Neutrino damping rate at finite temperature and density

Eduardo S. Tututi*
Escuela de Ciencias Físico Matemáticas, Apartado Postal 2-71, 58040, Universidad Michoacana, Morelia Mich., México

Manuel Torres†
Instituto de Física, Apartado Postal 20-364, 01000, Universidad Nacional Autónoma de México, D.F., México

Juan Carlos D'Olivo‡
Instituto de Ciencias Nucleares, Apartado Postal 70-534, 04510, Universidad Nacional Autónoma de México, D.F., México

(Received 21 March 2002; published 20 August 2002)

A first principles derivation is given of the neutrino damping rate in real-time thermal field theory. Starting from the discontinuity of the neutrino self-energy at the two loop level, the damping rate can be expressed as integrals over phase space of amplitudes squared, weighted with statistical factors that account for the possibility of particle absorption or emission from the medium. Specific results for a background composed of neutrinos, leptons, protons, and neutrons are given. Additionally, for the real part of the dispersion relation we discuss the relation between the results obtained from the thermal field theory and those obtained by the thermal average of the forward scattering amplitude.

DOI: 10.1103/PhysRevD.66.043001 PACS number(s): 95.30.Cq, 11.10.Wx, 98.80.Cq

I. INTRODUCTION

A neutrino propagating in matter no longer respects the vacuum energy momentum relation. The modification of the neutrino dispersion relation is caused by their coherent interaction with the particles in the background and can be accounted for in terms of an index of refraction [1] or an effective potential [2]. The topic of neutrino propagation in matter became of prime relevance after Wolfenstein [3] studied the neutrino refractive index in matter, and later when Mikheyev-Smirnov [4] recognized the resonant neutrino flavor oscillations triggered by matter effects. The Mikheyev-Smirnov effect has become the most popular explanation of the solar neutrino deficit [5].

In general, the neutrino dispersion relation is a complex function $\omega_\kappa = \omega(\vec{\kappa})$. According to the thermal field theory (TFT) matter contributions to the real and imaginary parts of $\omega(\vec{\kappa})$ arise from the temperature and density-dependent part of the neutrino self-energy. To leading order in (g^2/M_W^2) the real part of the dispersion relation is proportional to the particle-antiparticle asymmetry in the background. If the asymmetry is small or zero the imaginary part of $\omega(\kappa)$ and corrections of order g^2/M_W^4 to the real part may be important because they do not depend on the differences between the number of particles and antiparticles. This may be the case in the early Universe, when the medium was probably nearly CP symmetric.

Special attention has been given to the calculations of the $O(g^2/M_W^4)$ corrections to the real part of the neutrino dispersion relations [6–8]. Within the framework of the TFT these corrections arise from the momentum-dependent terms of the boson propagator in the self-energy diagrams.

The imaginary part of the index of refraction for neutrinos propagating in a CP-symmetric plasma composed of electrons, neutrinos, and their antiparticles has been considered in Refs. [6,9,10]. These calculations have been based upon the computation of the neutrino reaction rates, assuming massless background fermions. In our opinion the relation of the neutrino damping rate to the self-energy discontinuities deserves further consideration. Additionally this work addresses the effects of the nucleons' background contributions and the fermion mass correction.

A systematic method to compute the damping rate from the imaginary part of the self-energy can be formulated in terms of the Cuttosky thermal rules. Weldon [11] and Kobes [12] proved that the imaginary part of the self-energy can be organized in a form that includes the square of amplitudes of the various processes obtained from the cuts of the self-energy and weighted with the appropriate statistical factors. The examples discussed in those papers are always at the one loop level. As it shall be discussed, the calculation of the neutrino damping rate requires one to consider the self-energy at the two loop level, the interpretation in terms of the square of amplitudes of the allowed processes will be proved to remain valid. The approximations required to recover the results obtained from the optical theorem will be clearly stated.

