Seesaw mechanism, baryon asymmetry, and neutrinoless double beta decay

D. Falcone
Dipartimento di Scienze Fisiche, Università di Napoli, Complesso di Monte S. Angelo, Via Cintia, Napoli, Italy
(Received 10 May 2002; published 10 September 2002)

A simplified but very instructive analysis of the seesaw mechanism is here performed. Assuming a nearly diagonal Dirac neutrino mass matrix, we study the forms of the Majorana mass matrix of right-handed neutrinos, which reproduce the effective mass matrix of left-handed neutrinos. As a further step, the important effect of a nondiagonal Dirac neutrino mass matrix is explored. The corresponding implications for baryogenesis via leptogenesis and for the neutrinoless double beta decay are reviewed. We propose two distinct models where the baryon asymmetry is enhanced.

DOI: 10.1103/PhysRevD.66.053001 PACS number(s): 13.35.Hb, 12.15.Ff

I. INTRODUCTION

The seesaw mechanism [1] is a simple framework to account for the small effective mass of the left-handed neutrino. It requires only a modest extension of the minimal standard model, namely the addition of the right-handed neutrino. As a consequence of this inclusion, both a Dirac mass term for the neutrino and a Majorana mass term for the right-handed neutrino are allowed. While the Dirac mass, m_ν, is expected to be of the same order of magnitude as the quark or charged lepton mass, the Majorana mass of the right-handed neutrino, m_R, is not constrained and may be very large. If this is the case, a small effective Majorana mass for the left-handed neutrino, $m_L \simeq (m_\nu/m_R) m_\nu$, is generated.

At the same time, the out-of-equilibrium decay of the heavy neutrino can produce a baryon asymmetry through the so-called baryogenesis via leptogenesis mechanism [2,3]. Hence, the existence of the heavy Majorana neutrino may explain both the smallness of the effective neutrino mass and the baryon asymmetry in the universe. Moreover, the Majorana nature of the light neutrino, generated by the Majorana nature of the heavy neutrino, allows the neutrinoless double beta decay, because of lepton number violation at high energy [4]. Thus the mass scale of the heavy neutrino could be a new fundamental scale in physics.

For three generations of fermions, the light neutrino mass matrix M_L as well as the baryon asymmetry Y_B depend on the Dirac neutrino mass matrix M_ν and the heavy neutrino mass matrix M_R. In this paper we describe, in a simplified but instructive approach, the structure of M_ν and M_R within the seesaw mechanism and the consequences for the baryon asymmetry generated in the baryogenesis via leptogenesis mechanism and for the neutrinoless double beta decay.

As a first approximation, we assume a diagonal form for the Dirac neutrino mass matrix, and then the effect of a nondiagonal form is analyzed. The prediction for the neutrinoless double beta decay can remain unchanged because for a fixed M_L and any choice of M_ν one can find a certain M_R which reproduces M_L through the seesaw formula, while the impact on the amount of baryon asymmetry is significant, because those M_R and M_ν determine Y_B through the leptogenesis formula.

The outline of the paper is the following. In Sec. II we give an approximate description of the effective neutrino mass matrix. In Secs. III and IV we briefly discuss the seesaw mechanism and the baryogenesis via leptogenesis, respectively. In Sec. V the forms of the heavy neutrino mass matrix, and their implications for the amount of baryon asymmetry and the rate for neutrinoless double beta decay are reviewed, according to different mass spectra of light neutrinos. In this section we assume a diagonal Dirac neutrino mass matrix. The important effect of a nondiagonal Dirac neutrino mass matrix on the amount of baryon asymmetry is explored in Sec. VI, where we also propose two different models of mass matrices, which produce an enhancement of the baryon asymmetry. Finally, in Sec. VII we summarize the subject.

II. THE EFFECTIVE NEUTRINO MASS MATRIX

The lepton mixing matrix U [called the Maki-Nakagawa-Sakata (MNS) matrix [5]], which relates mass eigenstates to flavor eigenstates, by means of the unitary transformation $\nu_\alpha = U_{\alpha i} \nu_i (\alpha = e, \mu, \tau; i = 1,2,3)$, can be parametrized as the standard form of the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [6] (including a phase δ) times a diagonal phase matrix $P = \text{diag}(e^{i\varphi_1/2}, e^{i\varphi_2/2}, 1)$. The two phases φ_1 and φ_2 are present only if the effective neutrino is a Majorana particle, and thus they are sometimes called the Majorana phases, in contrast with the phase δ which is called the Dirac phase. Moreover, contrary to quark mixings, which are small, lepton mixings can be large. In fact, the mixing of atmospheric neutrinos, related to $U_{\mu 3}$, is almost maximal, while the mixing of solar neutrinos, related to U_{e2}, may be large or small, although the large mixing is favored [7]. In the case of double large mixing, the lepton mixing matrix is given by

$$U \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \epsilon \\ -\frac{1}{2}(1+\epsilon) & \frac{1}{2}(1-\epsilon) & \frac{1}{\sqrt{2}} \\ \frac{1}{2}(1-\epsilon) & -\frac{1}{2}(1+\epsilon) & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1}$$

while for single large mixing it is given by

$$U \simeq \begin{pmatrix} 1 & 0 & \epsilon \\ -\frac{\epsilon}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{\epsilon}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}; \tag{2}$$

see for example Ref. [8]. As we said, the double large mixing is favored. The mixing $\epsilon = U_{e3}$ is very small, $\epsilon \lesssim 0.1$, according to the result of the Chooz experiment [9]. If we call D_L the diagonal matrix of light neutrino masses,

$$D_L = \text{diag}(m_1, m_2, m_3), \tag{3}$$

then, in the basis where the charged lepton mass matrix is diagonal, $M_e = D_e$, we get

$$M_L = U D_L U^T. \tag{4}$$

In the flavor basis we have $M_L = U_e U D_L U^T U_e^T$, where U_e diagonalizes M_e by $U_e^T M_e U_e$, where M_e is here supposed to be symmetric. We do not consider general phases in U. However, we allow the masses m_1 and m_2 to be both positive and negative, corresponding to phases $\varphi_{1,2} = 0$ and $\varphi_{1,2} = \pi$, respectively, in the lepton mixing matrix. In such a way, we consider the two extreme cases for $\varphi_{1,2}$ and the general case should be intermediate between them.

