization through the recoil hadron distribution. We will also consider the essential ingredients of the measured physical system which are necessary for a theoretically clean measurement, that is, a measurement which involves a minimal amount of hadronic dependence.

Since the photon helicity is odd under parity, and since one only measures the momenta of the photon and of the final hadronic decay products, helicity information cannot be obtained from two-body decays of the excited kaon. A hadronic quantity which is proportional to the photon helicity must be parity odd. The pseudoscalar quantity, which contains the smallest number of hadron momenta, is a triple product. Thus, one requires at least a three-body hadronic final state recoiling against the photon, in which one can form a parity-odd triple product $\vec{p}_\gamma \cdot (\vec{p}_1 \times \vec{p}_2)$ in the K resonance rest frame. Here $\vec{p}_\gamma$ is the photon momentum, while $\vec{p}_1$ and $\vec{p}_2$ are two of the final hadron momenta, all measured in the recoiling hadron (a K-resonance) rest frame. Applying parity, the average value of the triple product has one sign for a left-handed photon and an opposite sign for a right-handed photon.

But here there seems to be a theoretical difficulty. A triple product correlation is also odd under time reversal. Since time-reversal symmetry holds in K-resonance decays, the decay amplitude must involve a nontrivial phase due to final state interactions. Such a phase is usually suspected of being uncalculable and hard to measure. This strong phase originates from the interference of at least two amplitudes leading to a common three-body final state. Noting that the $K\pi\pi$ decay modes of excited resonance states around 1400 MeV (to which we will draw our attention) are dominated by $K^*\pi$ and ρK channels [20], let us list the three kinds of interference which one may encounter:

(1) Interference between two intermediate $K^*\pi$ states with different charges, for instance $K^{*+}\pi^0$ and $K^{*0}\pi^+$, decaying to a common $K^0\pi^+\pi^0$ state. These two amplitudes are related by isospin; consequently the strong phase is calculable purely in terms of Breit-Wigner forms.

(2) Interference between $K^*\pi$ and ρK amplitudes. In several cases these amplitudes can be related by SU(3), and SU(3) breaking can be obtained from measured decay branching ratios of excited kaons into $K^*\pi$ and ρK. In some cases relative strong phases are extracted from resonance production experiments.

(3) Interference between different partial waves into $K^*\pi$ or ρK. In certain cases the ratio of these partial wave amplitudes and their relative phases were measured in resonance production experiments.

Since we will consider a $K\pi\pi$ state with an invariant mass in a narrow band ($\pm$ 100 MeV) around 1400 MeV, we will neglect the direct nonresonant radiative $B \to K\pi\pi\gamma$ decay.

The decay rate for this phase space restricted process is much smaller than the decay rate through an excited resonance state. A simple estimate of the nonresonant contribution can be made by noting that the ratio of nonresonant to resonant $K\pi\pi\gamma$ events, with an invariant mass of width $\Gamma_{K_{res}}$ ~ 200 MeV is about $\Gamma_{K_{res}}/[M_B - (2M_\pi + M_K)] \sim 4\%$. We assumed here for simplicity equal total resonant and nonresonant branching ratios, with a flat distribution in the $K\pi\pi$ invariant mass for the latter.

We conclude that the theoretically cleanest calculation of a $K\pi\pi\gamma$ decay amplitude, in which final state interaction phases can be computed most reliably from pure isospin considerations, corresponds to cases in which only the first kind of interference exists. We will focus our attention on cases which are dominated by such interference, which permits a theoretically rather clean measurement of the photon polarization through decay distributions.

In Sec. IV we will study the decays of three K resonances, $K_1(1400)$, $K^*(1410)$ and $K_2^*(1430)$, with quantum numbers $J^P = 1^+, 1^-$ and 2^+, respectively. The most general result for the decay amplitude involves relative strong phases between $K^*\pi$ and ρK amplitudes and between different partial waves. In order to have a measurement which can be cleanly interpreted in terms of the photon polarization parameter, these phase differences must be known, at least crudely. This is the case in the decays $K_1(1400) \to K\pi\pi$, which are dominated by $K^*\pi$ intermediate states, and where some information is known both about the $S-D$ admixture in the $K^*\pi$ channel, and about the magnitude and phase of the smaller $K\rho$ amplitude.

Parametrizing resonance amplitudes in terms of Breit-Wigner forms, known to be a rather good approximation, yields a calculable strong phase. As mentioned, the remaining strong phases can be estimated in some cases using arguments based on flavor SU(3) symmetry, or can be extracted from resonance production experiments. In many respects, this method is similar to measuring the τ neutrino helicity in $\tau \to a_1 \nu_\tau$, where the corresponding phase difference is calculable in terms of the two interfering $a_1 \to \rho\pi$ amplitudes corresponding to two different $\rho\pi$ charge assignments [21–23].

Let us list the channels through which excited kaons may decay into distinct charged $K\pi\pi$ states:

$$K_{res}^+ \to \left\{ \begin{array}{c} K^{*+}\pi^0 \\ K^{*0}\pi^+ \\ \rho^+ K^0 \end{array} \right\} \to K^0\pi^+\pi^0, \tag{2}$$

$$\left\{ \begin{array}{c} K^{*0}\pi^+ \\ \rho^0 K^+ \end{array} \right\} \to K^+\pi^+\pi^-, \tag{3}$$

$$K_{res}^0 \to \left\{ \begin{array}{c} K^{*+}\pi^- \\ K^{*0}\pi^0 \\ \rho^- K^+ \end{array} \right\} \to K^+\pi^-\pi^0, \tag{4}$$

$$\left\{ \begin{array}{c} K^{*+}\pi^- \\ \rho^0 K^0 \end{array} \right\} \to K^0\pi^+\pi^-. \tag{5}$$

K_{res}^+ and K_{res}^0 occur in radiative B^+ and B^0 decays, respectively. No interference is present in the amplitudes for the two final states $K^+\pi^0\pi^0$ and $K^0\pi^0\pi^0$, which can be produced only through $K^{*+}\pi^0$ and $K^{*0}\pi^0$ modes, respectively. We note that, in the isospin and narrow resonance width limits, and assuming a vanishing ρK contribution, the partial decay rates of each of the modes (2) and (3) [(4) and (5)] for K_{res}^+ (K_{res}^0) is equal to 4/9 of the total decay branching ratio into $K\pi\pi$.

III. $B \to K\pi\pi\gamma$ IN TERMS OF A PHOTON POLARIZATION PARAMETER

Consider the radiative decays into a given kaon resonance state $\bar{B}(b\bar{q}) \to \bar{K}_{\text{res}}^{(i)}\gamma$. Let us denote the weak radiative amplitudes by $c_L^{(i)} \equiv A(\bar{B} \to K_{\text{res}}^{(i)}\gamma_L)$ and $c_R^{(i)} \equiv A(\bar{B} \to K_{\text{res}}^{(i)}\gamma_L)$, for left- and right-polarized photons, respectively. The photon polarization in $B \to K_{\text{res}}^{(i)}\gamma$ is given by

$$\lambda_\gamma^{(i)} = \frac{|c_R^{(i)}|^2 - |c_L^{(i)}|^2}{|c_R^{(i)}|^2 + |c_L^{(i)}|^2}. \tag{6}$$

Let us show that the ratios $|c_R^{(i)}/c_L^{(i)}|$, and therefore $\lambda_\gamma^{(i)}$, are equal for all K resonance states, and are given by fundamental couplings in the effective weak radiative Hamiltonian. The effective Hamiltonian has the general structure

$$\mathcal{H}_{\text{rad}} = -\frac{4G_F}{\sqrt{2}} V_{tb}V_{ts}^*(C_{7R}\mathcal{O}_{7R} + C_{7L}\mathcal{O}_{7L}),$$

$$\mathcal{O}_{7L,R} = \frac{e}{16\pi^2} m_b \bar{s} \sigma_{\mu\nu} \frac{1 \pm \gamma_5}{2} b F^{\mu\nu}, \tag{7}$$

where the Wilson coefficients C_{7L} and C_{7R} describe the amplitudes of $b \to s\gamma$ for left- and right-handed photons, respectively. Because of the chiral structure of the $W^\pm$ couplings to quarks in the SM, the amplitude for the emission of a left-handed photon in $b \to s\gamma$ is enhanced relative to that for a right-handed photon by $C_{7R}/C_{7L} \simeq m_s/m_b$. This property holds also when adding to Eq. (7) the four-quark operators $\mathcal{O}_{1,2}$, which have the same $(V-A) \times (V-A)$ chiral structure as the couplings producing the dominant penguin operator $\mathcal{O}_{7L}$. On the other hand, much larger C_{7R}/C_{7L} ratios are permitted in LR and MSSM extensions of the SM.

