Spectrum of a noncommutative formulation of the $D=11$ supermembrane with winding

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A regularized model of a noncommutative formulation of the double compactified $D=11$ supermembrane with nontrivial winding in terms of SU(N) valued maps is obtained. The condition of nontrivial winding is described in terms of a nontrivial line bundle introduced in the formulation of the compactified supermembrane. The multivalued geometrical objects of the model related to the nontrivial wrapping are described in terms of a SU(N) geometrical object, which in the $N\rightarrow\infty$ limit converges to the symplectic connection related to the area-preserving diffeomorphisms of the recently obtained noncommutative description of the compactified $D=11$ supermembrane [I. Martín, J. Ovalle, and A. Restuccia, Phys. Rev. D **64**, 096001 (2001)]. The SU(N) regularized canonical Lagrangian is explicitly obtained. The spectrum of the Hamiltonian of the double compactified $D=11$ supermembrane is discussed. Generically, it contains local string such as spikes with zero energy. However, the sector of the theory corresponding to a principle bundle characterized by the winding number $n\neq 0$, described by the SU(N) model we propose, is shown to have no local stringlike spikes and hence the spectrum of this sector should be discrete.

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I. INTRODUCTION

The matrix model for bosonic membranes was first introduced in [1], and its study was extended to the supersymmetric case in [2]. In that work it was shown that the supermembrane theory could also be understood as a supersymmetric gauge theory of the infinite group of area-preserving diffeomorphisms which appeared as a residual symmetry of the supermembrane in the light cone gauge. They also found a satisfactory SU(N) regularization of the model. The spectrum of the quantized model was found to be continuous [3], and was afterwards interpreted in terms of a multiparticle theory [4].

The $D=11$ supermembrane with winding was first analyzed in [5] in terms of multivalued maps from the world volume to the target space. Part of their study was based on a previous work on the area-preserving diffeomorphisms in [6]. In [5] the Hamiltonian of the theory was explicitly obtained. Its analysis in terms of a finite N regularization was performed, with the conclusion that the SU(N) regularization of the model, which was essential in the analysis of the spectrum of the noncompactified supermembrane in a $D=11$ Minkowski target space [3], was not possible because the structure constants associated with the presence of the nonexact modes did not fit in an SU(N) description of the model. It was also argued in [5] that the spectrum of the compactified $D=11$ supermembrane should also be continuous since the instability, caused by the stringlike spikes, is also present in the compactified case. The stringlike spikes are singular physical configurations (the determinant of the induced metric is zero at some points or open sets of the world volume) which may even change the topology of the world volume without changing the energy of the system. Together with the supersymmetry they render the spectrum of the $D=11$ supermembrane continuous from zero to infinity. A complete analysis of the spectrum for the compactified case similar to the one in [3], for the noncompactified case, has not been yet presented.

The analysis of the compactified D-brane was first approached from the matrix model point of view in [7]. The matrix models [8,9] describe the dynamics of the membranes in the light cone gauge in the approximation of finite number of oscillations modes. They provide an equivalent description, to the one in [5] of the supermembrane in terms of D0-branes. The formulation of compactified D-branes in [7] was done by considering the universal covering of the compactified target space. In that simply connected space the matrix model may be directly formulated in terms of the infinite set of the copies of the D-brane system restricted by the symmetry generated by the covering group. An interesting result was obtained in [10]. It was shown that the matrix model on a noncommutative torus is equivalent to M-theory compactified in a constant antisymmetric background field. The noncommutative geometry of the supermembrane in terms of matrix models was also described in several papers, see [11] and references therein, in particular in [12] and [13]. In [14,15] the analysis of the compactified $D=11$ supermembrane was performed following the original description [2] but the analysis emphasizes the global structure associated with the nontrivial wrapping of the supermembrane in terms of an associated principal bundle which is naturally constructed from the nontrivial central charge of the supersymmetric algebra. This analysis is best performed in the dual formulation of the theory. The double compactified $D=11$ supermembrane dual directly introduces the connection 1-form associated to the nontrivial principal bundle. In the

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formulation in [16,17] the canonical Lagrangian is expressed as a noncommutative gauge theory. The geometrical meaning of the noncommutativity was explained in that work in terms of symplectic fibrations over the world volume. The symplectomorphisms on the fibers are generated by the area-preserving diffeomorphisms on the world volume.

