Shear-free rotating inflation

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We demonstrate the existence of shear-free cosmological models with rotation and expansion which support inflationary scenarios. The corresponding metrics belong to the family of spatially homogeneous models with the geometry of the closed universe (Bianchi type IX). We show that the global vorticity does not prevent inflation and can even accelerate it.

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I. INTRODUCTION

Rotation is a universal physical phenomenon. All known objects from the fundamental particles to planets, stars, and galaxies are rotating. We then naturally come to the question of whether the largest physical system—the universe—has such a property. This problem comprises several aspects. If our world does not rotate, then why and how does this happen? Since the rotating models cannot be excluded from consideration *a priori*, it is necessary to reveal a physical mechanism which prevents universal rotation. On the other hand, if the world can and does rotate, then what are the corresponding observational manifestations of the cosmic rotation? Technically, this reduces to the study of the geometry of a rotating cosmological model and to the analysis of the motion of particles and light in such a spacetime manifold. And the ultimate question is of course about the dynamical realization of the rotating models, i.e., the description of realistic matter sources and the derivation of the solutions of the gravitational field equations.

Since the early work of Lanczos [1], Gamow [2], and Gödel [3], cosmological models with rotation have been studied in a great number of publications (see the overview in [4] and the exhaustive list of references therein). Quite strong upper limits for the cosmic vorticity were obtained from the analysis of the observed properties of the microwave background radiation [5]. However, all these works deal with models in which shear and vorticity are inseparable (in the sense that zero shear automatically implies zero vorticity). Correspondingly, the limits [5] are actually placed not on the vorticity, but rather on the shear induced by it within the specific geometrical models. One thus needs a separate analysis of cosmological models with trivial shear but nonzero rotation and expansion.

Earlier [4] we studied the wide class of spatially homogeneous models described by the metric

$$ds^2 = dt^2 - 2Rn_a dx^a dt - R^2\gamma_{ab}dx^a dx^b. \tag{1}$$

Here the indices $a,b,c=1,2,3$ label the spatial coordinates, $R=R(t)$ is the scale factor, and

$$n_a = \nu_A e_a^A, \qquad \gamma_{ab} = \beta_{AB} e_a^A e_b^B, \tag{2}$$

with the constant coefficients ν_A, β_{AB} $(A,B=1,2,3)$. The one-forms $e^A = e_a^A(x)dx^a$ are invariant with respect to the action of a three-parameter group of motion which is admitted by the space-time (1). The action of this group is simply transitive on the spatial ($t=\mathrm{const}$) hypersurfaces. There exist nine types (Bianchi types) of such manifolds, distinguished by the Killing vectors ξ_A and their commutators $[\xi_A,\xi_B] = f_{AB}^C \xi_C$.

The models (1) are shear free but the vorticity and expansion are nontrivial, in general. The *kinematic* analysis [4] of the models (1) reveals their several attractive properties: the complete causality (no timelike closed curves), the absence of parallax effects, and the isotropy of the microwave background radiation. As a result, these shear-free models satisfy all the known observational criteria for cosmic rotation. In particular, it is worthwhile to note that the vorticity bounds [5] are not applicable to the class of metrics (1). The satisfactory observational properties suggest that the shear-free homogeneous models can be considered as viable candidates for the description of cosmic rotation.

The aim of the present paper is to address the *dynamic* aspect of the theory, namely, to study the realization of the models (1) as exact solutions of the gravitational field equations. This represents a nontrivial problem, in general, as it is notoriously difficult to combine the expansion with vorticity in a realistic cosmological model. In technical terms, the most important thing needed is to determine the physically reasonable matter content of such cosmologies.

In this paper we continue the study of Bianchi type IX models belonging to the class (1). The Bianchi type IX type is distinguished among the other spatially homogeneous models by the fact that its geometry describes a spatially *closed* world. Many very interesting questions related to the Mach principle arise in this connection. In particular, it is a matter of principal importance to know whether Einstein's field equations admit truly anti-Machian solutions or not. A first example of such a solution was given by the stationary model of Ozsváth and Schücking [6]. However, later its anti-Machian nature was questioned by King [7] who developed the idea that the total angular velocity of the closed world is ultimately zero because the cosmic vorticity is compensated

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by rotating gravitational waves. Recently, we demonstrated the existence of another stationary rotating closed Bianchi IX world in which the cosmic vorticity is balanced by the spin of the cosmological matter [8].

