

## Variable-speed-of-light cosmology and second law of thermodynamics

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We examine whether cosmologies with a varying speed of light (VSL) are compatible with the second law of thermodynamics. We find that the VSL cosmology with a varying fundamental constant is severely constrained by the second law of thermodynamics, whereas the bimetric cosmological models are less constrained.

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### I. INTRODUCTION

Variable-speed-of-light (VSL) cosmological models were proposed [1,2] as an alternative to inflation [3–5] for solving initial value problems in the standard big bang model. It is assumed in the VSL models that the speed of light initially took a larger value and then decreased to the present day value during an early period of cosmic evolution. VSL models have attracted some attention, not only because various cosmological problems that are solved by the inflationary models but also because the cosmological constant problem can be solved [1,2,6–15] by VSL models. Furthermore, the recent study of quasar absorption line spectra in comparison with laboratory spectra shows that the fine structure constant  $\alpha = e^2/(4\pi\hbar c)$  varies over cosmological time scales [16–18], indicating that the speed of light may indeed vary with time. Also, it has been shown [19–26] that brane world models, which have been in vogue recently, manifest Lorentz violation, which is a necessary requirement for the VSL models. (Note that the recent work [27] studies the experimental limits which are permitted for the graviton's speed in brane world scenarios.)

It is the purpose of this paper to examine the compatibility of the VSL models with the second law of thermodynamics. (Previous related work can be found in Ref. [28].) Recently, there has been active interest in the holographic principle in cosmology, after the initial work by Fischler and Susskind (FS) [29]. The cosmological holographic bound originally formulated by FS had the problem of being violated by the closed Friedmann-Robertson-Walker (FRW) universe. Later works attempted to circumvent this problem through various modifications. In particular, it was proposed in Refs. [30,31] that the FS holographic bound has to be replaced by the generalized second law of thermodynamics. The generalized second law states that the total entropy  $S$  of the universe should not decrease with time during cosmological evolution:  $dS \geq 0$ . In order to be compatible with the holographic principle, VSL cosmological models therefore have to obey the generalized second law of thermodynamics. In Sec. II, we consider the original VSL model, where the speed of light  $c$  in the action is just assumed to vary with time. In Sec. III, we consider the bimetric cosmology of Clayton and Moffat.

### II. VSL COSMOLOGY WITH VARYING FUNDAMENTAL CONSTANT

First, we consider the original VSL cosmology [1,2], in which the fundamental constant  $c$  of nature is assumed to vary just with time during the early period of cosmic evolution, and thereby the Lorentz symmetry is explicitly broken. In such VSL theories, it is postulated that there exists a preferred Lorentz frame in which the laws of physics simplify with the action taking a standard form with the constant  $c$  replaced by a field  $c(x^\mu)$ , *the principle of minimal coupling*; namely, the action in the preferred frame takes the form

$$S = \int d^4x \left[ \sqrt{-g} \left\{ \frac{c^4}{16\pi G} (\mathcal{R} - 2\Lambda) + \mathcal{L} \right\} + \mathcal{L}_c \right], \quad (1)$$

where  $\mathcal{L}_c$  controls the dynamics of  $c$  and  $\mathcal{L}$  is the action for the fields in the universe. It is required that  $\mathcal{L}_c$  should be explicitly independent of the other fields, including the metric, so that the principle of minimal coupling continues to hold for the equations of motion.

The general metric ansatz for a four-dimensional homogeneous and isotropic universe is given by the following Robertson-Walker metric:

$$g_{\mu\nu} dx^\mu dx^\nu = -c^2 dt^2 + a^2 \gamma_{ij} dx^i dx^j, \quad (2)$$

with a time-varying  $c$ . Here,  $a(t)$  is the cosmic scale factor and  $\gamma_{ij}(x^k)$  is given by

$$\begin{aligned} \gamma_{ij} dx^i dx^j &= \left( 1 + \frac{k}{4} \delta_{mn} x^m x^n \right)^{-2} \delta_{ij} dx^i dx^j \\ &= \frac{dr^2}{1 - kr^2} + r^2 d\Omega_2^2, \end{aligned} \quad (3)$$

where  $k = -1, 0, 1$ , respectively, for the open, flat, and closed universes.

