Variations of alpha in space and time

John D. Barrow
DAMTP, Centre for Mathematical Sciences, Cambridge University, Wilberforce Road, Cambridge CB3 0WA, United Kingdom

João Magueijo and Håvard Bunes Sandvik
Theoretical Physics, Blackett Laboratory, Imperial College, Prince Consort Road, London SW7 2BZ, United Kingdom

(Received 6 February 2002; published 19 August 2002)

We study inhomogeneous cosmological variations in the fine structure ''constant'' α in Friedmann universes. Inhomogeneous motions of the scalar field driving changes in α display spatial oscillations that decrease in amplitude with increasing time. The inhomogeneous evolution quickly approaches that found for exact Friedmann universes. We prove a theorem to show that oscillations of α in time (or redshift) cannot occur in Friedmann universes in the Bekenstein-Sandvik-Barrow-Magueijo theories considered here.

DOI: 10.1103/PhysRevD.66.043515 PACS number(s): 98.80.Cq, 98.80.Es

I. INTRODUCTION

Elsewhere [1,2] we have discussed the behavior of a class of cosmologies in an exact theory in which the fine structure ''constant'' varies in time. This theory of Sandvik, Barrow and Magueijo is an extension, to include the self-gravitation of the dielectric medium, of Bekenstein's prescription [3] for generalizing Maxwell's equations to incorporate varying electron charge. We will refer to it as the Bekenstein-Sandvik-Barrow-Magueijo (BSBM) theory. The fine structure ''constant'' α varies through the space-time dynamics of a scalar ''dielectric'' field ψ (where $\alpha=\exp[2\psi]$) in these theories. However the overall behavior is significantly affected by the form of the coupling. Even though the requirement that the energy in ψ be positive definite fixes the sign of the coupling constant ω, we find that ψ is driven by a term of the form $\mathcal{L}_{em}/\omega$, where $\mathcal{L}_{em}$ is the electromagnetic Lagrangian. In general, $\mathcal{L}_{em}$ can be positive or negative, a fact we parametrize in terms of $\zeta=\mathcal{L}_{em}/\rho$, where ρ is the energy density. The sign of ζ for the dark matter in the universe turns out to be of exceptional significance.

In our earlier studies [1,2] we have focused on the case where $\zeta<0$ and the dark matter in the universe is dominated by magnetic field couplings. This was motivated by the discovery that $\zeta<0$ matter leads to a slow (logarithmic) *increase* in the value of α with cosmic time during the matter era of the universe and constant behavior during any period in which the expansion is curvature dominated, accelerates, or is dominated by radiation in universes with a matter-radiation balance such as our own. Thus for $\zeta<0$ we find slow time-evolution of the fine structure 'constant' that is consistent with the observations of Webb et al. [4–6] of $\Delta\alpha/\alpha=(-0.72\pm0.18)\times10^{-5}$ at $z=1-3.5$. They are also consistent with the low-redshift, $z\sim0.1$, upper limits on time variation of α provided by Oklo [7,8], and high-redshift constraints imposed by the microwave background temperature fluctuations [9,10], and primordial nucleosynthesis [11]. Other hints of varying constants in astronomical studies have recently been reported by Ivanchik et al. [12].

All of the studies described above have been performed in the context of an exact isotropic and homogeneous Friedmann universe. All variations in the fine structure ''constant'' therefore depend only on cosmic time. However, the rate of variation that is suggested by recent astronomical observations of quasar spectra, or allowed by geophysical data at recent times, is very small, $\Delta\alpha/\alpha\sim10^{-5}$, and spatial variations in the rate of time variation could easily be of similar order [13]. It is therefore important to determine if spatial variations in the rate of change of α are significant in the BSBM theory and whether they allow different modes of time variation to occur in addition to those studied in the purely homogeneous variations found in Refs. [1,2].