The above mentioned results refer to stationary models, which are clearly of the academic interest only because of the absence of expansion. Here we give two explicit examples of more physically realistic nonstationary closed Bianchi IX worlds with nontrivial rotation and expansion. After the description of the spacetime geometry in Sec. II, we present the rotating version of the de Sitter solution in Sec. III. Further, in Sec. IV we demonstrate that shear-free models with rotation and expansion arise in the standard inflationary scheme.

II. CLOSED WORLD GEOMETRY

Closed spatially homogeneous Bianchi type IX worlds are constructed with the help of the triad of invariant one-forms e^A which satisfy the structure equations

$$de^A = f^A_{BC} e^B \wedge e^C \tag{3}$$

with

$$f^1_{23} = f^2_{31} = f^3_{12} = 1.$$

Denoting the spatial coordinates $x = x^1, y = x^2, z = x^3$, one can choose them in the following explicit realization:

$$e^1 = \cos y \cos z\, dx - \sin z\, dy,$$

$$e^2 = \cos y \sin z\, dx + \cos z\, dy,$$

$$e^3 = -\sin y\, dx + dz. \tag{4}$$

We assume the diagonal β_{AB} and can write the ansatz for the line element (1) as

$$ds^2 = g_{\alpha\beta} \vartheta^\alpha \vartheta^\beta, \quad g_{\alpha\beta} = \mathrm{diag}(1,-1,-1,-1), \tag{5}$$

where the orthonormal coframe one-forms ϑ^α read

$$\vartheta^{\hat 0} = dt - R\, \nu_A\, e^A,\ \vartheta^{\hat 1} = R\, k_1\, e^1,$$

$$\vartheta^{\hat 2} = R\, k_2\, e^2,\ \vartheta^{\hat 3} = R_3\, e^3. \tag{6}$$

Here, k_1, k_2, k_3 are positive constant parameters. The greek indices $\alpha, \beta, \ldots = 0,1,2,3$ hereafter label objects with respect to the orthonormal frame; the carets over indices denote the separate frame components of these objects.

The kinematic properties of the spacetime geometry are described by the vorticity $\omega_{\mu\nu} = h^\alpha_\mu h^\beta_\nu \nabla_{[\alpha} u_{\beta]}$, the shear $\sigma_{\mu\nu} = h^\alpha_\mu h^\beta_\nu \nabla_{(\alpha} u_{\beta)} - \frac{1}{3}\, h_{\mu\nu} \nabla_\lambda u^\lambda$, and the volume expansion $\theta = \nabla_\lambda u^\lambda$. Here $u = \partial_t$ is the comoving velocity (normalized by $u_\alpha u^\alpha = 1$) and $h_{\mu\nu} = g_{\mu\nu} - u_\mu u_\nu$ is the standard projector on the rest three-space. A direct calculation yields

$$\sigma_{\mu\nu} = 0, \quad a^{\hat 1} = \frac{\dot R \nu_1}{R k_1}, \ \ a^{\hat 2} = \frac{\dot R \nu_2}{R k_2},$$

$$a^{\hat 3} = \frac{\dot R \nu_3}{R k_3}, \quad \theta = 3\frac{\dot R}{R}, \tag{7}$$

$$\omega_{\hat 2 \hat 3} = -\frac{\nu_1}{2 R k_2 k_3}, \quad \omega_{\hat 3 \hat 1} = -\frac{\nu_2}{2 R k_1 k_3},$$

$$\omega_{\hat 1 \hat 2} = -\frac{\nu_3}{2 R k_1 k_2}. \tag{8}$$

III. ROTATING de SITTER WORLD

First we study the case when matter is represented by just the cosmological constant. An equivalent physical model is given by the ideal fluid with the vacuum equation of state. The total Lagrangian reads

$$L = -\frac{1}{2\kappa}(R + 2\Lambda), \tag{9}$$

and the left-hand side of the Einstein field equations $R_{\alpha\beta} - \frac{1}{2}\, R\, g_{\alpha\beta} = \Lambda\, g_{\alpha\beta}$, take the form (A1)–(A10) given in the Appendix.

