

Extension of boundary string field theory on disc and $\mathbb{R}P^2$ worldsheet geometries

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We present an extension of boundary string field theory on disc and $\mathbb{R}P^2$ worldsheet geometries. Finding an appropriate Becchi-Rouet-Stora (BRS) operator in the case of the $\mathbb{R}P^2$ geometry, we generalize the background independent open string field theory (or boundary string field theory) of Witten on a unit disc. The space of two-dimensional field theories on $\mathbb{R}P^2$ is spanned by the scalar operator inserted at the nontrivial loop of $\mathbb{R}P^2$. We discuss the off-shell extension of the crosscap states, which provides an interpolation of orientifold planes of various dimensions.

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I. INTRODUCTION

For a long time, a number of physicists have suspected that the “space of all two-dimensional field theories,” if it is to be defined, should be a natural configuration space of strings on which string field theory is constructed with an appropriate gauge invariant action. Ultraviolet divergences associated with irrelevant operators carrying arbitrarily high dimensions have hampered progress toward this goal. It was shown by Witten [1],¹ however, that, modulo this difficulty, it is possible to find a clear-cut framework based on the Batalin-Vilkovisky (BV) formalism [3,4] in the case of open-string field theory. This framework materializes the idea of “theory space” on a unit disc. String field theory² of this kind, originally called background independent open-string field theory, provides a self-consistent framework for off-shell processes in which relevant and marginal operators are involved.

More recently, this theory has been successfully applied to the problem of open-string tachyon condensation [6,7]. It provides an off-shell interpolation between the Neumann and the Dirichlet boundary conditions of an open string. An exact tachyon potential [6,7] with the right normalization coefficient [8] has been obtained and this has provided a proof of Sen’s conjecture [9] that the difference between the two extremes just cancels the tension of the D-brane in question. It allows us to consider the decay of a higher dimensional brane to a lower one as well. Agreement on the tensions of branes of various dimensions has been obtained. See [10–16] for some of the subsequent developments. We will refer to the background independent open-string field theory of Witten as boundary string field theory (BSFT), following the current nomenclature. An extension of this framework to closed-string field theory appears to be formidable as we do not yet understand well enough the processes under which

the matter central charge is changed by the presence of Becchi-Rouet-Stora (BRS) noninvariant operators located on the bulk of a Riemann surface.

One may note in this last regard that there are some processes of a closed string and geometries represented by a closed string which do not involve operators living in the entire bulk of a Riemann surface. In particular, orientifold planes of various dimensions are characterized as crosscap states on a complex plane representing the $\mathbb{R}P^2$ worldsheet geometry and can be discussed in parallel to D-brane boundary states representing the disc worldsheet geometry. These points have prompted us to consider an $\mathbb{R}P^2$ generalization of BSFT, an extension of the idea of theory space on the unit disc and hence of open-closed-string field theory from the disc and the $\mathbb{R}P^2$ worldsheet geometries, which share the same Euler number. For recent discussions of other aspects of closed-string geometries and closed-string tachyons, see, for example, [17–23].

In this paper, we present an extension of BSFT on disc and $\mathbb{R}P^2$ worldsheet geometries for bosonic strings.³ Our construction is based on the BV formalism applied to both the correlators on the boundary of the disc and the ones on the nontrivial loop \mathcal{C} of $\mathbb{R}P^2$ [$\pi_1(\mathbb{R}P^2) = \mathbf{Z}_2$]. The former part is a repetition of [1] except for the insertion of Chan-Paton space. As for the latter, let us first note that this nontrivial loop can be identified as a source of the crosscap contributing to the Euler number when $\mathbb{R}P^2$ is represented as a part of the complex plane. We find that the scalar operators located on this nontrivial loop accomplish the desirable interpolation between the orientifold planes of various dimensions in much the same way as the operators of BSFT do between D-branes of various dimensions. The off-shell extension [26,27] (see also [28]) of D-brane boundary states and that of crosscap states [29–31] provide an expedient tool for this. The latter will be given in this paper.

The family of off-shell crosscap states considered in our paper corresponds to the background field of the dilaton. The

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¹See also [2].²For a recent review on string field theory, see, for example, [5].³See [24] for a recent discussion. For earlier references on the open-closed mixed system, see, for example, [25].

reason that this is the case is the following. It is well known that the dilaton is the only background field which couples directly to the worldsheet two-dimensional curvature. We choose the worldsheet metric on \mathbb{RP}^2 to be flat everywhere except on the nontrivial loop, that is, the upper half of a unit circle. The curvature is then concentrated on this loop or fringe as a delta function singularity. This justifies our construction that the background field that deforms the theory lives only on the nontrivial loop of \mathbb{RP}^2 .

We find an appropriate BRS operator which acts on the operators on this loop on \mathbb{RP}^2 . Finding such an operator is mandatory for us to materialize the BV formalism on \mathbb{RP}^2 . The total action resulting from our discussion is simply a sum of two terms, one from the disc geometry and the other from \mathbb{RP}^2 . In this sense, our construction generalizes that of the first quantized unoriented string theory at Euler number 1. Each part of the action separately obeys a first order differential equation with respect to the couplings of operators inserted. The relative normalization between the two terms is fixed by appealing to the dilaton tadpole cancellation at Euler number 1 [33–35]. The work of [8] tells us that the normalization of the disc part is in fact the D-brane tension.

In the next section, we briefly summarize essential ingredients of BSFT, which are the BV formalism, the action of the BRS operator on the operators located on the boundary of a disc, and the defining differential equation for the action S^{disc} . We recall the off-shell boundary states introduced in [26,27]. In Sec. III, the \mathbb{RP}^2 extension of BSFT is given. We find an appropriate BRS operator acting properly on operators on the nontrivial loop represented as half of a unit circle in the complex plane. We compute the action of the BRS operator and derive a differential equation for $S^{\mathbb{RP}^2}$. We introduce off-shell crosscap states which manage to interpolate states representing orientifold planes of various dimensions. In Sec. IV, we discuss the total action. The total action consists of the contributions from the two geometries. We discuss how to fix the relative normalization.

