Texture of neutrino mass matrix in view of recent neutrino experimental results

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In view of recent neutrino experimental results such as SNO, Super-Kamiokande (SK), CHOOZ and neutrinoless double beta decay $(\beta\beta_{0\nu})$, we consider a texture of neutrino mass matrix which contains three parameters in order to explain those neutrino experimental results. We first fitted the parameters in a model independent way with solar and atmospheric neutrino mass squared differences and a solar neutrino mixing angle which satisfies the LMA solution. The maximal value of the atmospheric neutrino mixing angle comes out naturally in the present texture. Most interestingly, the fitted parameters of the neutrino mass matrix considered here also marginally satisfy the recent limit on the effective Majorana neutrino mass obtained from the neutrinoless double beta decay experiment. We further demonstrate an explicit model which gives rise to the texture investigated by considering an $SU(2)_L\times U(1)_Y$ gauge group with two extra real scalar singlets and a discrete $Z_2\times Z_3$ symmetry. Majorana neutrino masses are generated through higher dimensional operators at the scale M. We have estimated the scales at which singlets get VEV's and M by comparing with the best fitted results obtained in the present work.

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I. INTRODUCTION

A recent global analysis [1] including SNO experimental results containing neutral current data of the solar neutrino flux, day night effect, and higher statistics data of charged current neutrino electron scattering rate [2], has disfavored the well known conjecture of bimaximal neutrino mixing [3,4] by considering the solution of the solar neutrino problem through the two flavor neutrino oscillation scenario. It has been shown that the best fit global oscillation parameters with all solar neutrino experimental data strongly in favor of the large mixing angle (LMA) Mikheyev-Smirnov-Wolfenstein (MSW) oscillation solution of the solar neutrino deficit and the best fit result comes out as $\Delta m^2_{\odot}=5.0\times 10^{-5}\ e\mathrm{V}^2$, $\tan^2\theta_{\odot}=4.2\times 10^{-1}$ with the value of $\chi^2_{\rm min}=45.5$ and g.o.f. $=49\%$. Although the atmospheric neutrino oscillation mixing angle $\theta_{\rm atm}$ is maximal as observed by Super-Kamiokande (SK) [5,6], the LMA solution for the solar neutrino oscillation is best fitted with a considerably lower value of $\theta_{\odot}$. If we identify the $\theta_{\odot}$ as θ_{12} and $\theta_{\rm atm}$ as θ_{23} then the CHOOZ [7] experiment has also constrained the third mixing angle $\theta_{13}<13^o$. Furthermore, recent result from neutrinoless double beta decay $(\beta\beta_{0\nu})$ experiment [8] has reported the bound on the effective Majorana neutrino mass (by considering uncertainity of the nuclear matrix elements up to $\pm 50\%$ and the contribution to this process due to particles other than Majorana neutrino is negligible [9]) as

$$\langle m\rangle=(0.05-0.84)\ \text{eV at 95\% C.L.} \tag{1.1}$$

Although there is much literature investigating the implications of the above experimental result, the claim is still controversial [10]. In the present work, we go optimistically with the result and it is important to note that our analyses crucially depend on the future results of MOON, EXO, and 1 ton and 10 ton GENIUS double beta decay experiments. If the lower limit of the Majorana neutrino mass goes below the value presented in Eq. (1), the texture and the model considered here will be ruled out provided there must not be significant changes in the present solar and atmospheric neutrino experimental results. Two parameter texture of neutrino mass matrix [4,11] which naturally gives bimaximal neutrino mixing is disfavored by the recent SNO experimental results. It needs more parameters in the neutrino mass matrix in order to explain the present neutrino experimental results [12]. Moreover, in order to satisfy the limit on effective Majorana neutrino mass the $\nu_e\nu_e$ element of the neutrino mass matrix should be nonzero. All these results need further modification of two parameter neutrino mass matrix.

In the present work, we consider a three parameter texture of neutrino mass matrix which can accommodate the present experimental results. These three parameters are fixed by defining a function χ^2_p (see later) as the sum of squares of the differences between the calculated values of neutrino oscillation parameters (with the texture considered) and the best fitted values of the same (obtained from different analyses) and then minimizing this function. We then propose an explicit model based on an $SU(2)_L\times U(1)_Y$ gauge group with two additional singlet real scalar fields and discrete $Z_2\times Z_3$ symmetry. Neutrino mass is generated in our model through higher dimensional terms, the scale of which is fixed through the best fit result.

