Convention-independent study of *CP*-violating asymmetries in $B \to \pi\pi$

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(Received 6 June 2002; published 19 September 2002)

CP-violating asymmetries in the decay $B^0(t) \to \pi^+\pi^-$ are a potentially rich source of information about both strong and weak phases. In a previous treatment by the present authors use was made of an assumption about the relative magnitude of tree and penguin amplitudes contributing to this process. This assumption involved an ambiguity in relating the tree amplitude to the amplitude for $B \to \pi l \nu$. It is shown here that one can avoid this assumption, which adopted a particular parametrization of tree and penguin amplitudes, and that the results are convention independent.

DOI: 10.1103/PhysRevD.66.053003 PACS number(s): 12.15.Hh, 12.15.Ji, 13.25.Hw, 14.40.Nd

I. INTRODUCTION

The study of *CP*-violating asymmetries in the decays $B^0(t) \to \pi^+\pi^-$ has reached an interesting stage. Two collaborations working at asymmetric B factories, the Babar Collaboration at the SLAC e^+e^- storage ring PEP-II (Stanford) [1] and the Belle Collaboration at KEK-B (Tsukuba, Japan) [2] have both reported measurements of time-dependent asymmetries in this process and its charge conjugate which are potentially rich sources of information on both strong and weak phases. The weak phases are those of elements in the Cabibbo-Kobayashi-Maskawa (CKM) matrix describing the weak charge-changing couplings of quarks. At present these phases provide a satisfactory description of all observed *CP*-violating phenomena in both K and B decays.

In a previous article [3] (for a more complete discussion, see also [4]), we analyzed these *CP*-violating asymmetries using assumptions which included knowledge of the ratio of tree and penguin amplitudes [5,6]. This knowledge was obtained from other processes using the factorization hypothesis. However, the nature of the tree amplitude and the value of the above ratio depended on our parametrization of the tree and penguin amplitudes, leading to some indeterminacy in the result. Certain aspects of ambiguities following from the penguin amplitude parametrization were discussed earlier in [7–9], and recently in [10].

In the present paper we find that one can obtain useful information from *CP*-violating asymmetries in $B^0 \to \pi^+\pi^-$ *independently* of the penguin amplitude parametrization, and without prior knowledge of the tree/penguin ratio. Some sacrifice in statistical power unavoidably occurs, so that determination of the weak phase $\alpha = \phi_2$ to better than 10° is difficult without additional assumptions. Thus, $\Delta\alpha \simeq 10°$ seems to be an estimate of the theoretical systematic error of the present method. This would still represent an improvement with respect to the present situation, in which we estimated α to be determined only within a 50° range [3].

The data which we use in the present determination consist of the charge-averaged branching ratio $\overline{\mathcal{B}}_{\pi\pi}$, the time-dependent asymmetries $S_{\pi\pi}$ and $C_{\pi\pi}$ which are coefficients of $\sin\Delta mt$ and $\cos\Delta mt$, and the charge-averaged branching ratio $\mathcal{B}(B^\pm \to K\pi^\pm)$. Similar inputs were also advocated in a previous analysis by Charles [11], which differs in details of correction factors and which presents results in terms of the ρ and η variables of the CKM matrix [12] rather than in terms of the phase α.

The paper is organized as follows. We introduce two different amplitude conventions in Sec. II. We show that, while the tree amplitudes in the two parametrizations are different, the corresponding penguin amplitudes are essentially the same, up to a simple CKM factor. We write down a dictionary relating the magnitudes and strong phases of corresponding tree amplitudes. In Sec. III we specify our assumptions and explain the method for determining the weak phases γ or α, as well as the relevant strong phase, by including information about the penguin amplitude in $B^+ \to K^0\pi^+$. The only required assumptions are penguin dominance of this amplitude and factorization of penguin amplitudes. We also summarize the present relevant experimental data. In Sec. IV we then plot the two measured *CP*-violating asymmetries as functions of strong and weak phases. We also plot relations between strong phases in the two parametrizations. While no use is made in this study of a prior knowledge of the ratio of tree and penguin amplitudes, this ratio could be used as a cross check and could resolve a possible discrete ambiguity in determining the weak phase. Section V qualitatively compares uncertainties in evaluating this ratio in the two conventions using other experimental inputs. Experimental prospects and conclusions are contained in Sec. VI.

II. NOTATION AND CONVENTIONS

The expressions for the decay amplitudes of $B^0 \to \pi^+\pi^-$ and $\overline{B^0} \to \pi^+\pi^-$ depend on the convention employed. We now describe two different parametrizations used in the literature, denoted c and t conventions, where c and t represent appropriate CKM factors governing penguin amplitudes.