With the assumption of the principle of minimal coupling, the Einstein equations with the metric ansatz (2) lead to the Friedmann equations:

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{c^2}{3} \Lambda - \frac{kc^2}{a^2}, \quad (4)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3\frac{p}{c^2} \right) + \frac{c^2}{3} \Lambda, \quad (5)$$

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where the overdot denotes the derivative with respect to  $t$ . From the Friedmann equations, we obtain the following generalized conservation equation:

$$\dot{\rho} + 3\left(\rho + \frac{p}{c^2}\right)\frac{\dot{a}}{a} = \frac{3kc\dot{c}}{4\pi Ga^2} - \frac{c\dot{c}}{4\pi G}\Lambda. \quad (6)$$

We now study the thermodynamics of the VSL cosmology. We assume that the universe satisfies the first law of thermodynamics. When applied to the comoving volume element of unit coordinate volume and physical volume  $v = a^3$ , the first law of thermodynamics takes the form

$$Tds = d(\rho c^2 v) + p dv, \quad (7)$$

where  $s = s(v, T)$  is the entropy density of the universe at temperature  $T$  within the volume  $v$ , and  $\rho = \rho(T)$  and  $p = p(T)$  are the mass density and the pressure of matter in the universe. In this paper, we assume that  $c$  is a function of  $a$ , just as in Ref. [8]. Since  $v = a^3$ , we have  $c'(a) = 3a^2 dc/dv = 3v^{2/3} dc/dv$ , where the prime denotes the derivative with respect to  $a$ . Then Eq. (7) can be rewritten as

$$\begin{aligned} Tds &= c^2 v d\rho + \left(\rho c^2 + p + 2\rho c v \frac{dc}{dv}\right) dv \\ &= c^2 v d\rho + \left(\rho c^2 + p + \frac{2}{3}\rho c c' v^{1/3}\right) dv. \end{aligned} \quad (8)$$

So the partial derivatives of  $s(v, T)$  are given by

$$\frac{\partial s(v, T)}{\partial v} = \frac{1}{T} \left(\rho c^2 + p + \frac{2}{3}\rho c c' v^{1/3}\right), \quad (9)$$

$$\frac{\partial s(v, T)}{\partial T} = \frac{c^2 v}{T} \frac{d\rho}{dT}. \quad (10)$$

From the integrability condition  $\partial^2 s / (\partial v \partial T) = \partial^2 s / (\partial T \partial v)$ , we obtain

$$\begin{aligned} \frac{d\rho}{dT} &= \frac{1}{T} \left(\rho c^2 + p + \frac{2}{3}\rho c c' v^{1/3}\right) \\ &= \frac{1}{T} \frac{d}{dv} [(\rho c^2 + p)v]. \end{aligned} \quad (11)$$

Making use of this equation, we can put Eq. (7) into the following form:

$$ds = d\left[\frac{v}{T}(\rho c^2 + p)\right] - \frac{2v^2}{T^2} \rho c \frac{dc}{dv} dT, \quad (12)$$

from which we see that the usual Euler relation  $s = (v/T) \times (\rho c^2 + p)$  does not hold for VSL theories with varying fundamental constant.

To obtain the time derivative of  $s$ , we express the conservation equation (6) in the following form, making use of Eq. (11):

$$\begin{aligned} \frac{d}{dt} \left[ \frac{v}{T} (\rho c^2 + p) \right] &= \left( 2\rho \frac{\dot{c}}{c} - \frac{c\dot{c}}{4\pi G} \Lambda + \frac{3kc\dot{c}}{4\pi Ga^2} \right) \frac{c^2 v}{T} \\ &\quad + 2\rho c \frac{dc}{dv} v^2 \frac{\dot{T}}{T^2}. \end{aligned} \quad (13)$$

Equation (12) along with Eq. (13) yields<sup>1</sup>

$$\dot{s} = \left( 2\rho - \frac{c^2}{4\pi G} \Lambda + \frac{3kc^2}{4\pi Ga^2} \right) \frac{c\dot{c}a^3}{T}. \quad (14)$$

This equation could also have been obtained directly from Eqs. (6) and (7).

Since  $\dot{c} < 0$  for the VSL cosmology with varying fundamental constant, the terms in the parentheses of Eq. (14) have to be nonpositive in order to be compatible with the second law of thermodynamics  $dS \geq 0$ :

$$2\rho - \frac{c^2}{4\pi G} \Lambda + \frac{3kc^2}{4\pi Ga^2} \leq 0. \quad (15)$$

