Aspects of tachyonic inflation with an exponential potential

M. Sami,* Pravabati Chingangbam,† and Tabish Qureshi‡
Department of Physics, Jamia Millia Islamia, New Delhi 110025, India
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We consider issues related to tachyonic inflation with an exponential potential. We find an exact solution of the evolution equations in the slow roll limit in FRW cosmology. We also carry out a similar analysis in the case of brane assisted tachyonic inflation. We investigate the phase space behavior of the system and show that the dustlike solution is a late time attractor. The difficulties associated with reheating in the tachyonic model are also indicated.

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I. INTRODUCTION

M or string theory inspired models are under active consideration in cosmology at present. It has recently been suggested that a rolling tachyon condensate, in a class of string theories, may have interesting cosmological consequences. Sen [1,2] has shown that decay of D-branes produces a pressureless gas with a finite energy density that resembles classical dust. Rolling tachyon matter associated with unstable D-branes has an interesting equation of state which smoothly interpolates between -1 and 0. As the tachyon field rolls down the hill, the universe undergoes accelerated expansion and at a particular epoch the scale factor passes through the point of inflection marking the end of inflation. At late times the energy density of tachyon matter scales as a^{-3}. Tachyonic matter, therefore, might provide an explanation for inflation at early epochs and could contribute to some new form of cosmological dark matter at late times [3–16]. The evolution of Friedmann-Robertson-Walker (FRW) cosmological models and linear perturbations of tachyon matter rolling towards a minimum of its potential is discussed by Frolov, Kofman and Starobinsky [17]. An effective potential for the tachyon field is computed in Ref. [18]; the expression is exact in α' but at the tree level in g_s. Sen [19] has shown that the choice of an exponential potential for the tachyonic field leads to the absence of plane-wave solutions around the tachyon vacuum and exponential decay of the pressure at late times. A homogeneous tachyon field evolves towards its ground state without oscillating about it. Therefore, the conventional reheating mechanism in the tachyonic model does not work. Quantum mechanical particle production during inflation provides an alternative mechanism by means of which the universe could reheat. However, the energy density of radiation so created is much smaller than the energy density of the field. In the case of a nontachyonic field, this led to the requirement of a steep inflaton potential so that the field energy density could decay faster than the radiation energy density and radiation domination could commence. Brane assisted inflation was then invoked to support inflation with steep potentials. A detailed account of reheating via gravitational particle production in brane world scenario is discussed in Refs. [20,21].

In this paper we discuss issues of inflation with tachyon rolling down an exponential potential. We find exact solution of slow roll equations in the usual 4-dimensional FRW cosmology as well as on the brane. We carry out the phase space analysis for the system under consideration and find out the fixed point and discuss its stability. We also indicate the problems of reheating in the tachyonic model. Effects of tachyons in the context of brane world cosmology are discussed in Ref. [22]. Dynamics of gauge fields with rolling tachyon on unstable D-branes is studied in Refs. [23–25].

As recently demonstrated by Sen [1,2] a rolling tachyon condensate in a spatially flat FRW cosmological model is described by an effective fluid with energy momentum tensor $T^{\mu}_{\nu} = \mathrm{diag}(-\rho, p, p, p)$, where the energy density ρ and pressure p are given by

$$\rho = \frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}, \tag{1}$$

$$p = -V(\phi)\sqrt{1-\dot{\phi}^2}. \tag{2}$$

The Friedmann equation takes the form

$$H^2 = \frac{1}{3M_p^2}\rho \equiv \frac{1}{3M_p^2}\frac{V(\phi)}{\sqrt{1-\dot{\phi}^2}}. \tag{3}$$

The equation of motion of the tachyon field minimally coupled to gravity is

$$\frac{\ddot{\phi}}{1-\dot{\phi}^2} + 3H\dot{\phi} + \frac{V_{,\phi}}{V(\phi)} = 0. \tag{4}$$

The conservation equation equivalent to Eq. (4) has the usual form

$$\frac{\dot{\rho}_\phi}{\rho_\phi} + 3H(1+\omega) = 0,$$

*Electronic address: sami@iucaa.ernet.in; Present address: Inter-university Centre for Astronomy and Astrophysics, Post Bag 4, Ganeshkhind, Pune 411007, India.
†Electronic address: prava@jamia.net
‡Electronic address: tabish@jamia.net

where $\omega \equiv p_\phi/\rho_\phi = \dot{\phi}^2 - 1$ is the equation of state for the tachyon field. Thus a universe dominated by a tachyon field would go under accelerated expansion as long as $\dot{\phi}^2 < \frac{2}{3}$ which is very different from the condition of inflation for a non-tachyonic field, $\dot{\phi}^2 < V(\phi)$. It is obvious that the tachyon field should start rolling with a small value of $\dot{\phi}$ in order to have a long period of inflation.