Let us call the elements of M_L as $M_{\alpha\beta}$ with $\alpha = e, \mu, \tau$ and $\beta = e, \mu, \tau$. Then, for the double large mixing we have the approximate expressions

$$M_{ee} = \frac{m_1}{2} + \frac{m_2}{2} + \epsilon^2 m_3,$$

$$M_{e\mu} = -\frac{m_1}{2\sqrt{2}}(1+\epsilon) + \frac{m_2}{2\sqrt{2}}(1-\epsilon) + \frac{\epsilon m_3}{\sqrt{2}},$$

$$M_{e\tau} = \frac{m_1}{2\sqrt{2}}(1-\epsilon) - \frac{m_2}{2\sqrt{2}}(1+\epsilon) + \frac{\epsilon m_3}{\sqrt{2}},$$

$$M_{\mu\mu} = \frac{m_1}{4}(1+\epsilon)^2 + \frac{m_2}{4}(1-\epsilon)^2 + \frac{m_3}{2},$$

$$M_{\mu\tau} = -\frac{m_1}{4}(1-\epsilon^2) - \frac{m_2}{4}(1-\epsilon^2) + \frac{m_3}{2},$$

$$M_{\tau\tau} = \frac{m_1}{4}(1-\epsilon)^2 + \frac{m_2}{4}(1+\epsilon)^2 + \frac{m_3}{2},$$

and for the single large mixing the corresponding expressions

$$M_{ee} = m_1 + \epsilon^2 m_3,$$

$$M_{e\mu} = -\frac{\epsilon m_1}{\sqrt{2}} + \frac{\epsilon m_3}{\sqrt{2}},$$

$$M_{e\tau} = -\frac{\epsilon m_1}{\sqrt{2}} + \frac{\epsilon m_3}{\sqrt{2}},$$

$$M_{\mu\mu} = \frac{\epsilon^2 m_1}{2} + \frac{m_2}{2} + \frac{m_3}{2},$$

$$M_{\mu\tau} = \frac{\epsilon^2 m_1}{2} - \frac{m_2}{2} + \frac{m_3}{2},$$

$$M_{\tau\tau} = \frac{\epsilon^2 m_1}{2} + \frac{m_2}{2} + \frac{m_3}{2}.$$

Of course, both M_L and M_R are symmetric matrices. Note that for the single large mixing we have $M_{e\mu} = M_{e\tau}$ and $M_{\mu\mu} = M_{\tau\tau}$.

The element $M_{ee} = U_{ei}^2 m_i$ is involved in the neutrinoless double beta decay. The experimental upper bound for $|M_{ee}|$, obtained from nonobservation of the process, is $|M_{ee}| < 0.38h$ [10], where the factor $h = 0.6 - 2.8$ [11] is present because of the uncertainty in the calculation of the nuclear matrix element. The recent positive evidence [12] is controversial [11,13] and is not used here. The parameter M_{ee} is the unique element in M_L which can be tested in a direct way, because for the other elements the theoretical prediction is very much below the experimental data [14].

From the study of oscillations of atmospheric and solar neutrinos we know that $|m_3^2 - m_2^2| \gg |m_2^2 - m_1^2|$, so that there are three main mass spectra for the light neutrino, the normal hierarchy $m_3^2 \gg m_2^2, m_1^2$, the inverse hierarchy $m_1^2 \simeq m_2^2 \gg m_3^2$, and the nearly degenerate spectrum $m_1^2 \simeq m_2^2 \simeq m_3^2$ (see Ref. [15]). In particular, from atmospheric oscillations we get $|m_3^2 - m_2^2| \sim 10^{-3}$ eV^2, while from solar oscillations $|m_2^2 - m_1^2| \sim 10^{-5}$ eV^2 for the large mixing and $|m_2^2 - m_1^2| \sim 10^{-6}$ eV^2 for the small mixing. Then, with the normal hierarchy we get $m_3^2 \sim 10^{-3}$ eV^2, and with the inverse hierarchy $m_{1,2}^2 \sim 10^{-3}$ eV^2. For the nearly degenerate spectrum we expect $m_{1,2,3}^2$ around 1 eV^2. In fact, the experimental upper bound on the parameter $m_{\nu_e} = (U_{ei}^2 m_i^2)^{1/2}$, obtained from the end point energy of electrons in single beta decay, is $m_{\nu_e} < 2.5$ eV [16]. In contrast with M_{ee}, in m_{ν_e} cancellations cannot occur. If for the normal hierarchy also $m_2^2 \gg m_1^2$ is assumed, then we get $m_2^2 \sim 10^{-5}$ eV^2 for the large mixing and $m_2^2 \sim 10^{-6}$ eV^2 for the small mixing.

As a matter of fact, for solar neutrinos there are at least three oscillation solutions with a large mixing angle [7]: the large mixing angle (LMA) matter oscillation with $|m_2^2 - m_1^2| \sim 10^{-5}$ eV^2, the low-mass (LOW) matter oscillation with $|m_2^2 - m_1^2| \sim 10^{-7}$ eV^2, and the vacuum oscillation (VO) with $|m_2^2 - m_1^2| \sim 10^{-10}$ eV^2. In our paper we refer mainly to the LMA solution which is the most favored [7].

Zeroth order forms for M_L can be obtained by setting $\epsilon = 0$ and (m_1, m_2, m_3) equal to (0,0,1), (1,−1,0), (1,1,0), (1,1,1), (−1,1,1), (1,−1,1), (−1,−1,1). We call these mass spectra A, B_1, B_2, C_0, C_1, C_2, C_3, respectively. Of course, type A is the normal hierarchy, type B the inverse hierarchy, and type C nearly degenerate. In the present paper we write only the zeroth order form of M_L, although we study the full element M_{ee}. For a more detailed description of M_L see for example [8].