When the cosmological constant is nonpositive ( $\Lambda \leq 0$ ), this condition can never be satisfied by the flat ( $k=0$ ) and the closed ( $k=1$ ) universes. Although it can be satisfied by the open universe ( $k=-1$ ), the condition is very restrictive about the possible type of matter in the universe and the time variation of  $c$ . On the other hand, when the cosmological constant is positive ( $\Lambda > 0$ ) and is large enough, the restriction becomes less severe. We now examine the explicit conditions under which the constraint (15) can be satisfied. Assuming that  $c$  varies gradually as  $c(t) = c_0 a^n$  and the equation of state is of the form  $p = (\gamma - 1)\rho c^2$ , by solving Eq. (6) one obtains

$$\rho = \frac{B}{a^{3\gamma}} + \frac{3kc_0^2 n a^{2(n-1)}}{4kG(3\gamma + 2n - 2)} - \frac{\Lambda n c_0^2 a^{2n}}{4\pi G(3\gamma + 2n)}, \quad (16)$$

where  $B$  is a positive constant. So the constraint (15) reduces to

$$\frac{2B}{a^{3\gamma+2n}} + \frac{3(3\gamma+4n-2)kc_0^2}{4\pi G(3\gamma+2n-2)a^2} \leq \frac{(3\gamma+4n)c_0^2\Lambda}{4\pi G(3\gamma+2n)}. \quad (17)$$

First of all, we see from this that for the flat universe ( $k=0$ ) the constraint is always violated for some values of  $a$ . [When  $3\gamma+2n=0$ , the last term on the right-hand side (RHS) of Eq. (16) behaves with  $a$  as  $\sim a^{-3\gamma} \ln a$  and therefore the RHS of Eq. (17) behaves with  $a$  as  $\sim \ln a$ .] In order for Eq. (17) to be satisfied for any values of  $a$ , the LHS has to have a maximum value and its value must not be greater than the RHS. From this condition, we obtain the following constraint on the constants  $n$  and  $\gamma$ :

<sup>1</sup>The previous related work [28] does not take into account the effects on  $\dot{s}$  of the modification of the usual Euler relation due to the time-varying  $c$  and the nonzero cosmological constant  $\Lambda$ .

$$k(3\gamma+4n-2)>0, \quad (3\gamma+2n)(3\gamma+2n-2)<0, \quad (18)$$

along with the following lower limit on the cosmological constant:

$$\Lambda \geq \frac{3(3\gamma+4n-2)k}{3\gamma+4n} \times \left[ -\frac{3(3\gamma+4n-2)kc_0^2}{4\pi(3\gamma+2n)(3\gamma+2n-2)GB} \right]^{2/(3\gamma+2n-2)}. \quad (19)$$

One can always choose the integration constant  $B$  in such a way that the bound (19) is compatible with the observed value of  $\Lambda$ . However, the constraint (18) severely restricts the allowed value of  $\gamma$ . For the closed universe ( $k=1$ ), Eq. (18) implies  $\gamma < 2/3$  and therefore the radiation-dominated universe ( $\gamma=4/3$ ) and the matter-dominated universe ( $\gamma=1$ ) are not allowed. As for the open universe ( $k=-1$ ), Eq. (18) leads to the less severe constraint  $\gamma > -2/3$ , which allows the  $\gamma=4/3, 1$  cases.

### III. SCALAR-TENSOR BIMETRIC COSMOLOGY

In this section, we consider the bimetric VSL model, proposed by Clayton and Moffat [10]. The bimetric models achieve a time-variable speed of light in a diffeomorphism invariant manner and without explicitly breaking the Lorentz symmetry by introducing bimetric structure into spacetime. (It was recently found in Ref. [32] that the fine-structure constant  $\alpha = e^2/(4\pi\hbar c)$  in bimetric models is constant in spacetime although the speed of light varies with time, due to the compensating time variation of the electric charge.) It is usually assumed in bimetric models that graviton and the biscalar (or the bivector) propagate on the geometry described by the ‘‘gravity metric,’’ whereas all the matter fields (including photons) propagate on the geometry described by the ‘‘matter metric.’’ In the case of a scalar-tensor bimetric model, the gravity metric  $g_{\mu\nu}$  and the matter metric  $\hat{g}_{\mu\nu}$  are related by the biscalar field  $\Phi$  as

$$\hat{g}_{\mu\nu} = g_{\mu\nu} - B\partial_\mu\Phi\partial_\nu\Phi, \quad (20)$$

where the dimensionless constant  $B$  is assumed to be positive. Since these two metrics are nonconformally related, a photon and a graviton propagate at different speeds. The action for the scalar-tensor bimetric model has the form

$$S = \int d^4x \sqrt{-g} \left[ \frac{c^4}{16\pi G} (\mathcal{R} - 2\Lambda) + \mathcal{L}_\Phi \right] + \int d^4x \sqrt{-\hat{g}} \mathcal{L}_{\text{mat}}, \quad (21)$$

where  $\mathcal{L}_{\text{mat}}$  is the Lagrangian density for matter fields and the Lagrangian density  $\mathcal{L}_\Phi$  for the biscalar is given by

$$\mathcal{L}_\Phi = -\frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - V(\Phi). \quad (22)$$

