

## Deviation of neutrino mixing from bimaximal mixing

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We study how observables of the neutrino mixing matrix can link up with those in the quark sector. The deviation from the bimaximal flavor mixing is parametrized using a  $3 \times 3$  unitary matrix. The neutrino mixings are investigated supposing this unitary matrix to be hierarchical like the quark mixing matrix. We obtain the remarkable prediction  $|U_{e3}| \geq 0.03$  from the experimentally allowed range  $\tan^2 \theta_{\text{sol}} = 0.24 - 0.89$ . The  $CP$  violation in neutrino oscillations is expected to be very small.

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Recent data from the Super-Kamiokande [1,2] and SNO (Sudbury Neutrino Observatory) [3] experiments have provided model independent evidence in favor of oscillations of atmospheric and solar neutrinos. The disappearance of atmospheric  $\nu_\mu$ 's measured in the Super-Kamiokande experiment [1] has been confirmed by the MACRO [4] and Soudan 2 [5] experiments, and by the K2K long-baseline experiment [6]. The Super-Kamiokande [7] and MACRO [4] atmospheric neutrino data favor the  $\nu_\mu \rightarrow \nu_\tau$  process. The solar neutrino data of the Super-Kamiokande [2] and SNO [3] experiments show in a model independent way that  $\nu_e \rightarrow \nu_{\mu,\tau}$  transitions take place. This evidence agrees with the comparison of the solar standard model [8] predictions with the data of the other solar neutrino experiments (Homestake, GALLEX, SAGE, GNO [9]). The global analysis of all solar neutrino data in terms of  $\nu_e \rightarrow \nu_{\mu,\tau}$  oscillations [10,11] favor strongly the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) solution [12].

These experimental results indicate that neutrinos are massive and mixed particles [13,14] and the flavor mixing of neutrinos is bilarge, i.e., close to bimaximal [15]. This means that the neutrino flavor mixing is very different from the one in the quark sector. It is therefore important to investigate how the observables of the neutrino mixing matrix [16] can link up with those in the quark sector [17].

In the recent experimental data, the neutrino flavor mixings deviate from the bimaximal flavor mixing as follows [11,18]:

$$\begin{aligned} \sin^2 2\theta_{\text{atm}} &> 0.83 \quad (99\% \text{ C.L.}), \\ \tan^2 \theta_{\text{sol}} &= 0.24 - 0.89 \quad (99.73\% \text{ C.L.}). \end{aligned} \quad (1)$$

One may consider seriously the deviation [19] from bimaximal flavor mixing [15].

In this paper, we discuss the deviation from bimaximal flavor mixing of neutrinos by linking it up phenomenologically with the quark flavor mixing. In our naive understanding it is natural that the charged lepton mass matrix has a structure similar to the quarks mass matrices. On the other hand, the neutrino mass matrix has special structure, like the possibility to be Majorana particles. Therefore the neutrino mixing matrix is the very different from the Cabibbo-Kobayashi-Maskawa (CKM) matrix. In the standpoint of this naive understanding, the deviation from the bimaximal links up with the quark mixing.

Let us consider the bimaximal flavor mixing as follows:

$$\nu_\alpha = U_{\alpha i}^{(0)} \nu_i, \quad (2)$$

where

$$U^{(0)} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (3)$$

In general, it is not natural to expect a bimaximal flavor mixing without some model or symmetry. We suppose that the bimaximal flavor mixing in Eq. (3) is guaranteed by an appropriate flavor symmetry,<sup>1</sup> although we do not discuss such symmetry in this paper.

One can parametrize the deviation  $U^{(1)}$  in  $\nu_\alpha = [U^{(1)\dagger} U^{(0)}]_{\alpha i} \nu_i$  as follows:<sup>2</sup>

<sup>1</sup>For example, one may consider  $L_e - L_\mu - L_\tau$  symmetry [20].

<sup>2</sup>We take  $U^{(1)\dagger}$  in order to compare with quark mixings.