As a first step, we specialize to the case

$$\nu_1 \neq 0, \quad \nu_2 = \nu_3 = 0. \tag{10}$$

Then there remains only the "01" nontrivial off-diagonal equation which reduces to

$$-\frac{\ddot R}{R} + \frac{\dot R^2}{R^2} + \frac{k_1^2}{4R^2\, k_2^2 k_3^2} = 0. \tag{11}$$

The analysis of the four diagonal Einstein equations [see Eqs. (A1)–(A4)] shows that they are consistent under the algebraic conditions

$$k_3 = k_2 \quad \text{and} \quad k_2^2 = k_1^2 - \nu_1^2. \tag{12}$$

Then the diagonal equations, using Eq. (11), reduce to the first order equation

$$3\left(\frac{\dot R^2}{R^2} + \frac{k_1^2}{4R^2\, k_2^4}\right) = \frac{k_1^2}{k_2^2}\Lambda. \tag{13}$$

This can be straightforwardly integrated, yielding the solution

$$R(t) = \frac{1}{2k_2}\sqrt{\frac{3}{\Lambda}} \cosh\left(\frac{k_1}{k_2}\sqrt{\frac{\Lambda}{3}}t\right). \tag{14}$$

One can check that Eq. (11) is then identically fulfilled. The metric (5),(6) with the scale factor (14) represents the rotating version of the de Sitter world. A slightly different form of that solution was obtained in [10]. Another rotating generalization of the de Sitter model is described in [9], which is also shear-free Bianchi type IX, although it does not belong to the class (1).

IV. ROTATING INFLATIONARY MODELS

Inspired by the above preliminary demonstration that rotation can coexist with inflation, we now consider the general inflationary model (see [11–14], for example) which is described by the Lagrangian with the scalar field

$$L=-\frac{1}{2\kappa}R+\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi)-V(\phi). \tag{15}$$

The Einstein equations now read $R_{\alpha\beta}-\frac{1}{2}R\,g_{\alpha\beta}=\kappa\,T_{\alpha\beta}$, where the explicit form of the energy-momentum components is given in the Appendix [see Eqs. (A11)–(A16)].

Again specializing to the case (10), we find that all the off-diagonal equations are trivially fulfilled except for the "01" component. The latter reads

$$2\left(-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}+\frac{k_1^2}{4R^2\,k_2^2k_3^2}\right)=\kappa\,\dot{\phi}^2. \tag{16}$$

Substituting $\kappa\,\dot{\phi}^2$ from Eq. (16) into the four diagonal Einstein equations [use Eqs. (A1)–(A4) and (A11)–(A14)], we again discover the consistency condition (12). As a result, the diagonal equations reduce to

$$\frac{\ddot{R}}{R}+2\frac{\dot{R}^2}{R^2}+\frac{k_1^2}{2R^2\,k_2^4}=\frac{k_1^2}{k_2^2}\kappa V. \tag{17}$$

In addition to the Einstein equations, we have the Klein-Gordon equation for the scalar field, $D_\mu D^\mu\phi+V'=0$ [where $V':=dV(\phi)/d\phi$]. For the metric (5), (6) it reads

$$\ddot{\phi}+3\frac{\dot{R}}{R}\dot{\phi}+\frac{k_1^2}{k_2^2}V'=0. \tag{18}$$

Only two of the three equations (16)–(18) are independent. In order to see this, let us take, instead of Eqs. (16) and (17), their sum and difference. This yields

$$\frac{\dot{R}^2}{R^2}+\frac{k_1^2}{4R^2\,k_2^4}=\frac{\kappa}{3}\left(\frac{1}{2}\dot{\phi}^2+\frac{k_1^2}{k_2^2}V\right), \tag{19}$$

$$\frac{\ddot{R}}{R}=\frac{\kappa}{3}\left(-\dot{\phi}^2+\frac{k_1^2}{k_2^2}V\right). \tag{20}$$

We can take as the independent dynamical equations either (19) together with (18), or (19) together with (20). Then, correspondingly, the third equation will be derived from the first two, provided $\dot{\phi}\neq 0$.

We thus have recovered the system of the usual inflationary model in which the spatial curvature K and the inflation potential are "corrected" by the rotation parameters

$$K\to\frac{k_1^2}{4k_2^4},\qquad V\to\frac{k_1^2}{k_2^2}V. \tag{21}$$

The form of the exact or approximate solutions of the final system depends on the inflation potential $V(\phi)$, and we refer to the relevant analysis of the standard inflationary system [11–14] (see also [15] and references therein) which is completely applicable to our rotating world after we make the redefinitions (21).