II. A TEXTURE

Keeping in mind the charged lepton mass matrix is diagonal, consider the following texture of the Majorana neutrino mass matrix:

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$$M_\nu=\lambda\begin{pmatrix} b & a & a \\ a & 1 & 1 \\ a & 1 & 1 \end{pmatrix}, \tag{2.1}$$

where λ, a, and b are all real. It is to be noted that a more general form of the above neutrino mass matrix is presented in Ref. [11] from which under certain conditions of model parameters the above form can be obtained. Next, particularly, due to the choice of a and b parameters as real, the above neutrino mass matrix gives no CP violation effects in the leptonic sector. Furthermore, the $\nu_e\nu_e$ element of the M_ν is nonzero hence, it should give rise to $\beta\beta_{0\nu}$ decay. We will estimate the constraint on the $\nu_e\nu_e$ element from $\beta\beta_{0\nu}$ decay after fitting the solar and atmospheric neutrino experimental results. Moreover, the above neutrino mass matrix can be generated either by radiative way or by nonrenormalizable operators.

Diagonalizing the above neutrino mass matrix M_ν by an orthogonal transformation as

$$O^T M_\nu O=\mathrm{Diag}(m_1,m_2,m_3), \tag{2.2}$$

where

$$O=\begin{pmatrix} c_{31}c_{12} & c_{31}s_{12} & s_{31} \\ -c_{23}s_{12} & c_{23}c_{12} & s_{23}c_{31} \\ -s_{23}s_{31}c_{12} & -s_{23}s_{31}s_{12} & \\ s_{23}s_{12} & -s_{23}c_{12} & c_{23}c_{31} \\ -c_{23}s_{31}c_{12} & -c_{23}s_{31}s_{12} & \end{pmatrix}, \tag{2.3}$$

we obtain the following mixing angles:

$$\theta_{23}=-\pi/4, \theta_{31}=0, \tag{2.4a}$$

$$\tan^2\theta_{12}=\frac{\lambda b-m_1}{m_2-\lambda b} \tag{2.4b}$$

and the eigenvalues are

$$m_1=\frac{\lambda}{2}\{(2+b)+\sqrt{(2-b)^2+8a^2}\},$$

$$m_2=\frac{\lambda}{2}\{(2+b)-\sqrt{(2-b)^2+8a^2}\},$$

$$m_3=0. \tag{2.5}$$

Furthermore, in terms of these eigenvalues the mixing matrix O can be written as

$$O=\begin{pmatrix} c_{12} & s_{12} & 0 \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_{12}}{\sqrt{2}} & \frac{c_{12}}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$=\begin{pmatrix} \sqrt{\frac{m_2-\lambda b}{m_2-m_1}} & \sqrt{\frac{\lambda b-m_1}{m_2-m_1}} & 0 \\ -\frac{1}{\sqrt{2}}\sqrt{\frac{\lambda b-m_1}{m_2-m_1}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_2-\lambda b}{m_2-m_1}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}}\sqrt{\frac{\lambda b-m_1}{m_2-m_1}} & \frac{1}{\sqrt{2}}\sqrt{\frac{m_2-\lambda b}{m_2-m_1}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{2.6}$$