III. SEESAW MECHANISM

Since in the standard model with right-handed neutrinos the Dirac neutrino mass is generated on the same footing as the up quark masses, and the charged lepton masses on the same footing as the down quark masses, for the corresponding mass matrices we first assume the relations $M_\nu \simeq M_u$ and $M_e \simeq M_d$, that is

$$M_\nu \simeq \mathrm{diag}(m_u, m_c, m_t), \tag{5}$$

and $M_e \simeq \mathrm{diag}(m_e, m_\mu, m_\tau)$, where we neglect the small Dirac lepton mixing analogous to the quark mixing. In other terms, we set $U_e \simeq \mathbb{1}$ and $U_\nu \simeq \mathbb{1}$, where U_e diagonalizes M_e and U_ν diagonalizes M_ν. As a matter of fact, the mass hierarchy of charged leptons is similar to that of down quarks. See Ref. [17] for a summary of quark and lepton mass matrices. Because of the seesaw mechanism, we have

$$M_L \simeq M_\nu M_R^{-1} M_\nu, \tag{6}$$

where the heavy neutrino masses M_1, M_2, M_3 have to be much larger than the elements of the matrix M_ν, which is here supposed to be symmetric. Inverting such a formula, the heavy neutrino mass matrix can be achieved,

$$M_R \simeq M_\nu M_L^{-1} M_\nu. \tag{7}$$

Since $M_L^{-1} = U D_L^{-1} U^T$, we can get M_L^{-1} from M_L by exchanging m_i with $1/m_i$ in Eq. (4). We must keep in mind that for the inverse hierarchy the seesaw mechanism implies a cancellation of the Dirac hierarchy for the third and second generations, and for the nearly degenerate spectrum also with the first generation, which seems unnatural, especially for the VO solution. Note that for zero mixing we have $m_1 = m_u^2/M_1$, $m_2 = m_c^2/M_2$ and $m_3 = m_t^2/M_3$.

In this paper we follow a kind of inverse or bottom-up approach, namely we will determine M_R from M_L (and M_ν). In the alternative direct or top-down approach both M_R and M_ν are obtained from a theoretical framework and the inferred M_L is matched to neutrino phenomenology. We would like to stress that for any precise choice of M_ν we can reproduce any form of M_L by adjusting M_R. However, as we will see in the following sections, the form of both M_ν and M_R has a crucial impact on the amount of baryon asymmetry generated in the leptogenesis mechanism, so that we can use the constraint from baryogenesis to study M_R and M_ν. Unfortunately, we are not yet able to determine the mass matrices with precision. Nevertheless, some important considerations, involving also the neutrinoless double beta decay, can be made and this is a central issue of our paper.

IV. BARYOGENESIS FROM LEPTOGENESIS

A baryon asymmetry can be generated from a lepton asymmetry [2]. In fact, this lepton asymmetry is produced by the out-of-equilibrium CP-violating decay of heavy neutrinos. The electroweak sphalerons [18], which violate $B+L$ but conserve $B-L$, transform part of this asymmetry into a baryon asymmetry. Then, the baryon asymmetry, defined as $Y_B = (n_B - n_{\bar{B}})/7n_\gamma = \eta/7$, where $n_B, n_{\bar{B}}, n_\gamma$ are number densities and η is the baryon-to-photon ratio, can be written as (see Refs. [19,20] and references therein)

$$Y_B \simeq \frac{1}{2}\,\frac{1}{g^*}\;d\;\;\epsilon_1, \tag{8}$$

with the CP violating asymmetry in the decay of the lightest heavy neutrino with mass $M_1 \ll M_2 < M_3$ given by

$$\epsilon_1 \simeq \frac{3}{16\pi v^2}\left[\frac{[(M_D^\dagger M_D)_{12}]^2}{(M_D^\dagger M_D)_{11}}\,\frac{M_1}{M_2} + \frac{[(M_D^\dagger M_D)_{13}]^2}{(M_D^\dagger M_D)_{11}}\,\frac{M_1}{M_3}\right], \tag{9}$$

where $M_D = M_\nu U_R$ and $U_R^T M_R U_R = D_R$, and $v \simeq m_t$ is the vacuum expectation value of the Higgs doublet. The lightest heavy neutrino is in equilibrium during the decays of the two heavier ones, washing out the lepton asymmetry generated by them. The factor 1/2 represents the part of the lepton asymmetry converted into baryon asymmetry [21]. The parameter $g^* \simeq 100$ is the number of light degrees of freedom in the theory. Finally, the quantity d is a dilution factor which mostly depends on the mass parameter

$$\tilde{m}_1 = \frac{(M_D^\dagger M_D)_{11}}{M_1}, \tag{10}$$

although for high values of $\tilde{m}_1$ some dependence on M_1 shows up. Minor dilution d of order 10^{-1}, is obtained for 10^{-5} eV$\lesssim \tilde{m}_1 \lesssim 10^{-2}$ eV, while outside this range the dilution grows (that is d diminishes) [22,23]. In fact, if $\tilde{m}_1$ is too small, it is not possible to produce a sufficient number of heavy neutrinos at high temperature, while if $\tilde{m}_1$ is too large, the washout effect of lepton number violating scatterings is too strong and destroys the generated asymmetry. In order to be consistent with primordial nucleosynthesis, the baryon asymmetry Y_B must be in the range 10^{-11}–10^{-10} [24]. At best, Y_B is smaller than ϵ_1 by three orders of magnitude. It is clear that when we obtain M_R from M_ν and M_L through the inverse seesaw formula (7), a determination of the baryon asymmetry is also achieved. In the following two sections we try a partial selection of mass matrices using the bound on the amount of baryon asymmetry.

V. SIMPLIFIED ANALYSIS OF THE SEESAW MECHANISM

In this section we determine the structure of the heavy neutrino mass matrix by assuming the diagonal form (5) for the Dirac neutrino mass matrix. In the next section the effect of a nondiagonal form is discussed. In any subsection we write the zeroth order form of M_L, according to the three possible hierarchies of light neutrino masses, and then we study the heavy neutrino mass matrix and the implications for the baryon asymmetry and for the neutrinoless double beta decay.

A. Normal hierarchy

For both kinds of mixing the zeroth order form for M_L is given by

$$M_L \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_3, \tag{11}$$

where a dominant block in the μ-τ sector appears (see for example [25]). The overall scale is $m_3 \sim 10^{-2} - 10^{-1}$ eV.