The gravity metric for the universe has the form

$$g_{\mu\nu}dx^\mu dx^\nu = -c^2dt^2 + a^2\gamma_{ij}dx^i dx^j, \quad (23)$$

with constant speed of the graviton  $c_{\text{grav}} = c$  and  $\gamma_{ij}$  given by Eq. (3). Due to the requirement of homogeneity and isotropy of the universe, the biscalar field  $\Phi$  is independent of the spatial coordinates  $x^i$ . According to Eq. (20), the matter metric is therefore given by

$$\hat{g}_{\mu\nu}dx^\mu dx^\nu = -(c^2 + B\dot{\Phi}^2)dt^2 + a^2\gamma_{ij}dx^i dx^j, \quad (24)$$

where the overdot stands for the derivative with respect to  $t$ . So, the speed of the photon  $c_{\text{ph}} = c\sqrt{1 + B\dot{\Phi}^2/c^2} \equiv c\sqrt{I}$  varies with  $t$ , taking a larger value than  $c_{\text{grav}} = c$  while  $\dot{\Phi} \neq 0$ .

In obtaining the energy-momentum tensor for the purpose of deriving the Einstein’s equations, one has to keep in mind that the matter fields and the biscalar are coupled to different metrics. Since  $\hat{g}_{\mu\nu}$  is the physical metric for the matter fields, the energy-momentum tensor for the matter fields is defined in terms of  $\hat{g}_{\mu\nu}$ :

$$\hat{T}^{\mu\nu} \equiv \frac{2}{\sqrt{-\hat{g}}} \frac{\delta(\sqrt{-\hat{g}}\mathcal{L}_{\text{mat}})}{\delta\hat{g}_{\mu\nu}} = (\hat{\rho}c_{\text{ph}}^2 + \hat{p})\hat{U}^\mu\hat{U}^\nu + \hat{p}\hat{g}^{\mu\nu}, \quad (25)$$

where  $\hat{\rho}$  and  $\hat{p}$  are the mass density and the pressure of the matter fields and  $\hat{U}^\mu$  is the four-velocity of a matter perfect fluid normalized as  $\hat{g}_{\mu\nu}\hat{U}^\mu\hat{U}^\nu = -1$ . Since the nonzero component of the four-velocity vector in the comoving coordinates is  $\hat{U}^t = 1/c_{\text{ph}}$ , the nonzero components of the energy-momentum tensor for the matter fields are

$$\hat{T}^{tt} = \hat{\rho}, \quad \hat{T}^{ij} = \frac{\hat{p}}{a^2}\gamma^{ij}. \quad (26)$$

On the other hand, since the biscalar field is coupled to the gravity metric  $g_{\mu\nu}$ , its energy-momentum tensor is defined in terms of  $g_{\mu\nu}$ :

$$T_\Phi^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_\Phi)}{\delta g_{\mu\nu}} = g^{\mu\alpha}g^{\nu\beta}\partial_\alpha\Phi\partial_\beta\Phi - \frac{1}{2}g^{\mu\nu}\partial_\alpha\Phi\partial^\alpha\Phi - V(\Phi)g^{\mu\nu} = (\rho_\Phi c^2 + p_\Phi)U^\mu U^\nu + p_\Phi g^{\mu\nu}, \quad (27)$$

where the four-velocity  $U^\mu$  for the biscalar is normalized as  $g_{\mu\nu}U^\mu U^\nu = 1$ , so its nonzero component is  $U^t = 1/c$ . The mass density and the pressure of the biscalar field are therefore

$$\rho_\Phi = \left( \frac{1}{2} \frac{\dot{\Phi}^2}{c^2} + V \right) \frac{1}{c^2}, \quad p_\Phi = \frac{1}{2} \frac{\dot{\Phi}^2}{c^2} - V. \quad (28)$$

Taking the variation of the action  $S$  with respect to the metric, we obtain the Einstein equations

$$\mathcal{G}^{\mu\nu} + \Lambda g^{\mu\nu} = \frac{8\pi G}{c^4} \mathcal{T}^{\mu\nu}, \quad (29)$$

where  $\mathcal{G}_{\mu\nu}$  is the Einstein tensor for  $g_{\mu\nu}$  and the energy-momentum tensor  $\mathcal{T}_{\mu\nu}$  has the form

$$\mathcal{T}^{\mu\nu} = T_{\Phi}^{\mu\nu} + \hat{T}^{\mu\nu} \frac{\sqrt{-\hat{g}}}{\sqrt{-g}}. \quad (30)$$