A. Dynamics of tachyonic inflation in FRW cosmology

The exponential potential

$$V(\phi) = V_0 e^{-\alpha\phi}$$

has played an important role within the inflationary cosmology. Sen [19] has recently argued in support of this potential for the tachyonic system. The field equations (3) and (4) for tachyonic matter with the exponential potential can be solved exactly in the slow roll limit. In this limit

$$\frac{\dot{a}(t)}{a(t)} \simeq \sqrt{\frac{1}{3M_p^2} V(\phi)}, \tag{5}$$

and Eq. (4) becomes

$$3H\dot{\phi} \simeq -\frac{V_{,\phi}}{V(\phi)} = \alpha. \tag{6}$$

Substitution of Eq. (5) in Eq. (6) leads to

$$\dot{\phi} = \frac{\alpha}{3\beta} e^{\alpha\phi/2}, \tag{7}$$

where $\beta = \sqrt{V_0/3M_p^2}$. Equation (7) immediately integrates to yield

$$\phi(t) = -\frac{2}{\alpha} \ln\left[C - \frac{\alpha^2}{6\beta} t\right], \tag{8}$$

where $C = e^{-\alpha\phi_i/2}$ and $\phi(t = t_i = 0) \equiv \phi_i$. Putting this expression for $\phi(t)$ in Eq. (5) we get

$$\frac{a(t)}{a_i} = e^{\beta t(C - (\alpha^2/12\beta)t)}. \tag{9}$$

This equation tells us that the scale factor passes through an inflection point at

$$t = t_{end} = \frac{6\beta}{\alpha^2}\left(C - \frac{\alpha}{\sqrt{6}\beta}\right).$$

Thus t_{end} marks the end of inflation. From Eq. (8) we find

$$\dot{\phi}_{end} = \sqrt{\frac{2}{3}}, \phi_{end} = -\frac{1}{\alpha}\ln\left(\frac{\alpha^2}{6\beta^2}\right), V_{end} = \frac{\alpha^2 M_p^2}{2}. \tag{10}$$

The expression for the number of inflationary e-foldings can easily be established

$$\mathcal{N} = \ln\frac{a(t)}{a_i} = \int_{t_i=0}^{t} H(t')\,dt' = \beta t\left(C - \frac{\alpha^2}{12\beta} t\right). \tag{11}$$

Using Eqs. (10) and (11) we find

$$V_{end} = \frac{V_i}{2\mathcal{N}+1}. \tag{12}$$

It is interesting to note that the expression (12) is very similar to what one gets in case of a normal scalar field with exponential potential propagating on the brane. As the condition $\dot{\phi}^2 < 2/3$ is very different from the condition of inflation for a normal scalar field, the slow roll parameters ϵ and η assume an unusual form in the case of tachyonic field, for instance

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \frac{1}{V} \tag{13}$$

which resembles the slow roll parameter in brane world cosmology [27]. As noted by Fairbairn and Tytgat [4] for small $\dot{\phi}$ near the top of the potential we can use the canonically normalized field $\phi_c = \sqrt{V_0}\phi$ which brings the parameters to their usual form [28]. We will also use this crude approximation to estimate the magnitude of perturbations. The detailed description of perturbations is given by Hwang and Noh [29]. The Cosmic Background Explorer (COBE) normalized value for the amplitude of scalar density perturbations

$$\delta_H^2 \simeq \frac{1}{75\pi^2} \frac{V_i^2}{\alpha^2 M_p^2} \simeq 4 \times 10^{-10} \tag{14}$$

can be used to estimate V_{end} as well as α. Here V_i refers to the value of the potential at the commencement of inflation. Using Eqs. (12) and (14) with $\mathcal{N} = 60$ we obtain

$$V_{end} \simeq 4 \times 10^{-11} M_p^4. \tag{15}$$

At the end of inflation, apart from the field energy density, a small amount of radiation is also present due to particles being produced quantum mechanically during inflation [30]

$$\rho_r = 0.01 \times g_p H_{end}^4 \qquad (10 \leqslant g_p \leqslant 100) \tag{16}$$

which shows that the field energy density far exceeds the density in the radiation

$$\frac{\rho_r}{\rho(\phi)} \simeq 0.01 \times g_p \frac{V_{end}}{9M_p^2} \simeq 4 \times g_p \times 10^{-14}. \tag{17}$$