We set the solar and atmospheric neutrino mass squared differences as

$$\Delta m^2_{\mathrm{sol}}=\Delta m^2_{21}=m_1^2-m_2^2,$$

$$=\lambda^2(2+b)\sqrt{(2-b)^2+8a^2}, \tag{2.7a}$$

and

$$\Delta m^2_{\mathrm{atm}}=\Delta m^2_{23}=m_2^2-m_3^2=m_2^2. \tag{2.7b}$$

The best fit values of oscillation parameters from solar neutrino experiments and atmospheric neutrino experiments are used to obtain the values of a, b, and λ. For this purpose, we consider Δm^2_{12}, the difference of the square of mass eigenstates m_1 and m_2 and the mixing angle θ_{12} are responsible for solar neutrino oscillation and Δm^2_{23} and θ_{23} are responsible for oscillation of atmospheric neutrinos. A recent global analysis by Bahcall *et al.* [1] of the solar neutrino data from all solar neutrino experiments, namely, chlorine, gallium, Super-Kamiokande, and SNO charged current including the recently published SNO neutral current data, shows that large mixing angle solution is most favored for solar neutrino oscillation. According to this analysis $\Delta m^2_\odot(\equiv\Delta m^2_{12}$ for our model) $=5\times10^{-5}$ and $\tan^2\theta_\odot(\equiv\tan^2\theta_{12}$ for our model) $=0.42$. From the analysis of atmospheric neutrino oscillation data [5] we have the best fit values of $\Delta m^2_{\mathrm{atm}}(\equiv\Delta m^2_{23}$ for our model) $=3.1\times10^{-3}$ eV^2 and θ_{atm} maximal. This value of θ_{atm} has already been obtained in our model for θ_{23}. Thus treating a, b, and λ as parameters we can obtain different values of Δm^2_{12}, Δm^2_{23}, and $\tan^2\theta_{12}$ and compare them with best fit values of those quantities, namely, $\Delta m^2_\odot$, $\Delta m^2_{\mathrm{atm}}$, and $\tan^2\theta_\odot$ obtained from solar and atmospheric neutrino analysis of data (discussed above) to fix a, b, and λ. To this end we define a function

$$\chi^2_p=(\Delta m^2_{12}-\Delta m^2_\odot)^2+(\Delta m^2_{23}-\Delta m^2_{\mathrm{atm}})^2$$
$$+(\tan^2\theta_{12}-\tan^2\theta_\odot)^2. \tag{2.8}$$

The function χ^2_p as defined above is calculated for a wide range of values of a, b, and λ and the minimum of the function is obtained. The corresponding values of a, b, and λ are given below.

$$\text{Minimum } \chi^2_p=1.4\times10^{-8},$$

$$a=0.0142,$$

TABLE I. Best fitted values for neutrino oscillation parameters obtained in the present work. The values estimated in Refs. [1,5] are also given.

Present work	Bahcall *et al.* [1]
Δm^2_{12}	$\Delta m^{2}_{\odot}$
(eV^2)	(eV^2)
1.39×10^{-4}	5.0×10^{-5}
Δm^2_{23}	$\Delta m^2_{\mathrm{atm}}$
(eV^2)	(eV^2)
3.1×10^{-3}	3.1×10^{-3} [5]
$\tan^2\theta_{12}$	$\tan^2\theta_{\odot}$
0.42	0.42

$$b=2.018,$$

$$\lambda=0.028 \ \ \mathrm{eV}. \tag{2.9}$$

Δm^2_{12}, Δm^2_{23}, and $\tan^2\theta_{12}$ obtained from the above values of a, b and λ and their comparison with the best fit values for $\Delta m^2_{\odot}$, $\Delta m^2_{\mathrm{atm}}$, and $\tan^2\theta_{\odot}$ obtained from recent analysis of the solar and atmospheric neutrino data are shown in Table I. In order to find out the range of values of a and b that satisfy the 3σ limits of $\Delta m^2_{\odot}$ and $\tan^2\theta_{\odot}$ for LMA solution [Eqs. (2.1) and (2.3) of Ref. [1]] of combined analysis of recent solar neutrino data (including SNO neutral current data), we have fixed the value of λ at 0.028 [see Eq. (2.9)] and varied a and b so that Δm^2_{12} and $\tan^2\theta_{12}$ satisfy the allowed LMA solution range mentioned above. This range as given in Ref. [1] is $2.3\times10^{-5}<\Delta m^2_{\odot}<3.7\times10^{-4}$ and $0.24<\tan^2\theta_{\odot}<0.89$. In doing this Δm^2_{23} remains fixed at 3.1×10^{-3}, the value for atmospheric neutrino solution. The allowed region in parameter space of a and b is shown in Fig. 1. We find that the allowed parameter space is very sensitive on λ, and there is not much freedom to vary λ within a wide range. Next, we consider the bound on the $\nu_e\nu_e$ matrix element of M_ν from $\beta\beta_{0\nu}$ decay experiment. In the present work, after fitting all three parameters with solar and atmospheric neutrino experimental results, we find that the value of effective neutrino mass comes out as

$$\langle m_\nu\rangle=\lambda b=0.05 \ \ \mathrm{eV} \tag{2.10}$$

which is marginally at the lower end of the experimental value. Such value may be accidental or may have some deeper meaning; however, it is quite interesting to note that the testability of the present texture crucially lies on the future result of the $\beta\beta_{0\nu}$ experiment.