In Appendix A, we construct a holomorphic field with weight p on the disc and that on \mathbb{RP}^2 , using the construction on the plane as the double of the surfaces. These are found to be useful in the calculation in the text. In Appendix B, we recall the tachyon vertex operator of an unoriented bosonic string with intercept -4 .

II. BACKGROUND INDEPENDENT OPEN-STRING FIELD THEORY OF WITTEN AND OFF-SHELL BOUNDARY STATES

A. BV formalism

Let us first recall the BV formalism briefly. The basic ingredients of this formalism can be summarized as a triplet

$$(\mathcal{M}, \omega, V). \tag{2.1}$$

Here \mathcal{M} is a supermanifold equipped with local coordinates $\{\{u^I\}\}$. We denote by ω a nondegenerate odd (fermionic) symplectic two-form which is closed,

$$d\omega = 0. \tag{2.2}$$

Finally, \mathcal{M} possesses $U(1)$ symmetry generated by the fermionic vector field $V = V^I(d/du^I)$. This $U(1)$ symmetry is identified with the ghost number.

The action functional (or zero-form) S in this formalism is introduced through

$$dS = i_V \omega, \tag{2.3}$$

where i_V is an interior contraction by V . That V generates a symmetry of ω translates into

$$\mathcal{L}_V \omega = 0, \tag{2.4}$$

where $\mathcal{L}_V \equiv di_V + i_V d$ is the Lie derivative. We see that Eqs. (2.2) and (2.4) ensure the existence of a scalar functional S obeying Eq. (2.3) modulo global problems on “theory space.” That the antibrackets

$$\{S, S\}_{AB} \tag{2.5}$$

be constant provides the nilpotency of V ,

$$V^2 = 0, \tag{2.6}$$

and V is naturally identified with the BRS charge. In the next section, we will find that this framework is realized on the \mathbb{RP}^2 worldsheet geometry as well. Let us first review the realization on the disc by Witten.

B. $\{Q_{BRS}, \mathcal{O}\}$

We will first obtain the BRS transformation of a generic operator \mathcal{O} with ghost number 1 located on the boundary $\partial\Sigma$ of the unit disc Σ . Let

$$\mathcal{O}(w, \bar{w}) = \frac{1}{i w} c^z(w) \mathcal{V}(w, \bar{w}) = c^\sigma(\sigma) \mathcal{V}(w, \bar{w}), \tag{2.7}$$

where \mathcal{V} is a generic scalar operator with ghost number 0 and the last equality holds only on the unit circle. Note that $c^\sigma(\sigma)$, the tangent component of the ghost field along the boundary of the disc, is the only nonvanishing component on the boundary. See Eq. (A9) in Appendix A. It is safe to obtain the BRS transformation

$$\delta_{BRS} \mathcal{O} = i \epsilon \{Q_{BRS}, \mathcal{O}\}, \tag{2.8}$$

using a free open string:

$$Q_{BRS} = \frac{1}{2\pi i} \oint dz j_{BRS}, \tag{2.9}$$

$$j_{BRS}(z) = c^z(z) T_{zz}^m(z) + :b_{zz}(z) c^z(z) \partial_z c^z(z): + \frac{3}{2} \partial_z^2 c^z(z),$$

$$T_{zz}^m(z) = -\frac{1}{\alpha'} : \partial_z X^\mu \partial_z X_\mu :. \tag{2.10}$$

We find

$$\{Q_{BRS}, \mathcal{O}(w, \bar{w})\} = c^\sigma \partial_\sigma c^\sigma(w) \left(\alpha' \frac{\partial^2}{\partial X^\mu \partial X_\mu} + 1 \right) \mathcal{V}(w, \bar{w}). \quad (2.11)$$

We confirm that an on-shell open-string tachyon with intercept -1 is represented by the vertex operator $\mathcal{V} = \exp\{ik \cdot X\}$ with $\alpha' M^2 = -\alpha' k^\mu k_\mu = -1$.

$$\mathbf{C} \cdot dS^{disc} = i_V \boldsymbol{\omega}$$

The defining differential equation (2.3) for S^{disc} is written as

$$\frac{\delta S^{disc}}{\delta \lambda_\alpha} = \frac{K}{2} \int \int \text{tr} \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} \langle \mathcal{O}^\alpha(\sigma) \{Q_{BRS}, \mathcal{O}\}(\sigma') \rangle_{\mathcal{V}, disc}^{ghost}, \quad (2.12)$$

$$\begin{aligned} \mathcal{O}(\sigma) &= c^\sigma(\sigma) \mathcal{V}(\sigma) = \sum_\alpha \lambda_\alpha c^\sigma(\sigma) \mathcal{V}^\alpha(\sigma) \\ &= \sum_\alpha \lambda_\alpha \mathcal{O}^\alpha(\sigma). \end{aligned} \quad (2.13)$$

Here $\langle \dots \rangle_{\mathcal{V}, disc}^{ghost}$ is the unnormalized path integral with respect to the worldsheet matter action

$$I^{disc} = I^{bulk} + \int_{\partial\Sigma} \frac{d\sigma}{2\pi} \mathcal{V}(\sigma), \quad (2.14)$$

$$I^{bulk} = \frac{1}{4\pi\alpha'} \int_\Sigma \sqrt{\det \eta} \eta^{ij} \partial_i X^\mu \partial_j X^\nu g_{\mu\nu} d^2\sigma. \quad (2.15)$$