In this paper we present an SU(N) regularization of that formulation. All the multivalued objects related to the nontrivial wrapping are handled by the connections 1-form. If the theory is restricted to a principle bundle characterized by the winding number n, any connection on that bundle may be expressed in terms of a fix one $\hat{\Pi}$ plus a uniform 1-form $\mathcal{A}$:

$$\hat{\mathcal{A}}=\hat{\Pi}+\mathcal{A}. \tag{1}$$

We consider $\hat{\Pi}$ to be the connection 1-form which minimizes the Hamiltonian of the double compactified supermembrane. Although $\mathcal{A}$ is a uniform 1-form it has a transformation law, under the gauge symmetry of the theory,

$$\mathcal{A}\rightarrow\mathcal{A}+d\epsilon+\{\mathcal{A},\epsilon\}=\mathcal{A}+\mathcal{D}\epsilon, \tag{2}$$

corresponding to a symplectic connection preserving the symplectic structure of the fibers under holonomies. Its regularization in terms of SU(N) valued objects has consequently a very different behavior compared to the geometrical objects in the other approaches.

II. THE HAMILTONIAN OF THE COMPACTIFIED $D=11$ SUPERMEMBRANE

In this section we describe the Hamiltonian of the compactified $D=11$ supermembrane on $(R^9\times S^1\times S^1)$. It seems that the best approach, from a global point of view, is to consider its dual formulation since as discussed previously the global features are geometrically handled in terms of a connection 1-form over a nontrivial principle bundle on the world volume which is intrinsically introduced in the formulation.

The Hamiltonian for the double compactified $D=11$ supermembrane was obtained in [16,17] starting from the Lagrangian formulation of the $D=11$ supermembrane. It was important to follow step by step the dualization procedure in order to show that the nontrivial winding of the supermembrane was indeed described by the nontrivial bundle over which the gauge field, dual to the compactified coordinates, is defined. Having that geometrical structure one may introduce in an intrinsic way a symplectic structure on the world volume. One finally may formulate the double compactified $D=11$ supermembrane as a symplectic noncommutative gauge theory [16,17]. The final form of the Hamiltonian is

$$\begin{aligned}H=\int_\Sigma \frac{1}{2\sqrt{W}}[(P^m)^2+(\Pi_r)^2+1/2W\{X^m,X^n\}^2\\+W(\mathcal{D}_rX^m)^2+1/2W(\mathcal{F}_{rs})^2]+\int_\Sigma[1/8\sqrt{W}n^2\\-\Lambda(\mathcal{D}_r\Pi^r+\{X^m,P_m\})]-\frac{1}{4}\int_\Sigma\sqrt{W}n*\mathcal{F}\end{aligned} \tag{3}$$

together with its supersymmetric extension

$$\int_\Sigma\sqrt{W}[-\overline{\theta}\Gamma_-\Gamma_r\mathcal{D}_r\theta+\overline{\theta}\Gamma_-\Gamma_m\{X^m,\theta\}+\Lambda\{\overline{\theta}\Gamma_-,\theta\}] \tag{4}$$

in terms of the original Majorana spinors of the $D=11$ formulation, which may be decomposed in terms of a complex 8-component spinor of $SO(7)\times U(1)$.

$m=1,\ldots,7$ are the indexes denoting the scalar fields once the supermembrane is formulated in the light cone gauge. $r,s=1,2$ are the indexes related to the two compactified directions of the target space, where Σ is the spatial part of the world volume which is assumed to be closed Riemann surface of topology g. P_M and Π_r are the conjugate momenta to X^M and the connection 1-form $\mathcal{A}_r$, respectively. The covariant derivative is

$$\mathcal{D}_r=D_r+\{\mathcal{A}_r,\} \tag{5}$$

and the field strength

$$\mathcal{F}_{rs}=D_r\mathcal{A}_s-D_s\mathcal{A}_r+\{\mathcal{A}_r,\mathcal{A}_s\}. \tag{6}$$