A. *c* convention

In the convention of Refs. [3,4], one writes the decay amplitudes in terms of a color-favored tree amplitude T_c and a penguin amplitude P_c as

$$A(B^0\to\pi^+\pi^-)=-(|T_c|e^{i\delta_c^T}e^{i\gamma}+|P_c|e^{i\delta_c^P}),$$

$$A(\overline{B^0}\to\pi^+\pi^-)=-(|T_c|e^{i\delta_c^T}e^{-i\gamma}+|P_c|e^{i\delta_c^P}), \tag{1}$$

where we use the definitions in [13] of weak phases $\alpha=\phi_2$, $\beta=\phi_1$, and $\gamma=\phi_3$. The strong phases of the tree and penguin amplitudes are δ_c^T and δ_c^P, while $\delta_c\equiv\delta_c^P-\delta_c^T$. Here the subscript c refers to the convention in which the weak phase of the strangeness-preserving ($\Delta S=0$) penguin amplitude in $\overline{b}\to\overline{d}q\overline{q}$ is defined to be that of $V_{cb}^*V_{cd}$. The top quark in the $\overline{b}\to\overline{d}$ loop diagram has been integrated out and the unitarity relation $V_{tb}^*V_{td}=-V_{cb}^*V_{cd}-V_{ub}^*V_{ud}$ has been employed. The term $-V_{ub}^*V_{ud}$ has been included in the tree amplitude, which has the same weak phase.

B. *t* convention

A different convention has been commonly employed in the past [14] and also quite recently [15]. In this parametrization, one uses the unitarity relation in the form $V_{cb}^*V_{cd}=-V_{tb}^*V_{td}-V_{ub}^*V_{ud}$ and assumes the penguin amplitude to be dominated by the t quark term $V_{tb}^*V_{td}$. The tree amplitude, again, absorbs a penguin contribution proportional to $V_{ub}^*V_{ud}$, but it is different from that in the previous case. For this convention we shall use a subscript t on all quantities. The expressions for the decay amplitudes are then

$$A(B^0\to\pi^+\pi^-)=-(|T_t|e^{i\delta_t^T}e^{i\gamma}+|P_t|e^{i\delta_t^P}e^{-i\beta}),$$

$$A(\overline{B^0}\to\pi^+\pi^-)=-(|T_t|e^{i\delta_t^T}e^{-i\gamma}+|P_t|e^{i\delta_t^P}e^{i\beta}), \tag{2}$$

where one denotes $\delta_t\equiv\delta_t^P-\delta_t^T$.

C. Equivalence of the two conventions

It is obvious that the c and t conventions are equivalent. However, since in general they imply different tree and penguin amplitudes, an assumption about the tree amplitude in one parametrization is not equivalent to the same assumption in the other. On the other hand, as we will show now, the penguin amplitudes in the two cases are equal, up to a trivial CKM factor. Let us write the amplitude for $B^0\to\pi^+\pi^-$ in a most general form in terms of the three CKM factors and three corresponding hadronic weak amplitudes $A_i(i=u,c,t)$ involving strong phases:

$$A(B^0\to\pi^+\pi^-)=V_{ub}^*V_{ud}A_u+V_{cb}^*V_{cd}A_c+V_{tb}^*V_{td}A_t. \tag{3}$$

Using unitarity, this can be written in the c and t conventions as

$$A(B^0\to\pi^+\pi^-)=V_{ub}^*V_{ud}(A_u-A_t)+V_{cb}^*V_{cd}(A_c-A_t) \tag{4}$$

$$=V_{ub}^*V_{ud}(A_u-A_c)+V_{tb}^*V_{td}(A_t-A_c). \tag{5}$$

Comparing the second terms in Eqs. (1) and (2) with the corresponding terms in Eqs. (4) and (5), one finds a simple relation between the two penguin amplitudes:

$$\frac{|P_t|}{|P_c|}=\frac{|V_{tb}^*V_{td}|}{|V_{cb}^*V_{cd}|}=\frac{\sin\gamma}{\sin\alpha},\quad \delta_t^P=\delta_c^P; \tag{6}$$

namely, the penguin amplitudes in the two parametrizations involve a common hadronic matrix element A_t-A_c but different CKM factors.