The nonzero components of  $\mathcal{T}^{\mu\nu}$  are

$$T^t = \rho_{\Phi} + \hat{\rho} \frac{c_{\text{ph}}}{c}, \quad T^{ij} = \left( p_{\Phi} + \hat{p} \frac{c_{\text{ph}}}{c} \right) \frac{1}{a^2} \gamma^{ij}. \quad (31)$$

The Einstein equations lead to the Friedmann equations

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{c^2}{3} \Lambda - \frac{kc^2}{a^2}, \quad (32)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3 \frac{p}{c^2} \right) + \frac{c^2}{3} \Lambda, \quad (33)$$

where

$$\rho \equiv \rho_{\Phi} + \hat{\rho} \frac{c_{\text{ph}}}{c}, \quad p \equiv p_{\Phi} + \hat{p} \frac{c_{\text{ph}}}{c}. \quad (34)$$

From these Friedmann equations, we obtain the following conservation equation:

$$\dot{\rho} + 3 \left( \rho + \frac{p}{c^2} \right) \frac{\dot{a}}{a} = 0. \quad (35)$$

The factors of  $c_{\text{ph}}/c_{\text{grav}} = c_{\text{ph}}/c$  in Eq. (34) can be understood from the fact that the definition of the energy-momentum tensor depends on the choice of metric. Generally, the following two energy-momentum tensors, associated with the same Lagrangian density  $L$  but defined with respect to the two different metrics  $g_{\mu\nu}$  and  $\hat{g}_{\mu\nu}$ :

$$T_{\text{grav}}^{\mu\nu} \equiv \frac{2}{\sqrt{-g}} \frac{\delta L}{\delta g_{\mu\nu}}, \quad T_{\text{ph}}^{\mu\nu} \equiv \frac{2}{\sqrt{-\hat{g}}} \frac{\delta L}{\delta \hat{g}_{\mu\nu}}, \quad (36)$$

are related to each other as

$$T_{\text{grav}}^{\mu\nu} = \frac{\sqrt{-\hat{g}}}{\sqrt{-g}} T_{\text{ph}}^{\mu\nu} = \frac{c_{\text{ph}}}{c_{\text{grav}}} T_{\text{ph}}^{\mu\nu}. \quad (37)$$

Here, the subscripts ‘‘grav’’ and ‘‘ph’’ signify that the quantity under consideration is defined with respect to the gravity and matter metrics, respectively. The mass densities and the pressures in the two different definitions are defined by

$$T_{\text{grav}}^{\mu\nu} = (\rho_{\text{grav}} c_{\text{grav}}^2 + p_{\text{grav}}) U^{\mu} U^{\nu} + p_{\text{grav}} g^{\mu\nu},$$

$$T_{\text{ph}}^{\mu\nu} = (\rho_{\text{ph}} c_{\text{ph}}^2 + p_{\text{ph}}) \hat{U}^{\mu} \hat{U}^{\nu} + p_{\text{ph}} \hat{g}^{\mu\nu}, \quad (38)$$

where the four-velocities are normalized as  $g_{\mu\nu} U^{\mu} U^{\nu} = -1$  and  $\hat{g}_{\mu\nu} \hat{U}^{\mu} \hat{U}^{\nu} = -1$ . From Eqs. (37) and (38), we see that the mass densities and the pressures in the two different definitions are related as

$$\rho_{\text{grav}} = \frac{c_{\text{ph}}}{c_{\text{grav}}} \rho_{\text{ph}}, \quad p_{\text{grav}} = \frac{c_{\text{ph}}}{c_{\text{grav}}} p_{\text{ph}}. \quad (39)$$

The factor of  $c_{\text{grav}}/c_{\text{ph}}$ , multiplying the matter field mass density and pressure in Eq. (34), arose due to the fact that the matter field mass density and pressure, which are defined with respect to the matter metric, have to be transformed to ‘‘gravity metric’’ quantities, since the Friedmann equations are defined with respect to the gravity metric. [In the Einstein equations (29), the total energy-momentum tensor  $\mathcal{T}^{\mu\nu}$  is defined with respect to the gravity metric, i.e.,  $\mathcal{T}^{\mu\nu} \equiv (2/\sqrt{-g}) \delta L / \delta g_{\mu\nu}$  where the Lagrangian density  $L$  is the sum of the matter field Lagrangian density  $L_{\text{mat}} = \sqrt{-\hat{g}} \mathcal{L}_{\text{mat}}$  and the biscalar field Lagrangian density  $L_{\Phi} = \sqrt{-g} \mathcal{L}_{\Phi}$ .]