In this work we use the method of real-time thermal field theory to carry out a careful calculation of the imaginary part of the neutrino dispersion relation in a medium composed of electrons, protons, neutrons, neutrinos, and their antiparticles. As already mentioned the contributions to the imaginary part of the neutrino self-energy vanishes at the one loop level, so we have to consider the contributions at the two loop level.

*Email address: tututi@zeus.ccu.umich.mx
†Email address: torres@fisica.unam.mx
‡Email address: dolivo@nuclecu.unam.mx

The paper is organized as follows. In the next section we briefly review those ingredients of the TFT formalism that are required to accomplish our calculations. In Sec. III we rederive the real part of the dispersion relation of a massless neutrino that propagates in a thermal background. We prove that utilizing the methods of the finite-temperature field theory (at the g^2 order), the neutrino effective potential reduces to the thermal average of the neutrino forward scattering amplitude. The calculation of the imaginary part of the dispersion relation is presented in Sec. IV. The neutrino damping rate is extracted from the discontinuity of self-energy at the two loop level; it is expressed in terms of integrals over phase space of amplitudes squared, weighted with statistical factors that account for the possibility of particle absorption or emission from the medium. Specific results for a background composed of neutrinos, leptons, protons, and neutrons are given. The last section contains a summary of our main results.

II. BASIC FORMALISM

The relevant quantity is the self-energy Σ, which embodies the effects of the background on a neutrino that propagates through it. According to the real-time formalism of the TFT [13–15], the real and imaginary parts of Σ are given by the formulas

$$\mathrm{Re}\,\Sigma = \mathrm{Re}\,\Sigma_{11}, \tag{2.1}$$

$$\mathrm{Im}\,\Sigma = \frac{\Sigma_{12}}{2i[\eta_f(k\cdot u) - \theta(-k\cdot u)]}, \tag{2.2}$$

where Σ_{ab} $(a,b=1,2)$ are the complex elements of the 2×2 self-energy matrix to be computed utilizing the Feynman rules of the theory. θ is the step function and

$$\eta_f(k\cdot u) = [\theta(k\cdot u)n_f(x_k) + \theta(-k\cdot u)n_f(-x_k)], \tag{2.3}$$

where the thermal distribution is given by

$$n_f(x_k) = \frac{1}{e^{x_k}+1}, \tag{2.4}$$

with $x_k = \beta(k\cdot u - \mu_f)$. Here, β is the inverse of the temperature and μ_f is the chemical potential associated with the fermion *f*. We have introduced the velocity four-vector of the background u^μ. In its own rest frame $u^\mu = (1,\mathbf{0})$ and the components of the neutrino momentum k^μ are $k^\mu = (\omega, \vec{\kappa})$.

In the presence of a medium the chiral nature of the neutrino interactions implies that the self-energy of a massless (left-handed) neutrino is of the form [16]

$$\Sigma(k) = (a\not{k} + b\not{u})L, \tag{2.5}$$

where $L = (1-\gamma_5)/2$ and a,b are complex functions of the scalars

$$\omega = k\cdot u, \qquad \kappa = \sqrt{\omega^2 - k^2}. \tag{2.6}$$

In this case, the Dirac equation for the spinor U of the neutrino mode in the medium is

$$(\not{k} - \Sigma)U = 0, \tag{2.7}$$

which has nontrivial solutions only for those values of ω and κ such that $V^2=0$, with $V_\mu = (1-a)k_\mu - bu_\mu$. Then, the dispersion relations ω_κ of the neutrino and antineutrino modes are obtained as the solutions of

$$f(\omega_\kappa, \kappa) = 0 \tag{2.8}$$

and

$$\overline{f}(\omega_\kappa, \kappa) = 0, \tag{2.9}$$

where

$$f(\omega,\kappa) = (1-a)(\omega-\kappa) - b,$$
$$\overline{f}(\omega,\kappa) = (1-a)(\omega+\kappa) - b. \tag{2.10}$$