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$$U^{(1)} = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\phi} \\ -c_{23}s_{12} - s_{23}s_{13}c_{12}e^{i\phi} & c_{23}c_{12} - s_{23}s_{13}s_{12}e^{i\phi} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}s_{13}c_{12}e^{i\phi} & -s_{23}c_{12} - c_{23}s_{13}s_{12}e^{i\phi} & c_{23}c_{13} \end{pmatrix}, \quad (4)$$

where  $s_{ij} \equiv \sin \theta_{ij}$  and  $c_{ij} \equiv \cos \theta_{ij}$  denote the mixing angles in the bimaximal basis and  $\phi$  is the  $CP$  violating Dirac phase. The mixings  $s_{ij}$  are expected to be small since these are deviations from the bimaximal mixing. Here, the Majorana phases are absorbed in the neutrino mass eigenvalues.

Let us assume the mixings  $s_{ij}$  to be hierarchical like the ones in the quark sector,  $s_{12} \gg s_{23} \gg s_{13}$ . Then, taking the leading contribution due to  $s_{12}$ , we have

$$\begin{aligned} |U_{e1}| &\approx \frac{1}{\sqrt{2}} \left( c_{12} + \frac{1}{\sqrt{2}} s_{12} \right), \\ |U_{e2}| &\approx \frac{1}{\sqrt{2}} \left( c_{12} - \frac{1}{\sqrt{2}} s_{12} \right), \quad |U_{e3}| \approx \frac{1}{\sqrt{2}} s_{12}, \end{aligned} \quad (5)$$

which lead to

$$\begin{aligned} \tan^2 \theta_{\text{sol}} &\approx \left( \frac{c_{12} - \frac{1}{\sqrt{2}} s_{12}}{c_{12} + \frac{1}{\sqrt{2}} s_{12}} \right)^2 \\ &= 1 - 2\sqrt{2}s_{12} + \mathcal{O}(s_{12}^2). \end{aligned} \quad (6)$$

Thus, the solar neutrino mixing is somewhat reduced due to  $s_{12}$ . Taking the limits in Eq. (1), we get the allowed region  $s_{12} = 0.04 - 0.43$ .<sup>3</sup> On the other hand, the experimental upper bound  $|U_{e3}| < 0.2$  obtained from the results of the CHOOZ experiment [21] (see Ref. [22]) gives the limit  $s_{12} < 0.28$ . In conclusion, we get the allowed region

$$s_{12} = 0.04 - 0.28, \quad (7)$$

which implies

$$|U_{e3}| = 0.03 - 0.2. \quad (8)$$

Let us emphasize the lower bound for  $|U_{e3}|$ , which implies that  $|U_{e3}|$  could be measured in the JHF-Kamioka long-baseline neutrino oscillation experiment [23,24], which has a planned sensitivity of  $|U_{e3}| \approx 0.04$  at 90% C.L. in the first phase with the Super-Kamiokande detector and  $|U_{e3}| < 10^{-2}$  in the second phase with the Hyper-Kamiokande detector [23].

Next, taking the leading term due to  $s_{23}$  and neglecting  $s_{13}$ , we have

$$\begin{aligned} |U_{\mu 3}| &\approx \frac{1}{\sqrt{2}} c_{12} (c_{23} - s_{23}), \\ |U_{\tau 3}| &\approx \frac{1}{\sqrt{2}} (c_{23} + s_{23}), \end{aligned} \quad (9)$$

which give  $\sin \theta_{\text{atm}}$  and  $\cos \theta_{\text{atm}}$  as follows:

$$\sin^2 \theta_{\text{atm}} = \frac{|U_{\mu 3}|^2}{1 - |U_{e3}|^2}, \quad \cos^2 \theta_{\text{atm}} = \frac{|U_{\tau 3}|^2}{1 - |U_{e3}|^2}. \quad (10)$$

Then,  $\sin^2 2\theta_{\text{atm}}$  is given as

$$\begin{aligned} \sin^2 2\theta_{\text{atm}} &\approx [1 - s_{12}^2 (1 - (c_{23} - s_{23})^2)] (1 - 2s_{23}^2)^2 \\ &= 1 - \mathcal{O}(s_{12}^4 \sim s_{23}^2). \end{aligned} \quad (11)$$

Since we have  $s_{12}^4 \leq 6 \times 10^{-3}$  from the upper bound in Eq. (7), we predict in practice

$$\sin^2 2\theta_{\text{atm}} = 1. \quad (12)$$

Thus the quarklike mixing of  $U^{(1)}$  is nicely consistent with the experimental data.