1. Double large mixing

If there is full hierarchy of light neutrino masses, $m_3 \gg m_2 \gg m_1$, the matrix M_L^{-1} takes a nearly democratic form [26], so that

$$M_R \simeq \begin{pmatrix} m_u^2 & -m_u m_c & m_u m_t \\ -m_u m_c & m_c^2 & -m_c m_t \\ m_u m_t & -m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_1}. \tag{12}$$

This matrix is diagonalized by the rotation

$$U_R \simeq \begin{pmatrix} 1 & -\frac{m_u}{m_c} & \frac{m_u}{m_t} \\ \frac{m_u}{m_c} & 1 & -\frac{m_c}{m_t} \\ \frac{m_u}{m_t} & \frac{m_c}{m_t} & 1 \end{pmatrix}, \tag{13}$$

with eigenvalues $M_1 \simeq m_u^2/m_1$, $M_2 \simeq m_c^2/m_1$, $M_3 \simeq m_t^2/m_1$. For the full hierarchy and large solar mixing we have $m_2^2 \sim 10^{-5}$ eV^2; hence $m_1 \lesssim 10^{-3}$ eV and the overall scale is $M_R \sim m_t^2/m_1 \gtrsim 10^{16}$ GeV. In this section we take the full hierarchy as reference model.

If $m_3 \gg m_2 \simeq m_1$ (partially degenerate spectrum), we get

$$M_R \simeq \begin{pmatrix} m_u^2 & -\epsilon m_u m_c & -\epsilon m_u m_t \\ -\epsilon m_u m_c & m_c^2 & -m_c m_t \\ -\epsilon m_u m_t & -m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_1}, \tag{14}$$

where very small elements M_{R12} and M_{R13} appear. The overall scale and the heavy masses do not change with respect to the full hierarchical case.

The further condition $m_2 < 0$ leads to the form

$$M_R \simeq \begin{pmatrix} \epsilon^2 \frac{m_1}{m_3} m_u^2 & -m_u m_c & m_u m_t \\ -m_u m_c & \left(\frac{m_1}{m_3} + \epsilon\right) m_c^2 & \frac{m_1}{m_3} m_c m_t \\ m_u m_t & \frac{m_1}{m_3} m_c m_t & \left(\frac{m_1}{m_3} - \epsilon\right) m_t^2 \end{pmatrix} \frac{1}{m_1}. \tag{15}$$

If $m_1 \gg \epsilon m_3$ the overall scale is lowered to $M_R \sim m_t^2/m_3 \sim 10^{14}$ GeV. For $m_1 \simeq \epsilon m_3$ we get the interesting form

$$M_R \simeq \begin{pmatrix} \epsilon^3 m_u^2 & -m_u m_c & m_u m_t \\ -m_u m_c & 2\epsilon m_c^2 & \epsilon m_c m_t \\ m_u m_t & \epsilon m_c m_t & 0 \end{pmatrix} \frac{1}{m_1}. \tag{16}$$

We stress the sharp difference between matrix (16) and matrix (12) or (14). While in Eqs. (12) and (14) the largest element is M_{R33}, in Eq. (16) it is M_{R13} or M_{R23}. In the first case the structure of M_R is roughly similar to the Dirac neutrino mass matrix M_ν in Eq. (5), that is, a nearly diagonal form. In the second case, M_R is roughly off diagonal. As a consequence, also the overall mass scale is different, $m_t^2/m_1 \gtrsim 10^{16}$ GeV for the nearly diagonal form and $m_u m_t/m_1 \gtrsim 10^{11}$ GeV for the nearly off-diagonal form.

Let us discuss the implications for baryogenesis via leptogenesis and for the neutrinoless double beta decay. The baryon asymmetry for the full normal hierarchy is $Y_B \sim 10^{-16}$. Due to the suppression of M_{R12} and M_{R13}, the baryon asymmetry is smaller in the partially degenerate spectrum than in the full normal hierarchy. The suppression of Y_B is of order ϵ^2. In fact, the relation $M_D^\dagger M_D \simeq M_R m_1$ holds in both cases, since $M_D^\dagger M_D$ and $M_R m_1$ are diagonalized by the same U_R with the same eigenvalues. For the nearly off-diagonal form, the baryon asymmetry is enhanced to a sufficient level, due to the moderate hierarchy within M_R. In the neutrinoless double beta decay we get $M_{ee} \simeq m_2 \sim 10^{-3}$–$10^{-2}$ eV if $m_2 > 0$, and $M_{ee} \simeq (m_2^2 - m_1^2)/m_{1,2} \sim 10^{-4}$–$10^{-3}$ eV if $m_2 < 0$. Therefore, the nearly off-diagonal form for M_R tends to enhance Y_B but to suppress M_{ee}. Thus we have found that if the diagonal M_ν in Eq. (5) is assumed then in order to get sufficient baryon asymmetry the matrix M_R must be roughly off diagonal, which leads to a negative m_2 and an approximate prediction for M_{ee}, smaller by one order with respect to the case of a roughly diagonal M_R. The negative value for m_2 could be an indication that phases φ_1 and φ_2 are very different from each other.

2. Single large mixing

In this case we have the form

$$M_R \simeq \begin{pmatrix} m_u^2 & -\epsilon m_u m_c & -\epsilon m_u m_t \\ -\epsilon m_u m_c & \left(\frac{m_1}{m_2}+\epsilon^2\right) m_c^2 & \left(\frac{m_1}{m_2}-\epsilon^2\right) m_c m_t \\ -\epsilon m_u m_t & \left(\frac{m_1}{m_2}-\epsilon^2\right) m_c m_t & \left(\frac{m_1}{m_2}+\epsilon^2\right) m_t^2 \end{pmatrix} \frac{1}{m_1}, \tag{17}$$

and for $m_1 \gg \epsilon^2 m_2$ the structure of M_R is nearly diagonal with the overall scale given by $m_t^2/m_2 \sim 10^{15}$ GeV. The baryon asymmetry can get a moderate enhancement, because of the factor m_2/m_1, and $M_{ee} \simeq m_1 \lesssim 10^{-4}$ eV. If $m_1 \simeq \epsilon^2 m_2$ the element M_{R23} vanishes. For $m_2 < 0$ and $m_1 \simeq -\epsilon^2 m_2$, we have a vanishing M_{R33}, leading to a nearly off-diagonal form at a lower scale.