The bracket $\{,\}$ is defined as

$$\{*,\diamond\}=\frac{2\epsilon^{sr}}{n}(D_r^*)(D_s\diamond), \tag{7}$$

where n denotes the integer which characterizes the nontrivial principle bundle under consideration. D_r is a tangent space derivative

$$D_r\diamond=\frac{\hat{\Pi}_r^a\partial_a\diamond}{\sqrt{W}}=\{\hat{\Pi}_r,\diamond\},\quad r,s=1,2,\quad a=1,2, \tag{8}$$

where ∂_a denotes derivatives with respect to the local coordinates of the world volume while $\hat{\Pi}_r^a=\epsilon^{au}\partial_u\Pi_r$ is a zweivein defined from the minimal solution of the Hamiltonian of the theory. It satisfies

$$\epsilon^{rs}\hat{\Pi}_r^a\hat{\Pi}_s^b\epsilon_{ab}=n\sqrt{W}, \tag{9}$$

equivalently

$$\{\hat{\Pi}_r,\hat{\Pi}_s\}=1/2n\epsilon_{sr}. \tag{10}$$

We consider now an expansion of the geometrical objects in the formulation in terms of an orthonormal basis in the space L^2 or functions over the world volume. They are uniform functions over the manifold. The X^M,P_M may be expressed in the standard way since they are uniform maps from the world volume Σ to the target space,

$$\begin{aligned}X^m(\sigma^1,\sigma^2,\tau)=\sum X^{mA}(\tau)Y_A(\sigma^1,\sigma^2),\\P_m(\sigma^1,\sigma^2,\tau)=\sum\sqrt{W}P_m^A(\tau)Y_A(\sigma^1,\sigma^2).\end{aligned} \tag{11}$$

The multivalued maps defining the nontrivial winding of the membrane are now expressed in terms of the connection and its conjugate momenta. In this sector we impose the global condition

$$\int_{\Sigma}\sqrt{W}{}^{*}\mathcal{F}=0. \tag{12}$$

If $\mathcal{F}$ were the curvature of a $U(1)$ connection 1-form, this condition would imply the connection has no transitions on the $U(1)$ bundle. However, we have a symplectic connection $\mathcal{A}$, instead. We will assume that $\mathcal{A}$ has no transitions over Σ, which of course implies Eq. (12) since $\sqrt{W}{}^{*}\mathcal{F}$ is a total derivative. The central charge Z of the supersymmetric (SUSY) algebra becomes then

$$Z=\int_{\Sigma}\sqrt{W}\epsilon_{sr}\{\hat{\Pi}_r,\hat{\Pi}_s\}=n\times area_{\Sigma}. \tag{13}$$

We may now decompose $\mathcal{A}_r$ and its canonical conjugate momenta under the same basis as before:

$$\mathcal{A}_r(\sigma^1,\sigma^2,\tau)=\sum \mathcal{A}_r^A(\tau)Y_A(\sigma^1,\sigma^2),$$
$$\Pi^r(\sigma^1,\sigma^2,\tau)=\sum \sqrt{W}\,\Pi^{r,A}(\tau)Y_A(\sigma^1,\sigma^2). \tag{14}$$

There is, however, a main difference between X^A and $\mathcal{A}_r^A$. It is their transformation law under the symmetry generated by the first class constraint. To analyze this point we introduce as in [6] the structure constants g_{AB}^C

$$\{Y_A,Y_B\}=\frac{2\epsilon^{sr}}{n}D_rY_AD_sY_B=g_{AB}^CY_C, \tag{15}$$

where Y_A is a complete orthonormal basis of functions over the spatial part of the world volume, and g_{AB}^C are the structure constants associated with the group of area-preserving diffeomorphisms in this basis. That is

$$g_{AB}^C=\int d^2\sigma\sqrt{W}\{Y_A,Y_B\}Y_{-C}, \tag{16}$$

where we use the normalization condition

$$\int d^2\sigma\sqrt{W}Y_CY_B=\delta_{B+C}. \tag{17}$$

We then have the infinitesimal gauge transformations

$$\delta X^C=\sum_{A,B} g_{AB}^C\epsilon^AX^B,$$
$$\delta\mathcal{A}_r^C=-\lambda_{rA}^C\epsilon^A-\sum_{A,B} g_{AB}^C\mathcal{A}_r^A\epsilon^B, \tag{18}$$