III. A MODEL

We consider an $SU(2)_L\times U(1)_Y$ model with two additional singlet real scalar fields and discrete $Z_2\times Z_3$ symmetry. The representation content of the leptonic and scalar fields is given in Table II. Apart from the standard model (SM) Higgs doublet, the extra singlets considered in the present model give rise to three parameters in the neutrino mass matrix. The charged lepton masses generated in the present model are similar to those in SM. To make the charged lepton mass matrix flavor diagonal we consider a reflection symmetry on the lepton-Higgs Yukawa coupling f_{ij} ($i,j=1,2,3$ flavor indices) as

$$f_{ij}\leftrightarrow f_{ji}, i\neq j. \tag{3.1}$$

We consider soft discrete symmetry breaking terms (Dim $\leqslant 3$) in the scalar potential of the model, and, hence, none of the vacuum expectation values (VEV) are zero upon minimization of the scalar potential [4,13,14]. In the present model, Majorana neutrino masses are obtained through higher di-

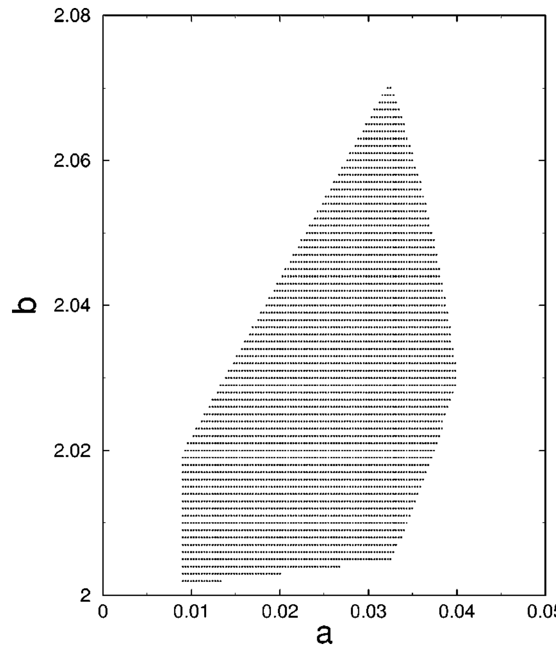

FIG. 1. The region (shaded area) of the parameters a and b that produce the values of Δm^2_{12} and $\tan^2\theta_{12}$ within 3σ range of the best fitted values from global solar neutrino data analysis [1]. The value for λ is kept fixed at the best fit value in the present calculation. Δm^2_{23} remains fixed at the best fit value for $\Delta m^2_{\mathrm{atm}}$. See text for details.

TABLE II. Representation content of the lepton and scalar fields considered in the present model. The elements of Z_2 and Z_3 are given by $\{1,-1\}$, $\{1,\omega,\omega^2\}$ respectively. In general, elements of Z_n are given by $e^{2\pi i/n}$.

Fields	$SU(2)_L\times U(1)_Y$	Z_2	Z_3
Leptons			
l_{1L}	$(2,-1)$	1	ω
l_{2L}	$(2,-1)$	-1	1
l_{3L}	$(2,-1)$	-1	1
e_R	$(1,-2)$	1	ω
μ_R	$(1,-2)$	1	1
τ_R	$(1,-2)$	-1	1
Scalars			
ϕ_1	(2,1)	1	1
η_1	(1,0)	1	ω
η_2	(1,0)	-1	ω^2

mensional terms due to explicit violation of lepton number. The most general lepton-scalar Yukawa interaction in the present model generating Majorana neutrino masses is given by