B. Inverse hierarchy

The zeroth order form of M_L for spectrum B_1 is given by

$$M_L \sim \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & 0 & 0 \\ \frac{1}{\sqrt{2}} & 0 & 0 \end{pmatrix} m_1, \tag{18}$$

$$M_L \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{2} & \frac{1}{2} \\ 0 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix} m_1, \tag{19}$$

according to double or single large mixing. For spectrum B_2 we have

$$M_L \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} \end{pmatrix} m_1 \tag{20}$$

for both cases. The overall scale is $m_{1,2} \sim 10^{-2}$–10^{-1} eV. This is the same as the normal hierarchy, because both are determined by atmospheric oscillations.

1. Double large mixing

The heavy neutrino mass matrix for spectrum B_2 is given by

$$M_R \simeq \begin{pmatrix} \frac{m_3}{m_1} m_u^2 & \epsilon m_u m_c & \epsilon m_u m_t \\ \epsilon m_u m_c & m_c^2 & m_c m_t \\ \epsilon m_u m_t & m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_3}. \tag{21}$$

If $m_2 < 0$, spectrum B_1, we get

$$M_R \simeq \begin{pmatrix} \epsilon^2 m_u^2 & -\frac{m_3}{m_1} m_u m_c & \frac{m_3}{m_1} m_u m_t \\ -\frac{m_3}{m_1} m_u m_c & m_c^2 & m_c m_t \\ \frac{m_3}{m_1} m_u m_t & m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_3}. \tag{22}$$

In the inverse pattern there is stronger hierarchy in M_R with respect to the full normal pattern; see the first row and column in Eqs. (21) and (22). The form of M_R is always nearly diagonal and the off-diagonal form cannot be realized. The mass scale is $M_R \sim m_t^2/m_3 \gtrsim 10^{15}$ GeV. Note also the difference between the inverse hierarchy and the partially degenerate spectrum in the element M_{R11}, which is responsible for the inversion of the light neutrino masses $m_{1,2}$ and m_3.

As a consequence, the baryon asymmetry is even smaller than in the partially degenerate spectrum with $m_2 > 0$, because M_1 is also suppressed. In fact, the suppression of Y_B is of order m_3/m_1 for spectrum B_2 and ϵ^2 for spectrum B_1. Instead, the rate for neutrinoless double beta decay, related to M_{ee}, can be enhanced. In particular, for spectrum B_2 we have $M_{ee} \simeq m_{1,2} \sim 10^{-2}$–$10^{-1}$ eV, which is by one order-higher than for the full normal hierarchy. For spectrum B_1 we obtain $M_{ee} \simeq (m_2^2 - m_1^2)/m_{1,2} \sim 10^{-4}$–$10^{-3}$ eV.

2. Single large mixing

In this case the heavy neutrino mass matrix is the same as for the previous case with $m_2 > 0$, and $M_{ee} \simeq m_1 \sim 10^{-2}$–$10^{-1}$ eV.

C. Nearly degenerate spectrum

Here the light masses are around 1 eV. The zeroth order form of M_L for spectrum C_0 is diagonal,

$$M_L \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_1, \tag{23}$$

for both kinds of mixing. For spectrum C_1 we have

$$M_L \sim \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_1, \tag{24}$$

$$M_L \sim \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m_1, \tag{25}$$

and for spectrum C_2

$$M_L \sim \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m_1, \tag{26}$$

$$M_L \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_1, \tag{27}$$

according to double or single large mixing. Finally, for spectrum C_3

$$M_L \sim \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m_1 \tag{28}$$

for both kinds of mixing. The overall scale is $m_{1,2,3} \sim 0.1$–1 eV, determined by single beta decay. In the nearly degenerate spectrum several delicate cancellations among the terms of M_{ee} may occur and our study of the heavy neutrino mass matrix is only indicative.

1. Double large mixing

The heavy neutrino mass matrix for the spectra C_0, C_1, C_2, C_3 is respectively given by the forms

$$M_R \simeq \begin{pmatrix} m_u^2 & \rho m_u m_c & \rho m_u m_t \\ \rho m_u m_c & m_c^2 & \epsilon^2 m_c m_t \\ \rho m_u m_t & \epsilon^2 m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_1}, \tag{29}$$

$$M_R \simeq \begin{pmatrix} \epsilon^2 m_u^2 & m_u m_c & -m_u m_t \\ m_u m_c & m_c^2 & m_c m_t \\ -m_u m_t & m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_1}, \tag{30}$$

$$M_R \simeq \begin{pmatrix} \epsilon^2 m_u^2 & -m_u m_c & m_u m_t \\ -m_u m_c & m_c^2 & m_c m_t \\ m_u m_t & m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_1}, \tag{31}$$

$$M_R \simeq \begin{pmatrix} -m_u^2 & \epsilon m_u m_c & \epsilon m_u m_t \\ \epsilon m_u m_c & -\epsilon^2 m_c^2 & m_c m_t \\ \epsilon m_u m_t & m_c m_t & -\epsilon^2 m_t^2 \end{pmatrix} \frac{1}{m_1}. \tag{32}$$

In Eq. (29) and Eq. (34) below, we write ρ whenever a cancellation at the level of ϵ occurs. We see that matrices (29), (30), (31) are roughly close to the diagonal form, while Eq. (32) is not. The overall mass scale is $M_R \sim 10^{13}$ GeV.

For spectra C_0 and C_3 the baryon asymmetry is generally suppressed or much suppressed with respect to the full normal hierarchy, while for spectra C_1 and C_2 it is comparable. The suppression of Y_B for spectra C_0 and C_3 is of order ρ^2 and ϵ^2, respectively. Instead, the rate for neutrinoless double beta decay can be further enhanced with respect to the inverse hierarchy. In particular, for spectra C_0 and C_3 we have $M_{ee} \simeq m_1 \sim 0.1$–$1$ eV, by one order higher than for the spectrum B_2. For spectra C_1 and C_2 we have cancellations leading to $M_{ee} \sim 10^{-5}$–10^{-4} eV.