On the other hand, the relation between tree amplitudes in the two conventions is more complicated. It can be obtained by subtracting the first terms in Eqs. (1) and (2) from each other and comparing with Eq. (4) or (5), in which the corresponding difference is proportional to the penguin amplitudes, A_t-A_c,

$$|T_t|e^{-i\delta_t}-|T_c|e^{-i\delta_c}=\frac{|V_{ub}^*V_{ud}|}{|V_{tb}^*V_{td}|}|P_t|$$

$$=\frac{\sin\beta}{\sin\gamma}|P_t|$$

$$=\frac{\sin\beta}{\sin\alpha}|P_c|. \tag{7}$$

As a consequence of these relations, one has a "dictionary" relating the two parametrizations, with

$$|P_t|\sin\alpha=|P_c|\sin\gamma,\quad |T_t|\sin\delta_t=|T_c|\sin\delta_c, \tag{8}$$

$$X_t\cos\delta_t\sin\gamma-X_c\cos\delta_c\sin\alpha=\sin\beta, \tag{9}$$

where we have defined $X_c\equiv|T_c/P_c|, X_t\equiv|T_t/P_t|$. One consequence of these relations is

$$\cot\delta_t=\cot\delta_c+\frac{\sin\beta}{X_c\sin\alpha\sin\delta_c}, \tag{10}$$

which we shall use when relating δ_t to δ_c.

III. MEASURABLES IN TERMS OF WEAK AND STRONG PHASES

In the present section we derive expressions for the two CP asymmetries in $B^0(t)\to\pi^+\pi^-$, $S_{\pi\pi}$ and $C_{\pi\pi}$, in terms of a strong and a weak phase. For completeness, expressions are given in the two equivalent parametrizations, which imply identical constraints on α. These constraints do not require knowledge of the tree/penguin ratio. Information about this ratio, which could resolve a certain discrete ambiguity in these constraints, can be more useful in one convention than in the other. This question is discussed in Sec. V.

The time-dependent rate of an initially produced B^0 decaying to $\pi^+\pi^-$ at time t is given by [16]

$$\Gamma(B^0(t)\to\pi^+\pi^-)\propto e^{-\Gamma_d t}[1+C_{\pi\pi}\cos\Delta(m_d t) - S_{\pi\pi}\sin(\Delta m_d t)]. \tag{11}$$

The coefficients of $\sin\Delta m_d t$ and $\cos\Delta m_d t$, measured in time-dependent CP asymmetries of $\pi^+\pi^-$ states produced in asymmetric e^+e^- collisions at the $\Upsilon(4S)$ are

$$S_{\pi\pi}\equiv\frac{2\,\mathrm{Im}(\lambda_{\pi\pi})}{1+|\lambda_{\pi\pi}|^2},\quad C_{\pi\pi}\equiv\frac{1-|\lambda_{\pi\pi}|^2}{1+|\lambda_{\pi\pi}|^2}, \tag{12}$$

where

$$\lambda_{\pi\pi}\equiv e^{-2i\beta}\frac{A(\overline{B}^0\to\pi^+\pi^-)}{A(B^0\to\pi^+\pi^-)}. \tag{13}$$

The extraction of phases from data on $S_{\pi\pi}$ and $C_{\pi\pi}$ now proceeds in the following manner. As in Ref. [3], we define the charge-averaged branching ratio

$$\overline{\mathcal{B}}_{\pi\pi}\equiv[\mathcal{B}(B^0\to\pi^+\pi^-)+\mathcal{B}(\overline{B}^0\to\pi^+\pi^-)]/2. \tag{14}$$

We use the convention

$$\mathcal{B}(B^0\to\pi^+\pi^-)=|A(B^0\to\pi^+\pi^-)|^2|\vec{p}_{\pi\pi}|\tau_0, \tag{15}$$

where $|\vec{p}_{\pi\pi}|$ is the pion center-of-mass momentum and τ_0 is the B^0 lifetime.

However, in contrast to the approach of Ref. [3], we no longer normalize this branching ratio with respect to the corresponding tree value, which is convention dependent. Instead, we normalize all amplitudes by the penguin amplitude P_c or P_t, which we have shown to be convention independent, up to a CKM factor.

Using broken flavor SU(3) [17] and factorization, the magnitude of the penguin amplitude is obtained from the $|\Delta S|=1$ penguin amplitude P' which dominates the decay $B^+\to K^0\pi^+$ [18]. That is, our approach relies on neglecting both rescattering effects in $B^+\to K^0\pi^+$ and nonfactorizable contributions in penguin amplitudes. Several ways of testing the first assumption were discussed in [19]. We note that this assumption is also made in two detailed theoretical schemes for calculating weak hadronic matrix elements [20,21]. In the first scheme [20] factorization of penguin amplitudes is assumed to hold to a good approximation and strong phases are small. In the second framework [21] nonfactorizable terms in penguin amplitudes are strongly suppressed, but strong phases are sizable. Thus, while it may seem natural to combine the assumption of factorization of penguin amplitudes with small strong phases, we will not rely on the latter assumption.