We now study the thermodynamics of the bimetric VSL cosmology. Since we are considering the frame associated with a gravity metric of the form (23), the physical quantities in the thermodynamic laws should be ‘‘gravity metric’’ quantities. The first law of thermodynamics, applied to the comoving volume element of unit coordinate volume and physical volume  $v = a^3$ , therefore takes the form

$$T ds = d(\rho c^2 v) + p dv, \quad (40)$$

where  $\rho$  and  $p$  are given by Eq. (34). Note that the square of the speed of graviton  $c_{\text{grav}} = c$  multiplies  $\rho$  to yield the energy density, because we are considering the frame associated with the gravity metric. From Eqs. (35) and (40), we obtain  $ds/dt = 0$ , namely, the total entropy  $S = \int d^3x \sqrt{-g}$  of the universe remains constant during the cosmic evolution. Therefore, unlike the VSL cosmology with varying fundamental constant considered in the previous section, the second law of thermodynamics  $dS \geq 0$  is always obeyed regardless of the value of  $k$ .

We comment on the thermodynamics associated with the matter fields only. Since it is assumed that the matter field action is constructed out of  $\hat{g}_{\mu\nu}$ , the equations of motion of the matter fields imply the conservation law for the matter field energy-momentum tensor [12]

$$\hat{\nabla}_{\mu} \hat{T}^{\mu\nu} = 0, \quad (41)$$

where  $\hat{\nabla}_{\mu}$  is the covariant derivative defined with respect to  $\hat{g}_{\mu\nu}$ . The  $t$  component  $\hat{\nabla}_{\mu} \hat{T}_t^{\mu} = 0$  of the conservation equation, where  $\hat{T}_t^{\mu} \equiv \hat{T}^{\mu\rho} \hat{g}_{\rho\nu}$ , takes the following form:

$$\hat{\rho} + 3 \left( \hat{\rho} + \frac{\hat{p}}{c_{\text{ph}}^2} \right) \frac{\dot{a}}{a} = -2 \hat{\rho} \frac{\dot{c}_{\text{ph}}}{c_{\text{ph}}}. \quad (42)$$

From this equation we see that matter fields are created while the speed of the photon decreases with time to the present day value  $c$ . In the frame associated with the matter metric, the first law of thermodynamics takes the form

$$Tds_{\text{mat}} = d(\hat{\rho}c_{\text{ph}}^2v) + \hat{p}dv, \quad (43)$$

where  $s_{\text{mat}}$  is the entropy density associated with the matter fields. From Eqs. (42) and (43), we see that  $\dot{s}_{\text{mat}} = 0$ , namely, that the entropy for the matter fields remains constant despite the fact that the matter fields are created when  $\dot{c}_{\text{ph}} < 0$ . This apparent paradox can be understood from the fact that in the matter metric frame the past light cone contracts and thereby less information is collected by the observer. Next, we consider the frame associated with the gravity metric. In this frame, the mass density and the pressure of the matter fields are transformed to  $\hat{\rho}c_{\text{ph}}/c$  and  $\hat{p}c_{\text{ph}}/c$ , and the square of the speed of the graviton  $c$  should be multiplied by the mass density to obtain the energy density. So the first law of thermodynamics takes the form

$$Tds_{\text{mat}} = d(\hat{\rho}cc_{\text{ph}}v) + \frac{c_{\text{ph}}}{c}p dv. \quad (44)$$

From Eqs. (42) and (44), we have

$$\dot{s}_{\text{mat}} = \left( 3 \frac{c_{\text{ph}}^2 - c^2}{c^2 c_{\text{ph}}^2} \hat{p} \frac{\dot{a}}{a} - \hat{\rho} \frac{\dot{c}_{\text{ph}}}{c_{\text{ph}}} \right) \frac{cc_{\text{ph}}a^3}{T}. \quad (45)$$

So, in the gravity frame, the entropy for the matter fields varies with time while  $\dot{s}_{\text{mat}} \neq 0$ . This time variation of the entropy of the matter fields is due to the exchange of entropy with the biscalar sector, since we have seen that the total entropy density  $s$  remains constant in the gravity metric frame.