Parity invariance of the strong interactions relates the amplitude of $b \to s\gamma$ for emitting a left-handed photon through $\mathcal{O}_{7L}$ to the amplitude for emitting a right-handed photon through $\mathcal{O}_{7R}$,

$$\langle K_{\text{res}}^{(i)R}\gamma_R|\mathcal{O}_{7R}|\bar{B}\rangle = (-1)^{J_i-1} P_i \langle K_{\text{res}}^{(i)L}\gamma_L|\mathcal{O}_{7L}|\bar{B}\rangle, \tag{8}$$

where J_i and P_i are the resonance spin and parity, and $K_{\text{res}}^{(i)R,L}$ denote states with helicities ± 1.

To prove this relation, let us assume for definiteness that the photon moves along the $+z$ axis and the K resonance along the opposite direction. Under a parity transformation $\mathcal{P}$

the operators $\mathcal{O}_{7R}$ and $\mathcal{O}_{7L}$ transform to each other, $\mathcal{P}\mathcal{O}_{7R}\mathcal{P}^\dagger = \mathcal{O}_{7L}$. This gives

$$\langle K_{\text{res}}^{(i)R}(\downarrow)\gamma_R(\uparrow)|\mathcal{O}_{7R}|\bar{B}\rangle = \langle K_{\text{res}}^{(i)R}(\downarrow)\gamma_R(\uparrow)|\mathcal{P}^\dagger\mathcal{P}\mathcal{O}_{7R}\mathcal{P}^\dagger\mathcal{P}|\bar{B}\rangle$$

$$= P_i\langle K_{\text{res}}^{(i)L}(\uparrow)\gamma_L(\downarrow)|\mathcal{O}_{7L}|\bar{B}\rangle, \tag{9}$$

where the arrows denote particle momenta relative to the z direction. Under a rotation $\mathcal{R}$, around the y axis through $180°$, the states transform as $\mathcal{R}|J,M\rangle = (-1)^{J-M}|J,-M\rangle$. Applying this rotation to the right-hand side of Eq. (9) gives Eq. (8).

Together with the equality

$$\langle K_{\text{res}}^{(i)L}\gamma_L|\mathcal{O}_{7R}|\bar{B}\rangle = \langle K_{\text{res}}^{(i)R}\gamma_R|\mathcal{O}_{7L}|\bar{B}\rangle = 0, \tag{10}$$

Eq. (8) shows that, for a given resonance, the weak amplitudes $c_R^{(i)}$ and $c_L^{(i)}$ are proportional, up to a sign, to the Wilson coefficients C_{7R} and C_{7L}, respectively, and to a common hadronic matrix element of $\mathcal{O}_{7R}$, $g_+^{(i)}(0)$

$$\begin{Bmatrix} c_R^{(i)} \\ c_L^{(i)} \end{Bmatrix} = -\frac{4G_F}{\sqrt{2}} \begin{Bmatrix} C_{7R} \\ P_i(-1)^{J_i-1}C_{7L} \end{Bmatrix} V_{tb}V_{ts}^* g_+^{(i)}(0). \tag{11}$$

This implies

$$\frac{|c_R^{(i)}|}{|c_L^{(i)}|} = \frac{|C_{7R}|}{|C_{7L}|} \Rightarrow \lambda_\gamma^{(i)} = \frac{|C_{7R}|^2 - |C_{7L}|^2}{|C_{7R}|^2 + |C_{7L}|^2} \equiv \lambda_\gamma. \tag{12}$$

Namely, the photon polarization in $\bar{B} \to \bar{K}_{\text{res}}^{(i)}\gamma$ is common to all K resonance states and is given purely in terms of Wilson coefficients. We will refer to the quantity λ_γ, defined by Wilson coefficients, as the photon polarization parameter. We note that the above argument does not depend on the form factor matrix elements $g_+^{(i)}(0)$ between B and $K_{\text{res}}^{(i)}$, which were calculated in several models [17].

Now, consider the decays $\bar{B}(b\bar{q}) \to \bar{K}\pi\pi\gamma$, to which several overlapping kaon resonances $K_{\text{res}}^{(i)}$ contribute. Let us denote the strong decay amplitudes for $\bar{K}_{\text{res}}^{(i)} \to \bar{K}\pi\pi$ by $A_L^{(i)}$ and $A_R^{(i)}$, corresponding to a left- and right-polarized resonance, respectively. The radiative differential decay rate can be written as a sum of contributions from left- and right-polarized photons,

$$d\Gamma(\bar{B} \to \bar{K}\pi\pi\gamma) = \left|\sum_i \frac{c_R^{(i)}A_R^{(i)}}{s - M_i^2 - iM_i\Gamma_i}\right|^2 + \left|\sum_i \frac{c_L^{(i)}A_L^{(i)}}{s - M_i^2 - iM_i\Gamma_i}\right|^2, \tag{13}$$

where $s = (p_K + p_{\pi_1} + p_{\pi_2})^2$ is the invariant mass of the hadronic $K\pi\pi$ state. The two terms do not interfere in the partial decay rate since *in principle* the photon polarization is measurable. Using Eqs. (11) and (12), one finds

$$d\Gamma(\bar{B} \to \bar{K}\pi\pi\gamma) \propto (|\mathcal{A}_R|^2 + |\mathcal{A}_L|^2) + \lambda_\gamma(|\mathcal{A}_R|^2 - |\mathcal{A}_L|^2), \tag{14}$$
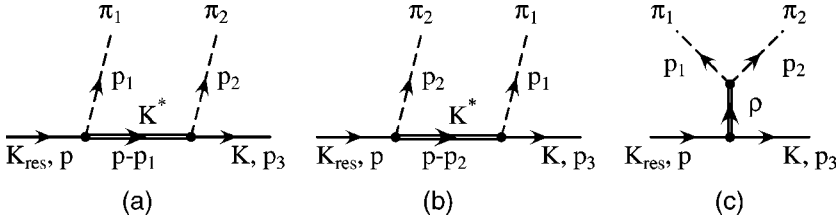
FIG. 1. Resonant contributions to the decay $K_{\text{res}} \rightarrow K\pi\pi$, proceeding through $K^*\pi$ and ρK intermediate states.

where

$$\mathcal{A}_R \equiv \sum_i g_+^{(i)} A_R^{(i)} B_i(s),$$

$$\mathcal{A}_L \equiv \sum_i P_i(-1)^{J_i-1} g_+^{(i)} A_L^{(i)} B_i(s),$$

(15)

and where

$$B_i(s) \equiv \frac{1}{s - M_i^2 - i\Gamma_i M_i} \qquad (16)$$

are Breit-Wigner forms involving the mass M_i and width Γ_i of a resonance K_{res}^i.

The term in the decay distribution (14), which is sensitive to λ_γ, involves the difference $|\mathcal{A}_R|^2 - |\mathcal{A}_L|^2$. For a single resonance $K_{\text{res}}^{(i)}$, it is proportional to $|A_R^{(i)}|^2 - |A_L^{(i)}|^2$. Therefore, a measurement of the photon polarization parameter λ_γ is sensitive to asymmetries between the resonance decay amplitudes $|A_R^{(i)}|$ and $|A_L^{(i)}|$. Such a measurement can be directly translated into information about the ratio of Wilson coefficients $|C_{7R}/C_{7L}|$, which is given in the SM by m_s/m_b implying $\lambda_\gamma = -1 \ (+1) + \mathcal{O}(m_s^2/m_b^2)$, for $\bar{B}$ (B) decays, respectively. We will show that such a determination is essentially free of hadronic uncertainties, and can be performed even in the absence of any information about the strong matrix elements $g_+^{(i)}$.

It is interesting to note the relation between the photon polarization parameter λ_γ, which is defined in (12) in terms of Wilson coefficients, and the photon polarization in $\bar{B} \rightarrow \bar{K}\pi\pi\gamma$. The latter can be defined by

$$P_\gamma = \frac{\Gamma(\bar{B} \rightarrow \bar{K}\pi\pi\gamma_R) - \Gamma(\bar{B} \rightarrow \bar{K}\pi\pi\gamma_L)}{\Gamma(\bar{B} \rightarrow \bar{K}\pi\pi\gamma_R) + \Gamma(\bar{B} \rightarrow \bar{K}\pi\pi\gamma_L)}$$

$$= \frac{\int dPS(|\mathcal{A}_R|^2 - |\mathcal{A}_L|^2) + \lambda_\gamma \int dPS(|\mathcal{A}_R|^2 + |\mathcal{A}_L|^2)}{\int dPS(|\mathcal{A}_R|^2 + |\mathcal{A}_L|^2) + \lambda_\gamma \int dPS(|\mathcal{A}_R|^2 - |\mathcal{A}_L|^2)},$$

(17)

where $\int dPS$ denotes an integral over entire phase space. For a single resonance, one has

$$\int dPS(|\mathcal{A}_R|^2 - |\mathcal{A}_L|^2)$$

$$= |g_+^{(i)} B_i(s)|^2 \int dPS(|A_R^{(i)}|^2 - |A_L^{(i)}|^2), \quad (18)$$

and parity conservation in the resonance decay implies that the right-hand side vanishes. In this case one would have $P_\gamma = \lambda_\gamma$. This is not true for several interfering resonances for which a relation similar to Eq. (18) does not hold in general. Namely, the photon polarization parameter λ_γ coincides with the photon polarization P_γ in $\bar{B} \rightarrow \bar{K}\pi\pi\gamma$ only for a process proceeding through a single resonance.