Within the above assumptions, one obtains for the penguin amplitude $|P_i|$ $(i=c,t)$ an expression in terms of measurable quantities,

$$|P_i|=\frac{f_\pi}{f_K}\left|\frac{V_{ib}^*V_{id}}{V_{ib}^*V_{is}}\right||P'|,\quad |P'|=|A(B^+\to K^0\pi^+)|. \tag{16}$$

Here we use a convention similar to Eq. (15),

$$\mathcal{B}(B^+\to K^0\pi^+)\equiv|A(B^+\to K^0\pi^+)|^2|\vec{p}_{K\pi}|\tau_+, \tag{17}$$

where $|\vec{p}_{K\pi}|$ is the π or K center-of-mass momentum and τ_+ is the B^+ lifetime.

Applying Eqs. (15), (16) and (17), one finds for the normalized rates [22]

$$b_i\equiv\frac{|A(B^0\to\pi^+\pi^-)|^2+|A(\overline{B}^0\to\pi^+\pi^-)|^2}{2|P_i|^2} = \frac{\overline{\mathcal{B}}_{\pi\pi}}{\mathcal{B}(B^+\to K^0\pi^+)}\left|\frac{V_{ib}^*V_{is}}{V_{ib}^*V_{id}}\right|^2\frac{f_K^2}{f_\pi^2}\frac{|\vec{p}_{K\pi}|}{|\vec{p}_{\pi\pi}|}\frac{\tau_+}{\tau_0}. \tag{18}$$

The three measurables $S_{\pi\pi}$, $C_{\pi\pi}$ and $\overline{\mathcal{B}}_{\pi\pi}/\mathcal{B}(B^+\to K^0\pi^+)$ can then be expressed in terms of the three parameters X_i, δ_i and a weak phase. We now display these expressions for the two mentioned conventions.

A. *c* convention

In this case one has

$$|P_c|=\frac{f_\pi}{f_K}\left|\frac{V_{cb}^*V_{cd}}{V_{cb}^*V_{cs}}\right||P'| = \frac{f_\pi}{f_K}\frac{\lambda}{1-\lambda^2/2}|A(B^+\to K^0\pi^+)|, \tag{19}$$

where $\lambda=0.22$ is the parameter describing the hierarchy of CKM elements [12]. Then, noting the weak and strong phases of T_c and P_c, and substituting $\alpha=\pi-\beta-\gamma$ when convenient, we have

$$\lambda_{\pi\pi}=e^{2i\alpha}\left(\frac{X_c+e^{i\delta_c}e^{i\gamma}}{X_c+e^{i\delta_c}e^{-i\gamma}}\right), \tag{20}$$

$$b_c=X_c^2+2X_c\cos\delta_c\cos\gamma+1, \tag{21}$$

$$b_cS_{\pi\pi}=X_c^2\sin2\alpha+2X_c\cos\delta_c\sin(\beta-\alpha)-\sin2\beta, \tag{22}$$

$$b_cC_{\pi\pi}=2X_c\sin\delta_c\sin\gamma. \tag{23}$$

One can use Eq. (21) to eliminate X_c using the experimental values of b_c. Since b_c is a number significantly greater than 1 [see Eq. (33) below], only one solution of the quadratic equation is relevant, and one finds

$$X_c=-\cos\delta_c\cos\gamma+\sqrt{(\cos\delta_c\cos\gamma)^2+b_c-1}. \tag{24}$$

This value can then be substituted into Eqs. (22) and (23) for $S_{\pi\pi}$ and $C_{\pi\pi}$ and the resulting values plotted against one another, e.g., as curves for specific values of α parametrized by δ_c. We shall exhibit such curves in the next section.

TABLE I. Values of $S_{\pi\pi}$ and $C_{\pi\pi}$ from Refs. [1,2] and their averages.

Collaboration	$S_{\pi\pi}$	$C_{\pi\pi}$
BaBar	$-0.01\pm0.37\pm0.07$	$-0.02\pm0.29\pm0.07$
Belle	$-1.21^{+0.38+0.16}_{-0.27-0.13}$	$-0.94^{+0.31}_{-0.25}\pm0.09$
Average	-0.64 ± 0.26	-0.49 ± 0.21