Here η^{ij} is a worldsheet metric which is taken to be flat, and $g_{\mu\nu}$ is a spacetime metric with Minkowski signature. We denote by “tr” the trace over the Chan-Paton space. This simply gives us a factor n of $so(n)$ Lie algebra. Separating the ghost Hilbert space, we find

$$\begin{aligned} \frac{\delta S^{disc}}{\delta \lambda_\alpha} &= Kn \int \int \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} [1 - \cos(\sigma - \sigma')] \\ &\quad \times \langle c_1 c_0 c_{-1} \rangle_{disc} \\ &\quad \times \left\langle \mathcal{V}^\alpha(\sigma) \left(\alpha' \frac{\partial^2}{\partial X^\mu \partial X_\mu} + 1 \right) \mathcal{V}(\sigma') \right\rangle_{\mathcal{V}, disc}. \end{aligned} \quad (2.16)$$

D. Off-shell boundary state

The unnormalized matter path integral $\langle \dots \rangle_{\mathcal{V}, disc}$ can be represented as a matrix element

$$\langle \dots \rangle_{\mathcal{V}, disc} = \langle B | \dots | 0 \rangle_{\mathcal{V}, disc}. \quad (2.17)$$

The bra vector $\langle B |$ obeys

$$\langle B | \left[\frac{1}{2\pi\alpha'} \left(z \frac{\partial}{\partial z} + \bar{z} \frac{\partial}{\partial \bar{z}} \right) X^\mu + \frac{1}{2\pi} \frac{\partial \mathcal{V}}{\partial X^\mu} \right] \Bigg|_{z=e^{i\sigma}, \bar{z}=e^{-i\sigma}} = 0. \quad (2.18)$$

We refer to $\langle B |$ as an off-shell boundary state of the disc. The condition (2.18) is a consequence of the correspondence between the matrix element (2.17) and the end point of the path integral. The latter obeys the boundary condition derived from the worldsheet action I^{disc} on the disc. Equation (2.18) tells us that, at the initial point of the coupling constant flow, the system obeys the Neumann boundary condition:

$$\langle B | (\alpha_n + \tilde{\alpha}_{-n}) = 0. \quad (2.19)$$

The end point of the flow is described by the zero of $\partial \mathcal{V} / \partial X^\mu$, namely, the Dirichlet boundary condition:

$$\langle B | (\alpha_n - \tilde{\alpha}_{-n}) = 0. \quad (2.20)$$

An explicit interpolation between the two ends can be done in solvable cases. In the case of quadratic profiles, the proper form of the Green's function has been given in [26,27]. For analyses which utilize the boundary sine-Gordon model and the g function = $\langle B | 0 \rangle$, see [36,37].

III. RP² EXTENSION

We now proceed to construct the RP² extension of BSFT. RP² is a nonorientable Riemann surface of Euler number 1 with no hole, no boundary, and one crosscap. It is a geometry swept by a closed string. The external physical states of this nonorientable closed string are represented by the vertex operators inserted at any point on the surface. The physical state conditions are stated as $\{Q_{BRS}, \mathcal{O}^{closed}\} = 0$. Here Q_{BRS} is the BRS operator on RP² and \mathcal{O}^{closed} is a generic vertex operator. In the case of the ground state scalar of a closed unoriented bosonic string, this leads us to the on-shell condition $\alpha' M^2 = -\alpha' k_\mu k^\mu = -4$, namely, the closed-string tachyon with intercept -4 . For completeness of our discussion, we will include its derivation in Appendix B.

Another property of RP², which is of vital importance to us, is $\pi_1(\text{RP}^2) = \mathbf{Z}_2$. This permits us to consider a nontrivial loop \mathcal{C} which is represented by a path connecting any two conjugate points (for example, 1 and -1) on the complex plane, i.e., the double of this Riemann surface. As a reference loop, we consider half of a unit circle $z = e^{i\sigma}$ ($0 \leq \sigma < \pi$).

A. Glossary of notation

Let us first summarize our notation in the RP² case. (See Appendix A for more detail.) The tangential component and the normal component of the ghost field c^σ and c^r along the loop are, respectively,

$$\begin{aligned} c^\sigma &\equiv \frac{\partial \sigma}{\partial z} c^z + \frac{\partial \sigma}{\partial \bar{z}} c^{\bar{z}} = -\frac{i}{2} \left(\frac{c^z(z)}{z} - \frac{\bar{c}^{\bar{z}}(\bar{z})}{\bar{z}} \right) \\ &= -i \frac{c^z(z) - c^z(-z)}{2z}, \end{aligned} \quad (3.1)$$

$$c^r \equiv \frac{\partial r}{\partial z} c^z + \frac{\partial r}{\partial \bar{z}} c^{\bar{z}} = \frac{1}{2r} [\bar{z} c^z(z) + z \bar{c}^{\bar{z}}(\bar{z})]$$

$$= \frac{c^z(z) + c^z(-z)}{2z}. \quad (3.2)$$

In both equations, the last equality holds only when $|z|=1$. We also note that

$$c^\sigma \partial_\sigma c^\sigma(w)$$

$$= -i \frac{c^z(w) - c^z(-w)}{2w} \frac{\partial_z c^z(w) + (\partial_z c^z)(-w)}{2}, \quad (3.3)$$

$$c^\sigma(w) \partial_\sigma \mathcal{V}(X^\mu)$$

$$= \frac{c^z(w) - c^z(-w)}{2} [\partial_z X^\mu(w) + (\partial_z X^\mu)(-w)] \frac{\partial}{\partial X^\mu} \mathcal{V}. \quad (3.4)$$

Here the argument $-w$ should be substituted after the derivatives are taken and \mathcal{V} is an arbitrary function of X^μ .