II. THE BSBM THEORY

There are a number of possible theories allowing for the variation of the fine structure constant, α. In the simplest cases one takes c and $\hbar$ to be constants (see however [15,16]) and attributes variations in α to changes in the electron charge, e, or the permittivity of free space (see Ref. [14] for a discussion of the meaning of this choice). This is done by letting e take on the value of a real scalar field which varies in space and time (for more complicated cases, resorting to complex fields undergoing spontaneous symmetry breaking, see the case of fast tracks discussed in [15]). Thus $e_0\rightarrow e=e_0\epsilon(x^\mu)$, where ϵ is a dimensionless scalar field and e_0 is a constant denoting the present value of e. This operation implies that some well established assumptions, like charge conservation, must give way [17]. Nevertheless, the principles of local gauge invariance and causality are maintained, as is the scale invariance of the ϵ field (under a suitable choice of dynamics). In addition there is no conflict with local Lorentz invariance or covariance. With this setup in mind, the dynamics of our theory is then constructed as follows. Since e is the electromagnetic coupling, the ϵ field couples to the gauge field as ϵA_μ in the Lagrangian and the gauge transformation which leaves the action invariant is $\epsilon A_\mu\rightarrow\epsilon A_\mu+\chi_{,\mu}$, rather than the usual $A_\mu\rightarrow A_\mu+\chi_{,\mu}$. The gauge-invariant electromagnetic field tensor is therefore

$$F_{\mu\nu}=\frac{1}{\epsilon}[(\epsilon A_\nu)_{,\mu}-(\epsilon A_\mu)_{,\nu}], \tag{1}$$

which reduces to the usual form when ϵ is constant. The electromagnetic part of the action is still

$$S_{em}=-\frac{1}{4}\int d^4x\sqrt{-g}F^{\mu\nu}F_{\mu\nu}, \tag{2}$$

and the dynamics of the ϵ field are controlled by the kinetic term

$$S_{\epsilon}=-\frac{1}{2}\frac{\hbar c}{l^2}\int d^4x\sqrt{-g}\,\frac{\epsilon_{,\mu}\epsilon^{,\mu}}{\epsilon^2}, \tag{3}$$

as in dilaton theories. Here, l is the characteristic length scale of the theory, introduced for dimensional reasons. This constant length scale gives the scale down to which the electric field around a point charge is accurately of Coulomb type. The corresponding energy scale, $\omega=\hbar c/l$, has to lie between a few tens of MeV and Planck scale, $\sim 10^{19}$ GeV, to avoid conflict with experiment.

Our generalization of the scalar theory proposed by Bekenstein [3] described in Ref. [1] includes the gravitational effects of $\psi=\log\epsilon$. It gives the field equations

$$G_{\mu\nu}=8\pi G(T^{m}_{\mu\nu}+T^{\psi}_{\mu\nu}+T^{em}_{\mu\nu}e^{-2\psi}), \tag{4}$$

where the various $T_{\mu\nu}$ are the matter, ψ and electromagnetic stress energy tensors. Recall the ψ Lagrangian is $\mathcal{L}_{\psi}=-(\omega/2)\partial_{\mu}\psi\partial^{\mu}\psi$ and the ψ field obeys the equation of motion

$$\Box\psi=\frac{2}{\omega}e^{-2\psi}\mathcal{L}_{em}, \tag{5}$$

where we have defined the coupling constant $\omega=(\hbar c)/l^2$. This constant is of order ~ 1 if, as in [1], the energy scale is similar to the Planck scale. It is clear that $\mathcal{L}_{em}$ vanishes for a sea of pure radiation since then $\mathcal{L}_{em}=(E^2-B^2)/2=0$. We therefore expect the variation in α to be driven by electrostatic and magnetostatic energy-components rather than electromagnetic radiation. In order to make quantitative predictions we need to know how much of the nonrelativistic matter contributes to the right-hand side (RHS) of Eq. (5). This is parametrized by $\zeta\equiv\mathcal{L}_{em}/\rho$, where ρ is the energy density, and for baryonic matter $\mathcal{L}_{em}=E^2/2$. For protons and neutrons, ζ_p and ζ_n can be *estimated* from the electromagnetic corrections to the nucleon mass, 0.63 MeV and -0.13 MeV, respectively [18,19]. This correction contains the $E^2/2$ contribution (always positive), but also terms of the form $j_{\mu}a^{\mu}$ (where j_{μ} is the quarks' current) and so cannot be used directly. Hence we take a guiding value $\zeta_p\approx\zeta_n\sim 10^{-4}$. Furthermore the cosmological value of ζ (denoted ζ_m) has to be weighted by the fraction of matter that is nonbaryonic, a point ignored in the literature [3,20]. Hence, ζ_m depends strongly on the nature of the dark matter and can take both positive and negative values depending on whether the Coulomb-energy or magnetostatic energy dominates the dark matter of the Universe. It could be that $\zeta_{CDM}\approx -1$ (superconducting cosmic strings, for which $\mathcal{L}_{em}\approx -B^2/2$), or $\zeta_{CDM}\ll 1$ (neutrinos). Big-bang nucleasynthesis (BBN) predicts an approximate value for the baryon density of $\Omega_B\approx 0.03$ with a Hubble parameter of $h_0\approx 0.6$, implying $\Omega_{CDM}\approx 0.3$. Thus depending on the nature of the dark matter, ζ_m can be virtually anything between -1 and $+1$. The uncertainties in the underlying quark physics and especially the constituents of the dark matter make it difficult to impose more certain bounds on ζ_m.