2. Single large mixing

For spectra C_0 and C_3 we have the same matrices as for the previous mixing. For spectra C_1 and C_2 we get

$$M_R \simeq \begin{pmatrix} -m_u^2 & \epsilon m_u m_c & \epsilon m_u m_t \\ \epsilon m_u m_c & m_c^2 & -\epsilon^2 m_c m_t \\ \epsilon m_u m_t & -\epsilon^2 m_c m_t & m_t^2 \end{pmatrix} \frac{1}{m_1}, \tag{33}$$

$$M_R \simeq \begin{pmatrix} m_u^2 & \rho m_u m_c & \rho m_u m_t \\ \rho m_u m_c & \epsilon^2 m_c^2 & m_c m_t \\ \rho m_u m_t & m_c m_t & \epsilon^2 m_t^2 \end{pmatrix} \frac{1}{m_1}. \tag{34}$$

In these two cases the baryon asymmetry is suppressed or much suppressed with respect to the full normal hierarchy, and $M_{ee}\sim 0.1-1$ eV. The suppression of Y_B is of order ϵ^2 and ρ^2, respectively.

VI. SEESAW MECHANISM AND BARYON ASYMMETRY

In this section, instead of the diagonal form, we take the realistic mass matrices, expressed in terms of the Cabibbo parameter $\lambda=0.22$ and the overall mass scale,

$$M_e\sim\begin{pmatrix}\lambda^6 & \lambda^3 & \lambda^5\\ \lambda^3 & \lambda^2 & \lambda^2\\ \lambda^5 & \lambda^2 & 1\end{pmatrix} m_b, \tag{35}$$

$$M_\nu\sim\begin{pmatrix}\lambda^{12} & \lambda^6 & \lambda^{10}\\ \lambda^6 & \lambda^4 & \lambda^4\\ \lambda^{10} & \lambda^4 & 1\end{pmatrix} m_t. \tag{36}$$

These forms can be motivated by an $U(2)$ horizontal symmetry; see Ref. [27] and references therein. Again we neglect U_e with respect to U. However, the effect of $U_\nu\neq\mathbb{1}$ is crucial. Of course, M_L^{-1} can be obtained from the previous section by deleting the quark masses in M_R. In the following calculation we will assume that no cancellations occur between two quantities of the same order in λ.

For the full normal hierarchy and double large mixing we get

$$M_R\sim\begin{pmatrix}\lambda^{12} & \lambda^{10} & \lambda^6\\ \lambda^{10} & \lambda^8 & \lambda^4\\ \lambda^6 & \lambda^4 & 1\end{pmatrix}\frac{m_t^2}{m_1}, \tag{37}$$

diagonalized by

$$U_R\sim\begin{pmatrix}1 & \lambda^2 & \lambda^6\\ -\lambda^2 & 1 & \lambda^4\\ \lambda^6 & -\lambda^4 & 1\end{pmatrix}, \tag{38}$$

with eigenvalues $M_3\sim m_t^2/m_1$, $M_2\sim\lambda^8 M_3$, $M_1\sim\lambda^{12}M_3$, and consistent with the $U(2)$ horizontal symmetry [27]. The baryon asymmetry is enhanced with respect to the diagonal case but remains too small, $Y_B\sim 10^{-14}$. In fact, the relation $M_D^\dagger M_D\sim M_R m_1$ holds, and one obtains

$$\epsilon_1\sim\frac{3}{16\pi}\left(\frac{\lambda^{20}}{\lambda^{12}}\,\lambda^4+\frac{\lambda^{12}}{\lambda^{12}}\,\lambda^{12}\right)\sim 10^{-10}, \tag{39}$$

and $\tilde{m}_1\sim m_1$. The same M_R and Y_B come out for the partially degenerate spectrum with $m_2>0$, the inverse hierarchy, and the nearly degenerate spectra C_1 and C_2, although the scale of M_R is changed accordingly.

For the partially degenerate spectrum with $m_2<0$, assuming for example both $\epsilon\sim\lambda^4$ and $M_{R33}\sim\lambda^{12}m_t^2/m_1$ in matrices (15),(16), we obtain,

$$M_R\sim\begin{pmatrix}\lambda^{16} & \lambda^{12} & \lambda^{10}\\ \lambda^{12} & \lambda^{10} & \lambda^6\\ \lambda^{10} & \lambda^6 & \lambda^8\end{pmatrix}\frac{m_t^2}{m_1}. \tag{40}$$

By considering $M_R^\dagger M_R$, one finds that M_R is diagonalized by

$$U_R\sim\begin{pmatrix}1 & \lambda^4 & \lambda^6\\ -\lambda^4 & 1 & \lambda^2\\ \lambda^6 & -\lambda^2 & 1\end{pmatrix}, \tag{41}$$

with eigenvalues $M_3\sim\lambda^6 m_t^2/m_1$, $M_2\sim M_3$, $M_1\sim\lambda^4 M_3$. Then we obtain

$$\epsilon_1\sim\frac{3}{16\pi}\left(\frac{\lambda^{16}}{\lambda^{12}}\,\lambda^4+\frac{\lambda^{12}}{\lambda^{12}}\,\lambda^4\right)\sim 10^{-4}, \tag{42}$$

and $\tilde{m}_1\sim\lambda^2 m_1$, so that a sufficient amount of baryon asymmetry can be easily achieved. Note that here the dominant term in the leptogenesis formula is the second one. The mass scale in Eq. (37) is given by $m_t^2/m_1\gtrsim 10^{16}$ GeV, and in Eq. (40) by $\lambda^6 m_t^2/m_1\gtrsim 10^{12}$ GeV.

For spectrum C_0 we have the nearly diagonal form

$$M_R\sim\begin{pmatrix}\lambda^{12} & \lambda^{10} & \lambda^{10}\\ \lambda^{10} & \lambda^8 & \lambda^4\\ \lambda^{10} & \lambda^4 & 1\end{pmatrix}\frac{m_t^2}{m_1}, \tag{43}$$

and for spectrum C_3

$$M_R\sim\begin{pmatrix}\lambda^{16} & \lambda^{10} & \lambda^6\\ \lambda^{10} & \lambda^8 & \lambda^4\\ \lambda^6 & \lambda^4 & \lambda^4\end{pmatrix}\frac{m_t^2}{m_1}. \tag{44}$$

The baryon asymmetry is very small in the case C_0 and moderate in the case C_3.