In general, the solutions ω_κ are complex; as usual we write

$$\omega_\kappa = \omega_r - i\gamma/2, \tag{2.11}$$

where both $\omega_r = \mathrm{Re}\,\omega_\kappa$ and $\gamma = -2\,\mathrm{Im}\,\omega_\kappa$ are real functions of κ. A consistent interpretation in terms of the dispersion relation for a mode propagating through a medium is possible only if the imaginary part of ω_κ is small compared to its real part. In this case the mode can be visualized as a particle (or antiparticle) with an energy ω_r and a damping γ. Under such assumptions, each one of Eqs. (2.8) and (2.9) yields two distinct solutions, one with positive energy and the other with negative energy, whose physical interpretation has been discussed in detail in Ref. [16]. Here we will concentrate on the solution of Eq. (2.8) having a positive real part, which corresponds to the neutrino mode with energy ω_r, but similar considerations and results apply to the other solutions.

It is convenient to make the decompositions $a = a_r + ia_i$ and $b = b_r + ib_i$. Then, using Eqs. (2.10) and (2.11), expanding in powers of γ, and retaining only terms that are at most linear in γ, a_i, and b_i, from Eq. (2.8) we obtain

$$f_r(\omega_r, \kappa) = 0 \tag{2.12}$$

and

$$\frac{\gamma}{2} = \left[\frac{f_i(\omega,\kappa)}{\dfrac{\partial f_r}{\partial \omega}}\right]_{\omega=\omega_r}, \tag{2.13}$$

with

$$f_r = (1-a_r)(\omega-\kappa) - b_r,$$
$$f_i = -a_i(\omega-\kappa) - b_i. \tag{2.14}$$

Only approximate analytical solutions of Eq. (2.12) are known [16]. At the one-loop level both $b_r(\omega_r,\kappa)$ and

$a_r(\omega_r,\kappa)$ are of order g^2/M_W^2. Therefore to this order the energy of a massless neutrino is

$$\omega_r \cong \kappa + b_r(\omega_r,\kappa)|_{\omega_r=\kappa}. \tag{2.15}$$

On the other hand, as we will show, the imaginary part of Σ vanishes to $O(g^2)$ and to this order there are no contributions to $\mathrm{Im}\,\omega_\kappa$. For the perturbative solution of Eq. (2.13) around $\omega_r=\kappa$, we have

$$\frac{\gamma}{2} \cong -b_i(\omega_r,\kappa)|_{\omega_r=\kappa}, \tag{2.16}$$

with $b_i(\omega_r,\kappa)$ of $O(g^4/M_W^4)$.

The matter effects on the neutrino oscillations are conveniently incorporated by means of the effective potential V. This can be introduced by subtracting the (vacuum) kinetic energy from the real part of the neutrino dispersion relation [8]:

$$V \equiv \omega_r - \kappa \cong b_r(\omega_r,\kappa)|_{\omega_r=\kappa}. \tag{2.17}$$

In the literature it is also customary to use a refraction index, which is defined by

$$n(\kappa) \equiv \frac{\kappa}{\omega_\kappa}. \tag{2.18}$$

In the approximation we are working on, and utilizing Eqs. (2.11)–(2.17), it follows that its real and imaginary parts are related to the effective potential and the damping rate by

$$\mathrm{Re}\,n(\kappa) \cong 1-\frac{V}{\kappa}, \qquad \mathrm{Im}\,n(\kappa) \cong \frac{\gamma}{2\kappa}, \tag{2.19}$$

with V and γ given by Eqs. (2.17) and (2.16), respectively.