We should not confuse this theory with other similar variations. Bekenstein's theory [3] does not take into account the stress energy tensor of the dielectric field in Einstein's equations. Dilaton theories predict a global coupling between the scalar and all other matter fields. As a result they predict variations in other constants of nature, and also a different dynamics to all the matter coupled to electromagnetism. This model may be seen as a more conservative alternative to varying-speed-of-light scenarios [14,22–26]. An interesting application of our approach has also recently been made to braneworld cosmology by Youm [21]. Assuming a homogeneous and isotropic Friedmann metric with expansion scale factor $a(t)$ and curvature parameter k in Eq. (4), we obtain the field equations ($c\equiv 1$)

$$\left(\frac{\dot a}{a}\right)^2=\frac{8\pi G}{3}\left(\rho_m(1+|\zeta_m|\exp[-2\psi])+\rho_r\exp[-2\psi]+\frac{\omega}{2}\dot\psi^2\right)-\frac{k}{a^2}+\frac{\Lambda}{3}, \tag{6}$$

where Λ is the cosmological constant. For the scalar field we have the propagation equation

$$\ddot\psi+3H\dot\psi=-\frac{2}{\omega}\exp[-2\psi]\zeta_m\rho_m, \tag{7}$$

where $H\equiv\dot a/a$ is the Hubble expansion rate. Note that the sign of the evolution of ψ is dependent on the sign of ζ_m. Since the observational data are consistent with a *smaller* value of α in the past, we will in this paper confine our study to *negative* values of ζ_m, in line with our recent discussion in Ref. [1]. The conservation equations for the noninteracting radiation and matter densities are

$$\dot\rho_m+3H\rho_m=0 \tag{8}$$

$$\dot\rho_r+4H\rho_r=2\dot\psi\rho_r, \tag{9}$$

and so $\rho_m\propto a^{-3}$ and $\rho_r e^{-2\psi}\propto a^{-4}$, respectively. If additional noninteracting perfect fluids satisfying the equation of state $p=(\gamma-1)\rho$ are added to the universe then they contribute density terms $\rho\propto a^{-3\gamma}$ to the RHS of Eq. (6) as usual.

III. INHOMOGENEOUS SOLUTIONS WITH VARYING α

The Friedmann models with varying α have shown that when $\zeta_m<0$ the homogeneous motion of the ψ does not in general create significant metric perturbations at late times and we can safely assume that the expansion scale factor is that of the usual Friedmann universe for the appropriate fluid. The behavior of ψ then follows from a solution of the ψ conservation equation in which the expansion scale factor is taken to be that of the Friedmann universe for matter with the same equation of state in general relativity ($\psi=\zeta=0$).

Our earlier analyses found that ψ is approximately constant during the radiation era, and α increases as $2N\ln(t)$ during the dust dominated era when spatial curvature is negligible, and tends to a constant in any subsequent era dominated by negative spatial curvature or a positive cosmological constant [2]. When $\zeta<0$ we can use the same test-motion approach to investigate inhomogeneous variations in ψ and α as the universe expands.

We assume that the expansion scale factor is that of the Friedmann model

$$a=t^n \tag{10}$$

and solve the wave equation in one of its appropriate forms:

$$\Box\psi=-\frac{2\zeta}{\omega}\rho_m\exp[-2\psi] \tag{11}$$

$$\ddot{\psi}+\frac{3\dot{a}}{a}\dot{\psi}-\frac{1}{a^2}\nabla^2\psi=-\frac{2\zeta}{\omega}\rho_m\exp[-2\psi] \tag{12}$$

$$\frac{d}{dt}(\dot{\psi}a^3)-a\nabla^2\psi=N\exp[-2\psi] \tag{13}$$

where N is a constant, defined by

$$N\equiv-\frac{2\zeta_m}{\omega}\rho_m a^3>0.$$

We can see in a general way that the effects of small inhomogeneities in the density of electromagnetically coupled matter will create a spatial variation in $N(\vec{x})$ and this will create small spatial variations in $\alpha\sim 2N(\vec{x})\ln t$ during the dust era. It is also possible for inhomogeneity in N to be created by variations in the value of ζ for the form of matter that dominates on any particular scale. Our assumption of $\zeta_m<0$ applies only to the dominant dark matter. On small scales the luminous matter might dominate and there will be a variation in the effective value (and even the sign) of ζ but we will not explore these possibilities further here.