B. *t* convention

Here one has

$$|P_t|=\frac{f_\pi}{f_K}\left|\frac{V^*_{tb}V_{td}}{V^*_{tb}V_{ts}}\right||P'|=\left|\frac{\sin\gamma}{\sin\alpha}\right||P_c|\Rightarrow \quad b_t=b_c\left(\frac{\sin\alpha}{\sin\gamma}\right)^2, \tag{25}$$

$$\lambda_{\pi\pi}=\frac{X_t e^{i\alpha}-e^{i\delta_t}}{X_t e^{-i\alpha}-e^{i\delta_t}}, \tag{26}$$

$$b_t=X_t^2-2X_t\cos\delta_t\cos\alpha+1, \tag{27}$$

$$b_t S_{\pi\pi}=X_t^2\sin 2\alpha-2X_t\cos\delta_t\sin\alpha, \tag{28}$$

$$b_t C_{\pi\pi}=2X_t\sin\delta_t\sin\alpha. \tag{29}$$

In solving Eq. (27) for X_t one again takes the positive square root:

$$X_t=\cos\delta_t\cos\alpha+\sqrt{(\cos\delta_t\cos\alpha)^2+b_t-1}. \tag{30}$$

Here it is convenient to use the relation $b_t=b_c(\sin\alpha/\sin\gamma)^2$ since b_c is most directly related to an experimental input.

Again, one may substitute the value of X_t into the equations for $S_{\pi\pi}$ and $C_{\pi\pi}$ and plot them against one another. Moreover, in this convention one may also eliminate both X_t and δ_t, thereby obtaining an equation for α alone in terms of measurable quantities:

$$b_t S_{\pi\pi}=\frac{1}{2}\sin 4\alpha+(b_t-1)\sin 2\alpha \pm\cos 2\alpha\sqrt{\sin^2 2\alpha+4(b_t-1)\sin^2\alpha-(b_t C_{\pi\pi})^2}. \tag{31}$$

This equation is derived in an analogous manner to one obtained recently for the phase γ in terms of measurables in $B_s(t)\to K^+K^-$ and $B_s\to K^0\overline{K^0}$ [24]. One may, of course, obtain an analogous equation for α by eliminating X_c and δ_c from Eqs. (22)–(24) and assuming that β is known, which is quite a good approximation. This result is equivalent to substituting Eq. (25) into Eq. (31) and noting that $\sin\gamma=\sin(\alpha+\beta)$.

C. Experimental inputs

The most recent measurements of $S_{\pi\pi}$ and $C_{\pi\pi}$ [1,2], together with our average of them, are shown in Table I. (We have corrected the BaBar entry for $S_{\pi\pi}$ misquoted by us in Ref. [3].)

The present world averages of $\overline{\mathcal{B}}_{\pi\pi}$ and $\mathcal{B}(B^+\to K^0\pi^+)$, combining measurements from the CLEO, Belle and BaBar Collaborations, are [25]

$$\overline{\mathcal{B}}_{\pi\pi}=(5.2\pm0.6)\times10^{-6},$$
$$\mathcal{B}(B^+\to K^0\pi^+)=(17.9\pm1.7)\times10^{-6}. \tag{32}$$

Adding errors in quadrature, using $f_\pi=130.7$ MeV, $f_K=159.8$ MeV and $\tau_+/\tau_0=1.068\pm0.016$ [23], we find for the normalized rate in Eq. (18)

$$b_c=9.04\pm1.36. \tag{33}$$

IV. *CP*-VIOLATING ASYMMETRIES

For a given value of b_c, Eqs. (22)–(24) [or Eqs. (28)–(30)] can be used to plot $S_{\pi\pi}$ and $C_{\pi\pi}$ as functions of α and δ_c (or δ_t). The values of $S_{\pi\pi}$ and $C_{\pi\pi}$ for the central and $\pm1\sigma$ values of the ratio b_c in (33), and for values of α mostly lying within the physical range [26] $\alpha=(97^{+30}_{-21})°$, are plotted in Fig. 1. (For other values of α see, e.g., Ref. [3].) We use $\beta=26°$ based on the most recent average $\sin 2\beta=0.78\pm0.08$ of Belle [2] and BaBar [27] values; the $\pm4°$ error on β has little effect [3]. The large plotted point corresponds to the average in Table I. As expected, the curves are identical in the two conventions. The existence of two solutions for $S_{\pi\pi}$, for given values of b_c,α and $C_{\pi\pi}$, can be easily understood. This follows from the $\pm$ sign in Eq. (31).