The  $CP$  violation originates from the phase  $\phi$  in  $U^{(1)}$ . Keeping  $s_{13}$  in the expression of  $U_{e3}$ , we get

$$\arg[U_{e3}] \approx -\arctan \left[ \frac{s_{13} \sin \phi}{s_{12}} \right], \quad (13)$$

which is the  $CP$  violating phase in the standard parametrization.<sup>4</sup> This phase is very small as far as  $s_{12} \gg s_{13}$ . Let us estimate the Jarlskog invariant as a measure of  $CP$  violation [25]:

$$J = \text{Im}[U_{e2}^* U_{e3} U_{\mu 2} U_{\mu 3}^*], \quad (14)$$

which is written as

$$\begin{aligned} J &= \frac{1}{4\sqrt{2}} c_{13} s_{13} (c_{23}^2 - s_{23}^2) [(c_{12}^2 - s_{12}^2) (c_{23} + s_{23}) \sin \phi \\ &\quad + c_{12} s_{12} s_{13} (c_{23} - s_{23}) \sin 2\phi]. \end{aligned} \quad (15)$$

If we assume the hierarchy of mixing angles

$$s_{13} \ll s_{23} \ll s_{12} \ll 1, \quad (16)$$

<sup>3</sup>The  $\mathcal{O}(s_{12}^2)$  terms are taken in order to estimate the upper bound because those becomes important in the case of  $s_{12} \geq 0.3$ .

<sup>4</sup>The mixing matrix  $U^{(1)\dagger} U^{(0)}$  can be reduced to the standard form with real  $U_{e1}$ ,  $U_{e2}$ ,  $U_{\mu 3}$ ,  $U_{\tau 3}$  through a rephase of the charged lepton and neutrino fields that does not change the phase of  $U_{e3}$ .

as in the quark sector, the leading contribution to  $J$  is given by

$$J \simeq \frac{1}{4\sqrt{2}} s_{13} \sin \phi. \quad (17)$$

If  $s_{13}$  and  $\phi$  are fixed, one can quantify the smallness of the  $CP$  violation comparing  $J$  with its upper limit [26]

$$J \leq \frac{1}{6\sqrt{3}}. \quad (18)$$

Let us present numerical predictions of the mixings and the  $CP$  violating phase. If the deviation is comparable to the flavor mixing of the quark sector, the Wolfenstein parametrization is useful [27]:

$$U^{(1)} = \begin{pmatrix} 1 - \frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (19)$$

where  $\lambda$ ,  $A$ ,  $\rho$  and  $\eta$  are independent of ones in the quark sector.

In order to estimate the neutrino mixings, we try to take the same values of the quark mixings. Putting typical values of the CKM matrix elements [17],

$$\lambda = 0.22, \quad A = 0.83, \quad \rho = 0.2, \quad \eta = 0.4, \quad (20)$$

we predict the neutrino mixing matrix  $U = U^{(1)\dagger} U^{(0)}$  as

$$|U| = \begin{pmatrix} 0.80 & 0.58 & 0.15 \\ 0.35 & 0.66 & 0.66 \\ 0.48 & 0.48 & 0.74 \end{pmatrix}, \quad (21)$$

which leads to

$$\sin^2 2\theta_{\text{atm}} = 0.99, \quad \tan^2 \theta_{\text{sol}} = 0.45. \quad (22)$$

These predictions are nicely consistent with the experimental bounds in Eq. (1). The solar neutrino mixing is reduced due to  $s_{12}$ , while the atmospheric neutrino mixing is not reduced as seen in Eq. (11). The prediction  $|U_{e3}| = 0.15$  is not much below the experimental upper bound  $\simeq 0.2$ . Therefore, it will not be difficult to test this prediction in the near future, for example in the JHF-Kamioka neutrino experiment [23,24].

Let us present the  $\lambda$  dependence of our results. We show in Fig. 1 the predictions for  $\tan^2 \theta_{\text{sol}}$  and  $\sin^2 2\theta_{\text{atm}}$  as functions of  $|U_{e3}|$ , obtained varying  $\lambda$  from 0 to 0.28 and keeping the values of the other parameters given in Eq. (20). One can see that  $|U_{e3}|$  is predicted to be larger than about 0.03 [see Eq. (8)], under the condition that  $\tan^2 \theta_{\text{sol}} \leq 0.89$ .