We seek a general solution of Eq. (13) of the form

$$\psi=\psi_h+\delta(\vec{x},t) \tag{14}$$

where $\psi_h(t)$ is the solution to the space-independent problem $(\nabla\psi\equiv 0)$, so by definition $\psi_h(t)$ is an exact solution of

$$\frac{d}{dt}(\dot{\psi}_h a^3)=N\exp[-2\psi_h].$$

We note immediately an important general property of this equation, that applies to all Friedmann universes with varying α:

No-oscillation theorem: In the BSBM theory, α cannot display oscillatory behavior in time in a Friedmann universe of any curvature.

The proof is simple: When N is positive (negative) the right-hand side of Eq. (7) is positive (negative); ψ_h cannot have an expansion maximum (minimum) since $\dot{\psi}_h=0$ and $\ddot{\psi}_h<0(>0)$ there. Therefore ψ_h cannot oscillate in time and so neither can $\alpha=\exp[2\psi]$.

We see that in the case of interest, when $N>0$, ψ can have a minimum but thereafter it must always increase irrespective of the behavior of the expansion scale factor. However, if the equation is linearized in ψ_h this is no longer true if attention is not confined to the small ψ regime where $\exp[-2\psi_h]\approx 1-2\psi_h>0$ and spurious oscillations of ψ (and α) in time can appear to arise at late times if ψ grows. It is of particular interest that this proof that ψ_h cannot have a maximum applies to recollapsing universes $(k=+1)$ as well as to ever-expanding universes $(k\leqslant 0)$. It also means that oscillations of α with redshift should not be observed in Friedmann universe. This might prove an interesting prediction for future observations to test.

Substituting Eq. (14) into Eq. (13) we get

$$\frac{d}{dt}(\dot{\delta}a^3)-a\nabla^2\delta=N\exp[-2\psi_h]\{\exp[-2\delta]-1\}.$$

So for small δ

$$\frac{d}{dt}(\dot{\delta}a^3)-a\nabla^2\delta=-2N\delta\exp[-2\psi_h]+O(\delta^2).$$

Now look for separable solutions

$$\delta=T(t)D(\vec{x})$$

and we have

$$\frac{\ddot{T}}{T}a^2+3a\dot{a}\frac{\dot{T}}{T}+\frac{2N}{a}\exp[-2\psi_h]=-\mu^2=\frac{\nabla^2 D}{D} \tag{15}$$

where μ^2 is a separation constant with a sign chosen to ensure nongrowing, oscillatory, inhomogeneity in $D(\vec{x})$ at spatial infinity. In this equation we can always neglect $2Na^{-1}\exp[-2\psi_h]$ with respect to μ^2 as $t\to\infty$ because ψ_h never falls with time (in the dust era ψ_h grows as $\frac{1}{2}\ln[2N\ln(t)]$ as $t\to\infty$, for example, [2]). This is an important feature of the variation of ψ, and α, in BSBM varying-α theories when $\zeta<0$. It ensures that the kinetic term and the $\zeta_m\exp[-2\psi]$ terms can be neglected in the Friedmann equation asymptotically and the expansion scale factor can self-consistently be assumed to be of the same form as when α does not vary (this is *not* true if $\zeta>0$). Note that in the Friedmann case $(\delta=0)$ we can evaluate the corrections to the test-motion approximation by calculating the leading order corrections to the Friedmann equation if we use the solution for ψ found from the solution of the wave equation. These corrections are largest for the dust universe but even there we find the next-order correction to the first-order assumption that $a(t)=t^{2/3}$ is $a(t)=t^{2/3}(\ln t)^{|\zeta|/3}$ with $|\zeta|/3\sim 0.3-0.03$ and so is small.