In the above, we considered the equations in the comoving frame for the gravity metric  $g_{\mu\nu}$ . Since all the matter fields in the universe are coupled to the matter metric  $\hat{g}_{\mu\nu}$ , it would be more natural to consider the comoving frame for the matter metric in studying the expansion of the universe. By defining the cosmic time  $\tau$  for the matter metric in the following way:

$$d\tau^2 \equiv (1 + B\Phi^2/c^2) dt^2, \quad (46)$$

we can bring the matter metric into the following standard comoving frame form for the Robertson-Walker metric:

$$\hat{g}_{\mu\nu} dx^\mu dx^\nu = -c^2 d\tau^2 + a^2(\tau) \gamma_{ij} dx^i dx^j. \quad (47)$$

In this new frame, the gravity metric (23) takes the form

$$g_{\mu\nu} dx^\mu dx^\nu = -(c^2 - B\Phi^2) d\tau^2 + a^2(\tau) \gamma_{ij} dx^i dx^j, \quad (48)$$

where the overdot from now on stands for the derivative with respect to  $\tau$ . So, in this new frame, a photon travels with a constant speed  $c_{\text{ph}} = c$  and a graviton travels with a time-variable speed  $c_{\text{grav}} = \sqrt{c^2 - B\Phi^2} = c/\sqrt{I}$ , taking smaller

value than  $c$  while  $\dot{\Phi} \neq 0$ . Note that  $I = 1/(1 - B\Phi^2/c^2)$  when the overdot stands for the derivative with respect to  $\tau$ .

In obtaining the Friedmann equations in the new frame, we do not just apply the change of time coordinate (46) in the Friedmann equations (32) and (33) in the old frame, unlike the previous work on bimetric VSL cosmology. The reason is that the definitions for the mass density and the pressure depend on the choice of time coordinate. We consider the Einstein equations (29) with  $\mathcal{G}_{\mu\nu}$  now being the Einstein tensor for the gravity metric given by Eq. (48). The energy-momentum tensor  $\mathcal{T}_{\mu\nu}$  is still given by Eq. (30) but now with

$$\hat{T}^{\mu\nu} = (\hat{\rho}c^2 + \hat{p}) \hat{U}^\mu \hat{U}^\nu + \hat{p} \hat{g}^{\mu\nu}, \quad (49)$$

$$T_{\Phi}^{\mu\nu} = (\rho_{\Phi} c_{\text{grav}}^2 + p_{\Phi}) U^\mu U^\nu + p_{\Phi} g^{\mu\nu}, \quad (50)$$

where the nonzero components of the four-velocities are  $\hat{U}^t = 1/c$  and  $U^t = 1/c_{\text{grav}}$ . Note that the mass densities and the pressures in Eqs. (49) and (50) are different from those in Eqs. (25) and (27), although they are denoted with the same notation. In the comoving frame for the matter metric, the Friedmann equations therefore take the forms

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho + \frac{c_{\text{grav}}^2}{3} \Lambda - \frac{kc_{\text{grav}}^2}{a^2}, \quad (51)$$

$$\frac{\ddot{a}}{a} - \frac{\dot{c}_{\text{grav}}}{c_{\text{grav}}} \frac{\dot{a}}{a} = -\frac{4\pi G}{3} \left( \rho + 3 \frac{p}{c_{\text{grav}}^2} \right) + \frac{c_{\text{grav}}^2}{3} \Lambda, \quad (52)$$

where

$$\rho \equiv \rho_{\Phi} + \hat{\rho} \frac{c}{c_{\text{grav}}}, \quad p \equiv p_{\Phi} + \hat{p} \frac{c}{c_{\text{grav}}}. \quad (53)$$

Note that although we are now considering the comoving frame for the matter metric, the total energy momentum tensor  $\mathcal{T}^{\mu\nu}$  is still defined with respect to the gravity metric. It is just that the time coordinate  $\tau$  in the gravity metric is the comoving frame time coordinate for the matter metric. So the mass density and pressure of the matter fields still have the factor of  $c/c_{\text{grav}}$  in Eq. (53). From these Friedmann equations, we obtain the following conservation equation:

$$\begin{aligned} \dot{\rho} + 3 \left( \rho + \frac{p}{c_{\text{grav}}^2} \right) \frac{\dot{a}}{a} = & \frac{3}{4\pi G} \frac{\dot{c}_{\text{grav}}}{c_{\text{grav}}} \left( \frac{\dot{a}}{a} \right)^2 - \frac{c_{\text{grav}} \dot{c}_{\text{grav}}}{4\pi G} \Lambda \\ & + \frac{3}{4\pi G} \frac{kc_{\text{grav}} \dot{c}_{\text{grav}}}{a^2}. \end{aligned} \quad (54)$$

