pp waves and holography

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We consider aspects of holography in the *pp*-wave limit of $AdS_5\times S^5$. This geometry contains two R^4's, one obtained from AdS_5 directions, and the other from the S^5. We argue that the holographic direction in the *pp*-wave background can be taken to be *r*, the radial direction in the first R^4. Normalizable modes correspond to states, and non-normalizable modes correspond to deformations of the boundary theory. In the strict *pp*-wave limit, there are additional non-normalizable modes in the second R^4, which have no apparent super-Yang-Mills interpretation. We outline the procedure for calculating correlation functions holographically.

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I. INTRODUCTION

Recently a study of *pp*-wave backgrounds from the point of view of the AdS-CFT correspondence was initiated [1,2]. Namely, the *pp* wave is realized as a certain limit of $AdS_5\times S^5$, where one considers a large boost along one of the S^5 directions. The light cone string theory in this background contains eight massive bosons and superpartners; it is exactly solvable, with a spectrum

$$H=\sum_{n=-\infty}^{\infty} N_n \sqrt{\mu^2+\frac{4n_{osc}^2}{(\alpha' p_-)^2}}. \tag{1.1}$$

BMN identified the oscillator modes of this string within the dual $N=4$ Super-Yang-Mills (SYM) theory.

In addition to providing another example of a holographic dual, the exciting new feature of [2] is a more or less direct connection between the boundary theory and the string theory in this background, including non-BPS massive string modes. Examining this duality can then provide a clue to the nature of holography in a more generic setting. Previous work on *pp* waves include Refs. [3], and subsequent work includes [4–7].

Here we consider in more detail aspects of holography for the *pp* wave. We would like to establish more clearly the holographic map. To do so, we will concentrate in this paper, on the supergravity modes (we are making an assumption that there is a decoupling here, $\alpha' \mu p_- \gg 1$). The map between normalizable supergravity modes and states, and non-normalizable modes and sources is well known in AdS-CFT. We explore this map in the present context. In a sense, we may expect that this is determined by the known $AdS_5\times S^5$ results, since the *pp*-wave geometry is obtained in a scaling limit. On the other hand, questions such as ''where does the SYM theory live'' are somewhat confusing. The *pp*-wave background has an $SO(4)\times SO(4)\subset SO(8)$ isometry; one might well wonder what this means from the SYM point of view.

In this paper we take the point of view that describing the holographic dual in terms of the original SYM theory amounts to making certain choices that are arbitrary from the bulk viewpoint. This seems to choose one of the two copies of R^4 in the geometry as the base space of the SYM theory, and retains the memory of the origin of the other copy from a compact space (the original S^5). Perhaps this is the most surprising aspect of our analysis—some aspects of the *pp*-wave background (namely non-normalizable modes in directions originating from the sphere) do not seem to be described by the original SYM theory.

Retaining features coming from the seemingly decoupled asymptotically AdS region, we formulate the holographic relation between this background and SYM theory. Up to some important differences we outline below, the correspondence is the familiar one, with the radial coordinate playing the role of the holographic one.

We discuss supergravity modes, normalizable and non-normalizable, in the next section. We find that the spectrum of p_- span the positive real axis, and there exist the two types of modes for all such values. We conclude by formulating the holographic calculation of correlation functions. We hope to return to this calculation in the near future.

There are several interesting questions not answered by the present work. The string theory in this background is simple, and one would hope to be able to calculate more general amplitudes directly on the world sheet. However, presently the string theory involves a Green-Schwarz formulation of the worldsheet theory, which is more difficult to work with. It is also of interest to compare the gravity (or string) calculations to the corresponding SYM calculations. This would be a check of the holographic correspondence we

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suggest, and can perhaps serve to formulate a holographic dual that is more directly connected with the pp-wave background.

While this manuscript was in final stages of preparation, we became aware of the reports in [8]. These papers discuss similar topics from a somewhat different viewpoint. In particular, we emphasize here that as long as one uses the original SYM theory as the boundary theory, the holographic coordinate which classifies gravity modes is the radial one. Indeed, we find that this classification simply descends from the corresponding classification in the original AdS space.