where λ_{rA}^C is defined by

$$D_rY_A=\lambda_{rA}^CY_C. \tag{19}$$

We consider Y_A to be a complete basis of eigenfunctions of the operator D_rD_r. Then we have

$$D_rD_rY_A=\omega_AY_A, \tag{20}$$

where no summation in the index A is performed. We assume without losing generality that

$$\overline{Y_A}=Y_{-A}, \tag{21}$$

where $\overline{Y_A}$ denotes the complex conjugate to Y_A. We notice the following property of the derivatives D_r:

$$D_s\hat{\Pi}_r=\frac{\hat{\Pi}_s^a}{\sqrt{W}}\partial_a\hat{\Pi}_r=\frac{\epsilon^{ab}}{\sqrt{W}}\partial_b\hat{\Pi}_s\partial_a\hat{\Pi}_r=\{\hat{\Pi}_s,\hat{\Pi}_r\}=\frac{n\epsilon_{rs}}{2}. \tag{22}$$

$\hat{\Pi}_r$ may be identified with the angles of the compactified directions of the target space.

We may introduce $\hat{\Pi}_r$ as local coordinates over Σ. We will assume from now on Σ to be of genus 1, although everything can be extended to arbitrary genus. We then have

$$Y_A=\mathrm{e}^{iA_r\hat{\Pi}_r},$$
$$\omega_A=-\frac{n^2}{4}A_r^2=-\frac{n^2}{4}(A_1^2+A_2^2), \tag{23}$$
$$\lambda_{rA}^C=iA_s\frac{n}{2}\epsilon_{sr}\delta_A^C\equiv\lambda_{rA}\delta_A^C,$$

where $\mathcal{A}_r, r=1,2$ is a pair of integral numbers associated to Y_A. The structure functions may then be expressed as

$$g_{AB}^C=\frac{n}{2}(A\times B)\delta_{A+B}^C \tag{24}$$

and

$$\lambda_{rA}^C=-i\frac{n}{2}(V_r\times A)\delta_A^C, \tag{25}$$

$$V_r=\begin{cases}(1,0), & r=1\\(0,1), & r=2.\end{cases} \tag{26}$$

Then

$$\lambda_{rA}^C=-ig_{V_r,(A-V_r)}^C \tag{27}$$

and also satisfies

$$\frac{2}{n}\epsilon^{sr}\lambda_{rA}\lambda_{sB}d_{AB}^C=g_{AB}^C$$
$$\text{with}\ \ d_{AB}^C=\int d^2\sigma\sqrt{W}Y_AY_B\overline{Y}_C, \tag{28}$$

with d_{AB}^C related to the invariant symmetric three index tensor of SU(N).

The Hamiltonian may then be expressed as in [18]:

$$H = H_{bosonic} + H_{Fermionic},$$

$$\begin{aligned} H_{Bosonic} = {} & \frac{1}{2}(P^{0m}P^{0}_{m} + P^{Am}P^{-A}_{m}) + \frac{1}{4}(g^{C}_{AB}X^{mA}X^{nB})^2 \\ & + \frac{1}{2}(\lambda_{rA}X^{mA} + g^{A}_{BC}\mathcal{A}^{B}_{r}X^{mC})^2 + \frac{1}{2}(\Pi^{rA}\Pi^{-A}_{r} \\ & + \Pi^{r0}\Pi^{-0}_{r}) + \frac{1}{4}[(\lambda_r\mathcal{A}_s - \lambda s\mathcal{A}_r)^A \\ & + (g^{A}_{BC}\mathcal{A}^{B}_{r}\mathcal{A}^{C}_{s})]^2 + \frac{1}{8}n^2 + \Lambda^{(-A)}[g^{A}_{BC}(X^{mB}P^{C}_{m} \\ & + \mathcal{A}^{B}_{r}\Pi^{rC}) + \lambda_{rA}\Pi^{rA}], \\ H_{Fermionic} = {} & -g^{A}_{BC}\overline{\Psi}^{(-A)}\gamma_{-}\gamma_{m}X^{mB}\Psi^{C} \\ & + g^{C}_{AB}\mathcal{A}^{A}_{r}\overline{\psi}^{(-C)}\gamma_{-}\gamma_{r}\Psi^{B} + \lambda_{rB}\overline{\Psi}^{(-B)}\gamma_{-}\gamma_{r}\Psi^{B} \\ & - g^{A}_{BC}\Lambda^{(-A)}\overline{\Psi}^{B}\gamma_{-}\Psi^{C}, \end{aligned} \tag{29}$$

where $(,)^2$ is understood as

$$(\star,\diamondsuit)^2 = (\star,\diamondsuit)\overline{(\star,\diamondsuit)} = (\star,\diamondsuit)^A(\star,\diamondsuit)^{-A}. \tag{30}$$