V. DISCUSSION AND CONCLUSION

The results of Sec. III represent a particular case of the general inflationary model when $\dot{\phi}=0$ with V playing the role of the cosmological constant. However, we found it more instructive to consider that special case separately, in particular because then it is possible to make a direct comparison with the earlier results of [9]. With an account of the algebraic conditions (10) and (12), we have constructed the exact solution of the Einstein equations in the form of the line element

$$\begin{aligned}ds^2={}&dt^2-2\,\nu_1\,R\,dt\,e^1\\&-k_2^2\,R^2[(e^1)^2+(e^2)^2+(e^3)^2],\end{aligned} \tag{22}$$

where the scale factor R is determined from Eq. (14) or from the inflationary system (18)–(20). This model is shear free, and the results obtained thus contribute to studies of the shear-free conjecture (see [16], for example). The volume expansion is $\theta=3\dot{R}/R$ and the vorticity is decreasing in the expanding universe with the only nontrivial component

$$\omega_{\hat{2}\hat{3}}=-\frac{\nu_1}{2Rk_2^2}. \tag{23}$$

During the de Sitter era (14) the cosmic rotation rapidly decays.

Our results confirm and extend the conclusions of Grøn [9] (see also [17,18]) in that cosmic rotation does not prevent inflation, whereas the latter yields a quick decrease of vorticity. The preliminary and qualitative conclusions of [17,18] were derived on the basis of the conservation law of angular momentum without analyzing Einstein's equations. The behavior of our exact solution now provides direct evidence in support of these results. Moreover, because of Eq. (21), we can see now that the cosmic vorticity in fact enhances the inflation: when the vorticity is large ($\nu_1\to\infty$ for a fixed value of k_2) the coefficient $k_1/k_2>1$ makes the inflation rate much bigger than in the vorticity-free case ($k_1/k_2=1$ for $\nu_1=0$).

Summarizing, we have demonstrated the existence of a realistic cosmological model with rotation and expansion: The exact Bianchi type IX solution (22) is determined by the standard inflationary system (19)–(20). Here we do not specify the explicit form of the inflation potential which represents a separate complicated subject in modern cosmology. However, for each given $V(\phi)$ the evolution of the scalar field and the cosmological scale factor can be found straightforwardly.

In our final remark, let us come back to the Mach principle. Since our model describes a *closed* world, its existence again raises the question whether the true anti-Mach cosmology is possible. The earlier discussion of stationary models [6–8] has revealed some mechanisms of compensation of the global vorticity by gravitational waves or by local spin of matter. As far as we can see, such a compensation does not exist for the new solution. This means that the shear-free rotating inflational Bianchi type IX model describes a true (and far more realistic due to the nontrivial expansion) anti-Machian model. In this connection, it would also be interesting to study Bianchi type V rotating models, which contain open standard cosmology as a particular case.

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APPENDIX

The left-hand side of the Einstein gravitational field equations is described by the Einstein tensor $G_{\alpha\beta}=R_{\alpha\beta}-\frac{1}{2}R\,g_{\alpha\beta}$. For the metric (5),(6), it reads

$$G_{\hat{0}\hat{0}}=-\left(2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\left(\frac{\nu_1^2}{k_1^2}+\frac{\nu_2^2}{k_2^2}+\frac{\nu_3^2}{k_3^2}\right)+3\frac{\dot{R}^2}{R^2}+\frac{-k_1^4-k_2^4-k_3^4+2(k_1^2k_2^2+k_1^2k_3^2+k_2^2k_3^2)+3(k_1^2\nu_1^2+k_2^2\nu_2^2+k_3^2\nu_3^2)}{4R^2\,(k_1k_2k_3)^2}, \tag{A1}$$

$$G_{\hat{1}\hat{1}}=\left(2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\left(-1+\frac{\nu_2^2}{k_2^2}+\frac{\nu_3^2}{k_3^2}\right)+3\frac{\dot{R}^2}{R^2}\,\frac{\nu_1^2}{k_1^2}+\frac{3k_1^4-k_2^4-k_3^4+2(-k_1^2k_2^2-k_1^2k_3^2+k_2^2k_3^2)-k_1^2\nu_1^2+k_2^2\nu_2^2+k_3^2\nu_3^2}{4R^2\,(k_1k_2k_3)^2}, \tag{A2}$$

$$G_{\hat{2}\hat{2}}=\left(2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\left(-1+\frac{\nu_1^2}{k_1^2}+\frac{\nu_3^2}{k_3^2}\right)+3\frac{\dot{R}^2}{R^2}\,\frac{\nu_2^2}{k_2^2}+\frac{-k_1^4+3k_2^4-k_3^4+2(-k_1^2k_2^2+k_1^2k_3^2-k_2^2k_3^2)+k_1^2\nu_1^2-k_2^2\nu_2^2+k_3^2\nu_3^2}{4R^2\,(k_1k_2k_3)^2}, \tag{A3}$$