III. EFFECTIVE POTENTIAL

According to Eq. (2.17), for a perturbative solution of the dispersion relation around the vacuum value $\omega_r=\kappa$, the effective potential is given by the real part of the coefficient of $\not{u}$ in the neutrino self-energy. This is only true in the lowest order, in general, V will also receive contributions from a_r. From Eq. (2.5), it is easy to see that

$$b_r(\omega,\kappa) = \frac{1}{4}\mathrm{Tr}\{\not{X}\,\mathrm{Re}\,\Sigma\} \tag{3.1}$$

with

$$\not{X} = \frac{1}{\kappa^2}(\omega\not{k} - k^2\not{u}). \tag{3.2}$$

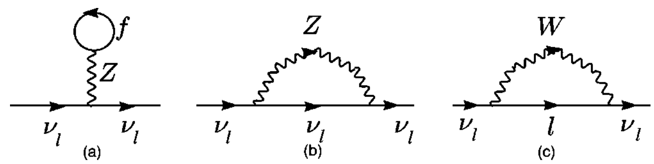

FIG. 1. One-loop diagrams for the self-energy of a neutrino in a thermal background of charged leptons, nucleons, and neutrinos. In (a), f stands for any fermion in the background. In (c), the charged lepton l is of the same flavor as the neutrino.

In Ref. [8] the real part of the neutrino self-energy was calculated in a general gauge up to terms of order g^2/M_W^4. It was shown that although the self-energy depends on the gauge parameter, the dispersion relation is independent of it. Taking this result into account, for simplicity we will work in the unitary gauge. Furthermore, for physical situations where the temperature is much lower than the masses of the gauge bosons, the propagators for the W and Z can be taken the same as in the vacuum, namely,

$$\Delta_B^{\alpha\beta} = \frac{1}{k^2-M_B^2+i\epsilon}\left(g^{\alpha\beta} - \frac{k^\alpha k^\beta}{M_B^2}\right), \tag{3.3}$$

with $B=W,Z$.

We shall assume that neutrinos are massless, so at the one-loop level the only contributions to Σ arise from the diagrams depicted in Fig. 1.

For a neutrino ν_l ($l=e,\mu,\tau$) that propagates through a medium with a momentum k, we split the different contributions to b^l according to the processes in Fig. 1 as follows:

$$b_r^l = b_{tad}^l + b_Z^l + b_W^l, \tag{3.4}$$

corresponding to the tadpole, Z-exchange, and W-exchange contributions. In this case, the background dependent parts of each term in the right-hand side of the previous equation can be worked out as

$$\begin{aligned}
b_{tad}^l &= \frac{1}{4}\mathrm{Tr}\{\not{X}\Gamma^Z_{\nu_l\alpha}\}\Delta_Z^{\alpha\beta}(0)\sum_f \int \frac{d^4p}{(2\pi)^3} \\
&\quad \times \mathrm{Tr}\{(\not{p}+m_f)\Gamma^Z_{f\beta}\}\,\delta(p^2-m_f^2)\,\eta_f(p\cdot u), \\
b_Z^l &= -\frac{1}{4}\int \frac{d^4p}{(2\pi)^3}\mathrm{Tr}\,\{\not{X}\,\Gamma^Z_{\nu_l\alpha}\,\not{p}\,\Gamma^Z_{\nu_l\beta}\} \\
&\quad \times \Delta_Z^{\alpha\beta}(k-p)\,\delta(p^2)\,\eta_{\nu_l}(p\cdot u), \\
b_W^l &= -\frac{1}{4}\int \frac{d^4p}{(2\pi)^3}\mathrm{Tr}\,\{\not{X}\Gamma^W_\alpha(\not{p}+m_l)\Gamma^W_\beta\} \\
&\quad \times \Delta_W^{\alpha\beta}(k-p)\,\delta(p^2-m_l^2)\,\eta_l(p\cdot u).
\end{aligned} \tag{3.5}$$

In b_W the charged lepton in the internal line has the same flavor as that of ν_l, while in the tadpole contribution b_{tad}, the summation is over all the fermion species present in the thermal background. In the previous expressions the vertices are given by