Using the following definitions, $H_{bosonic}$ can be directly reexpressed in a simpler way, which may be useful to compare with Eq. (3):

$$\begin{aligned} \tilde{\mathcal{D}}_r &= \lambda_r + [\mathcal{A}_r,], \\ \mathcal{F}^{A}_{rs} &= \lambda_{Ar}\mathcal{A}^{A}_{s} - \lambda_{sA}\mathcal{A}^{A}_{r} + [\mathcal{A}_r, \mathcal{A}_s]^A \quad \text{with} \\ \lambda_r(Y_A) &\equiv \lambda_{rA}Y_A \quad \text{no summation over index } A, \\ [\star,\diamondsuit]^A &\equiv g^{A}_{BC}\star^{B}_{r}\diamondsuit^{C}_{s}. \end{aligned} \tag{31}$$

Then the bosonic part of the Hamiltonian appears as

$$\begin{aligned} H_{Bosonic} = {} & \frac{1}{2}P^{0m}P^{0}_{m} + \frac{1}{2}(P^{Am}P^{-A}_{m}) + \frac{1}{4}[X^m, X^n]^2 \\ & + \frac{1}{2}(\tilde{\mathcal{D}}_r X^{mA})^2 + \frac{1}{8}n^2 + \frac{1}{2}\Pi^{r0}\Pi^{-0}_{r} \\ & + \frac{1}{2}\Pi^{rA}\Pi^{-A}_{r} + \frac{1}{4}(\mathcal{F}^{A}_{rs})^2 \\ & + \Lambda^{A}([X^m, P_m] + \tilde{\mathcal{D}}_r\Pi_r)^{-A}, \end{aligned} \tag{32}$$

where summation over the A index is performed.

III. THE HEISENBERG-WEYL GROUP AND THE $N\to\infty$ LIMIT

We follow in the first part of this section standard results concerning the Heisenberg-Weyl group [12]. We do so since in the literature there are some minor misprints that we would like to avoid.

The relevant Hilbert space $H(\Gamma)$ of functions on a torus $\Gamma = \mathcal{C}/L$ of complex modulus $\tau = \tau_1 + i\tau_2$ with integer lattice $L = \{m_1 + \tau m_2 | (m_1, m_2) \in Z\times Z\}$ is defined as the space of functions of complex argument $z = \sigma_1 + i\sigma_2$

$$f(z) = \sum_{n\in Z} C_n e^{i\pi(n^2) + 2\pi(inz)} \tag{33}$$

with the norm

$$\|f\|^2 = \int_{\Sigma} d^2\sigma e^{-2\pi(y^2)/\tau_2}|f(z)|^2. \tag{34}$$

The subspace $H_N(\Gamma)$ of $H(\Gamma)$ is defined by the periodicity condition

$$C_n = C_{n+N} \tag{35}$$

for a fixed natural number N. In the subspace $H_N(\Gamma)$ the discrete Heisenberg group with generators P and Q,

$$\begin{aligned} Qf(z) &= \sum_{n\in Z} C_n e^{2\pi in/N} e^{2\pi inz + \pi(in^2\tau)}, \\ Pf(z) &= \sum_{n\in Z} C_{n-1} e^{2\pi in + \pi(in^2 z)}. \end{aligned} \tag{36}$$