For strong phases δ_c or δ_t of 0 or π, the predictions for $S_{\pi\pi}$ and $C_{\pi\pi}$ depend only on b_c and α. These points are marked with diamonds and squares, respectively. A strong phase of π would signify a relative sign of tree and penguin amplitudes opposite to that obtained from factorization. Such a phase is strongly disfavored relative to a zero phase. For non-zero strong phases, the curves are identical in the two conventions, but points on them correspond to different values of δ_c and δ_t. Examples are shown for $\delta_c=\pi/2$ (crosses) and $\delta_t=\pi/2$ (fancy + signs). These parametric plots can also be used to find the values of the strong phases (modulo discrete ambiguities) once $S_{\pi\pi}$ and $C_{\pi\pi}$ have been determined experimentally. Alternatively, one may eliminate X_c from Eqs. (21) and (23) or X_t from Eqs. (27) and (29) and solve the resulting equations for the strong phases numerically.

If $C_{\pi\pi}$ is indeed small, as suggested by the BaBar data [1], α can be uncertain by as much as about 30°, depending on whether the strong phase is near 0 or π. This is seen in Fig. 1, where for $b_c=7.7$ the curves for $\alpha=90°$ and $\alpha=120°$ intersect near the horizontal axis. In that case, additional theoretical input [20,21] on strong phases can help resolve the ambiguity. Theoretically, it is much more likely that the strong phase is near 0 than near π. If the central value of $C_{\pi\pi}$ remains as large as suggested by the present experimental average, the discrete ambiguity becomes less of a problem. Nonetheless, as one can see from neighboring curves, even a very tiny error ellipse in the $(S_{\pi\pi},C_{\pi\pi})$ plane will not be able to resolve values of α differing by 10°. This

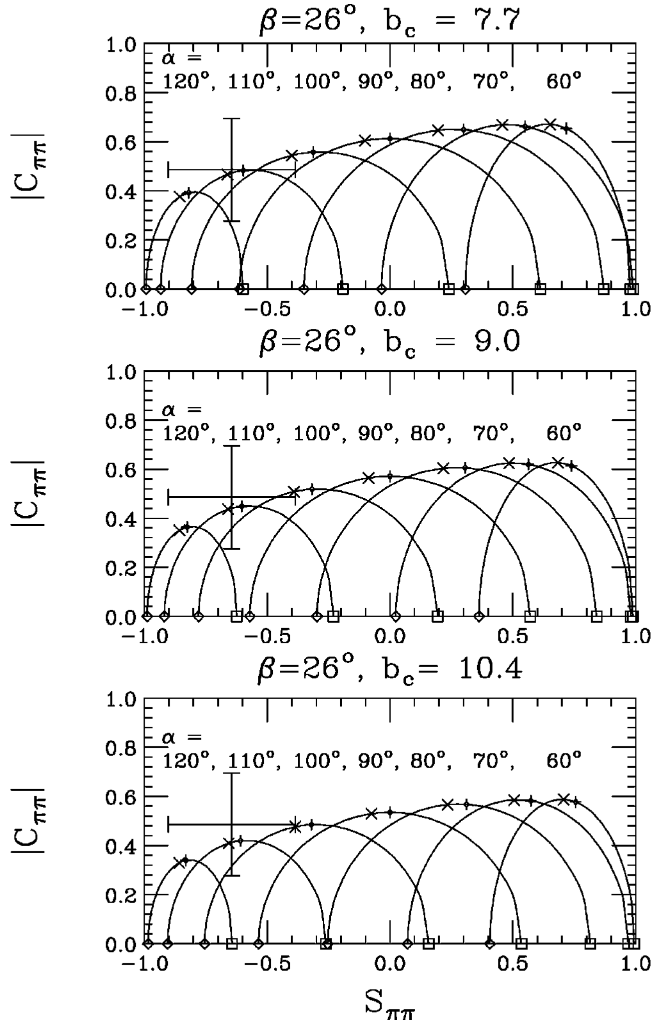

FIG. 1. Plots of $|C_{\pi\pi}|$ versus $S_{\pi\pi}$ for various values of b_c. Top panel: $b_c=7.7$. Middle panel: $b_c=9.0$. Bottom panel: $b_c=10.4$. Curves correspond, from left to right, to values of α in 10° steps ranging from 120° to 60°. The value $\beta=26°$ has been chosen. Large plotted point corresponds to present average of BaBar and Belle data (see text). Small plotted points: $\delta_c=\delta_t=0$ (diamonds), $\delta_c=\delta_t=\pi$ (squares), $\delta_c=\pi/2$ (crosses), $\delta_t=\pi/2$ (fancy+signs).

is a necessary price for giving up prior information on the tree/penguin ratio.

The values of δ_c and δ_t do not differ very much from one another. When they are close to $\pi/2$, their difference is close to maximal, but rarely exceeds 10°, as shown in Fig. 2. We used Eq. (10) in making these plots.