B. BRS charge $Q^{(\sigma)}$

Our first objective is to obtain a BRS charge which corresponds to the vector field V in the BV formalism for the $\mathbb{R}P^2$ case. Let us consider the following expression:

$$Q_{BRS}^{(\sigma)} \equiv \oint \frac{dz}{2\pi i} j_z^{BRS}(z),$$

$$j_z^{BRS}(z) \equiv 2c_{\text{even}}^z(z) \left(-\frac{1}{\alpha'} \right) :(\partial_z X^\mu)_{\text{even}}(z):$$

$$\times (\partial_z X_\mu)_{\text{even}}(z):$$

$$+ :b_{zz \text{ even}}(z) c_{\text{even}}^z(z) \partial_z c_{\text{even}}^z(z):$$

$$+ \text{total derivatives}, \quad (3.5)$$

where the subscript ‘‘even’’ implies that the modes are restricted to the even ones. For example, $c_{\text{even}}^z(z)$

$\equiv \sum_n \text{even} c_n z^{-n+1} = [c^z(z) - c^z(-z)]/2$. We will now show that our $Q_{BRS}^{(\sigma)}$, when acted upon the conformal fields on $|z|=1$, generates the BRS transformations representing the diffeomorphisms in the σ direction.

Let us first consider the ghost part of our BRS current:

$$j_z^{(g)}(z) \equiv \frac{1}{8} [b_{zz}(z) + b_{zz}(-z)] [c^z(z) - c^z(-z)]$$

$$\times [\partial_z c^z(z) + (\partial_z c^z)(-z)]. \quad (3.6)$$

We obtain

$$j_z^{(g)}(z) c^\sigma(w) \sim -\frac{i}{2} \left(\frac{1}{z-w} - \frac{1}{z+w} \right)$$

$$\times \frac{c^z(z) - c^z(-z)}{2w} \frac{\partial_z c^z(z) + (\partial_z c^z)(-z)}{2}, \quad (3.7)$$

so that

$$\delta_B c^\sigma(w) \equiv i \epsilon \left\{ \oint \frac{dz}{2\pi i} j_z^{(g)}(z), c^\sigma(w) \right\}$$

$$= i \epsilon c^\sigma \partial_\sigma c^\sigma(w). \quad (3.8)$$

One can easily show that $\delta_B c^r(w) = 0$. These are the desirable BRS transformation laws for the σ diffeomorphisms. (Again $|w|=1$ is understood.)

Next, we consider the matter part of the BRS current:

$$j_z^{(m)}(z) \equiv 2 \frac{c^z(z) - c^z(-z)}{2} \left(-\frac{1}{\alpha'} \right)$$

$$\times :(\partial_z X^\mu)_{\text{even}}(z) (\partial_z X_\mu)_{\text{even}}(z):. \quad (3.9)$$

We find

$$j_z^{(m)}(z) \mathcal{V}'(X^\mu(w, \bar{w})) \sim \frac{c^z(z) - c^z(-z)}{2} \left(\frac{1}{z-w} + \frac{1}{z+w} \right) :(\partial_z X^\mu)_{\text{even}}(z) \frac{\partial}{\partial X^\mu} \mathcal{V}'(X^\mu):$$

$$- \frac{\alpha'}{8} \frac{c^z(z) - c^z(-z)}{2} \left(\frac{1}{(z-w)2} + \frac{2}{(z-w)(z+w)} + \frac{1}{(z+w)2} \right) : \frac{\partial^2}{\partial X^\mu \partial X_\mu} \mathcal{V}'(X) :, \quad (3.10)$$

so that

$$\left\{ \oint \frac{dz}{2\pi i} j_z^{(m)}(z), \mathcal{V}'(X^\mu(w, \bar{w})) \right\} = c^\sigma \partial_\sigma \mathcal{V}'(X^\mu) - \frac{\alpha'}{4} [\partial_\sigma c^\sigma(w) + 2i c^\sigma(w)] : \frac{\partial^2}{\partial X^\mu \partial X_\mu} \mathcal{V}'(X^\mu) :. \quad (3.11)$$

In particular, setting $\mathcal{V}'(X^\mu) = X^\mu$, we obtain

$$\delta_B X^\mu \equiv i\epsilon \left\{ \oint \frac{dz}{2\pi i} j_z^{(m)}(z), X^\mu \right\} = i\epsilon c^\sigma \partial_\sigma X^\mu(w). \quad (3.12)$$

This is again the desirable BRS transformation. Thus, the operator

$$Q_{BRS}^{(\sigma)} = \oint \frac{dz}{2\pi i} [j_z^{(g)}(z) + j_z^{(m)}(z)] \quad (3.13)$$

generates the BRS transformations δ_B associated with the σ diffeomorphisms on $|w| = 1$.

C. $\{Q_{BRS}^{(\sigma)}, \mathcal{O}\}$

It is now immediate to carry out the action of the BRS charge on a generic operator with ghost number 1. This also brings us an operator which is ‘‘on shell’’ with respect to $Q_{BRS}^{(\sigma)}$, that is, invariant under δ_B . Let \mathcal{O} be a scalar operator with ghost number 1,

$$\mathcal{O}(w, \bar{w}) = c^\sigma(w) \mathcal{V}'^\sigma(X^\mu(w, \bar{w})) + c^r(w) \mathcal{V}'^r(X^\mu(w, \bar{w})). \quad (3.14)$$

We find

$$\begin{aligned} \{Q_{BRS}^{(\sigma)}, \mathcal{O}(w, \bar{w})\} &= c^\sigma \partial_\sigma c^\sigma \left(1 + \frac{\alpha'}{4} \frac{\partial^2}{\partial X^2} \right) \mathcal{V}'^\sigma(X) \\ &\quad - c^r c^\sigma \left(\partial_\sigma \mathcal{V}'^r(X) - i \frac{\alpha'}{2} \frac{\partial^2}{\partial X^2} \mathcal{V}'^r(X) \right) \\ &\quad + \frac{\alpha'}{4} c^r \partial_\sigma c^\sigma \frac{\partial^2}{\partial X^2} \mathcal{V}'^r(X). \end{aligned} \quad (3.15)$$