IV. STRONG DECAYS $K_{\text{res}} \rightarrow K\pi\pi$

In the resonance mass region $M_{\text{res}} = 1300 - 1500$ MeV, which we consider, there exist several K resonances with different quantum numbers J^P, which decay to $K\pi\pi$. We list these states in Table I, specifying their masses, widths and decay branching ratios [20].

The strong decay of a kaon resonance K_{res} into a three-body $K\pi\pi$ final state proceeds through the graphs shown in Fig. 1, with intermediate $K^*\pi$ and ρK states. In the following subsections we use these diagrams to compute for the three resonances around 1400 MeV the strong decay amplitudes $A_L^{(i)}$ and $A_R^{(i)}$ appearing in the rate equations (14) and (15).

A. Decays of $K_1(1400)$ ($J^P = 1^+$)

The $K_1(1400)$ decays predominantly to $K^*(892)\pi$ with a branching ratio of $(94 \pm 6)\%$, and to ρK with a much smaller branching ratio, $(3.0 \pm 3.0)\%$. Both decays occur in a mixture of S and D waves. The D to S wave ratio of rates in the $K_1 \rightarrow K^*(892)\pi$ channel was measured to be small, 0.04 ± 0.01 [20,24]. No similar measurement exists for the ρK channel.

The invariant matrix element for $K_1 \rightarrow K^*\pi$ (and similarly for $K_1 \rightarrow \rho K$) can be parametrized in terms of two complex couplings, $A^{(K^*)}$ and $B^{(K^*)}$,

$$\mathcal{A}(K_1(p,\varepsilon) \rightarrow K^*(p_1,\varepsilon')\pi(p_\pi))$$

$$= A^{(K^*)}(\varepsilon \cdot \varepsilon'^*) + B^{(K^*)}(\varepsilon'^* \cdot p_\pi)(\varepsilon \cdot p_\pi),$$

(19)

where momenta and polarization 4-vectors are specified for each of the particles. The couplings $A^{(K^*)}$ and $B^{(K^*)}$ are related to the partial wave amplitudes c_S and c_D through

$$A^{(K^*)} = c_S + c_D \frac{M_{K_1} \vec{p}_\pi^2}{2M_{K^*} + E_{K^*}},$$

(20)

$$B^{(K^*)} = c_D + c_S \frac{E_{K^*} - M_{K^*}}{M_{K_1} \vec{p}_\pi^2}.$$

TABLE I. Kaon resonances with masses in the region 1250–1450 MeV decaying into $K\pi\pi$.

Resonance	J^P	$(M_{\rm res}, \Gamma_{\rm res})$ (MeV)	Decay mode	Br (%)
$K_1(1270)$	1^+	$(1273\pm7, 90\pm20)$	ρK	42 ± 6
			$K^*\pi$	16 ± 5
			$K_0^*(1430)\pi$	28 ± 4
$K_1(1400)$	1^+	$(1402\pm7, 174\pm13)$	$K^*\pi$	94 ± 6
			ρK	3.0 ± 3.0
$K^*(1410)$	1^-	$(1414\pm15, 232\pm21)$	$K^*\pi$	>40
			ρK	<7
$K_2^*(1430)$	2^+	$(1425.6\pm1.5, 98.5\pm2.7)$	$K^*\pi$	24.7 ± 1.5
		(charged K_2^*)	ρK	8.7 ± 0.8

In the nonrelativistic limit, $M_{K_1} - M_{K^*} \ll M_{K_1}$, this reduces to the more familiar couplings, $c_S(\vec{\varepsilon}\cdot\vec{\varepsilon}'^*) + c_D[(\vec{\varepsilon}\cdot\vec{p}_\pi) \times(\vec{\varepsilon}'^*\cdot\vec{p}_\pi) - \frac{1}{3}\vec{p}_\pi^2(\vec{\varepsilon}\cdot\vec{\varepsilon}'^*)]$. (A similar partial wave decomposition was performed in [22] within the context of $a_1 \to \rho\pi$ decays.) Leaving out isospin factors, the amplitude (19) implies the following result for the $\Gamma(K_1\to K^*\pi)$ width:

$$\Gamma(K_1\to K^*\pi) = |c_S|^2 \frac{|\vec{p}_\pi|}{8\pi M_{K_1}^2}$$

$$+ |c_D|^2 \frac{|\vec{p}_\pi|^5}{4\pi(E_{K^*}+2M_{K^*})^2}, \quad (21)$$

where the two terms correspond to the S-wave and D-wave partial widths, respectively. A similar parametrization can be given for the $K_1\to K\rho$ coupling.

Using the rate formula (21) and the measured value of the D to S ratio of width, one finds $|c_D/c_S| = (1.75 \pm 0.22)$ GeV^{-2}. The relative phase between the D and S wave amplitudes was measured in [24] to be $\delta_{D/S} \equiv \text{Arg}(c_D/c_S) = 260° \pm 20°$. These values can be used to determine $A^{(K^*)}$ and $B^{(K^*)}$ from Eq. (20).

The amplitude for $K_1(1400)\to K\pi\pi$ is obtained from the graphs shown in Fig. 1 by convoluting amplitudes such as Eq. (19) with the amplitude for $K^*\to K\pi$ which is proportional to $\varepsilon'\cdot(p_\pi - p_K)$. The modes (2) and (4) obtain contributions from all three graphs Figs. 1(a)–1(c), while in the modes (3) and (5) only the graphs Fig. 1(a) and Fig. 1(c) contribute. In the first case, isospin considerations imply that the two K^* contributions from Fig. 1(a) and Fig. 1(b) are antisymmetric under the exchange of the two pion momenta. We will write down general expressions for Eqs. (2) and (3), noting that the amplitudes of the processes (4) and (5) have correspondingly similar forms.

The amplitude for the process (2) can be summarized in the rest frame of the K_1 by the following expression:

$$\mathcal{M}(K_1^+(p,\varepsilon)\to\pi^+(p_1)\pi^0(p_2)K^0(p_3))$$

$$= C_1\vec{p}_1\cdot\vec{\varepsilon} - C_2\vec{p}_2\cdot\vec{\varepsilon}, \quad (22)$$

where

$$C_i(s_{13},s_{23}) = C_i^{(K^*)}(s_{13},s_{23}) + C_i^{(\rho)}(s_{13},s_{23}),$$
$$s_{ij} = (p_i+p_j)^2. \quad (23)$$

The explicit expressions for $C_i^{(K^*)}$ and $C_i^{(\rho)}$ are

$$C_1^{(K^*)}(s_{13},s_{23}) = \frac{\sqrt{2}}{3}g_{K^*K\pi}A^{(K^*)}\left[\left(1+\frac{m_2^2-m_3^2}{M_{K^*}^2}\right)B_{K^*}(s_{23}) - 2B_{K^*}(s_{13})\right]$$

$$+ \frac{\sqrt{2}}{3}g_{K^*K\pi}B^{(K^*)}\left[-\left(1+\frac{m_2^2-m_3^2}{M_{K^*}^2}\right)(M_{K_1}E_1-m_1^2)+2p_1\cdot p_2\right]B_{K^*}(s_{23}),$$

$$C_2^{(K^*)}(s_{13},s_{23}) = \frac{\sqrt{2}}{3}g_{K^*K\pi}A^{(K^*)}\left[\left(1+\frac{m_1^2-m_3^2}{M_{K^*}^2}\right)B_{K^*}(s_{13}) - 2B_{K^*}(s_{23})\right]$$

$$+ \frac{\sqrt{2}}{3}g_{K^*K\pi}B^{(K^*)}\left[-\left(1+\frac{m_1^2-m_3^2}{M_{K^*}^2}\right)(M_{K_1}E_2-m_2^2)+2p_1\cdot p_2\right]B_{K^*}(s_{13}), \quad (24)$$

$$C_1^{(\rho)}(s_{13},s_{23}) = \frac{1}{\sqrt{3}}g_{\rho\pi\pi}A^{(\rho)}B_\rho(s_{12}) - \frac{1}{\sqrt{3}}g_{\rho\pi\pi}B^{(\rho)}M_{K_1}(E_1-E_2)B_\rho(s_{12}),$$

$$C_2^{(\rho)}(s_{13},s_{23})=\frac{1}{\sqrt{3}}g_{\rho\pi\pi}A^{(\rho)}B_\rho(s_{12})+\frac{1}{\sqrt{3}}g_{\rho\pi\pi}B^{(\rho)}M_{K_1}(E_1-E_2)B_\rho(s_{12}),$$

where $B_{K*}(s_{ij})$ and $B_\rho(s_{ij})$ are Breit-Wigner functions

$$B_{K*}(s_{ij})=(s_{ij}-M_{K*}^2-iM_{K*}\Gamma_{K*})^{-1},$$
$$B_\rho(s_{ij})=(s_{ij}-m_\rho^2-im_\rho\Gamma_\rho)^{-1}. \tag{25}$$