II. BULK MODES: DEFORMATIONS AND STATES

The metric of the pp wave is obtained through a suitable scaling limit on $AdS_5 \times S^5$ in global coordinates [2]. In this coordinate system the initial background is dual to $N=4$ SYM theory on $R \times S^3$. The metric obtained in [2] is

$$ds_{pp}^2 = -4dx^+ dx^- - y_i^2 \mu^2 (dx^+)^2 + dy_i^2 \tag{2.1}$$

where $i=1,\ldots,8$, while the RR background is

$$F_{+1234} = F_{+5678} = \mu. \tag{2.2}$$

This background is clearly $SO(4) \times SO(4)$ invariant; as such, let us rewrite the metric in a suggestive form, in spherical coordinates in each of the two R^4 factors.

$$ds_{pp}^2 = -4dx^+ dx^- + dr^2 + r^2 (d\Omega_3^2 - \mu^2 (dx^+)^2) + d\tilde{r}^2 + \tilde{r}^2 (d\tilde{\Omega}_3^2 - \mu^2 (dx^+)^2). \tag{2.3}$$

We note that at large r, we have an $S^3 \times R$ coordinatized by Ω_3, x^+. Similarly, at large $\tilde{r}$, we also have an $S^3 \times R$ coordinatized by $\tilde{\Omega}_3, x^+$. In what follows, we will show that the holographic coordinate is indeed r. Equivalently, we could say that the holographic coordinate is $\tilde{r}$; thus there are apparently two distinct ''boundaries'' in the pp-wave geometry. For the duality to SYM theory, we focus on one or the other.

Let us consider a scalar mode which is massless in ten dimensions.[1] In the coordinates given in Eq. (2.1), the Laplacian is clearly

$$\Delta = -\partial_+ \partial_- + \frac{1}{4} \mu^2 y^2 \partial_-^2 + \Delta_8. \tag{2.4}$$

Solutions are of the form

$$\phi = e^{ip_+ x^+} e^{ip_- x^-} \prod_{j=1}^{8} e^{-\alpha_j y_j^2/4} H_{n_j}\left(\sqrt{\frac{\alpha_j}{2}} y_j \right) \tag{2.5}$$

with $p_+ = \Sigma_j (\alpha_j / p_-)(n_j + 1/2)$, and $\alpha_j = \pm \mu p_-$. For $\alpha > 0$, the H_n's are Hermite polynomials, and these solutions are clearly normalizable.

As is familiar from AdS-CFT, we should not be so quick to discard the non-normalizable solutions. Instead, we are faced with the task of finding a criterion for distinguishing allowed modes from forbidden ones.[2]

An analysis similar to the one leading to the Breitenlohner-Freedman bound suggests that we should take modes with p_- positive only. From the gauge theory side we have a variant of the familiar light-cone quantization; it is a familiar aspect of lightcone treatment that the spectrum of p_- is semi-infinite. We claim this should be the case both for (normalizable) states, and for (non-normalizable) operators, or sources.

The bulk treatment clarifies the need for positivity of p_-. In the familiar AdS story, there are two modes for each value of all quantum numbers, one normalizable, and one non-normalizable.[3] Those modes have different values of the Hamiltonian (scaling dimension). They are associated with each other since they carry the same quantum numbers; thus turning on the non-normalizable mode will inevitably excite the normalizable mode.

Now, for the normalizable modes the situation is clear—one should not allow a normalizable state with a negative value of the Hamiltonian; this would correspond to a runaway behavior of the vacuum. This is the origin of the Breitenlohner-Freedman bound in AdS space [10]. Excluding the normalizable state with a negative scaling dimension also eliminates the corresponding source with the same quantum numbers.

In the present case, the Hamiltonian is p_+, and one should not allow normalizable modes with negative eigenvalues. This excludes all normalizable modes with negative p_-. The non-normalizable mode with the same quantum numbers is also required to be absent. The quantum numbers in question correspond to oscillator numbers in all eight transverse directions, and to the light-cone momentum p_-.