They satisfy the Weyl relation [19]:

$$QP = \kappa PQ \quad \text{where } \kappa = \exp(2\pi i/N). \tag{37}$$

The Heisenberg group elements are defined by

$$T_{r,s} = N\kappa^{1/2rs}P^rQ^s. \tag{38}$$

They are SU(N) matrices which satisfy the following relations:

$$\begin{aligned} T^{\dagger}_{r,s} &= T_{-r,-s}, \\ (T_{r,s})^N &= N^N e^{i\pi rs(N-1)!} I_{N\times N}, \\ \mathrm{tr}\, T_{r,s} &= 0, \\ T_{r,s}T_{r',s'} &= N\kappa^{1/2(r's - rs')}T_{r+r',s+s'}, \\ T_{r+N,s} &= e^{i\pi s}T_{r,s}, \\ T_{r,s+N} &= e^{i\pi r}T_{r,s}. \end{aligned} \tag{39}$$

The SU(N) algebra may be realized in terms of the base T_A with $A = (a_1, a_2) = (r,s)$ with $a_1, a_2 = 0, \ldots, N$ with (0,0) excluded. We include $T_0 = NI_{N\times N}$ to have a complete set of matrices which close under multiplication [6],

$$[T_A, T_B] = -2iN\sin\left(\frac{(A\times B)\pi}{N}\right)T_{A+B}, \tag{40}$$

where $[,]$ is simply the commutator (do not confuse with the $[,]$ symbol used in the preceding section). The structure constants are then

$$f_{AB}^{C}\equiv\frac{1}{N^3}\mathrm{tr}([T_A,T_B],T_{-C})$$
$$=-2iN\sin\left(\frac{(A\times B)\pi}{N}\right)\delta^{C}_{A+B}. \tag{41}$$

When $N\to\infty$ one obtains the Poisson algebra of area-preserving diffeomorphisms

$$\frac{ni}{4\pi}f_{ABC}\to g_{ABC}\equiv\frac{n}{2}(A\times B)\delta_{A+B-C}. \tag{42}$$

We introduce $\widetilde{\lambda_{rA}^{B}}$ as a particular choice of the structure constants associated with the finite group:

$$\widetilde{\lambda_{rA}^{B}}=-if^{B}_{V_r(A-V_r)} \tag{43}$$

with

$$\widetilde{\lambda_{rA}^{B}}=\tilde{\lambda}_{rA}\delta^{B}_{A}. \tag{44}$$

$\tilde{\lambda}$ converges to

$$\frac{ni}{4\pi}\widetilde{\lambda_{rA}}\to\lambda_{rA}=\{\hat{\Pi}_r,Y_A\}=iA_s\frac{n}{2}\epsilon_{rs}. \tag{45}$$

IV. THE SU(N) FORMULATION OF THE THEORY

We may now introduce an SU(N) canonical Lagrangian which in the limit $N\to\infty$ converges to the formulation of Sec. II describing the dual of the double compactified $D=11$ supermembrane. The coordinates X^m as well as the connection $\mathcal{A}_r$ and their canonical conjugate momenta are valued over the SU(N) algebra. The Lagrangian contains unusual terms which indeed are necessary if $\mathcal{A}_r$ is going to converge to a connection in the $N\to\infty$ limit. In fact, a connection of a principle bundle allows us to translate the geometrical objects in the horizontal direction; however, in the SU(N) model all the dependence on the world volume coordinates has been removed. We then expect some unusual terms which in the $N\to\infty$ limit allow that property of the connection 1-form to be recovered.

We are going to consider the Hamiltonian (29) in a particular gauge. Since the first class constraint has been expressed as a generalized Gauss law, these are several interesting conditions we may impose. We will consider the gauge condition:

$$\begin{aligned}\mathcal{A}_1&=\mathcal{A}_1^{a_1,0}Y_{a_1,0},\\ \mathcal{A}_2&=\mathcal{A}_2^{a_1,a_2}Y_{a_1,a_2},\quad a_2\neq 0.\end{aligned} \tag{46}$$