$$G_{\hat{3}\hat{3}}=\left(2\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\left(-1+\frac{\nu_1^2}{k_1^2}+\frac{\nu_2^2}{k_2^2}\right)+3\frac{\dot{R}^2}{R^2}\,\frac{\nu_3^2}{k_3^2}+\frac{-k_1^4-k_2^4+3k_3^4+2(k_1^2k_2^2-k_1^2k_3^2-k_2^2k_3^2)+k_1^2\nu_1^2+k_2^2\nu_2^2-k_3^2\nu_3^2}{4R^2\,(k_1k_2k_3)^2}, \tag{A4}$$

$$G_{\hat{0}\hat{1}}=2\left(-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\frac{\nu_1}{k_1}+\frac{\dot{R}}{R^2}\,\frac{k_1\nu_2\nu_3(k_3^2-k_2^2)}{(k_1k_2k_3)^2}+\frac{k_1\nu_1}{2R^2\,k_2^2k_3^2}, \tag{A5}$$

$$G_{\hat{0}\hat{2}}=2\left(-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\frac{\nu_2}{k_2}+\frac{\dot{R}}{R^2}\,\frac{k_2\nu_1\nu_3(k_1^2-k_3^2)}{(k_1k_2k_3)^2}+\frac{k_2\nu_2}{2R^2\,k_1^2k_3^2}, \tag{A6}$$

$$G_{\hat{0}\hat{3}}=2\left(-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\frac{\nu_3}{k_3}+\frac{\dot{R}}{R^2}\,\frac{k_3\nu_1\nu_2(k_2^2-k_1^2)}{(k_1k_2k_3)^2}+\frac{k_3\nu_3}{2R^2\,k_1^2k_2^2}, \tag{A7}$$

$$G_{\hat{1}\hat{2}}=2\left(-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\frac{\nu_1\nu_2}{k_1k_2}+\frac{\dot{R}}{R^2}\,\frac{k_1k_2\nu_3(k_1^2-k_2^2)}{(k_1k_2k_3)^2}-\frac{\nu_1\nu_2}{2R^2\,k_1k_2k_3^2}, \tag{A8}$$

$$G_{\hat{1}\hat{3}}=2\left(-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\frac{\nu_1\nu_3}{k_1k_3}+\frac{\dot{R}}{R^2}\,\frac{k_1k_3\nu_2(k_3^2-k_1^2)}{(k_1k_2k_3)^2}-\frac{\nu_1\nu_3}{2R^2\,k_1k_2^2k_3}, \tag{A9}$$

$$G_{\hat{2}\hat{3}}=2\left(-\frac{\ddot{R}}{R}+\frac{\dot{R}^2}{R^2}\right)\frac{\nu_2\nu_3}{k_2k_3}+\frac{\dot{R}}{R^2}\,\frac{k_2k_3\nu_1(k_2^2-k_3^2)}{(k_1k_2k_3)^2}-\frac{\nu_2\nu_3}{2R^2\,k_1^2k_2k_3}. \tag{A10}$$

The right-hand side of Einstein's equations is the energy-momentum tensor of the scalar field $T_{\alpha\beta}=(D_\alpha\phi)(D_\beta\phi)-\frac{1}{2}(D_\mu\phi)(D^\mu\phi)\,g_{\alpha\beta}+V\,g_{\alpha\beta}$. For the metric (5),(6), we find

$$T_{\hat{0}\hat{0}}=\frac{\dot{\phi}^2}{2}\left(1+\frac{\nu_1^2}{k_1^2}+\frac{\nu_2^2}{k_2^2}+\frac{\nu_3^2}{k_3^2}\right)+V, \tag{A11}$$

$$T_{\hat{1}\hat{1}}=\frac{\dot{\phi}^2}{2}\left(1+\frac{\nu_1^2}{k_1^2}-\frac{\nu_2^2}{k_2^2}-\frac{\nu_3^2}{k_3^2}\right)-V, \tag{A12}$$

$$T_{\hat{2}\hat{2}}=\frac{\dot{\phi}^2}{2}\left(1-\frac{\nu_1^2}{k_1^2}+\frac{\nu_2^2}{k_2^2}-\frac{\nu_3^2}{k_3^2}\right)-V, \tag{A13}$$

$$T_{\hat{3}\hat{3}}=\frac{\dot{\phi}^2}{2}\left(1-\frac{\nu_1^2}{k_1^2}-\frac{\nu_2^2}{k_2^2}+\frac{\nu_3^2}{k_3^2}\right)-V, \tag{A14}$$