To more fully appreciate the significance of Eq. (2.5), let us consider the coordinatization given in Eq. (2.3). The scalar Laplacian is

$$\Delta = -\partial_+ \partial_- + \left[\frac{1}{4} \mu^2 r^2 \partial_-^2 + \Delta_{r,\Omega_3} \right] + \left[\frac{1}{4} \mu^2 \tilde{r}^2 \partial_-^2 + \Delta_{\tilde{r},\tilde{\Omega}_3} \right]. \tag{2.6}$$

Solutions can be written as

$$\phi = e^{ip_+ x^+} e^{ip_- x^-} f(r, \Omega_3) \tilde{f}(\tilde{r}, \tilde{\Omega}_3). \tag{2.7}$$

Acting on such modes, the Laplacian reduces to

$$\Delta = p_+ p_- + \left[-\frac{1}{4} \mu^2 p_-^2 r^2 + \Delta_{r,\Omega_3} \right] + \left[-\frac{1}{4} \mu^2 p_-^2 \tilde{r}^2 + \Delta_{\tilde{r},\tilde{\Omega}_3} \right]. \tag{2.8}$$

[1] Such a mode is dual to an operator which is a descendant in the SYM theory, of dimension $\Delta = J + 4$.

[2] As this contradicts some statements in the literature, we demonstrate the existence of the non-normalizable modes in the Appendix.

[3] There is a small window where both modes can be normalizable. This subtlety is well understood, and will play no role in our discussion.

Any given solution will satisfy

$$\Delta_{\tilde{r},\tilde{\Omega}_3}\tilde{f}-\frac{1}{4}\mu^2 p_-^2\tilde{r}^2\tilde{f}=-\tilde{E}\tilde{f} \tag{2.9}$$

$$\Delta_{r,\Omega_3}f-\frac{1}{4}\mu^2 p_-^2 r^2 f=-Ef \tag{2.10}$$

with

$$p_+=\frac{E+\tilde{E}}{p_-}. \tag{2.11}$$

This form is consistent with the $n_{osc}=0$ part of Eq. (1.1).

Solutions are readily obtained as

$$f=e^{-\alpha r^2/4}r^\beta L_n^{(\beta+1)}(\alpha r^2/2)Y_{\ell,m_1,m_2}(\Omega_3) \tag{2.12}$$

with a similar expression for $\tilde{f}$. Here $L_n^{(\nu)}(x)$ is the Laguerre polynomial.[4] If we want the solutions to be singularity-free in the interior, $r\sim 0$, then $\beta=\ell$. The differential equation then reduces to

$$E=\alpha(\ell+2+2n). \tag{2.13}$$

For each ℓ, there is a certain degeneracy, which is accounted for in the analysis in Cartesian coordinates (see the Appendix for details).

So the analysis of the gravity modes suggests that it is sensible to formulate question in the SYM theory regarding modes with positive p_-, and to answer those questions holographically. The gravity modes are divided in the familiar way: normalizable modes are states in the Hilbert space of the theory, and non-normalizable modes are sources (deformations) of the theory. We proceed next to outline the procedure to calculate the correlation functions of arbitrary modes of positive p_-.

The picture is less clear regarding the other R^4 contained in the *pp*-wave geometry. *A priori*, in considering propagation on the *pp*-wave background, one should consider both normalizable and non-normalizable modes in that R^4 as well. It is not clear to us, though, that the SYM formulation knows about both types of modes.

To clarify this statement, let us derive the modes in this background as limits of modes in the original $AdS_5\times S^5$ background. Let us consider the scalar field ϕ on AdS_5 with metric

$$ds^2_{AdS_5}=R^2(-\cosh^2\rho dt^2+d\rho^2+\sinh^2\rho d\Omega_3^2). \tag{2.14}$$

The Laplacian is

$$\Delta=\frac{1}{R^2}\left(-\frac{1}{\cosh^2\rho}\partial_t^2-\frac{\ell(\ell+2)}{\sinh^2\rho}+\frac{1}{\cosh\rho\sinh^3\rho}\partial_\rho(\cosh\rho\sinh^3\rho\partial_\rho)\right). \tag{2.15}$$

The solution of the Laplace equation $(\Delta-m^2)\phi=0$ is given by [see review [9], Eq. (2.34)]

$$\phi=e^{i\omega t}(\cosh\rho)^{-\lambda}(\tanh\rho)^\ell\,{}_2F_1(-n,\lambda+\ell+n,l+2,\tanh^2\rho)Y_{\ell,m_1,m_2}(\Omega_3) \tag{2.16}$$

where $(\omega R)^2=(\lambda+\ell+2n)^2$. In the AdS_5 case, λ is given by

$$\lambda=\Delta_\pm=2\pm\sqrt{4+m^2R^2}. \tag{2.17}$$

In the limit $\Delta\equiv\Delta_+\sim R^2\sim\sqrt{N}\gg 1$, the normalizable mode Δ_+ corresponds to $\lambda=\Delta$ and the non-normalizable mode $\Delta_-=4-\Delta_+\sim-\Delta$ corresponds to $\lambda=-\Delta$.