The corresponding amplitudes for the modes (3) and (5) are obtained from Eq. (22) by setting $B_{K*}(s_{13})\rightarrow0$, and multiplying the $K^*\pi$ part by $-\sqrt{2}$ and the $K\rho$ part by $-1/\sqrt{2}$. All amplitudes are symmetric under the exchange of the two pions, as expected from Bose symmetry.

Whereas the $K_1(1400)$ decays predominantly to $K^*\pi$, the small measured branching ratio into ρK, $3\pm3\%$, which only implies an upper limit, may have a nonnegligible effect on the decay distribution from which the photon polarization is measured. In order to estimate this effect, we parametrize

the relative contributions of the ρK and $K^*\pi$ amplitudes in terms of two complex ratios of corresponding partial wave amplitudes for S and D waves,

$$\kappa_{S,D}=|\kappa_{S,D}|e^{i\alpha_{S,D}}=\sqrt{\frac{3}{2}}\frac{c_{S,D}^{(\rho K)}}{c_{S,D}^{(K^*\pi)}}\cdot\frac{g_{\rho\pi\pi}}{g_{K*K\pi}}. \tag{26}$$

In the absence of experimental data on the individual partial widths for the ρK mode, these parameters cannot be determined at present. Some measure for the ρK contribution can be obtained by neglecting the D-wave admixture in this mode and assuming that the measured width (for which only an upper limit exists) is pure S wave. With this approximation, the coefficients C_i in Eq. (22) are given by

$$C_1(s_{13},s_{23})\propto\left(1+0.081\frac{c_D}{c_S}\right)\left[\left(1+\frac{m_2^2-m_3^2}{M_{K*}^2}\right)B(s_{23})-2B(s_{13})\right]+\left(0.384+\frac{c_D}{c_S}\right)$$

$$\times\left[-\left(1+\frac{m_2^2-m_3^2}{M_{K*}^2}\right)(M_{K_1}E_1-m_1^2)+2p_1\cdot p_2\right]B(s_{23})+\kappa_S[1-0.631(E_1-E_2)]B_\rho(s_{12}), \tag{27}$$

and $C_2(p_1,p_2)=C_1(p_2,p_1)$.

The absolute value of the ratio κ_S can be obtained from the measured widths for the respective modes

$$\frac{|c_S^{(\rho K)}|^2}{|c_S^{(K^*\pi)}|^2}=\frac{\mathcal{B}_S(K_1\rightarrow\rho K)}{\mathcal{B}_S(K_1\rightarrow K^*\pi)}\cdot\frac{|\vec{p}_{K*\pi}|}{|\vec{p}_{\rho K}|}\approx0.043, \tag{28}$$

$$\frac{|g_{\rho\pi\pi}|^2}{|g_{K*K\pi}|^2}=\frac{2\Gamma_\rho}{\Gamma_{K*}}\cdot\frac{|\vec{p}_{K\pi}|^3}{|\vec{p}_{\pi\pi}|^3}=3.16, \tag{29}$$

where we assumed that the measured ρK branching ratio is pure S-wave and is given by the central value. Thus we find from Eq. (26) $|\kappa_S|=0.45$. The relative phase α_S was measured in [24], and was found to lie in the range $20°\leq\alpha_S\leq60°$. In our subsequent numerical calculation we will use the range $|\kappa_S|=0.32\pm0.32$, corresponding to $\mathcal{B}(K_1\rightarrow\rho K)=0.03\pm0.03$, and will scan the values of the phase α_S in the above range.

B. Decays of $K_2^*(1430)$ $(J^P=2^+)$

The $K_2^*(1430)$ decays to $K^*\pi$ and ρK in a pure D wave, with branching ratios of $(24.7\pm1.5)\%$ and $(8.7\pm0.8)\%$, re-

spectively. The invariant amplitude for a $K^*\pi$ final state is written in terms of a single coupling,

$$\mathcal{A}(K_2^*(p,\varepsilon)\rightarrow K^*(p_1,\varepsilon_1)\pi(p_2))$$
$$=g_{K_2^*K*\pi}i\epsilon_{\alpha\beta\gamma\delta}\varepsilon_1^{*\alpha}p^\beta\varepsilon^{\gamma\rho}p_{2\rho}p_2^\delta, \tag{30}$$

and a similar expression can be written for $\mathcal{A}(K_2^*\rightarrow\rho K)$. The amplitude for $K_2^*\rightarrow K\pi\pi$ obtains contributions from the three graphs in Fig. 1. Their computation gives for charged K_2^* decays of the type (2):

$$\mathcal{M}(K_2^{*+}(p,\varepsilon)\rightarrow\pi^+(p_1)\pi^0(p_2)K^0(p_3))$$
$$\propto i\epsilon_{\alpha\beta\gamma\mu}p^\alpha p_1^\beta p_2^\gamma\varepsilon^{\mu\nu}[p_{1\nu}B_{K*}(s_{23})$$
$$+p_{2\nu}B_{K*}(s_{13})+\kappa(p_{1\nu}+p_{2\nu})B_\rho(s_{12})]. \tag{31}$$

A similar expression is obtained for neutral K_2^* decays of the type (4). Here again, Bose symmetry requires the amplitude to be symmetric under an exchange of the two pions.

The ratio of the ρK and $K^*\pi$ contributions is parametrized by a complex parameter κ, which is defined in a way similar to Eq. (26),

$$\kappa = |\kappa| e^{i\alpha} = \sqrt{\frac{3}{2}} \frac{g_{K_2^* \rho K}}{g_{K_2^* K^* \pi}} \cdot \frac{g_{\rho \pi \pi}}{g_{K^* K \pi}}. \qquad (32)$$

Using the measured branching ratios of these modes, one finds for the absolute value of the first ratio,

$$\frac{|g_{K_2^* \rho K}|^2}{|g_{K_2^* K^* \pi}|^2} = \frac{\mathcal{B}(K_2^* \to \rho K)}{\mathcal{B}(K_2^* \to K^* \pi)} \times \frac{|\vec{p}_{K^* \pi}|^5}{|\vec{p}_{\rho K}|^5} = 1.2, \qquad (33)$$

which gives $|\kappa| = 2.38$ when applying Eq. (29).

We will argue now that also the phase α of κ can be constrained using available experimental data. Let us consider first the phase α_1 of $g_{\rho \pi \pi}/g_{K^* K \pi}$, the second factor in κ. This ratio, given in Eq. (29), is predicted to be $\sqrt{8/3}$ in the SU(3) limit. The small SU(3) breaking in the measured value of this ratio (about 8%) suggests that its phase is also small. Now consider the first ratio in κ, $g_{K_2^* \rho K}/g_{K_2^* K^* \pi}$. Although SU(3) symmetry does not predict this ratio, it is possible to determine its phase α_2 by noting that in the SU(3) limit the amplitudes for the decays $K_2^* \to K^* \pi$, $K_2^* \to \rho K$ and $a_2(1320) \to \rho \pi$ satisfy a triangle relation,

$$A(K_2^{*+} \to K^{*+} \pi^0) + \frac{1}{\sqrt{2}} A(K_2^{*+} \to \rho^+ K^0)$$

$$= A(a_2^+(1320) \to \rho^+ \pi^0). \qquad (34)$$

α_2 is given by the relative phase of the amplitudes on the left-hand side. Using the measured widths for these modes one finds that, although the triangle does not close at the central values of the measured amplitudes [the right-hand side of Eq. (34) is slightly larger than the algebraic sum of the amplitudes on the left-hand side], it closes with a very small angle α_2 when errors are included. Namely, α_2 is close to zero in the SU(3) limit. Allowing for some SU(3) breaking effects, we will use in our numerical estimates below values for $\alpha = \alpha_1 + \alpha_2$ between $-30°$ and $+30°$. A small phase in this range was measured in a K_2^* resonance production experiment [24].