For single large mixing the results are roughly similar to those for double large mixing. Few changes can be easily shown and they are not discussed here. Thus we see that using nondiagonal Dirac neutrino mass matrices and doing an order-of-magnitude analysis leads to few forms for the heavy neutrino mass matrix. Then the two questions of determining the symmetry generating mass matrices and discovering their coefficient should be addressed. For example, horizontal $U(1)$ or $U(2)$ symmetries can be used. This is a fundamental subject that we will try to discuss elsewhere. We need not start from matrices (35), (36) but other forms are possible as well [28].

A simple way to enhance the baryon asymmetry is by means of a quite moderate hierarchy in the Dirac neutrino mass matrix, that is, for its eigenvalues and/or for its mixing angles [29,30]. This is quite evident from the leptogenesis formula. For example, instead of Eq. (36), one can adopt

$$M_\nu\sim\begin{pmatrix}\lambda^6 & \lambda^3 & \lambda^5\\ \lambda^3 & \lambda^2 & \lambda^2\\ \lambda^5 & \lambda^2 & 1\end{pmatrix} m_t, \tag{45}$$

that is a matrix similar to charged lepton masses but with the same overall scale as up quark masses. Let us discuss this issue. In Sec. III we have assumed $M_e \sim M_d$ and $M_\nu \sim M_u$. This is the simplest hypothesis within the standard model, and in the supersymmetric model can be motivated by the fact that the two pairs $M_{e,d}$ and $M_{\nu,u}$ are generated by two distinct Higgs doublets. However, Yukawa couplings for the Dirac neutrino can be very different from the Yukawa couplings for the up quarks. If this case occurs, the hierarchy of masses and mixings in the Dirac neutrino sector can be very different from the mass hierarchy of up quarks and the CKM quark mixing, respectively. When we take matrices (35) and (45) we obtain

$$M_R \sim \begin{pmatrix} \lambda^6 & \lambda^5 & \lambda^3 \\ \lambda^5 & \lambda^4 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \frac{m_t^2}{m_1}, \tag{46}$$

$$\epsilon_1 \sim \frac{3}{16\pi} \left(\frac{\lambda^{10}}{\lambda^6} \ \lambda^2 + \frac{\lambda^6}{\lambda^6} \ \lambda^6 \right) \sim 10^{-6}, \tag{47}$$

and a sufficient amount of baryon asymmetry, $Y_B \sim 10^{-10}$. Both the internal hierarchy and the overall scale of M_ν are important. In fact, if the overall scale in Eq. (45) is $m_{b,\tau}$ instead of m_t, the baryon asymmetry is again suppressed [29]. The relation $M_\nu \sim (\tan\beta) M_e$ can be obtained within supersymmetric left-right models [31]. For very large $\tan\beta$ we get a Dirac neutrino mass matrix similar to Eq. (45). Finally, assuming M_ν truly diagonal and the full normal hierarchy, we get $Y_B \sim 10^{-12}$, to be matched with the value $Y_B \sim 10^{-16}$ obtained in the previous section.

VII. DISCUSSION

Assuming a nearly diagonal mass matrix for Dirac neutrinos, we have studied the structure of the Majorana mass matrix of right-handed neutrinos within the seesaw framework, and its implications for baryogenesis via leptogenesis and for the neutrinoless double beta decay. Then we have explored the effect of a nondiagonal mass matrix for Dirac neutrinos. In this context, we find few possibilities to obtain a sufficient level of baryon asymmetry. The only case when the asymmetry is large should be the nearly off-diagonal form of M_R. Usually, the behavior of M_{ee} is opposite to Y_B, namely when Y_B is suppressed M_{ee} is enhanced and vice versa. For the off-diagonal form we get $M_{ee} \sim 10^{-4}$–10^{-3} eV. In the supersymmetric formula for leptogenesis, the baryon asymmetry is only slightly enhanced [19,22]. In fact, although there are new decay channels, also the washout process is stronger.

When a moderate hierarchy in M_ν is adopted, as in Eq. (45), then M_R need not be close to the off-diagonal form. Therefore, instead of $M_{ee} \sim 10^{-4}$–10^{-3} eV, we can yield the value $M_{ee} \sim 10^{-3}$–10^{-2} eV for the normal hierarchy and $M_{ee} \sim 10^{-2}$–10^{-1} eV for the inverse hierarchy. Note that these predictions cover three different ranges of values for M_{ee}, so that informations from the neutrinoless double beta decay could clarify the structure of fermion mass matrices, if the leptogenesis mechanism is valid.

As a conclusion, we find that, if Y_B has to be within the allowed range, then retaining quark-lepton mass relations $M_e \sim M_d$ and $M_\nu \sim M_u$ leads to a roughly off-diagonal form for M_R and the prediction $M_{ee} \sim 10^{-4}$–10^{-3} eV. If $M_\nu \sim M_u$ is not true, then M_R can be roughly close to the diagonal form and M_{ee} larger than 10^{-3} eV, with Y_B in the allowed range. This is the central result of our paper. We have proposed two such different kinds of model in the previous section.

It has been suggested [26] that the nearly off-diagonal form for M_R is consistent with the nonsupersymmetric unified model $SO(10)$ with an intermediate symmetry breaking scale, where the heavy neutrino mass is generated, while the nearly diagonal form for M_R is consistent with the supersymmetric version without such an intermediate scale, the heavy neutrino mass being generated at the unification scale. In both cases, quark-lepton symmetry is valid. Then, we ask if the framework described here could be embedded in unified models. On the one hand, the leptogenesis scenario could work within unified models [32]. On the other hand, unified models have not yet received decisive support from the experimental detection of proton decay, so that we find it attractive to keep the minimal scenario, as was the motivation of the original paper on the leptogenesis [2]. Within a unified framework, the leptogenesis constraint favors the nonsupersymmetric model.

The present paper focuses on the general structure of mass matrices and does not exclude that possible fine tuning could produce a sufficient amount of baryon asymmetry [19,33], even with nearly diagonal mass matrices. Other interesting papers on the relation between leptogenesis and mass matrices are reported in Ref. [34].