In the scaling limit

$$\rho=\frac{r}{R},\quad \lambda=\pm\Delta=\pm\frac{1}{2}(p_++p_-R^2),\quad R\to\infty, \tag{2.18}$$

with $r,p_+=\Delta-J,p_-=(\Delta+J)/R^2>0$ fixed, this AdS solution reduces to

$$\phi\to e^{i\omega t}\left(1+\frac{r^2}{2R^2}\right)^{\mp(1/2)p_-R^2}\left(\frac{r}{R}\right)^\ell {}_1F_1\left(-n,\ell+2;\pm\frac{1}{2}p_-R^2\tanh^2(r/R)\right)Y_{\ell,m_1,m_2}(\Omega_3)$$
$$\sim e^{i\omega t}e^{\mp(1/4)p_-r^2}r^\ell L_n^{(\ell+1)}(\pm p_-r^2/2)Y_{\ell,m_1,m_2}(\Omega_3). \tag{2.19}$$

One can introduce the parameter μ by the redefinition $x^\pm\to\mu^\pm x^\pm$ as usual. Then $p_\pm$ goes to $\mu^\mp p_\pm$. Therefore, these solutions agree with those we obtained in the *pp*-wave background. Moreover, the normalizability in the *pp*-wave limit, i.e., the sign of the Gaussian factor descends from the choice of Δ_+ or Δ_-.

Therefore we see that by scaling of the modes on $AdS_5\times S^5$ we obtain both types of modes in the AdS directions. However, we started with normalizable modes only in the sphere directions. The non-normalizable modes in directions originating from the sphere are then not expected to be included in the original SYM description.

The situation is analogous to the original derivation of AdS-CFT from the near horizon geometry of the 3-brane. One obtains $AdS_5\times S^5$ in Poincaré coordinates. In the decoupling limit a lot of the original structure of the asymptotically flat space is eliminated, of course. In our case the analogous statement is that most of the structure of the SYM theory will be irrelevant in the limit taken. This is apparent,

[4]We would like to thank Sumit Das for pointing out the existence of this factor.

for example, in the statement that most of the virtual processes appearing in SYM diagrams will have vanishing contributions to the amplitudes we are interested in.

However, in the AdS case, there were also additional structures in the theory which are only visible in global coordinates. This is visible only when one truly eliminates even the possibility of appending an asymptotically flat region to the geometry.

Therefore, it seems to us that insisting on describing the system by the *original* holographic dual retains some features of the original asymptotic region. It may well be the case that this eliminates the possibility of discussing non-normalizable modes in the other directions. Such discussion naively requires a higher dimensional holographic dual. Since a putative dual is absent, we proceed by assuming that the modes are normalizable in the directions originating from the sphere.

III. CONCLUSIONS AND CORRELATION FUNCTIONS

The next step in discussing the radial holography is a computation of the correlation functions. We sketch here the procedure for such a calculation.

The holographic calculation of the partition function with arbitrary sources is performed by fixing a surface near the boundary, and solving the Dirichlet problem, finding the fields with fixed sources on the surface. The partition function is then, in the supergravity approximation, the value of the action on shell.

A primary role in this calculation is played by the bulk to boundary propagator. The solution to the Dirichlet problem is the propagator convoluted with the given boundary source. In $AdS_5\times S^5$ the bulk to boundary operator $K_{m^2}(x,r;y)$ is defined for each mode of mass m, where x,y are boundary points, and r is the holographic coordinate (so that x,r specifies a bulk position). It is required to satisfy the Laplace equation:

$$(\partial^2-m^2)K_{m^2}=0 \tag{3.1}$$

with the boundary conditions

$$K(x,r;y)\to r^{\Delta}\delta(x-y)$$

$$\text{as } r \text{ approaches the boundary} \tag{3.2}$$

where Δ is determined by the behavior of the non-normalizable mode at the boundary—it depends only on m^2.