C. Decays of $K^*(1410)$ ($J^P = 1^-$)

The $K_1^*(1410)$ decays predominantly to $K^* \pi$ in a pure P wave, with a branching ratio larger than 40% (at 95% C.L.), while an upper bound of 7% exists for its decay branching ratio into ρK. The invariant amplitude describing the first decay is

$$\mathcal{A}(K_1^*(p, \varepsilon) \to K^*(p_1, \varepsilon_1) \pi(p_2))$$

$$= g_{K_1^* K^* \pi} i \epsilon(p, \varepsilon, \varepsilon_1^*, p_2). \qquad (35)$$

Using Figs. 1 to calculate three contributions, one finds the $K_1^* \to K \pi \pi$ amplitude in the rest frame of the decaying resonance,

$$\mathcal{M}(K_1^*(p, \varepsilon) \to \pi(p_1) \pi(p_2) K)$$

$$\propto \vec{\varepsilon} \cdot (\vec{p}_1 \times \vec{p}_2) [B_{K^*}(s_{13}) + B_{K^*}(s_{23})$$

$$+ \kappa' B_\rho(s_{12})], \qquad (36)$$

where κ' parametrizes the ratio of ρK and $K^* \pi$ contributions. The information on the magnitude of this ratio from measured branching ratios is limited to a rather weak upper bound.

It is quite simple to argue from a general principle that the decay amplitude (36) leads to a radiative decay distribution which is insensitive to the photon polarization. The only parity-invariant decay amplitude which can be constructed from the K_1^* polarization vector $\vec{\varepsilon}$ and the final mesons momenta is proportional to $\vec{\varepsilon} \cdot (\vec{p}_1 \times \vec{p}_2)$. Its square is invariant under $\vec{\varepsilon}_{+1} \leftrightarrow \vec{\varepsilon}_{-1}$ and therefore cannot be used to measure the photon polarization.

V. ANGULAR DISTRIBUTIONS

As explained in Sec. III, Eqs. (14) and (15) and the subsequent discussion, the sensitivity to the photon polarization parameter is manifested through an asymmetry between the decay distributions of right- and left-polarized K resonance states. In order to compute this asymmetry, one has to specify the orientation of the decay products $K \pi \pi$ with respect to the so-called helicity axis, defined to be along the photon momentum direction and opposite to it $\hat{e}_z = -\vec{p}_\gamma$. Working in the rest frame of the $K \pi \pi$ state, we define the normal $\vec{n}$ to the $K \pi \pi$ plane as $\vec{n} = (\vec{p}_1 \times \vec{p}_2)/|\vec{p}_1 \times \vec{p}_2|$. The orientation of the $K \pi \pi$ system with respect to the helicity axis $\vec{e}_z$ is given in the most general case by three angles (θ, ϕ, ψ). Two polar angles (θ, ϕ), with $\cos \theta = \vec{n} \cdot \vec{e}_z$, describe the orientation of $\vec{n}$ with respect to $\vec{e}_z$. A third angle ψ (unobservable in this case) parametrizes rotations around the $\vec{e}_z$ axis. In the subsequent study we will derive angular distributions in the angle θ, integrating over the azimuthal angle ϕ.

In Ref. [15] angular distributions were studied for $B \to K \pi \pi \gamma$ separately for the three resonances $K_1(1400)$, $K_2^*(1430)$ and $K^*(1410)$. Here we will study the most general decay distribution combining all three overlapping resonances including their interference. The structure of the amplitude of $\bar{B} \to \bar{K} \pi \pi \gamma_{R,L}$, given in Eq. (15) for right- and left-polarized photons, can be obtained by summing over contributions calculated in the preceding section for the three K resonance states. One finds, in the rest frame of the $K \pi \pi$ system,

$$\mathcal{A}_{R,L}(\bar{B} \to \bar{K} \pi \pi \gamma_{R,L})$$

$$= A(\vec{\varepsilon}_\pm \cdot \vec{J}) \pm B[(\vec{\varepsilon}_\pm \cdot \vec{n})(\vec{\varepsilon}_0 \cdot \vec{K})$$

$$+ (\vec{\varepsilon}_\pm \cdot \vec{K})(\vec{\varepsilon}_0 \cdot \vec{n})] \pm C(\vec{\varepsilon}_\pm \cdot \vec{n}), \qquad (37)$$

where the polarization vectors $\vec{\varepsilon}_i$ are defined in terms of $\vec{e}_z$ and two arbitrary unit vectors $\vec{e}_x$ and $\vec{e}_y$ in the plane perpendicular to $\vec{e}_z$,

$$\vec{\varepsilon}_{\pm} = \mp \frac{1}{\sqrt{2}}(\vec{e}_x \pm i\vec{e}_y), \quad \vec{\varepsilon}_0 = \vec{e}_z. \qquad (38)$$

The three terms in Eq. (37) are obtained from intermediate K_{res} states with quantum numbers $J^P = 1^+, 2^+$ and 1^-, respectively. Their strong decay amplitudes were given in Eqs. (22), (31) and (36), respectively. In order to obtain the second term from Eq. (31), we used the following expressions for polarization tensors corresponding to right- and left-handed K_2^* of helicity ± 1:

$$\varepsilon_{\pm 1}^{\mu\nu} = \frac{1}{\sqrt{2}}(\varepsilon_{\pm 1}^{\mu}\varepsilon_0^{\nu} + \varepsilon_0^{\mu}\varepsilon_{\pm 1}^{\nu}), \qquad (39)$$

where $\varepsilon_m^0 = 0$ $(m = \pm 1, 0)$.

The coefficients A, B, C include the strong matrix elements $g_+^{(i)}$ and the Breit-Wigner forms defined in Eq. (16). The vectors $\vec{J}, \vec{K}$ (lying in the decay plane of $K\pi\pi$) are functions of the Dalitz variables s_{13}, s_{23}. Explicit expressions for $\vec{J}$ and $\vec{K}$ are obtained from Eqs. (22) and (31), respectively,

$$\vec{J} = C_1\vec{p}_1 - C_2\vec{p}_2,$$

$$\vec{K} = |\vec{p}_1 \times \vec{p}_2|\{\vec{p}_1[B_{K^*}(s_{23}) + \kappa_\rho B_\rho(s_{12})]$$
$$+ \vec{p}_2[B_{K^*}(s_{13}) + \kappa_\rho B_\rho(s_{12})]\}. \qquad (40)$$

In the limit of isospin symmetry $\vec{K}$ is symmetric under $s_{13} \leftrightarrow s_{23}$, while $\vec{J}$ changes sign.

The dot products in Eq. (37) can be expressed in terms of the angles (θ, ϕ, ψ) described above,

$$\vec{\varepsilon}_{\pm} \cdot \vec{n} = -\frac{i}{\sqrt{2}}\sin\theta e^{\mp i\psi}, \quad \vec{\varepsilon}_0 \cdot \vec{n} = \cos\theta,$$

$$\vec{\varepsilon}_{\pm} \cdot \vec{J} = \mp \frac{1}{\sqrt{2}} e^{\mp i\psi}[(\cos\phi J_x + \sin\phi J_y)$$
$$\mp i\cos\theta(\sin\phi J_x - \cos\phi J_y)], \qquad (41)$$

$$\vec{\varepsilon}_0 \cdot \vec{J} = \sin\theta(\sin\phi J_x - \cos\phi J_y).$$

Similar expressions hold for $\vec{\varepsilon}_{\pm} \cdot \vec{K}$ and $\vec{\varepsilon}_0 \cdot \vec{K}$.

Squaring the amplitude (37), and integrating over ϕ, one finds

$$\frac{1}{2\pi}\int d\phi |\mathcal{A}_{R,L}|^2 = |A|^2 \left\{ \frac{1}{4}|\vec{J}|^2(1 + \cos^2\theta) \pm \frac{1}{2}\cos\theta\,\mathrm{Im}[\vec{n}\cdot(\vec{J}\times\vec{J}^*)] \right\} + |B|^2 \left\{ \frac{1}{4}|\vec{K}|^2(\cos^2\theta + \cos^2 2\theta) \right.$$

$$\left. \pm \frac{1}{2}\cos\theta\cos 2\theta\,\mathrm{Im}[\vec{n}\cdot(\vec{K}\times\vec{K}^*)] \right\} + |C|^2\frac{1}{2}\sin^2\theta$$

$$+ \left\{ \frac{1}{2}(3\cos^2\theta - 1)\mathrm{Im}[AB^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)] \pm \cos^3\theta\,\mathrm{Re}[AB^*(\vec{J}\cdot\vec{K}^*)] \right\}. \qquad (42)$$

The interference terms between the 1^- and the other resonances vanish identically upon integration over ϕ, but there remains a nonvanishing interference between the $J^P = 1^+$ and 2^+ contributions, manifested in the last term.