B. Tachyonic inflation on the brane

The prospects of inflation in brane world scenario [26] improve due to the presence of an additional quadratic density term in the Friedmann equation [20,27]. As a result, the class of steep potentials can successfully describe inflation on the brane. In the (4+1)-dimensional brane world scenario inspired by the Randall-Sundrum model [26], the Friedmann equation is modified to [17]

$$H^2=\frac{1}{3M_p^2}\rho_\phi\left(1+\frac{\rho_\phi}{2\lambda_b}\right)+\frac{\Lambda_4}{3}+\frac{\mathcal{E}}{a^4}, \tag{18}$$

where $\mathcal{E}$ is an integration constant which transmits bulk graviton influence onto the brane and λ_b is the brane tension. For simplicity we set Λ_4 equal to zero and also drop the last term as inflation would render it so, leading to the expression

$$H^2=\frac{1}{3M_p^2}\rho\left(1+\frac{\rho}{2\lambda_b}\right), \tag{19}$$

where ρ_ϕ is given by Eq. (1) if one is dealing with a universe dominated by a single tachyon field minimally coupled to gravity. The brane effects, in the context of inflation, are most pronounced in the high energy limit $V \gg \lambda_b$; the Friedmann equation in this limit becomes

$$H=\frac{\rho}{(6\lambda_b M_p^2)^{1/2}}. \tag{20}$$

Analogous to the preceding section, Eqs. (6) and (20) can be solved exactly to obtain

$$\phi(t)=-\frac{1}{\alpha}\ln\left[C-\frac{\alpha^2}{3\beta_b}t\right], \tag{21}$$

where $C=e^{-\alpha\phi_i}$ and $\beta_b=(V_0^2/6\lambda_b M_p^2)^{1/2}$. The scale factor is given by

$$\frac{a(t)}{a_i}=e^{\beta_b t(C-(\alpha^2/6\beta_b)t)}. \tag{22}$$

At the point of inflection $t=t_{end}=(3\beta_b/\alpha^2)(C-\alpha/\sqrt{3}\beta_b)$ we have

$$V_{end}=\frac{V_0\alpha}{\sqrt{3}\beta_b}=\alpha M_p\sqrt{2\lambda_b}, \quad \dot{\phi}_{end}=\sqrt{\frac{1}{3}}, \tag{23}$$

which is consistent with the expression of the slow roll parameter on the brane

$$\epsilon=M_p^2\left(\frac{V_{,\phi}}{V}\right)^2\frac{2\lambda_b}{V^2}.$$

The number of e-foldings is related to V_{end} and V_i as

$$V_{end}=\left(\frac{V_i^2}{2\mathcal{N}+1}\right)^{1/2}. \tag{24}$$

Similar to the previous section the COBE normalized value for the amplitude of scalar density perturbations can be used to estimate both V_{end} and λ_b. The ratio of the radiation density (created during inflation) to the field energy density turns out again to be a small number

$$\frac{\rho_r}{\rho(\phi)}\simeq 2\times g_p\times 10^{-16}$$

independent of α.

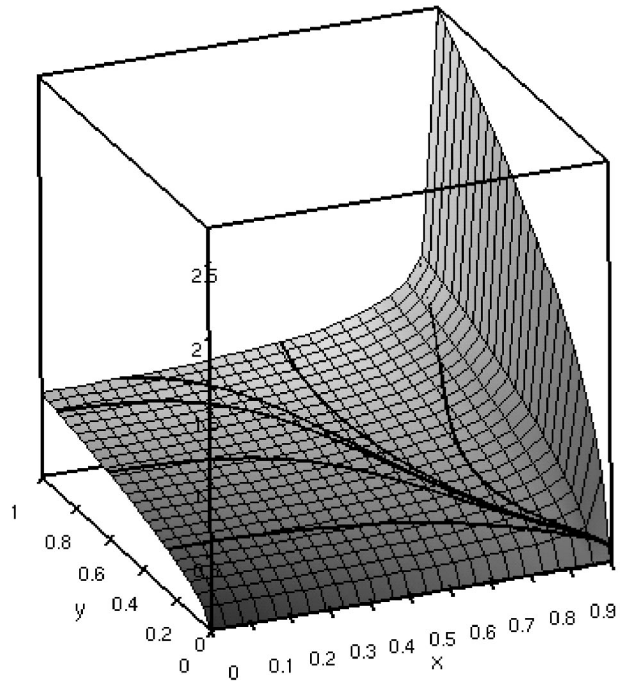

FIG. 1. Plot of the phase trajectories on the constraint surface. Trajectories starting anywhere in the phase space end up at the stable critical point (1,0).