Hence, in this approximation we have

$$\ddot{T}+\frac{3\dot{a}}{a}\dot{T}+\frac{\mu^2 T}{a^2}=0 \tag{16}$$

and

$$\nabla^2 D=-\mu^2 D$$

so we have the standard separable spherical oscillator solution in spherical polar coordinates:

$$D(r,\theta,\varphi)=\sum_{l=0}^{\infty} c_{\mu,l}Y_l(\theta,\varphi)r^{-1/2}Z_{l+(1/2)}(\mu r)$$

where Z is a cylindrical function and Y the spherical harmonic function. If we specialize to spatially flat cosmologies with perfect fluid equations of state for pressure p and density ρ of the form

$$p=(\gamma-1)\rho,$$

then the expansion scale factor will have a power law form

$$a=t^n \tag{17}$$

with $n=2/3\gamma$. In these cases we have

$$t\ddot{T}+3n\dot{T}+\mu^2 T t^{1-2n}=0. \tag{18}$$

We are interested in inhomogeneous solutions which introduce new behavior as a result of including inhomogeneity. Therefore we impose a boundary condition that $T=0$ for $\mu=0$ since when $D=0$ the time-dependent solution is already included in $\psi_h(t)$. Thus, for $n<1$,

$$T(t)=t^{(1-3n)/2}C_1 Z_\nu\left(\frac{\mu}{1-n}t^{1-n}\right), \tag{19}$$

$$\nu\equiv\frac{|1-3n|}{2(1-n)}, \tag{20}$$

where $Z(\ )$ is a cylindrical function, while for the curvature-dominated expansion with $n=1$:

$$T\propto t^q \tag{21}$$

$$q=-1\pm\sqrt{1-\mu^2} \tag{22}$$

and we choose the $+$ solution to satisfy the boundary condition. The late-time behavior is easily determined as $t\rightarrow\infty$:

$$T(t)\propto t^{-n}\ \times\ \text{oscillations};\quad n\neq 1, \tag{23}$$

$$T(t)\propto t^{-1+\sqrt{1-\mu^2}};\quad n=1 \tag{24}$$

and decays, $T\propto a^{-1}$, as $t\rightarrow\infty$. However, as we have already pointed out, the oscillatory behavior is an artifact of the linearization process and the Bessel-like oscillations are not reached by the solution for ψ.

In the radiation era we can find a solution of Eq. (15) for $T(t)$ without neglecting the term $2Na^{-1}\exp[-2\psi_h]$ since the radiation universe has the simple exact solution

$$\psi_h=\frac{1}{2}\log(8N)+\frac{1}{4}\log(t). \tag{25}$$

Substituting Eq. (25) in Eq. (15) we find

$$T(t)=\frac{1}{t^{1/4}}\{AJ_m(2\mu t^{1/2})+BJ_{-m}(2\mu t^{1/2})\}$$

where

$$m=\frac{i\sqrt{3}}{2}$$

and we see explicitly that there is agreement with the asymptote (23) of the approximated equation when $n=1/2$. The boundary condition for transition to the homogeneous problems requires that we put $B=0$ and again the late-time oscillations are recognized as arising purely from the linearization process. Similar exact solutions can be found for all universes with $1/3<n<2/3$.

The cosmological constant case of $\gamma=0$ is distinct, with

$$a=\exp[H_0 t]$$

which gives

$$0=\ddot{T}+3H_0\dot{T}+\mu^2 T\exp[-2H_0 t]\approx\ddot{T}+3H_0\dot{T}$$

as $t\rightarrow\infty$, so

$$T\rightarrow T_\infty-\frac{1}{3H_0}\exp[-3H_0(t+t_0)]\rightarrow T_\infty .$$

This behavior is in accord with the expectations of a cosmic no hair theorem. It means that if a period of inflation occurs in the very early universe then large scale inhomogeneity will appear increasingly negligible with time within the event horizon of a geodesically moving observer. In the late stages of a universe like our own, which displays evidence of being accelerated by the presence of a positive cosmological constant [27], it ensures that time variations in α will not grow. This is to be expected since the inhomogeneities in density are also prevented from growing by the effects of the cosmological constant.