The decay distribution for $\bar{B} \to \bar{K}\pi\pi\gamma$ is readily obtained as a function of the photon polarization parameter λ_γ,

$$\frac{d\Gamma}{ds_{13}ds_{23}d\cos\theta} = |A|^2 \left\{ \frac{1}{4}|\vec{J}|^2(1 + \cos^2\theta) + \frac{1}{2}\lambda_\gamma\mathrm{Im}[\vec{n}\cdot(\vec{J}\times\vec{J}^*)]\cos\theta \right\} + |B|^2 \left\{ \frac{1}{4}|\vec{K}|^2(\cos^2\theta \right.$$

$$\left. + \cos^2 2\theta) + \frac{1}{2}\lambda_\gamma\mathrm{Im}[\vec{n}\cdot(\vec{K}\times\vec{K}^*)]\cos\theta\cos 2\theta \right\} + |C|^2\frac{1}{2}\sin^2\theta$$

$$+ \left\{ \frac{1}{2}(3\cos^2\theta - 1)\mathrm{Im}[AB^*\vec{n}\cdot(\vec{J}\times\vec{K}^*)] + \lambda_\gamma\mathrm{Re}[AB^*(\vec{J}\cdot\vec{K}^*)]\cos^3\theta \right\}. \qquad (43)$$

This decay distribution is sensitive to λ_γ through the second terms in the $|A|^2$, $|B|^2$ and in the interference contributions. Each of these three terms introduces an asymmetry between the decay rates for right- and left-polarized photons. This asymmetry, from which λ_γ can be determined, describes an up-down asymmetry of the photon momentum with respect to the $K\pi\pi$ decay plane.

Noting the symmetry properties of $\vec{J}$ (odd) and $\vec{K}$ (even) under the exchange of s_{13} and s_{23}, one can see that after integrating over the entire Dalitz plot (or any symmetric part

of it), the asymmetry terms proportional to $|A|^2$, $|B|^2$ average to zero. In order to avoid this loss of polarization information, we introduce a new angle $\tilde{\theta}$ defined by $\cos\theta \equiv \mathrm{sgn}(s_{13}-s_{23})\cos\tilde{\theta}$. An equivalent definition of $\tilde{\theta}$ is the angle between $-\vec{p}_\gamma$ and the normal to the decay plane defined by $\vec{p}_{\mathrm{slow}}\times\vec{p}_{\mathrm{fast}}$, where $\vec{p}_{\mathrm{slow}}$ and $\vec{p}_{\mathrm{fast}}$ are the momenta of the slower and faster pions in the $K\pi\pi$ center of mass frame.

Expressed in terms of $\tilde{\theta}$, the radiative decay distribution (14), integrated over a region of the Dalitz plot, has the following general form:

$$\frac{d^2\Gamma}{ds\,d\cos\tilde{\theta}} = \frac{1}{4}|c_1|^2|B_{K_1}(s)|^2\langle|\vec{J}|^2\rangle\{1+\cos^2\tilde{\theta}+4\lambda_\gamma R_1\cos\tilde{\theta}\}$$

$$+\frac{1}{4}|c_2|^2|B_{K_2^*}(s)|^2\langle|\vec{K}|^2\rangle\{\cos^2\tilde{\theta}+\cos^2 2\tilde{\theta}$$

$$+12\lambda_\gamma R_2\cos\tilde{\theta}\cos 2\tilde{\theta}\}+|c_3|^2 B_{K_1^*}(s)\sin^2\tilde{\theta}$$

$$+\left\{\mathrm{Im}[c_1 c_2^* B_{K_1}(s)B_{K_2^*}^*(s)c_{12}']\frac{1}{2}(3\cos^2\tilde{\theta}-1)\right.$$

$$\left.+\lambda_\gamma \mathrm{Re}[c_1 c_2^* B_{K_1}(s)B_{K_2^*}^*(s)c_{12}]\cos^3\tilde{\theta}\right\}, \qquad (44)$$

where $\langle\cdots\rangle$ denotes integration over a region of the Dalitz plot. After integrating over $\tilde{\theta}$, the interference terms in the last line vanish, and the rate is given simply by a sum of Breit-Wigner forms corresponding to the three K resonance states,

$$\frac{d\Gamma}{ds} = \frac{2}{3}|c_1|^2\langle|\vec{J}|^2\rangle|B_{K_1}(s)|^2 + \frac{2}{5}|c_2|^2\langle|\vec{K}|^2\rangle|B_{K_2^*}(s)|^2$$

$$+\frac{4}{3}|c_3|^2|B_{K_1^*}(s)|^2. \qquad (45)$$

In Eq. (44) we introduced explicit expressions for A,B,C in terms of Breit-Wigner forms with the mass and width of the K resonances with quantum numbers $J^P=1^+,2^+$ and 1^-, respectively

$$A(s)=c_1 B_{K_1}(s), \quad B(s)=c_2 B_{K_2^*}(s),$$

$$C(s)=c_3 B_{K_1^*}(s), \qquad (46)$$

where $c_i\propto\langle K_{\mathrm{res}}^{(i)L}\gamma_L|\mathcal{O}_{7L}|\bar{B}\rangle$ are proportional to the decay amplitude into the resonance states $K_{\mathrm{res}}^{(i)}$.

The coefficients R_1 and R_2 multiplying the polarization-sensitive terms in the decay distributions are defined such that $\langle\cos\tilde{\theta}\rangle=R_1\lambda_\gamma$ and $\langle\cos\tilde{\theta}\rangle=R_2\lambda_\gamma$ for decay rates dominated by the $J^P=1^+$ or the 2^+ resonances, respectively. R_1 and R_2 are expressed in terms of Dalitz plot averages,

$$R_1=\frac{1}{2}\cdot\frac{\langle\mathrm{sgn}(s_{13}-s_{23})\mathrm{Im}[\hat{n}\cdot(\vec{J}\times\vec{J}^*)]\rangle}{\langle|\vec{J}|^2\rangle},$$

$$R_2=\frac{1}{6}\cdot\frac{\langle\mathrm{sgn}(s_{13}-s_{23})\mathrm{Im}[\hat{n}\cdot(\vec{K}\times\vec{K}^*)]\rangle}{\langle|\vec{K}|^2\rangle}, \qquad (47)$$

and are calculable quantities (see discussion below). The complex coefficients $c_{12}(s)$ and $c_{12}'(s)$, appearing in the expression for the 1^+-2^+ interference terms, are given by

$$c_{12}(s)=\langle\mathrm{sgn}(s_{13}-s_{23})\vec{J}\cdot\vec{K}^*\rangle,$$

$$c_{12}'(s)=\langle\hat{n}\cdot(\vec{J}\times\vec{K}^*)\rangle. \qquad (48)$$

Now let us describe a possible procedure which allows a measurement of the photon polarization parameter λ_γ in the presence of the three interfering K resonance contributions. First, one has to determine the three coefficients $|c_i|$ ($i=1,2,3$) parametrizing the relative radiative decay rates into the three resonance states. In principle, this can be achieved by fitting data to the $K\pi\pi$ invariant mass distribution (45), consisting of the sum of three Breit-Wigner forms. However, this may be difficult in practice, since the masses of the three resonances are too close to each other, and statistics may be insufficient for decomposing the distribution into a sum of three different Breit-Wigner widths. A complementary way of determining the coefficients $|c_i|$ for the three resonances is then to use their spin-parity characteristic angular decay distributions into two-body or quasi-two-body states [18,19]. This analysis can be applied to $K\pi$ final states for measuring $|c_2|$ and $|c_3|$ and to $K^*\pi$ states for measuring $|c_1|$, $|c_2|$ and $|c_3|$.