$$L_Y^\nu = \frac{l_{1L}l_{1L}\phi\phi\eta_1}{M^2} + \frac{l_{1L}l_{2L}\phi\phi\eta_2}{M^2} + \frac{l_{1L}l_{3L}\phi\phi\eta_2}{M^2} + \frac{l_{2L}l_{2L}\phi\phi}{M} + \frac{l_{2L}l_{3L}\phi\phi}{M} + \frac{l_{3L}l_{3L}\phi\phi}{M} \tag{3.2}$$

and the charged lepton masses are generated through the following interaction:

$$L_Y^E = f_{11}\overline{l}_{1L}e_R\phi + f_{22}\overline{l}_{2L}\mu_R\phi + f_{33}\overline{l}_{3L}\tau_R\phi + \text{H.c.} \tag{3.3}$$

All other terms in Eq. (3.2) are prohibited due to discrete $Z_2 \times Z_3$ symmetry and reflection symmetry mentioned in Eq. (3.1). Substituting VEV's of the scalar fields in Eqs. (3.3) and (3.2) we obtain, respectively,

$$M_E = \begin{pmatrix} f_{11}\langle\phi\rangle & 0 & 0 \\ 0 & f_{22}\langle\phi\rangle & 0 \\ 0 & 0 & f_{33}\langle\phi\rangle \end{pmatrix}, \tag{3.4}$$

$$M_\nu = \lambda \begin{pmatrix} b & a & a \\ a & 1 & 1 \\ a & 1 & 1 \end{pmatrix}, \tag{3.5}$$

with

$$\lambda = \frac{\langle\phi\rangle^2}{M}, b = \frac{\langle\eta_1\rangle}{M}, a = \frac{\langle\eta_2\rangle}{M}. \tag{3.6}$$

From our best fitted results given in Eq. (2.9) which satisfy the LMA solution of solar neutrino solution and atmospheric neutrino experimental results, we obtain the parameters M, $\langle\eta_1\rangle$, $\langle\eta_2\rangle$ as $M=3.5\times10^{14}$ GeV, $\langle\eta_1\rangle=7\times10^{14}$ GeV, $\langle\eta_2\rangle=4.97\times10^{12}$ GeV with $\langle\phi\rangle=100$ GeV. It is to be noted that although the value of $\langle\eta_1\rangle$ comes out to be greater than the effective scale M, the effective coupling is always less than unity due to the factor λ and the effective coupling is always within the perturbative limit. Apart from the electroweak scale, the present model contains three other mass scales, two of which are very near to supersymmetry (SUSY) unification scale and the third is little lower. One of the singlets gets VEV at little above the effective scale M of the theory. It has some analogy with the singlets getting VEV's between SUSY unification scale and Planck scale in supersymmetric theory.

IV. CONCLUSION

In view of the results from solar neutrino experiments including recent SNO neutral current experiment, results from atmospheric neutrino experiment and CHOOZ experimental results we consider a texture of neutrino mass matrix which contains three parameters. We have considered no CP violation effects by choosing these parameters as real. The atmospheric neutrino mixing angle θ_{23} comes out to be maximal and $\theta_{13}=0$. The latter satisfies CHOOZ experimental results. We have fixed the three parameters of the texture considered by fitting the mass squared differences corresponding to solar and atmospheric neutrinos and the mixing angles corresponding to solar neutrinos with the best fit values of those quantities (LMA solution for solar neutrinos). We also find that the best fitted parameters can accommodate very marginally the bound on effective neutrino mass from neutrinoless double beta decay experiment and thus the testability of the present texture lies crucially on the future result of $\beta\beta_{0\nu}$ experiment. We also demonstrate an explicit model based on an $SU(2)_L \times U(1)_Y$ gauge group with extended Higgs sector and discrete symmetry that gives rise to the texture of neutrino mass matrix investigated along with diagonal charged lepton mass matrix. Neutrino masses are generated through higher dimensional operators at the scale M. Comparing with the best fitted values obtained, we estimate the scale of the VEV's of the neutral singlet scalars as $\langle\eta_1\rangle=7\times10^{14}$ GeV, $\langle\eta_2\rangle=4.97\times10^{12}$ GeV, and the scale $M=3.5\times10^{14}$ GeV. We will further study how such texture can be realized within grand unified theory (GUT), SUSY, GUT scenarios.

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