We can predict the amount of  $CP$  violation in neutrino oscillations. If we assume that

$$s_{13} \sim s_{12}^3, \quad (23)$$

as in the quark sector, from the upper bound for  $s_{12}$  obtained from solar neutrino data ( $s_{12} < 0.28$ ), we obtain

$$J_{\text{max}} \sim 4 \times 10^{-3}, \quad (24)$$

which is about  $4 \times 10^{-2}$  times smaller than the maximum possible value of  $J$  in Eq. (18). It is interesting to compare the value of  $J_{\text{max}}$  that we have obtained in Eq. (24) with the maximum value of  $J$  that is possible in a general quasi-bimaximal mixing scheme, i.e., a scheme with

$$|U_{e1}| \simeq |U_{e2}| \simeq |U_{\mu 3}| \simeq |U_{\tau 3}| \simeq \frac{1}{\sqrt{2}}, \quad |U_{e3}| \ll 1. \quad (25)$$

In such scheme  $J$  is approximately given by

$$J^{(\text{bimax})} \simeq \frac{1}{4} \text{Im}[U_{e3}^*], \quad (26)$$

and its maximum possible value is

$$J_{\text{max}}^{(\text{bimax})} \simeq \frac{1}{4} |U_{e3}|. \quad (27)$$

Taking the bound

$$|U_{e3}| < 0.2, \quad (28)$$

obtained from CHOOZ data, we have

$$J_{\text{max}}^{(\text{bimax})} \simeq 5 \times 10^{-2}. \quad (29)$$

Comparing Eqs. (24) and (29) one can see that the maximum value of the Jarlskog invariant  $J$  in our scheme is about an order of magnitude smaller than the maximum value of  $J$  in a generic quasibimaximal scheme. Therefore, the  $CP$  violation seems too small to be measured in JHF [23,24], but maybe it can be measured in a neutrino factory [28]. On the other hand, the Majorana phases are not constrained, but un-

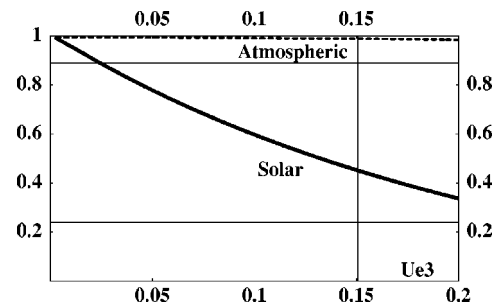


FIG. 1. Predictions in the  $|U_{e3}| - \tan^2 \theta_{\text{sol}}$  plane and  $|U_{e3}| - \sin^2 2\theta_{\text{atm}}$  plane. The thick solid curve corresponds to  $\tan^2 \theta_{\text{sol}}$ , while the dashed one to  $\sin^2 2\theta_{\text{atm}}$ . Horizontal lines delimit the experimental allowed regions for solar neutrinos. The parameter  $\lambda$  is varied from 0 to 0.28. The vertical line around  $|U_{e3}| = 0.15$  corresponds to the result in the case of  $\lambda = 0.22$ .

fortunately they are not measurable in neutrino oscillation experiments.

We summarize as follows. We have studied how observables of the neutrino mixing matrix can link up with the ones in the quark sector. The deviation from the bimaximal flavor mixing is parametrized by a  $3 \times 3$  unitary matrix. Supposing that this unitary matrix is similar to the quark mixing matrix, we predict the neutrino mixings, which are consistent with the experimental data. The element  $U_{e3}$  of the neutrino mixing matrix is predicted to be larger than 0.03 by using the

experimental bound on the solar neutrino mixing. When more solar neutrino data will be available in the near future, a more precise prediction will be given for  $U_{e3}$ . For instance, if we use  $\tan^2 \theta_{\text{sol}} \leq 0.58$  (90% C.L. at present), we predict  $|U_{e3}| \geq 0.11$ . The violation of  $CP$  is predicted to be very small. Thus, the measurements of the solar neutrino mixing and  $U_{e3}$  [23,24] will present a crucial test for our scheme.

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