One way of solving the equation is as follows. Consider a complete set of functions on the boundary, $\psi_n(x)$, and modes of the bulk Laplacian that satisfy

$$\Phi_n(r,x)\to r^{\Delta}\psi_n(x)$$

$$\text{as } r \text{ approaches the boundary.} \tag{3.3}$$

We note that the constant Δ is required to be identical for all modes, independent of n. It is uniquely specified by the mass m. In this case it is easy to check that

$$K(x,r;y)=\sum_n \Phi_n(r,x)\psi_n(y). \tag{3.4}$$

This is the bulk to boundary propagator. Since the set of boundary sources ψ_n is complete, one is able to turn on an arbitrary boundary source for any operator of a definite scaling dimension (which is related to m). The natural sources to consider then have definite values of the $SO(6)$ Casimir (m^2).

Similar procedure can be obtained in our case. A complete set of functions on the boundary, which is of the form $R\times S^3$, can be chosen to carry definite p_+ and $SO(4)$ quantum numbers. We have then $\psi_n(x_+,\Omega_3)=e^{ip_+X_+}Y_I(\Omega_3)$. Here the angles on the sphere are denoted by Ω_3, and $Y_I(\Omega_3)$ are scalar spherical harmonics.

The source function on the boundary determines the behavior of the corresponding non-normalizable modes. The internal quantum numbers in our case are p_-, and the angular and radial quantum numbers in the additional copy of R^4. The behavior of the non-normalizable mode only depends on p_-, so one has to work in a definite p_- basis. The quantum number p_- plays a role similar to the Casimir m^2 in the $AdS_5\times S^5$ case.

A slight complication here is the absence of coordinate system in which the consequences of conformal invariance are transparent. In AdS one usually works in Poincaré coordinates, where a surface near the boundary corresponds to an ultraviolet cutoff which manifests cleanly the consequences of conformal invariance. Here we are restricted to work in global coordinates, where the pp wave limit is done, so we are led to a more complicated procedure. We hope to report on progress in this direction in the near future.

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APPENDIX: FOUR DIMENSIONAL HARMONIC OSCILLATOR

We work out a simple quantum mechanics problem, considering the eigenmodes of a four dimensional harmonic oscillator. The questions we are interested in are naturally different from the conventional ones, for example the existence of non-normalizable modes and their behavior at radial infinity.

In Cartesian coordinates the problem is readily separable to four identical harmonic oscillators, satisfying

$$f''-\mu^2p_-^2x^2\,f(x)=E\,f(x). \tag{A1}$$

We can then write $f=e^{-(1/2)\alpha x^2}g(x)$. One gets

$$g''-2\alpha x g'+(E-\alpha)g=0 \tag{A2}$$

where we choose $\alpha = \pm \mu p_-$. We note that we reserve the choice of α being positive or negative, corresponding to normalizable or non-normalizable modes.

The equation for $g(x)$ is almost identical to the familiar Hermite equation. We need to rescale the coordinate $x = ay$ to get to the form

$$g'' - 2yg' + 2ng = 0. \tag{A3}$$

For this to be correct one has to choose $a^2 = \alpha$, so α has to be positive. In that case one obtains the equation for the Hermite polynomials, provided

$$E_n = \alpha\left(n + \frac{1}{2}\right). \tag{A4}$$

This yields normalizable solution for any sign of p_-. One simply has to choose $\alpha = \mu|p_-|$.

A different solution is obtained by choosing $\alpha = -\mu|p_-|$, which yields an exponentially growing mode. One can still do the above change of variables for $a^2 = -\alpha$. One gets a slightly different equation for g

$$-g'' - 2yg' + \left(\frac{E}{\alpha} - 1\right)g = 0. \tag{A5}$$

The solutions of this equation are polynomials of the form $P_n = i^n H_n(iy)$. These are real polynomials in y (the imaginary normalization factors for n odd do not matter for a linear equation). The modes we obtain are simply polynomials multiplied by an exponentially growing Gaussian. In particular they do not blow up at the origin.

The spectrum of E is the same for those non-normalizable modes. Note however that α is negative here, so the spectrum is negative definite.

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