IV. THE CASE OF $\zeta>0$

When the dark matter is dominated by electric field energy, we have $\zeta>0$, and the behavior of Eq. (7) is very different from that obtained when $\zeta<0$. Most crucially, the test-motion approximation used above to analyze the behavior of Eq. (7) does not apply, even for the purely time-dependent ψ evolution in a Friedmann universe. The solutions obtained for ψ by assuming the scale factor evolution $a=t^n$ of general relativity (with constant α) lead to solutions for ψ (and α) which do not increase with time. For example, we have $\alpha\propto t^{-1}$ in the curvature era and $\alpha\propto\ln(t_0/t)$ in the dust era. These contribute kinetic ($\dot{\psi}^2$) and magnetic contribution ($\zeta\exp[-2\psi]$) terms which dominate the underlying

Friedmann equation (6) at large times and the expansion of the universe is not well approximated by that obtained in general relativistic cosmologies with the same equation of state and constant α except over finite non-asymptotic intervals of time. This leads to problems accommodating observational constraints, notably the results of studies of the structure of the microwave background at last scattering [9,10] and big bang nucleosynthesis [11] in the radiation era because the value of α then is significantly different from today, unlike in the cases of $\zeta<0$. Cosmologies with $\zeta>0$ have been discussed in Ref. [19] in a theory that is similar in structure to the BSBM theory discussed here. We will discuss the $\zeta>0$ version of the theory in more detail elsewhere. It is less well behaved and does not seem to provide the smooth and simple perturbation of the standard cosmology with constant α as seen in the negative ζ case.

V. DISCUSSION

We have shown that the time-dependent solutions to the Friedmann model are stable against the effects of inhomogeneous motions of the ψ field. In the case of inhomogeneous variation the cosmological solutions in universes with scale factor $a(t)=t^n$ to leading order take the form

$$\psi(\vec{x},t)=\psi_h(t)+C_1 t^{(1-3n)/2}\left[J_\nu\left(\frac{\mu}{1-n}t^{1-n}\right)\right]\times\sum_{l=0}^{\infty} c_{\mu,l}Y_l(\theta,\varphi)r^{-1/2}Z_{l+(1/2)}(\mu r)$$

when $n\neq 1$, and

$$\psi(\vec{x},t)=\psi_h(t)+At^{-1+\sqrt{1-\mu^2}}\times\sum_{l=0}^{\infty} c_{\mu,l}Y_l(\theta,\varphi)r^{-1/2}Z_{l+(1/2)}(\mu r)$$

when $n=1$, while for the case of $a=\exp[H_0 t]$:

$$\psi(\vec{x},t)=\psi_h(t)+O(\exp[-3H_0 t]).$$

Thus in all cases we have

$$\psi(\vec{x},t)\to\psi_h(t)$$

as $t\to\infty$ and at late times spatial variations in the fine structure constant decay as

$$\alpha=\exp[2\psi_h]\left\{1+2t^{-n}\sum_{l=0}^{\infty} c_{\mu,l}Y_l(\theta,\varphi)\times r^{-1/2}Z_{l+(1/2)}(\mu r)+\cdots\right\} \tag{26}$$

for $n\neq 1$. Hence, denoting $\alpha_h\equiv\exp[2\psi_h]$, the spatial variation in α decays in time in the $n\neq 1$ universes as

$$\frac{\delta\alpha}{\alpha}\equiv\frac{\alpha-\alpha_h}{\alpha_h}\approx 2t^{-n}\sum_{l=0}^{\infty} c_{\mu,l}Y_l(\theta,\varphi)r^{-1/2}Z_{l+(12)}(\mu r).$$

Analogous expressions can be written down after the necessary changes have been made for $\delta\alpha/\alpha$ in the $n=0,1$ cases.