The right-hand side vanishes when

$$\begin{aligned} \mathcal{V}'^\sigma(X) &= \beta^\sigma \exp\{ik \cdot X(w, \bar{w})\}, \\ \alpha' M^2 &\equiv -\alpha' k^2 = -4, \\ \mathcal{V}'^r(X) &= \beta^r \mathbf{1}. \end{aligned} \quad (3.16)$$

Here β^σ and β^r are constants and $\mathbf{1}$ is the identity operator. Equation (3.16) includes the case

$$\mathcal{V}'^\sigma(X) = \beta^\sigma \exp\{ik \cdot X(w, \bar{w})\},$$

$$\alpha' M^2 \equiv -\alpha' k^2 = -4,$$

$$\mathcal{V}'^r(X) = 0 \quad (3.17)$$

as a special case. In what follows, we will consider an off-shell deformation of the following kind:

$$\mathcal{O} = c^\sigma \mathcal{V}'' + c^r \mathcal{V}', \quad (3.18)$$

where $\mathcal{V}', \mathcal{V}''$ are generic scalar relevant operators.

D. Nilpotency of δ_B

The condition we need in our formalism is $\delta_B^2 \mathcal{O} = 0$ with \mathcal{O} given by Eq. (3.18). Using $\delta_B^2 c^\sigma = 0$, we obtain

$$\delta_B^2 \mathcal{O} = c^\sigma \delta_B^2 \mathcal{V}'' + c^r \delta_B^2 \mathcal{V}'. \quad (3.19)$$

On the other hand, for both $\mathcal{V} = \mathcal{V}', \mathcal{V}''$

$$\begin{aligned} \delta_B^2 \mathcal{V} &= i\epsilon \delta_B \left\{ c^\sigma \partial_\sigma \mathcal{V} - \frac{\alpha'}{4} (\partial_\sigma c^\sigma + 2ic^\sigma) \frac{\partial^2}{\partial X^2} \mathcal{V} \right\} \\ &= i\epsilon \left\{ (\delta_B c^\sigma) \partial_\sigma \mathcal{V} - c^\sigma \partial_\sigma (\delta_B \mathcal{V}) - \frac{\alpha'}{4} (\partial_\sigma (\delta_B c^\sigma)) \right. \\ &\quad \left. + 2i \delta_B c^\sigma \frac{\partial^2}{\partial X^2} \mathcal{V} + \frac{\alpha'}{4} (\partial_\sigma c^\sigma + 2ic^\sigma) \delta_B \frac{\partial^2}{\partial X^2} \mathcal{V} \right\}. \end{aligned} \quad (3.20)$$

The terms in the right-hand side cancel with one another and we establish that $\delta_B^2 \mathcal{O} = 0$.

E. dS^{RP^2}

The analyses made and the properties established above provide the ingredients necessary for us to define dS in the framework of BV on $\Sigma' = \text{RP}^2$ as well. Following the disc case, we introduce a defining differential equation (2.3) for S^{RP^2} :

$$\frac{\delta S^{\text{RP}^2}}{\delta \lambda_\alpha} = \frac{K'}{2} \int_C \int_C \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} \langle \mathcal{O}^\alpha(\sigma) \{Q_{BRS}^{(\sigma)}, \mathcal{O}\}(\sigma') \rangle_{\mathcal{V}', \text{RP}^2}^{\text{ghost}} \quad (3.21)$$

$$= \frac{K'}{2} \int_C \int_C \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} \langle c^r(\sigma) (c^r c^\sigma)(\sigma') \rangle_{\text{RP}^2} \left\langle \mathcal{V}'^\alpha(\sigma) (-) \left(\partial_{\sigma'} \mathcal{V}'(\sigma') - i \frac{\alpha'}{2} \frac{\partial^2}{\partial X^\mu \partial X_\mu} \mathcal{V}'(\sigma') \right) \right\rangle_{\mathcal{V}', \text{RP}^2}$$

$$= K' \int_{\mathcal{C}} \int_{\mathcal{C}} \frac{d\sigma}{2\pi} \frac{d\sigma'}{2\pi} \sin(\sigma - \sigma') \langle c_1 c_0 c_{-1} \rangle_{\text{RP}^2} \left\langle \mathcal{V}'^\alpha(\sigma) \left(\partial_{\sigma'} \mathcal{V}'(\sigma') - i \frac{\alpha'}{2} \frac{\partial^2}{\partial X^\mu \partial X_\mu} \mathcal{V}'(\sigma') \right) \right\rangle_{\mathcal{V}', \text{RP}^2}, \quad (3.22)$$

$$\mathcal{V}' \equiv \sum_{\alpha} \lambda_{\alpha} \mathcal{V}'^{\alpha}. \quad (3.23)$$

Here the unnormalized path integral $\langle \dots \rangle_{\mathcal{V}'}$ is evaluated with respect to

$$I^{\text{RP}^2} = I'^{\text{bulk}} + \int_{\mathcal{C}} \frac{d\sigma}{\pi} \mathcal{V}'(\sigma), \quad (3.24)$$

$$I'^{\text{bulk}} = \frac{1}{4\pi\alpha'} \int_{\Sigma' - \mathcal{C}} \sqrt{\det \eta} \eta^{ij} \partial_i X^\mu \partial_j X^\nu g_{\mu\nu} d^2\sigma. \quad (3.25)$$

Note that, in Eq. (3.23), the allowed form of the nonvanishing ghost three-point function has selected \mathcal{V}' alone and \mathcal{V}'' has disappeared.