Next, we write the angular distribution (44), integrated over a range in s centered around the $K_1(1400)$ resonance, as a sum over the seven functions of $\tilde{\theta}$ appearing in Eq. (44),

$$F(\tilde{\theta})\equiv\int_{s_{\min}}^{s_{\max}}ds\frac{d^2\Gamma}{ds\,d\cos\tilde{\theta}}=\sum_{i=1}^{7}d_i f_i(\tilde{\theta}), \qquad (49)$$

where $f_1(x)=1+\cos^2 x$, $f_2(x)=\cos x$, $f_3(x)=\cos^2 x+\cos^2 2x$, $f_4(x)=\cos 2x\cos x$, $f_5(x)=\sin^2 x$, $f_6(x)=\frac{1}{2}(3\cos^2 x-1)$, $f_7(x)=\cos^3 x$. The three coefficients $d_{1,3,5}$ are obtained from the corresponding values of $|c_{1,2,3}|$. For example,

$$d_1=\frac{1}{4}|c_1|^2\int_{s_{\min}}^{s_{\max}}ds\frac{\langle|\vec{J}|^2\rangle}{(s-M_{K_1})^2+M_{K_1}\Gamma_{K_1}}. \qquad (50)$$

The extraction of the remaining coefficients is slightly complicated by the fact that two of the functions $f_i(x)$ are not linearly independent

$$f_6(x)=\frac{1}{2}f_1(x)-f_5(x),$$

$$f_7(x)=\frac{1}{2}[f_2(x)+f_4(x)]. \qquad (51)$$

Nevertheless, fitting the angular distribution $F(\tilde{\theta})$ in Eq. (49) to a sum of five independent functions of $\tilde{\theta}$, allows one to determine the linear combination d_2-d_4. [Alternatively, d_2-d_4 may be obtained by projecting out the part proportional

to $\cos\tilde{\theta}$ by integration with an appropriate weight function, $\phi(\tilde{\theta})=5\cos\tilde{\theta}-7\cos^3\tilde{\theta}$.] Since this combination is proportional to λ_γ, the photon polarization parameter can be extracted from the following ratio:

$$\lambda_\gamma=\frac{d_2-d_4}{4R_1d_1-12R_2d_3}. \qquad (52)$$

VI. CALCULATIONS OF R_1 AND R_2

The parameters R_1 and R_2 measure the up-down asymmetry of the photon momentum with respect to the $K\pi\pi$ decay plane in the $K_{\rm res}$ frame. For decays dominated by K_1 and K_2^* they are defined such that $\langle\cos\tilde{\theta}\rangle=R_1\lambda_\gamma$ and $\langle\cos\tilde{\theta}\rangle=R_2\lambda_\gamma$, respectively. In the case of K_1 the integrated up-down asymmetry is given by $(3/2)R_1\lambda_\gamma$, while in the case of K_2^* the integrated asymmetry vanishes. Here we will show that R_1 and R_2 can be computed quite reliably with some dependence on the hadronic parameters describing K_1 and K_2^* decays.

We start by considering the decays of the $K_1(1400)$ resonance discussed in Sec. IV A, and discuss first decays to final states involving a neutral pion, $K^0\pi^+\pi^0$ and $K^+\pi^-\pi^0$. The parameter R_1, defined in Eqs. (47), can be calculated by applying Eqs. (23), (24), (25) and (40). Its value depends somewhat on two pairs of hadronic parameters: $|c_D/c_S|$ and $\delta_{D/S}$, the magnitude and phase of the ratio of D- to S-wave amplitudes in $K_1\to K^*\pi$, and κ_S and α_S, the magnitude and phase of the ratio of S-wave amplitudes in $K_1\to\rho K$ and $K_1\to K^*\pi$. Varying these parameters in the measured ranges, $|c_D/c_S|=1.75\pm0.22$ GeV^{-2}, $\delta_{D/S}=260°\pm20°$, $\kappa_S=0.32$

±0.32, $20°\le\alpha_S\le60°$, we find values of R_1 in the range $R_1=0.22\pm0.03$, where in Eq. (47) we integrate over the entire Dalitz plot. The uncertainty in R_1 combines an uncertainty of ±0.02 due to the ratio of D- and S-wave amplitudes in the $K^*\pi$ channel, and an uncertainty of ±0.02 due to the ρK amplitude. We conclude that the integrated up-down asymmetry originating in K_1 alone is quite large, $(0.33\pm0.05)\lambda_\gamma$.

The corresponding asymmetry in the $K^+\pi^+\pi^-$ and $K^0\pi^+\pi^-$ channels is smaller. Here only one $K^*\pi$ intermediate state contributes, and the dominant asymmetry is due to interference between D and S wave amplitudes in this channel. It is proportional to $\sin\delta_{D/S}$ and, since $\delta_{D/S}$ is not far from $3\pi/2$, one finds $R_1\approx-0.07\sin\delta_{D/S}\approx0.07$. The correction from the ρK channel may change this value by about 50%, depending on the value of the strong phase α_S.

Next, consider the decays of the $K_2^*(1430)$ resonance discussed in Sec. IV B, from which the value of R_2 defined in Eq. (47) is calculated through Eqs. (25) and (40). This value depends on the magnitude $|\kappa|$ and phase α of the ratio of $K_2^*\to\rho K$ and $K_2^*\to K^*\pi$ amplitudes, which we take as specified in Sec. IV B, $|\kappa|=2.38$, $-30°\le\alpha\le30°$. The value obtained when integrating over the entire Dalitz plot is rather small, varying as a function of α in the range $R_2=0.01-0.05$.

Larger values for R_2, which are less sensitive to corrections from ρK, are obtained by restricting the region in the Dalitz plot over which one integrates. To be specific, let us consider a square region (A), 0.71 GeV$^2\le s_{13},s_{23}\le0.89$ GeV^2, centered at the K^* mass, $s_{13}=s_{23}=M_{K^*}^2$, of sides equal to twice the K^* width. The value of R_2 in region (A) is dominated by the K^* contributions and, when neglecting the ρ contribution, is given by

$$R_{2A}=-\frac{1}{3}\frac{\langle|\vec{p}_1\times\vec{p}_2|^3{\rm Im}[B_{K*}(s_{13})B_{K*}^*(s_{23})]{\rm sgn}(s_{13}-s_{23})\rangle_A}{\langle|\vec{p}_1\times\vec{p}_2|^2|\vec{p}_1B_{K*}(s_{23})+\vec{p}_2B_{K*}(s_{13})|^2\rangle_A}=0.091. \qquad (53)$$

Including the ρ contribution modifies this value only mildly to become $R_A=0.071\pm0.002$, where we use the above values of $|\kappa|$ and α.

One can easily see why the value of R_{2A} is positive. While the variable in the denominator of Eq. (53) is positive, the one in the numerator, containing a factor $(s_{23}-s_{13}){\rm sgn}(s_{13}-s_{23})$ is negative. The quantity ${\rm Im}[B_{K*}(s_{13})B_{K*}^*(s_{23})]$ contains the relatively narrow K^* width, $\Gamma_{K*}=51$ MeV. One may imagine a higher excited 2^+ resonance decaying to a pion and a wider 1^- K-resonance state, such as $K^*(1680)$, where subsequently $K^*(1680)\to K\pi$. In this case a larger value of R_2 can be obtained due to a larger width.

VII. FEASIBILITY OF THE METHOD

In order to estimate the number of B mesons required for a feasible measurement of the photon polarization parameter

λ_γ in $B\to K\pi\pi\gamma$, let us first assume for simplicity that one is able to measure separately decays through the $K_1(1400)$ 1^+ state. This will require the least number of B's, since this resonance state was shown to lead to much larger polarization effects than the other resonance states. For final states of the types $K^0\pi^+\pi^0$ and $K^+\pi^-\pi^0$, we calculated the integrated up-down asymmetry of the photon momentum with respect to the $K\pi\pi$ decay plane and found it to be $(0.33\pm0.05)\lambda_\gamma$. In order to measure at three standard deviations an asymmetry of -0.33, as expected in the SM (where $\lambda_\gamma\approx-1$), one needs to observe a total of about 80 charged and neutral B and $\bar{B}$ decays to $K\pi\pi\gamma$ in these two channels.

When estimating the branching ratio for these events, we will assume that the exclusive B decay branching ratio into $K_1(1400)\gamma$ is 0.7×10^{-5}, as calculated in some models [17]. The decays $K_1\to K\pi\pi$ are dominated by $K^*\pi$, $\mathcal{B}(K_1(1400)\to K^*\pi)=0.94\pm0.06$, where we will take the

central value. Using isospin, one finds that 4/9 of all $K^*\pi$ events in K_1^+ and K_1^0 decays occur in the two channels $K^0\pi^+\pi^0$ and $K^+\pi^-\pi^0$, respectively. One must also include a factor 1/3 for observing a K_S (from K^0) through its $\pi^+\pi^-$ decays. Overall, we estimate an observable branching ratio of $\mathcal{B} = 0.7 \times 10^{-5} \times (4/9)0.94 \simeq 0.3 \times 10^{-5}$ into $(K^+\pi^-\pi^0)_{K_1(1400)}$ and $\mathcal{B} \simeq 0.1 \times 10^{-5}$ into $(K_S\pi^+\pi^0)_{K_1(1400)}$. These branching ratios imply that, in order to observe the necessary 80 $K\pi\pi\gamma$ events and to measure their asymmetry at 3σ, one needs at least $2 \times 10^7 B\bar{B}$ pairs, including charged and neutrals. This estimate does not include factors of efficiency and background, which may increase the number of required B's by an order of magnitude.