The SU(N) model we introduce is the following:

$$\begin{aligned}H=\mathrm{tr}\Bigg[&\frac{1}{2N^3}[P^{0m}T_0P^0_mT_0+\Pi^{r0}T_0\Pi_r^{-0}T_0+(P^m)^2+(\Pi)^2_r]+\frac{n^2}{16\pi^2N^3}[X^m,X^n]^2+\frac{n^2}{8\pi^2N^3}\left(\frac{i}{N}[T_{V_r},X^m]T_{-V_r}-[\mathcal{A}_r,X^m]\right)^2\\&+\frac{n^2}{16\pi^2N^3}\left([\mathcal{A}_r,\mathcal{A}_s]+\frac{i}{N}([T_{V_s},\mathcal{A}_r]T_{-V_s}-[T_{V_r},\mathcal{A}_s]T_{-V_r})\right)^2+\frac{1}{8}n^2+\frac{n}{4\pi N^3}\Lambda\bigg([X^m,P_m]-\frac{i}{N}[T_{V_r},\Pi_r]T_{-V_r}\\&+[\mathcal{A}_r,\Pi_r]\bigg)+\frac{in}{4\pi N^3}\left(\bar{\Psi}\gamma_-\gamma_m[X^m,\Psi]-\bar{\Psi}\gamma_-\gamma_r[\mathcal{A}_r,\Psi]+\Lambda[\bar{\Psi}\gamma_-,\Psi]-\frac{i}{N}\bar{\Psi}\gamma_-\gamma_r[T_{V_r},\Psi]T_{-V_r}\right)\Bigg]\end{aligned} \tag{47}$$

subject to

$$\begin{aligned}\mathcal{A}_1&=\mathcal{A}_1^{(a_1,0)}T_{(a_1,0)},\\ \mathcal{A}_2&=\mathcal{A}_2^{(a_1,a_2)}T_{(a_1,a_2)}\quad\text{with } a_2\neq 0,\end{aligned} \tag{48}$$

where we used the following definitions:

$$\begin{aligned}X^m&=X^{mA}T_A,\quad P^m=P^{mA}T_A,\\ \mathcal{A}_r&=\mathcal{A}_r^AT_A,\quad \Pi^r=\Pi^{rA}T_A,\\ [T_B,T_C]&=f^A_{BC}T_A.\end{aligned} \tag{49}$$

It may be expressed in the form

$$\begin{aligned}H=\mathrm{tr}\Bigg(&\frac{1}{2N^3}[P^{0m}P^0_m+(P^m)^2]+\frac{1}{2N^3}(\Pi^0_r\Pi^0_r+(\Pi^r)^2)\\&+\frac{n^2}{16\pi^2N^3}\{[X^m,X^n]^2+2(\hat{\mathcal{D}}_rX^m)^2+(\tilde{\mathcal{F}}_{rs})^2\}\\&+\frac{1}{8}n^2+\frac{n}{4\pi N^3}\Lambda([X^m,P_m]+\hat{\mathcal{D}}_r\Pi_r)\\&+\frac{in}{4\pi N^3}\Lambda([\bar{\Psi}\gamma_-,\Psi]+\bar{\Psi}\gamma_-\gamma_m[X^m,\Psi]\\&+\hat{\mathcal{D}}_r\bar{\Psi}\gamma_-\gamma_r\Psi)\Bigg),\end{aligned} \tag{50}$$

where the identification of the terms in Eq. (50) are obvious from Eq. (47).

Each term of the above Hamiltonian density converges to the corresponding one in the formulation of the supermembrane of Sec. II. The condition (48) in the SU(N) model also converges to the gauge fixing condition (46) of the supermembrane.

V. ON THE SPECTRUM OF THE HAMILTONIAN

There are two properties of the Hamiltonian of the $D=11$ supermembrane on a Minkowski target space [3] which render the spectrum of the associated mass operator continuous.

(i) The existence of local stringlike configurations which may even change the topology of the membrane without changing its energy. This is a property of the bosonic sector of the supermembrane.

(ii) Supersymmetry. The supersymmetry cancels an effective potential coming from the quantization of the model. It is related to the zero point energy of the harmonic oscillators which is different from zero in the bosonic case and zero in the supersymmetric one.