F. The off-shell crosscap states

The unnormalized path integral on RP^2 geometry can be represented as a matrix element between the ket vector of the closed-string vacuum and the bra vector of the off-shell crosscap state $\langle C|$:

$$\langle \dots \rangle_{\mathcal{V}'} = \langle C | \dots | 0 \rangle_{\mathcal{V}'}. \quad (3.26)$$

The derivation of the equation which the bra vector $\langle C|$ obeys involves the closed and unoriented nature of the Riemann surface RP^2 . The fundamental domain can be represented by $\Sigma' = \{r < 1\} \cup \{r = 1, 0 \leq \sigma < \pi\}$ on the complex plane. The variation of the action I^{RP^2} must be carried out consistently with the antipodal identification $P(z) = -1/\bar{z}$. We find

$$\begin{aligned} \delta I^{\text{RP}^2} &= \frac{1}{2\pi\alpha'} \int_0^\pi d\sigma \delta X^\mu \left\{ \left(w \frac{d}{dw} + \bar{w} \frac{d}{d\bar{w}} \right) X^\mu(w, \bar{w}) + \alpha' \frac{\partial \mathcal{V}'(X^\mu(w, \bar{w}))}{\partial X^\mu} \right\} \Bigg|_{w=z, \bar{w}=\bar{z}} \\ &+ \frac{1}{2\pi\alpha'} \int_0^\pi d\sigma \delta X^\mu \left\{ \left(w \frac{d}{dw} + \bar{w} \frac{d}{d\bar{w}} \right) X^\mu(w, \bar{w}) + \alpha' \frac{\partial \mathcal{V}'(X^\mu(w, \bar{w}))}{\partial X^\mu} \right\} \Bigg|_{w=-1/\bar{z}, \bar{w}=-1/z} \\ &+ \text{the part proportional to the equation of motion.} \end{aligned} \quad (3.27)$$

Here, $z = (1 - \delta)e^{i\sigma}$ and δ is a small but nonvanishing number. Reflecting the fact that RP^2 is nonorientable, we have nonvanishing contributions from an open cover of $\mathcal{C} = \{r = 1, 0 \leq \sigma < \pi\}$, namely, the edge of the fundamental domain Σ' despite the fact that there is no boundary. Stokes theorem does not apply.

From the correspondence between the matrix element and the end point condition of the path integral, we conclude that the bra vector $\langle C|$ must obey

$$\langle C | K_\mu(z, \bar{z}; \delta) = 0,$$

$$K_\mu(z, \bar{z}; \delta) \equiv \left[\frac{1}{2\pi\alpha'} \left(w \frac{\partial}{\partial w} + \bar{w} \frac{\partial}{\partial \bar{w}} \right) X^\mu + \frac{1}{2\pi} \frac{\partial \mathcal{V}'}{\partial X^\mu} \right] \Bigg|_{w=z, \bar{w}=\bar{z}} + \left[\frac{1}{2\pi\alpha'} \left(w \frac{\partial}{\partial w} + \bar{w} \frac{\partial}{\partial \bar{w}} \right) X^\mu + \frac{1}{2\pi} \frac{\partial \mathcal{V}'}{\partial X^\mu} \right] \Bigg|_{w=-1/\bar{z}, \bar{w}=-1/z}. \quad (3.28)$$

Expansion of this condition in δ generates conditions which may be referred to as off-shell crosscap conditions. The initial point of the coupling constant flow is described as the ordinary crosscap condition:

$$\langle C | [\alpha_n + (-)^n \tilde{\alpha}_{-n}] = 0. \quad (3.29)$$

The end point of the flow is described by the zero of $(\partial \mathcal{V}' / \partial X^\mu)|_{w=z, \bar{w}=\bar{z}} + (\partial \mathcal{V}' / \partial X^\mu)|_{w=-1/\bar{z}, \bar{w}=-1/z}$, which is nothing but the orientifold condition

$$\langle C | [\alpha_n - (-)^n \tilde{\alpha}_{-n}] = 0. \quad (3.30)$$

We have derived the two-point function from Eq. (3.28) in

the case that \mathcal{V}' is quadratic in X^μ and checked that it takes the proper form on both ends of the flow.⁴

Let us finish this section by discussing the properties of orientifold planes in general. They are certainly nondynamical objects in string perturbation theory. Nonperturbatively, they may become dynamical, however. It is known in fact that the orientifold planes do become dynamical in Sen's description of F theory [32] which connects F to orientifolds and the Seiberg-Witten curve of the uv finite four-dimensional $N=2$ supersymmetric gauge theory. While we are not able to identify the nonperturbative dynamical origin which causes the dilaton condensation and hence orientifold planes to get deformed, our construction does provide a proper description once this happens.

IV. TOTAL ACTION

Our construction of the action for the open-closed string field theory is completed by adding S^{disc} and S^{RP^2} :

$$S = S^{disc} + S^{RP^2}. \quad (4.1)$$

The individual parts S^{disc} and S^{RP^2} obey the first order differential equations (2.16) and (3.23), respectively. Here we discuss the normalization constants appearing in these two quantities. It was shown in [1,2] that S^{disc} on shell (that is, with no operator insertion) is actually the matter partition function of a free open string up to an overall normalization. The same applies to S^{RP^2} as well: on shell it is the RP^2 matter partition function up to an overall normalization. We write these relations as

$$S^{disc}|_{on\ shell} = Z^{disc}/g_{disc}, \quad (4.2)$$

$$S^{RP^2}|_{on\ shell} = Z^{RP^2}/g_{RP^2}. \quad (4.3)$$

Here Z^{disc} and Z^{RP^2} are complete partition functions obtained by carrying out the path integrals over X^μ and the two-dimensional metric. [They are actually free (vacuum) energies of a string as a single string can produce only a connected graph.] We denote by g_{disc} and g_{RP^2} the contributions in the respective geometries from the path integrals of the two-dimensional metric in the Polyakov formulation. They are made of the ghost determinant divided by the volume of the conformal killing vector and the order of the disconnected diffeomorphisms.