In our simple approach we have not considered the effect of running masses and mixings from the low scale to the high M_R scale where the seesaw formula (6) applies. At our level of approximation, only for the supersymmetric version with large $\tan\beta$ can they be significant [35]. In particular, for spectrum C_0 both double and single large mixing are converted to nearly zero mixing, and for spectra B_2 and C_3 double large mixing is converted to single large mixing. Such spectra are characterized by degeneracy in mass and sign. However, it has been argued [36] that the running of the vacuum expectation values could improve the stability of the lepton mixing matrix.

[1] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe*, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); R.N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[2] M. Fukugita and T. Yanagida, Phys. Lett. B **174**, 45 (1986).

[3] P. Langacker, R.D. Peccei, and T. Yanagida, Mod. Phys. Lett. A **1**, 541 (1986); M.A. Luty, Phys. Rev. D **45**, 455 (1992); C.E. Vayonakis, Phys. Lett. B **286**, 92 (1992); R.N. Mohapatra and X. Zhang, Phys. Rev. D **46**, 5331 (1992); W. Buchmuller and M. Plumacher, Phys. Lett. B **389**, 73 (1996); R. Rangarajan and H. Mishra, Phys. Rev. D **61**, 043509 (2000).

[4] S. Weinberg, Phys. Rev. Lett. **43**, 1566 (1979).

[5] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962); S.M. Bilenky and B. Pontecorvo, Phys. Lett. **95B**, 233 (1980).

[6] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973); L.L. Chau and W.Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984).

[7] G.L. Fogli, E. Lisi, D. Montanino, and A. Palazzo, Phys. Rev. D **64**, 093007 (2001); J.N. Bahcall, M.C. Gonzalez-Garcia, and C. Pena-Garay, J. High Energy Phys. **08**, 014 (2001); A. Baudyopadhyay, S. Choubey, S. Goswami, and K. Kar, Phys. Lett. B **519**, 83 (2001); P.I. Krastev and A.Yu. Smirnov, Phys. Rev. D **65**, 073022 (2002); Q.R. Ahmad *et al.*, Phys. Rev. Lett. **89**, 011302 (2002).

[8] E.Kh. Akhmedov, Phys. Lett. B **467**, 95 (1999).

[9] M. Apollonio *et al.*, Phys. Lett. B **420**, 397 (1998); **466**, 415 (1999); see also the Palo Verde experiment: F. Boehm *et al.*, Phys. Rev. D **62**, 072002 (2000).

[10] H.V. Klapdor-Kleingrothaus *et al.*, Eur. Phys. J. A **12**, 147 (2001).

[11] F. Feruglio, A. Strumia, and F. Vissani, hep-ph/0201291.

[12] H.V. Klapdor-Kleingrothaus *et al.*, Mod. Phys. Lett. A **16**, 2409 (2001).

[13] C.E. Aalseth *et al.*, hep-ex/0202018.

[14] W. Rodejohann, Phys. Rev. D **62**, 013011 (2000).

[15] G. Altarelli and F. Feruglio, Phys. Rep. **320**, 295 (1999).

[16] J. Bonn *et al.*, Nucl. Phys. B (Proc. Suppl.) **91**, 273 (2001); V. Lobashev *et al.*, *ibid.* **91**, 280 (2001).

[17] D. Falcone, hep-ph/0105124.

[18] V.A. Kuzmin, V.A. Rubakov, and M.E. Shaposhnikov, Phys. Lett. **155B**, 36 (1985).

[19] D. Falcone and F. Tramontano, Phys. Rev. D **63**, 073007 (2001); Phys. Lett. B **506**, 1 (2001); F. Buccella, D. Falcone, and F. Tramontano, *ibid.* **524**, 241 (2002).

[20] L. Covi, E. Roulet, and F. Vissani, Phys. Lett. B **384**, 169 (1996).

[21] J.A. Harvey and M.S. Turner, Phys. Rev. D **42**, 3344 (1990); S.V. Khlebnikov and S.E. Shaposhnikov, Nucl. Phys. **B308**, 885 (1988).

[22] W. Buchmuller and M. Plumacher, Int. J. Mod. Phys. A **15**, 5047 (2000).

[23] M. Hirsch and S.F. King, Phys. Rev. D **64**, 113005 (2001).

[24] K.A. Olive, hep-ph/0202486.

[25] W. Buchmuller and T. Yanagida, Phys. Lett. B **445**, 399 (1999).

[26] D. Falcone, Phys. Lett. B **479**, 1 (2000); hep-ph/0006130.

[27] D. Falcone, Phys. Rev. D **64**, 117302 (2001).

[28] B.R. Desai and A.R. Vaucher, Phys. Rev. D **63**, 113001 (2001); J.L. Chkareuli and C.D. Froggatt, Phys. Lett. B **450**, 158 (1999).

[29] D. Falcone, Phys. Rev. D **65**, 077301 (2002).

[30] M.S. Berger and K. Siyeon, Phys. Rev. D **65**, 053019 (2002).

[31] K.S. Babu, B. Dutta, and R.N. Mohapatra, Phys. Rev. D **60**, 095004 (1999); Phys. Lett. B **458**, 93 (1999).

[32] A. Pilaftsis, Int. J. Mod. Phys. A **14**, 1811 (1999); S. Carlier, J.M. Frere, and F.S. Ling, Phys. Rev. D **60**, 096003 (1999); E. Ma, S. Sarkar, and U. Sarkar, Phys. Lett. B **458**, 73 (1999).

[33] G.C. Branco, R. Gonzalez Felipe, F.R. Joaquim, and M.N. Rebelo, hep-ph/0202030; E. Nezri and J. Orloff, hep-ph/0004227.

[34] J. Ellis, S. Lola, and D. Nanopoulos, Phys. Lett. B **452**, 87 (1999); M.S. Berger, Phys. Rev. D **62**, 013007 (2000); R. Barbieri, P. Creminelli, A. Strumia, and N. Tetradis, Nucl. Phys. **B575**, 61 (2000); W. Buchmuller and D. Wyler, Phys. Lett. B **251**, 291 (2001).

[35] N. Haba and N. Okamura, Eur. Phys. J. C **14**, 347 (2000); N. Haba, Y. Matsui, and N. Okamura, *ibid.* **17**, 513 (2000).

[36] N. Nimai Singh, Eur. Phys. J. C **19**, 137 (2001).