We will show in this section that there are no local stringlike configurations with zero energy density associated with the Hamiltonian of our SU(N) model (47). It is important to come back to the global condition which was imposed in order to obtain the Hamiltonian [see the comment after Eq. (12)] of the model under consideration. It was

$$\int_\Sigma \sqrt{W} {}^*\mathcal{F}=0. \tag{51}$$

The annihilation of that term (51), which is perfectly valid when we formulate our model over a fixed nontrivial line bundle, has important consequences with respect to the nonexistence of the local stringlike configurations with zero energy density. To analyze this point let us see first what occurs for the compactified membrane without that assumption. Without the assumption (51), there are local stringlike configurations arising from the following configurations:

$$\begin{aligned} X^m &= X^m(X(\sigma_1,\sigma_2)), \\ \mathcal{A}_r &= -\hat{\Pi}_r + f_r(X(\sigma_1,\sigma_2)). \end{aligned} \tag{52}$$

These configurations depend on an arbitrary uniform map $X(\sigma_1,\sigma_2)$. After some calculations one can show that the Hamiltonian density of Eq. (3) over those configurations becomes zero. Hence the compactified supermembrane allows local stringlike spikes with zero energy. Let us now discuss the sector of the theory arising from the imposition of the global condition (51). In the SU(N) model of Sec. IV, the singular configurations (52) do not arise because $\mathcal{A}_r$ is single valued in distinction to $\hat{\Pi}_r$ which is necessarily multivalued over Σ. More precisely, in order to have zero local energy density, the conditions

$$\begin{aligned} \mathcal{D}_r X &= 0, \\ \mathcal{F}_{rs} &= 0 \end{aligned} \tag{53}$$

must be satisfied. They impose severe restrictions to X^A and $\mathcal{A}_r^A$ which eliminates the possibility of having the stringlike configurations. In fact, the condition $\mathcal{F}_{rs}=0$ yields

$$k^{-1/2(V_r\times A)}\tilde{\lambda}_{rA}\mathcal{A}_s^A - k^{-1/2(V_s\times A)}\tilde{\lambda}_{sA}\mathcal{A}_r^A + f^A_{BC}\mathcal{A}_r^B\mathcal{A}_s^C=0 \tag{54}$$

and using the gauge fixing condition (48) we obtain, for any A

$$\begin{aligned} &k^{-1/2(V_1\times A)} N \sin\left(\frac{(V_1\times A)\pi}{N}\right)\mathcal{A}_2^A \\ &\quad - k^{-1/2(V_2\times A)} N \sin\left(\frac{(V_2\times A)\pi}{N}\right)\mathcal{A}_1^A \\ &\quad + iN\sin\left(\frac{(b_1V_1\times A)\pi}{N}\right)\mathcal{A}_1^{b_1,0}\mathcal{A}_2^{A-b_1V_1}=0, \end{aligned} \tag{55}$$

where b_1 are integers.

In particular, for $A=lV_1$ we get

$$\mathcal{A}_1^{lV_1}\equiv\mathcal{A}_1^{l,0}=0, \tag{56}$$

hence

$$\mathcal{A}_1^A=0. \tag{57}$$

We then obtain from Eq. (55) and the gauge fixing condition

$$\mathcal{A}_2^A=0. \tag{58}$$

The condition $\mathcal{D}_r X^m=0$ now reduces to

$$\tilde{\lambda}_{rA}X^{mA}=0, \quad r=1,2. \tag{59}$$

That is,

$$\begin{aligned} N\sin\left(\frac{V_1\times A}{N}\right)\pi X^{mA}&=0, \\ N\sin\left(\frac{V_2\times A}{N}\right)\pi X^{mA}&=0, \end{aligned} \tag{60}$$

which yields

$$X^{mA}=0. \tag{61}$$

Consequently, there are no local stringlike configurations with zero energy density for the SU(N) model of Sec. IV.

VI. CONCLUSIONS

We proposed a model described by SU(N) algebra valued geometrical objects which provides a regularization of the dual of the double compactified $D=11$ supermembrane with nontrivial winding or the equivalently noncommutative super-Maxwell theory over a Riemann surface of genus 1. It describes a supermembrane over a world volume of genus 1, with fixed winding $n \neq 0$ on a target space $M_9 \times S^1 \times S^1$. We showed explicitly the existence of local stringlike spikes in the general formulation of compactified supermembranes, in agreement with [5]. We then proved that in the proposed SU(N) model for supermembranes with fixed winding, which is only one sector of the full theory, there are no stringlike spikes and hence this sector should have discrete spectrum. We will analyze in more detail the properties of the spectrum elsewhere. It is important to remark that this sector is described by a global condition which eliminates completely the local stringlike spikes.

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