$$T_{\hat{0}\hat{1}}=\dot{\phi}^2\frac{\nu_1}{k_1},\quad T_{\hat{0}\hat{2}}=\dot{\phi}^2\frac{\nu_2}{k_2},\quad T_{\hat{0}\hat{3}}=\dot{\phi}^2\frac{\nu_3}{k_3}, \tag{A15}$$

$$T_{\hat{1}\hat{2}}=\dot{\phi}^2\frac{\nu_1\nu_2}{k_1k_2},\quad T_{\hat{1}\hat{3}}=\dot{\phi}^2\frac{\nu_1\nu_3}{k_1k_3},\quad T_{\hat{2}\hat{3}}=\dot{\phi}^2\frac{\nu_2\nu_3}{k_2k_3}. \tag{A16}$$

[1] K. Lanczos, Z. Phys. **21**, 73 (1924) [Gen. Relativ. Gravit. **29**, 361 (1997)].
[2] G. Gamow, Nature (London) **158**, 549 (1946).
[3] K. Gödel, Rev. Mod. Phys. **21**, 447 (1949); in *Proceedings of the International Congress of Mathematicians*, Cambridge, MA, 1950, edited by L. M. Graves, E. Hille, P. A. Smith, and O. Zariski (American Mathematical Society, Providence, RI, 1952), Vol. 1, p. 175.
[4] Yu. N. Obukhov, in *Colloquium on Cosmic Rotation*, Berlin, 1998, edited by M. Scherfner, T. Chrobok, and M. Shefaat (Wissenschaft und Technik Verlag, Berlin, 2000), pp. 23–96, astro-ph/0008106.
[5] S.W. Hawking, Mon. Not. R. Astron. Soc. **142**, 129 (1969); C.B. Collins and S.W. Hawking, *ibid.* **162**, 307 (1973); J.D. Barrow, R. Juszkiewicz, and D. Sonoda, *ibid.* **213**, 917 (1985); E. Bunn, P. Ferreira, and J. Silk, Phys. Rev. Lett. **77**, 2883 (1996); A. Kogut, G. Hinshaw, and A. Banday, Phys. Rev. D **55**, 1901 (1997).
[6] I. Ozsváth and E. Schücking, Nature (London) **193**, 1168 (1962); in *Recent Developments in General Relativity* (Pergamon, Oxford, 1962), p. 339; Ann. Phys. (N.Y.) **55**, 166 (1969); J. Geom. Phys. **24**, 303 (1998); I. Ozsváth, Class. Quantum Grav. **14**, A291 (1997).
[7] D. H. King, in *Einstein Studies, Vol. 6: Mach's Principle: From Newton's Bucket to Quantum Gravity*, edited by J. B. Barbour and H. Pfister (Birkhäuser, Boston, 1995), p. 237; Phys. Rev. D **44**, 2356 (1991).
[8] T. Chrobok, Yu.N. Obukhov, and M. Scherfner, Phys. Rev. D **63**, 104014 (2001).
[9] Ø. Grøn, Phys. Rev. D **33**, 1204 (1986).
[10] S. Weinberg, Diploma thesis, Institute of Theoretical Physics, Technical University, Berlin, 1999, pp. 37–39.
[11] L. F. Abbott and S.-Y. Pi, *Inflationary Cosmology* (World Scientific, Singapore, 1986). Reprints of all classical works on inflation are collected in this book.
[12] A. D. Linde, *Particle Physics and Inflationary Cosmology* (Harwood Academic, Chur, Switzerland, 1990).
[13] A. H. Guth, *The Inflationary Universe* (Addison-Wesley, Reading, MA, 1997).
[14] A. R. Liddle and D. H. Lyth, *Cosmological Inflation and Large-Scale Structure* (Cambridge University Press, Cambridge, England, 2000), p. 400.
[15] F.E. Schunck and E.W. Mielke, Phys. Rev. D **50**, 4794 (1994); E.W. Mielke and F.E. Schunck, *ibid.* **52**, 672 (1995); F.V. Kusmartsev, E.W. Mielke, Yu.N. Obukhov, and F.E. Schunck, *ibid.* **51**, 924 (1995).
[16] J.M.M. Senovilla, C.F. Sopuerta, and P. Szekeres, Gen. Relativ. Gravit. **30**, 389 (1998).
[17] J. Ellis and K. Olive, Nature (London) **303**, 679 (1983).
[18] Ø. Grøn and H.H. Soleng, Nature (London) **328**, 501 (1987).