Equation (4.2) and Eq. (4.3) can be related by invoking the cancellation between the dilaton tadpole of the disc and that of RP^2 [33–35]. The operator insertion of a zero momentum tadpole in the first quantized string can be done by taking a derivative of the partition function with respect to the tension of a fundamental string $T^{fund} = 1/2\pi\alpha'$.

On the other hand, the BV formalism guarantees that the right-hand side of the basic equation $dS = i_{\mathcal{V}}\omega$ is a total dif-

ferential at least locally. The integration constant is, therefore, an additive one and, up to this constant, S^{disc/RP^2} is completely determined:

$$\begin{aligned} S^{disc} &= K n f^{disc}(\lambda^\alpha), \\ S^{RP^2} &= K' f^{RP^2}(\lambda^\alpha), \end{aligned} \quad (4.4)$$

where $f^{disc}(\lambda^\alpha)$ and $f^{RP^2}(\lambda^\alpha)$ are both scalar functions, and can be determined by using solvable scalar profiles.⁵

Putting all these considerations together, we find that the cancellation of the dilaton tadpoles implies

$$\frac{\partial}{\partial T^{fund}} (K n g_{disc} f^{disc} + K' g_{RP^2} f^{RP^2})|_{on\ shell} = 0. \quad (4.5)$$

In [8], K was determined by demanding that a proper amplitude of the nearly on-shell three-point open-string tachyon be obtained. There is no argument in the RP^2 case which parallels the disc case. We see that Eq. (4.5) determines the normalization K' instead.

A final remark of this subsection is on the unoriented nature of our open and closed strings. The open-closed string field theory constructed above is nonorientable as our construction involves the nonorientable surface RP^2 . On shell, the state space of an unoriented open string is obtained by choosing \mathcal{O} vertex operators and Chan-Paton indices such that the product is even under the twist operation Ω . (We have nothing to say in this paper on the state space of an unoriented closed string.) Let us note, however, that Ω is a symmetry only at the two ends of the coupling constant flow. In the closed-string picture, this is easily seen in our construction of the off-shell boundary/crosscap states. Equations (2.19), (2.20), (3.29), and (3.30) stay invariant under the twist operation as

$$\Omega(\alpha_n \pm \tilde{\alpha}_{-n})\Omega^{-1} = \pm(\alpha_{-n} \pm \tilde{\alpha}_n),$$

$$\Omega(\alpha_n \pm (-)^n \tilde{\alpha}_{-n})\Omega^{-1} = \pm(\alpha_{-n} \pm (-)^n \tilde{\alpha}_n). \quad (4.6)$$

At a generic value of the coupling, neither of Eqs. (2.18),(3.28) stays invariant.

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APPENDIX A

In this appendix, the mode expansion of a conformal field with weight p is obtained both on a unit disc D_2 and on RP^2 .

⁴These will appear elsewhere together with other related issues.

⁵We have set $\langle c_1 c_0 c_{-1} \rangle_{disc} = \langle c_1 c_0 c_{-1} \rangle_{RP^2} = 1$.

D_2 and RP^2 are constructed from the complex plane by the respective identifications $z' = 1/\bar{z}$ and $z' = -1/\bar{z}$.

Let

$$\Phi(z) \equiv \Phi_{\underbrace{\begin{matrix} z \\ \vdots \\ z \end{matrix}}_p}^{\underbrace{\begin{matrix} z \\ \vdots \\ z \end{matrix}}_q}(z)$$

be a holomorphic field with weight $p-q$ and $\bar{\Phi}(\bar{z})$ be an antiholomorphic field with weight $p-q$. These mode expansions read, respectively,

$$\Phi(z) = \sum_{n \in \mathbb{Z}} \phi_n z^{-n-p+q}, \quad \bar{\Phi}(\bar{z}) = \sum_{n \in \mathbb{Z}} \bar{\phi}_n \bar{z}^{-n-p+q}. \quad (A1)$$

The modes of $\Phi(z)$ and those of $\bar{\Phi}(\bar{z})$ are related to each other on both D_2 and RP^2 and this relation can be made clear

by extending the domain of $\Phi(z')$ to $|z'| > 1$ through the respective involutions $z' = 1/\bar{z}$ (D_2) and $z' = -1/\bar{z}$ (RP^2):

$$\bar{\Phi}(\bar{z}) = \left(\frac{\partial z'}{\partial \bar{z}} \right)^{p-q} \Phi(z'). \quad (A2)$$

On $|z|=1$, this leads to the relation

$$(\bar{z})^{p-q} \bar{\Phi}(\bar{z})|_{|z|=1} = (\mp z)^{p-q} \Phi(\pm z)|_{|z|=1}. \quad (A3)$$

The upper and lower signs refer, respectively, to the D_2 and RP^2 cases. In terms of the modes, this equation reads

$$\bar{\phi}_n = \begin{cases} \phi_{-n} e^{-i(p-q)\pi}, & D_2 \\ \phi_{-n} e^{-i(p-q)\pi} (-)^n, & RP^2. \end{cases} \quad (A4)$$