It is important to compare the evolution of the fine structure constant $\alpha(t)$ in the BSBM theory in the homogeneous case with that for the situation admitting inhomogeneous motions of the fine structure "constant," $\alpha(t,\vec{x})$, here. To leading order, the overall pattern of time evolution studied in Refs. [1,2] is unaffected by the presence of small inhomogeneities. However, small spatial variations of an oscillatory character are expected to exist in the value of the fine structure constant over astronomical scales, reflecting the nonlinear self-interaction of the $\alpha(\psi)$ field which carries the variations in α. The spatial variation amplitudes, $\delta\alpha/\alpha$, are found to decay with time as the universe expands and will not be as significant as the overall variation in time of the mean value of $\alpha(t)\propto\ln(t)$ during the dust-dominated phase of a spatially flat universe. Inhomogeneous test motions of the ψ field will have been decaying in amplitude throughout the period when the universe was dominated by dust if $\zeta<0$. Therefore we would not expect any significant inhomogeneities to survive at the astronomically interesting epoch $z\sim 1-4$ where the value of the fine structure constant can be probed spectroscopically with high precision. However, our discussion has not considered three situations where more significant spatial variations might arise. The first is the situation within gravitationally bound matter inhomogeneities of large scale which separate out from the expansion of the Universe and collapse to form superclusters and clusters of galaxies. These behave in a manner similar to that expected of separate closed universes until deviations from spherical symmetry become significant. Our analysis is not applicable here because the dynamics of the bound inhomogeneities will differ significantly after they separate off from the background expansion. The second situation of interest is that in which perturbations of the Friedmann metric are included in the problem and allowed to couple to spatial variations in ψ, or α. This coupling will lead to small temperature fluctuations in the microwave background radiation. Finally, the variation in the value and sign of ζ with scale for the dominant form of matter could introduce a distinctive inhomogeneity. These problems will be discussed elsewhere.

ACKNOWLEDGMENTS

We would like to thank David Mota for discussions.

[1] H. Sandvik, J.D. Barrow, and J. Magueijo, Phys. Rev. Lett. **88**, 031302 (2002).
[2] J.D. Barrow, H. Sandvik, and J. Magueijo, Phys. Rev. D **65**, 063504 (2002); **65**, 123501 (2002).
[3] J.D. Bekenstein, Phys. Rev. D **25**, 1527 (1982).
[4] J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Rev. Lett. **82**, 884 (1999).
[5] J.K. Webb *et al.*, Phys. Rev. Lett. **87**, 091301 (2001).

[6] M.T. Murphy *et al.*, Mon. Not. R. Astron. Soc. **327**, 1208 (2001).
[7] A. Shlyakhter, Nature (London) **264**, 340 (1976); ATOMKI Report A/1 (1983).
[8] Y. Fujii *et al.*, Nucl. Phys. **B573**, 377 (2000).
[9] P.P. Avelino *et al.*, Phys. Rev. D **62**, 123508 (2000); *ibid.* **64**, 103505 (2001).
[10] R. Battye, R. Crittenden, and J. Weller, Phys. Rev. D **63**, 043505 (2001).
[11] J.D. Barrow, Phys. Rev. D **35**, 1805 (1987).
[12] A.V. Ivanchik, E. Rodriguez, P. Petitjean, and D.A. Varshalovich, Astron. Lett. **28**, 423 (2002).
[13] J.D. Barrow and C. O'Toole, Mon. Not. R. Astron. Soc. **22**, 585 (2001).
[14] A. Albrecht and J. Magueijo, Phys. Rev. D **59**, 043516 (1999).
[15] J. Magueijo, Phys. Rev. D **62**, 103521 (2000).
[16] J. Moffat, astro-ph/0109350.
[17] S. Landau, P. Sisterna, and H. Vucetich, Phys. Rev. D **63**, 081303(R) (2001).
[18] G. Dvali and M. Zaldarriaga, Phys. Rev. Lett. **88**, 091303 (2002).
[19] K. Olive and M. Pospelov, Phys. Rev. D **65**, 085044 (2002).
[20] M. Livio and M. Stiavelli, Astrophys. J. Lett. **507**, L13 (1998); L. Cowie and A. Songaila, Astrophys. J. **453**, 596 (1995); D.A. Varshalovich and A.Y. Potekhin, Space Sci. Rev. **74**, 259 (1995).
[21] D. Youm, hep-th/0108237.
[22] J. Moffat, Int. J. Mod. Phys. D **2**, 351 (1993).
[23] J.D. Barrow, Phys. Rev. D **59**, 043515 (1999).
[24] J.D. Barrow and J. Magueijo, Phys. Lett. B **447**, 246 (1999).
[25] J.D. Barrow and J. Magueijo, Phys. Lett. B **443**, 104 (1998).
[26] J.D. Barrow and J. Magueijo, Astrophys. J. Lett. **532**, L87 (2000).
[27] S. Perlmutter *et al.*, Astrophys. J. **517**, 565 (1999); *ibid.* **483**, 565 (1997); Nature (London) **391**, 51 (1998); P.M. Garnavich *et al.*, Astrophys. J. Lett. **493**, L53 (1998); B.P. Schmidt, Astrophys. J. **507**, 46 (1998); A.G. Riess *et al.*, Astron. J. **116**, 1009 (1998).