We have assumed factorization in obtaining the penguin amplitude. Any deviation from factorization would result in a corrected value for b_c, for which we have taken a 15% error arising from experimental errors in branching ratios. This would be equivalent to correcting the SU(3) breaking factor f_K/f_π in Eq. (18) by 7.5%. That is, even assuming perfect measurements of $\overline{\mathcal{B}}_{\pi\pi}$ and $\mathcal{B}(B^+\to K^0\pi^+)$, an irreducible uncertainty would be associated with the assumption of factorization for penguin amplitudes. If this uncertainty were 7.5%, we would obtain for perfect branching ratio measurements the range of possibilities shown in Fig. 1.

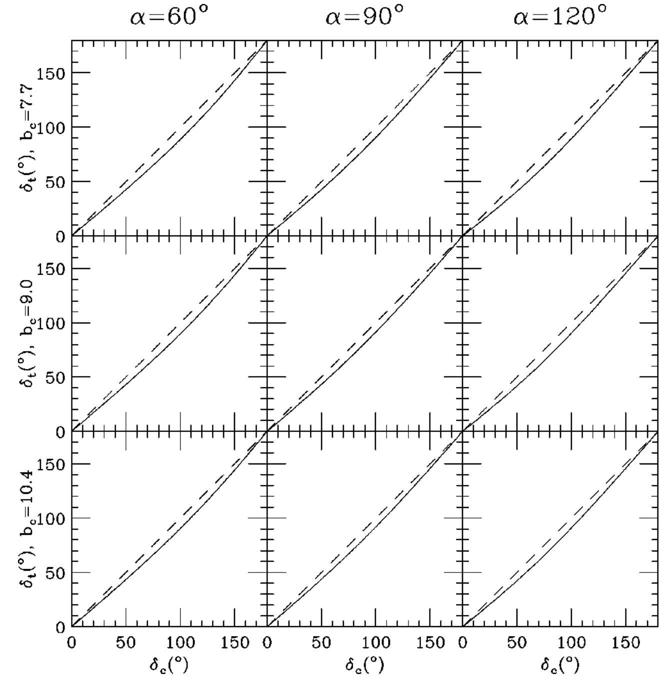

FIG. 2. Relations between δ_c and δ_t for various values of α and b_c.

Let us assume that this 7.5% is a reasonable estimate of the intrinsic possible deviation from factorization. By comparing the three panels of Fig. 1, one sees that if $C_{\pi\pi}$ is near its maximum, then $S_{\pi\pi}$ is not very sensitive to the value of b_c (and hence to the factorization assumption), while if $C_{\pi\pi}$ is near zero, a given value of $S_{\pi\pi}$ corresponds to values of α differing by only a few degrees depending on the value of b_c (aside from the much more serious discrete ambiguity mentioned earlier). In either case, the factorization assumption is not the source of the limiting error on α.

V. DEFINING AND USING A TREE/PENGUIN RATIO

Although we have shown that one does not need to know the tree/penguin ratio in order to extract useful information from $\overline{\mathcal{B}}_{\pi\pi}$, $S_{\pi\pi}$, and $C_{\pi\pi}$, the error on α and the strong phase δ_c or δ_t can be further reduced if one has some information on X_c or X_t. In the present section we first give an example of how improved information would help, and then discuss the more difficult questions of which parameter (X_c or X_t) is capable of being specified more precisely and how one would go about doing so.

Let us take as an example an ambiguity associated with curves for $\alpha=90°$ and 110° which intersect for the central value of $b_c=9.0$ around $S_{\pi\pi}=-0.4$ and $|C_{\pi\pi}|=0.4$. These correspond to different values of X_c or X_t, as illustrated in Table II. We also show two different values of α (90° and 119°) giving rise to the same values of $S_{\pi\pi}$ for $C_{\pi\pi}=0$.

From these examples, one sees that specification of X_c or X_t with an error of ± 0.3 would permit resolution of the ambiguity. In Ref. [3] we employed an estimate $X_c \simeq 3.6$ with about a 25% error. Reduction of this error to about $\pm 10\%$ is needed in order to have a significant impact on resolving the

TABLE II. Comparison of X_c and X_t values for pairs of α values giving the same $S_{\pi\pi}$ and $C_{\pi\pi}$. Here we have taken $b_c = 9.04$.

α	$S_{\pi\pi}$	$\lvert C_{\pi\pi}\rvert$	X_c	δ_c	X_t	δ_t
90°	−0.41	0.40	2.6	51°	3.2	44°
110°	−0.41	0.40	3.3	129°	4.1	122°
90°	−0.57	0.0	2.4	0°	3.2	0°
119°	−0.57	0.0	3.8	180°	4.9	180°

ambiguity exhibited in Table II. Is such accuracy achievable?