The radial, tangential, and mixed components are constructed as

$$\begin{aligned} \Phi_{\underbrace{\begin{matrix} r \cdots r \\ \vdots \\ r \end{matrix}}_{p_1}}^{\underbrace{\begin{matrix} \sigma \cdots \sigma \\ \vdots \\ \sigma \end{matrix}}_{q_1}}(z, \bar{z}) &\equiv \left(\frac{\partial z}{\partial r} \right)^{p_1} \left(\frac{\partial z}{\partial \sigma} \right)^{q_1} \Phi(z) + \left(\frac{\partial \bar{z}}{\partial r} \right)^{p_1} \left(\frac{\partial \bar{z}}{\partial \sigma} \right)^{q_1} \bar{\Phi}(\bar{z}) \\ &= \left(\frac{z}{r} \right)^{p_1} (iz)^{q_1} \left(\frac{\bar{z}}{2r} \right)^{q_1} \left(\frac{1}{2iz} \right)^{q_1} \Phi(z) + \left(\frac{\bar{z}}{r} \right)^{p_1} (-i\bar{z})^{q_1} \left(\frac{z}{2r} \right)^{q_1} \left(-\frac{1}{2i\bar{z}} \right)^{q_1} \bar{\Phi}(\bar{z}). \end{aligned} \quad (A5)$$

On $|z|=1$, we obtain

$$\begin{aligned} &\left. \Phi_{\underbrace{\begin{matrix} r \cdots r \\ \vdots \\ r \end{matrix}}_{p_1}}^{\underbrace{\begin{matrix} \sigma \cdots \sigma \\ \vdots \\ \sigma \end{matrix}}_{q_1}}(z, \bar{z}) \right|_{|z|=1} \\ &= (i)^{p_2-q_2} \frac{1}{2^q} z^{p-q} [\Phi(z)]|_{|z|=1} \\ &\quad + (-)^{p_2-q_2} (\mp)^{p-q} \Phi(\pm z)|_{|z|=1}. \end{aligned} \quad (A7)$$

For example,
 $p=1, q=0$:

$$\begin{aligned} \alpha_z(z) &\equiv i \sqrt{\frac{2}{\alpha'}} \partial_z X(z, \bar{z}) = \sum_n \alpha_n z^{-n-1}, \\ \bar{\alpha}_{\bar{z}}(\bar{z}) &\equiv i \sqrt{\frac{2}{\alpha'}} \bar{\partial}_{\bar{z}} X(z, \bar{z}) = \sum_n \tilde{\alpha}_n \bar{z}^{-n-1}, \end{aligned}$$

$$\alpha_r(z, \bar{z}) = \frac{z}{r} \alpha_z(z) + \frac{\bar{z}}{r} \bar{\alpha}_{\bar{z}}(\bar{z}),$$

$$\alpha_\sigma(z, \bar{z}) = iz \alpha_z(z) - i\bar{z} \bar{\alpha}_{\bar{z}}(\bar{z}),$$

$$\alpha_r(z, \bar{z})|_{|z|=1} = \begin{cases} 0, & D_2, \\ z(\alpha_z(z) + \alpha_z(-z))|_{|z|=1}, & RP^2, \end{cases}$$

$$\alpha_\sigma(z, \bar{z})|_{|z|=1} = \begin{cases} 2iz \alpha_z(z)|_{|z|=1}, & D_2, \\ iz(\alpha_z(z) - \alpha_z(-z))|_{|z|=1}, & RP^2, \end{cases} \quad (A8)$$

$p=0, q=1$:

$$c^z(z) = \sum_n c_n z^{-n+1}, \quad \bar{c}^{\bar{z}}(\bar{z}) = \sum_n \bar{c}_n \bar{z}^{-n+1},$$

$$c^r(z, \bar{z}) = \frac{1}{2r} [\bar{z} c^z(z) + z \bar{c}^{\bar{z}}(\bar{z})],$$

$$c^\sigma(z, \bar{z}) = -\frac{i}{2} \left(\frac{c^z(z)}{z} - \frac{\bar{c}^{\bar{z}}(\bar{z})}{\bar{z}} \right),$$

$$c^r(\sigma) \equiv c^r(z, \bar{z})|_{|z|=1} = \begin{cases} 0, & D_2, \\ \frac{1}{2z} [c^z(z) + c^z(e^{i\pi}z)]|_{|z|=1} \\ = \sum_{\text{nodd}} c_n e^{-in\sigma}, & \text{RP}^2, \end{cases}$$

$$c^\sigma(\sigma) \equiv c^\sigma(z, \bar{z})|_{|z|=1} = \begin{cases} \frac{1}{iz} c^z(z)|_{|z|=1} = -i \sum_n c_n e^{-in\sigma}, & D_2, \\ \frac{1}{2iz} [c^z(z) - c^z(e^{i\pi}z)]|_{|z|=1} \\ = -i \sum_{n \text{ even}} c_n e^{-in\sigma}, & \text{RP}^2. \end{cases} \quad (\text{A9})$$

Some of these formulas are exploited in the text.

APPENDIX B

Let $\mathcal{O}(z, \bar{z})$ be a generic scalar operator with ghost number 2 on RP^2 geometry. We write this as

$$\mathcal{O}(z, \bar{z}) = \frac{c^z(z)}{z} \frac{\bar{c}^{\bar{z}}(\bar{z})}{\bar{z}} \mathcal{V}(z, \bar{z}). \quad (\text{B1})$$

Let Q_{BRS} be the BRS charge obtained from the integration of the holomorphic BRS current. We find

$$[Q_{BRS}, \mathcal{O}(z, \bar{z})] = -[\partial_z c^z(z) + (\partial_z c^z)(-1/\bar{z})] \times \left(1 + \frac{\alpha'}{4} \frac{\partial^2}{\partial X^\mu \partial X_\mu} \right) \mathcal{O}(z, \bar{z}). \quad (\text{B2})$$

The on-shell ground state scalar is represented by the vertex operator

$$\mathcal{V}(z, \bar{z}) = e^{ik \cdot X(z, \bar{z})}, \quad (\text{B3})$$

$$\alpha' M^2 = -\alpha' k^2 = -4. \quad (\text{B4})$$

This latter condition is that of the tachyon of a closed unoriented bosonic string with intercept -4 .

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