The $K\pi\pi$ invariant mass range $1300-1500$ MeV which we considered obtains also a contribution from the upper tail of a lower 1^+ resonance at 1270 MeV, which decays to $K^*\pi$ and ρK with branching ratios of $(16\pm5)\%$ and $(42\pm6)\%$, respectively (see Table I). In order to suppress this contribution, which would interfere with the $K_1(1400)$ amplitude, one may study the upper-half mass range, $m(K\pi\pi) = 1400-1500$ MeV, and extend it to 1600 MeV for higher statistics. This range, which includes about half of all $B \to K_1(1400)\gamma$ decays, may obtain a small contribution from the lower tail of the wide 1^- $K^*(1680)$ resonance at 1717 MeV, decaying to $K^*\pi$ and ρK with branching ratios of about 30% each. This resonance does not interfere, however, with the $K_1(1400)$ state whose strong polarization effect one is using to measure λ_γ.

In the above estimate of the required number of B mesons we have assumed separation of events originating from the $K_1(1400)$ resonance, rather than basing our estimate on the calculated decay distribution which combines the three overlapping resonances. It would be interesting to study the efficiency of the method described in Sec. V, which extracts λ_γ from the decay distribution combining all resonances. This challenging task is beyond the scope of this paper, and

should be treated more professionally by experimental methods when more data become available.

VIII. CONCLUSIONS

We studied a method for measuring in $B \to K\pi\pi\gamma$ the photon polarization parameter λ_γ occurring in the effective weak Hamiltonian describing radiative b quark decays. The SM predicts that $\lambda_\gamma \approx -1$ $(+1)$ for $B^-(B^+)$ and $\bar{B}^0(B^0)$ decays. Different values, possibly with an opposite sign, can be obtained in extensions of the SM, such as the left-right model and minimal supersymmetry. The parameter λ_γ was shown to be measured through an up-down asymmetry of the photon direction relative to the $K\pi\pi$ decay plane in the $K\pi\pi$ center of mass frame. In the SM the photon prefers to move in the hemisphere of $\vec{p}_{\text{slow}} \times \vec{p}_{\text{fast}}$ in B^- and $\bar{B}^0$ decays, and in the opposite hemisphere in B^+ and B^0 decays.

We studied the amplitudes of $B \to K\pi\pi\gamma$ in the K resonance region for a few distinct charged modes. Combining contributions from several overlapping resonances in a mass range near 1400 MeV, $K_1(1400)$, $K_2^*(1430)$ and $K^*(1410)$, the general decay distribution was calculated. A method was proposed for using this distribution to determine the photon polarization parameter. Based on an up-down asymmetry of $(0.33 \pm 0.05)\lambda_\gamma$ from $K_1(1400)$ alone, we conclude that a first measurement of λ_γ can be performed with about 10^8 $B\bar{B}$ pairs, combining charged and neutrals. This study can be performed at currently operating B factories.

[1] CLEO Collaboration, M.S. Alam *et al.*, Phys. Rev. Lett. **74**, 2885 (1995); S. Chen *et al.*, *ibid.* **87**, 251807 (2001); ALEPH Collaboration, R. Barate *et al.*, Phys. Lett. B **429**, 169 (1998); Belle Collaboration, K. Abe *et al.*, *ibid.* **511**, 151 (2001).

[2] See, e.g., P. Gambino, J. Phys. G **27**, 1199 (2001), and references therein.

[3] M. Misiak, Proceedings of the XXXVIth Rencontres de Moriond, Les Arcs, 2001, and references therein, hep-ph/0106050.

[4] B. Grinstein and D. Pirjol, Phys. Rev. D **62**, 093002 (2000); M. Beneke, T. Feldmann, and D. Seidel, Nucl. Phys. **B612**, 25 (2001); S.W. Bosch and G. Buchalla, *ibid.* **B621**, 459 (2002); A. Ali and A.Y. Parkhomenko, Eur. Phys. J. C **23**, 89 (2002).

[5] K. Fujikawa and A. Yamada, Phys. Rev. D **49**, 5890 (1994); K.S. Babu, K. Fujikawa, and A. Yamada, Phys. Lett. B **333**, 196 (1994); P. Cho and M. Misiak, Phys. Rev. D **49**, 5894 (1994). For a review of B physics in LR models, see M. Gronau, in *B Decays*, 2nd ed., edited by S. Stone (World Scientific, Singapore, 1994), p. 644.

[6] L. Everett, G.L. Kane, S. Rigolin, L.T. Wang, and T.T. Wang, J. High Energy Phys. **01**, 022 (2002); for a review of B physics in SUSY models, see S. Bertolini, F. Borzumati, and A. Masiero, in *B Decays*, 2nd ed., edited by S. Stone (World Scientific, Singapore, 1994), p. 620.

[7] For a brief review, see M. Gronau, in Proceeding of the Ninth International Symposium on Heavy Flavor Physics, 2001, Pasadena, CA, hep-ph/0110316.

[8] D. Atwood, M. Gronau, and A. Soni, Phys. Rev. Lett. **79**, 185 (1997).

[9] CLEO Collaboration, R. Ammar *et al.*, Phys. Rev. Lett. **71**, 674 (1993); T.E. Coan *et al.*, *ibid.* **84**, 5283 (2000); BaBar Collaboration, B. Aubert *et al.*, *ibid.* **88**, 101805 (2002); Belle Collaboration, A. Ishikawa, talk at XXXVIIth Rencontres de Moriond on Electroweak Interactions and Unified Theories, Les Arcs, France, 2002.

[10] D. Melikhov, N. Nikitin, and S. Simula, Phys. Lett. B **442**, 381 (1998); F. Krüger, L.M. Sehgal, N. Sinha, and R. Sinha, Phys.

Rev. D **61**, 114028 (2000); C.S. Kim, Y.G. Kim, C.D. Lü, and T. Morozumi, *ibid.* **62**, 034013 (2000).

[11] Y. Grossman and D. Pirjol, J. High Energy Phys. **06**, 029 (2000).

[12] A. Ali, P. Ball, L.T. Handoko, and G. Hiller, Phys. Rev. D **61**, 074024 (2000) and references therein.

[13] T. Mannel and S. Recksiegel, Acta Phys. Pol. B **28**, 2489 (1997).

[14] G. Hiller and A. Kagan, Phys. Rev. D **65**, 074038 (2002).

[15] M. Gronau, Y. Grossman, D. Pirjol, and A. Ryd, Phys. Rev. Lett. **88**, 051802 (2002).

[16] T. Islam and A. Weinstein, CLEO Internal Note, 1999.

[17] T. Altomari, Phys. Rev. D **37**, 677 (1988); A. Ali, T. Ohl, and T. Mannel, Phys. Lett. B **298**, 195 (1993); D. Atwood and A. Soni, Z. Phys. C **64**, 241 (1994); S. Veseli and M.G. Olsson, Phys. Lett. B **367**, 309 (1996); D. Ebert, R.N. Faustov, V.O. Galkin, and H. Toki, *ibid.* **495**, 309 (2000); Phys. Rev. D **64**,

054001 (2001); A.S. Safir, Eur. Phys. J. C **15**, 1 (2001).

[18] CLEO Collaboration, T.E. Coan *et al.*, Phys. Rev. Lett. **84**, 5283 (2000).

[19] Belle Collaboration, A. Ishikawa, Ref. [9].

[20] Particle Data Group, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).

[21] J.H. Kühn and F. Wagner, Nucl. Phys. **B236**, 16 (1984); J.H. Kühn and A. Santamaria, Z. Phys. C **48**, 445 (1990).

[22] N. Isgur, C. Morningstar, and R. Leader, Phys. Rev. D **39**, 1357 (1989); M. Feindt, Z. Phys. C **48**, 681 (1990).

[23] ARGUS Collaboration, H. Albrecht *et al.*, Phys. Lett. B **250**, 164 (1990); OPAL Collaboration, K. Ackerstaff *et al.*, Z. Phys. C **75**, 593 (1997); DELPHI Collaboration, P. Abreu *et al.*, Phys. Lett. B **426**, 411 (1998); CLEO Collaboration, D.M. Asner *et al.*, Phys. Rev. D **61**, 012002 (2000).

[24] C. Daum *et al.*, Nucl. Phys. **B187**, 1 (1981).