We now study the compatibility of the bimetric VSL cosmology with the second law of thermodynamics. Although we are now considering the comoving frame for the matter metric, we first study thermodynamics in the frame of the gravity metric, since the conservation equation (54) is expressed in the gravity metric frame. The first law of thermo-

dynamics, applied to the comoving volume element of unit coordinate volume and physical volume  $v = a^3$ , takes the form

$$Tds = d(\rho c_{\text{grav}}^2 v) + pdv. \quad (55)$$

From Eqs. (54) and (55), we obtain

$$T\dot{s} = 4\rho a^3 c_{\text{grav}} \dot{c}_{\text{grav}}, \quad (56)$$

where we made use of Eq. (51) to simplify the RHS. So the total entropy  $S = \int dx^3 \sqrt{-\gamma}$  increases with time while  $c_{\text{grav}} = c/\sqrt{I}$  increases to the present day value. The second law of thermodynamics is therefore always satisfied for any value of  $k$ . Next, we consider the second law of thermodynamics in the frame of the matter metric. The mass density and the pressure in the matter metric frame are given by  $\rho c_{\text{grav}}/c$  and  $p c_{\text{grav}}/c$ , and the energy density is obtained by multiplying the mass density by  $c^2$ . So, the first law of thermodynamics takes the form

$$Tds = d(\rho c c_{\text{grav}} v) + \frac{c_{\text{grav}}}{c} pdv. \quad (57)$$

This along with Eq. (54) leads to

$$\dot{s} = 3 \left( \frac{c_{\text{grav}}^2 - c^2}{c^2 c_{\text{grav}}^2} p \frac{\dot{a}}{a} + \rho \frac{\dot{c}_{\text{grav}}}{c_{\text{grav}}} \right) \frac{c c_{\text{grav}} a^3}{T}. \quad (58)$$

So the terms in the parentheses of this equation have to be non-negative in order for the condition for the second law of thermodynamics to be satisfied. Making use of the explicit expressions for  $c_{\text{grav}}$ ,  $\rho$ , and  $p$ , one can put the condition in the form

$$\begin{aligned} \Phi^2 \left[ \frac{1}{2} \frac{I}{c^2} \Phi^2 - V + \hat{p} \sqrt{I} \right] \frac{\dot{a}}{a} \\ + \Phi \Phi \left[ \frac{1}{2} \frac{I}{c^2} \Phi^2 + V + \hat{\rho} c^2 \sqrt{I} \right] \leq 0. \end{aligned} \quad (59)$$

Making use of the equation of motion for the biscalar field  $\Phi$ , it can be shown that regardless of the value of  $k$  this condition can be satisfied by a perfect fluid obeying the equation of state  $\hat{p} = (\gamma - 1)\hat{\rho}c^2$  with  $\gamma \leq 2$ , provided the biscalar potential satisfies  $\partial_\tau V(\Phi) > 0$ .

We discuss the thermodynamics associated with matter fields only. As before, the equations of motion for the matter fields imply the conservation law (41) for the matter field energy-momentum tensor. Since the speed of a photon takes a constant value  $c_{\text{ph}} = c$  in the comoving frame for the matter metric, the  $t$  component of the matter field conservation equation takes the form

$$\hat{\rho} + 3 \left( \hat{p} + \frac{\hat{p}}{c^2} \right) \frac{\dot{a}}{a} = 0. \quad (60)$$

Unlike the case of the comoving frame for the gravity metric, the matter fields are observed to be conserved. Equation (60) along with the first law of thermodynamics for the matter fields

$$Tds_{\text{mat}} = d(\hat{\rho} c^2 v) + \hat{p} dv \quad (61)$$

yields  $\dot{s}_{\text{mat}} = 0$ . So the time variation of the entropy density  $s$  in the matter metric frame, as expressed in Eq. (58), is all due to the time variation of the entropy density of the biscalar field, and there is no entropy exchange between matter and the biscalar field sectors.

#### IV. CONCLUSIONS

We studied the compatibility of the VSL cosmological models with the second law of thermodynamics. We find that generally the VSL model with varying fundamental constant is severely constrained by the second law of thermodynamics. For the model with the speed of light gradually varying as a power law of the cosmic scale factor, only an open universe with positive cosmological constant is allowed to contain the cosmic perfect fluid satisfying a reasonable equation of state such as one including dust and radiation. On the other hand, the bimetric cosmological models are less constrained by the second law of thermodynamics. In the comoving frame associated with the gravity metric, there is no constraint on the model by the second law of thermodynamics. In the comoving frame associated with the matter metric, provided the biscalar potential satisfies a certain condition, regardless of the value of  $k$ , the second law of thermodynamics can be satisfied by a perfect fluid satisfying the equation of state  $\hat{p} = (\gamma - 1)\hat{\rho}c^2$  with  $\gamma \leq 2$ , which includes dust and radiation.

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