Our estimate of b_c involves a 15% error which consists of slightly less than 10% due to that in $\mathcal{B}(B^+ \to K^0\pi^+)$, and slightly more than 10% due to that in $\overline{\mathcal{B}}_{\pi\pi}$, added in quadrature. Clearly these errors will shrink with improved statistics. However, the determination of $|T_c|$ from $B \to \pi l \nu$ using factorization is problematic since $T_c \sim A_u - A_t$ [Eq. (4)] contains the short-distance penguin contribution involving the top quark loop. It might seem more reliable to estimate $T_t \sim A_u - A_c$ [Eq. (5)] using factorization since its penguin contribution does not contain a large logarithm of m_t. This is in fact the method advocated in Ref. [15], in which a determination of T_t with an accuracy of less than 6% was deemed feasible with about 500 $B \to \pi l \nu$ events. A corresponding accuracy for $|P_t|$ would require improved accuracy for $\mathcal{B}(B^+ \to K^0\pi^+)$ (which gives $|P_c|$, not $|P_t|$) and then using the relation (6), $|P_t| = |P_c| \sin\gamma / \sin\alpha$.

A potential problem with determining T_t using factorization is that while its contamination from the short-distance penguin amplitude is less than that in T_c, there is no corresponding guarantee for *long-distance* penguin contributions such as might be introduced by rescattering from tree amplitudes, for example via $B^0 \to D^{(*)+}D^{(*)-} \to \pi^+\pi^-$. Other processes, such as $B^0 \to K^+K^-$, are expected to proceed *mainly* via rescattering or else, if rescattering is unimportant, to be highly suppressed [19]. Present bounds on this last process are quite stringent [28]: $\mathcal{B}(B^0 \to K^+K^-) \leqslant 0.5 \times 10^{-6}$. It may be that one must rely on theoretical treatments of factorization (e.g., [20]) in order to specify $|T_t|$ (or perhaps $|T_c|$) more precisely.

VI. EXPERIMENTAL PROSPECTS AND CONCLUSIONS

We have shown that one can obtain useful information on weak and strong phases by studying the observables in $B^0(t) \to \pi^+\pi^-$ without having to define in advance the ratio of tree and penguin amplitudes, and in a manner which is independent of the convention adopted for the penguin amplitudes. These observables consist of the flavor-averaged branching ratio $\overline{\mathcal{B}}_{\pi\pi}$ normalized by $\mathcal{B}(B^+ \to K^0\pi^+)$ and the quantities $S_{\pi\pi}$ and $C_{\pi\pi}$ measured in time-dependent asymmetries. We consider only information based on the magnitude of $C_{\pi\pi}$; its sign determines the sign of the strong phase shift.

The degree of information obtainable without auxiliary tree/penguin information can be estimated from the curves in Fig. 1 and depends on whether $|C_{\pi\pi}|$ is near its maximum value (the envelope of the curves) or zero. If $|C_{\pi\pi}| \simeq 0$, important discrete ambiguities in α exist, amounting to up to about 30°, which must be resolved using additional information on the tree/penguin ratio or on the strong phase. If $|C_{\pi\pi}|$ is near its maximum, the error on α appears to depend roughly on the square root of the error in $|C_{\pi\pi}|$, as one can see by measuring how far from the envelope of the curves the intersection point of two curves for different α values lies. Thus, two curves for α differing by (10,20,30)° intersect at points about (0.04,0.08,0.18) below the envelope along the $|C_{\pi\pi}|$ axis. To take one example, if one wants to distinguish between two curves for α differing by 20° (as in the example of Table II), one should be prepared to measure $|C_{\pi\pi}|$ with an error of no more than ± 0.08, which is about 2.6 times less than the present error of ± 0.21. One thus would need $(2.6)^2$ times the data sample ($\simeq 100$ fb^{-1}) on which Table I was based, or about 700 fb^{-1} from the total of BaBar and Belle. This appears to be within the goals of the experiments. Errors on $S_{\pi\pi}$ in such a sample should be sufficiently small that they will not play a major role in the errors in α.

ACKNOWLEDGMENTS

M. G. wishes to thank The Enrico Fermi Institute at the University of Chicago for its kind hospitality. We thank A. Höcker for asking the question which led to this investigation, H. Jawahery for a communication regarding data, and H. N. Li, S. Olsen and L. Wolfenstein for helpful discussions. This work was supported in part by the United States Department of Energy through Grant No. DE FG02 90ER40560, by the Israel Science Foundation founded by the Israel Academy of Sciences and Humanities, and by the U.S.-Israel Binational Science Foundation through Grant No. 98-00237.

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