II. PHASE PORTRAIT

In this section we carry out phase space analysis [31] for the tachyonic field with exponential potential $V=V_0e^{-\alpha\phi}$ evolving in a spatially flat FRW universe. The tachyon field evolution equation and the Friedmann constraint equation can be cast in the form

$$x'=(1-x^2)(1-3zx), \tag{25}$$

$$y'=-xy, \tag{26}$$

$$z^2=\frac{y}{3}\frac{1}{\sqrt{(1-x^2)}}, \tag{27}$$

where we have introduced the dimensionless variables

$$x=\dot{\phi}, \quad y=\frac{V(\phi)}{\alpha^2M_p^2}, \quad z=\frac{H}{\alpha}$$

and prime denotes derivative with respect to the dimensional variable $\eta=\alpha t$. To find out the critical points we set the right-hand side (RHS) of Eqs. (25) and (26) to zero and we obtain two fixed points $(\pm 1,0)$ of which $(-1,0)$ is not physical. Phase trajectories live on the constraint surface defined by Eq. (27). Numerical solution of the equations show that trajectories starting anywhere on the constraint surface end up at the fixed point (1,0) as shown in Fig. 1.

In order to study the stability of the critical point we perturb about the critical point

$$x=1-u, \quad y=v,$$

where u and v are infinitesimally close to the critical point (1,0). Putting them in the coupled equations and keeping only linear terms we get

$$u' \simeq -2u,$$

$$v' \simeq -v,$$

whose solutions are

$$u = u_0 e^{-2\eta},$$

$$v = v_0 e^{-\eta},$$

which clearly demonstrates the stability of the critical point. Asymptotically, $\dot{\phi}(\infty)=1$ and $V(\infty)=0$ implying that $a(t) \propto t^{2/3}$ and that pressure tends to zero keeping the energy density finite and non-zero [6]. We, therefore, conclude that the dustlike solution in the tachyonic model is a late time attractor.

Reheating problem in the tachyonic model

The tachyonic field could play the dual role of inflaton at the early epochs and of some new form of dark matter at late times. However, as indicated by Kofman and Linde [32], reheating is problematic in these models. As emphasized by Sen [1,2] a homogeneous tachyon field evolves towards its ground state without oscillating about it. Hence the conventional reheating mechanism in the tachyonic model does not work. Quantum mechanical particle production during inflation provides an alternative mechanism by means of which the universe could reheat. But, the energy density of radiation so created is much smaller than the energy density in the field. In the case of the non-tachyonic field, this led to the requirement of a steep inflaton potential so that the field energy density could decay faster than the radiation energy density and radiation domination could commence. Unfortunately, from the general arguments in the case of the tachyonic matter, it seems that the field energy density after inflation always scales slower than radiation. Hence radiation domination will never commence in this model irrespective of the steepness of the tachyonic potential.

In fact, $\rho_\phi \propto a^{-3\dot{\phi}^2}$ for the tachyonic field, as $\omega = \dot{\phi}^2 - 1$ in this case. Thus immediately after inflation has ended ($\dot{\phi}^2 = 2/3$), the field energy density ρ_ϕ scales as a^{-2}. Gibbons has shown that $\dot{\phi}$ is a monotonically increasing function of time with maximum value equal to one. Therefore, at best ρ_ϕ can scale as a^{-3}. This argument is valid irrespective of the form of tachyonic potential provided it belongs to the class of potentials such that $V(\phi) \to 0$ as $\phi \to \infty$. Hence, radiation energy density created at the end of inflation would redshift faster than the energy density in the tachyonic field. We have evolved the tachyonic field equations for exponential potential numerically; Fig. 2 displays the field energy density versus the scale factor and shows that tachyonic matter comes to dominate very early after inflation has ended lending support to the analysis of Kofman and Linde [32].

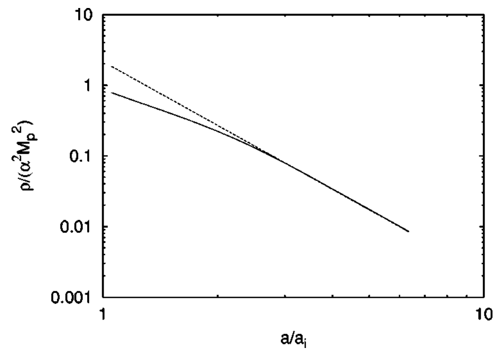

FIG. 2. The post inflation evolution of tachyonic field energy density (solid line) is shown as a function of the expansion factor. Immediately after inflation the field density scales as a^{-2}. The decay law $\rho_\phi \propto a^{-3}$ is shown by the dashed line. It is seen clearly that the energy density in the tachyonic field fast approaches a^{-3}.

To sum up, we have studied tachyonic inflation with exponential potential and found the exact solution of evolution equations in the slow roll limit in FRW cosmology as well as on the brane. We have shown that the dustlike solution is a late time attractor of the tachyonic system. We have also pointed out that reheating is problematic in the tachyonic model.

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