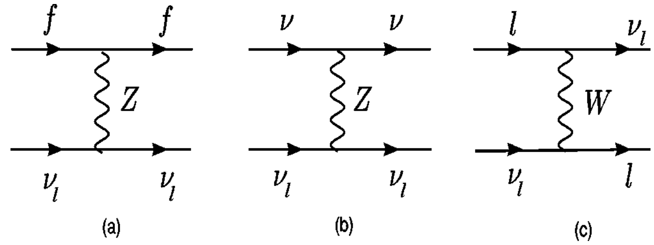

FIG. 2. Tree-level diagrams for $\nu_l f \to \nu_l f$ forward scattering; f represents the background fermions. These diagrams are obtained when the corresponding self-energy diagrams [Fig. 1] are cut along the internal fermion line.

$$\Gamma^W_\alpha = i\frac{g}{2\sqrt{2}}\gamma_\alpha(1-\gamma_5),$$
$$\Gamma^Z_{f\alpha} = i\frac{g}{2\cos\theta_W}\gamma_\alpha(X_f + Y_f\gamma_5). \tag{3.6}$$

With the vector X_l and axial couplings Y_l given for the charged leptons by

$$X_l = -\frac{1}{2} + 2\sin^2\theta_W,$$
$$Y_l = \frac{1}{2}; \tag{3.7}$$

for the neutrinos by

$$X_{\nu_l} = -Y_{\nu_l} = \frac{1}{2}; \tag{3.8}$$

and for the nucleons by

$$X_p = \frac{1}{2} - 2\sin^2\theta_W,$$
$$Y_p = X_n = -Y_n = -\frac{1}{2}. \tag{3.9}$$

According to Eq. (2.17) b^l_r has to be evaluated at $\omega=\kappa$; in this case the quantity $\kappa\not{\lambda}$ in Eq. (3.2) reduces to the energy projector for a massless particle in the vacuum $\not{k}$, and may be replaced by its usual expression in terms of the free spinors. In general for the external lines the spinors to be used are the solutions of the Dirac equation in the medium [18]. However, within the approximation we are using, they can be approximated by the vacuum solutions. For the internal fermionic lines we substitute $\not{p}+m_f$ by its corresponding energy projector. It is now straightforward to show that any of the b^l in Eq. (3.5) can be written in the form

$$b^l_r(\omega,\kappa)|_{\omega=\kappa} = \frac{1}{2\pi\kappa}\langle\mathcal{M}^l_f\rangle, \tag{3.10}$$

where $\mathcal{M}^l_f$ is the tree-level invariant amplitude for the forward scattering $\nu_l f \to \nu_l f$. The corresponding Feynman diagrams are shown in Fig. 2. Notice that these diagrams are obtained from the self-energy diagrams in Fig. 1 if we cut along the internal fermion line. Brackets in the previous expression represent the thermal average given by

$$\langle\mathcal{M}^l_f\rangle = \sum_f \int \frac{d^4p}{(2\pi)^3}\delta(p^2-m_f^2)\,\mathcal{M}^l_f\eta_f(p). \tag{3.11}$$

The previous result shows that utilizing the methods of finite-temperature field theory (at the g^2 order), the neutrino effective potential reduces to the thermal average of the neutrino forward scattering amplitude. In fact, it is interesting to combine Eqs. (2.17), (2.19), and (3.10) to write the real part of the index of refraction as

$$n = 1 + \frac{2\pi}{\kappa^2}\langle\mathcal{M}^l_f\rangle, \tag{3.12}$$

that generalizes the zero temperature result [17] $n=1+2\pi\mathcal{M}/\kappa^2$, by simply adding the thermal average of forward scattering amplitude, a result that proves that background does not spoil the coherent condition of the forward scattering processes.

In the rest of this section we derive the effective potential for a neutrino propagating in a thermal background composed of electrons, protons, neutrons, neutrinos, and their respective antiparticles. As previously mentioned, the Feynman diagrams in Fig. 2 are obtained by cutting the self-energy diagrams of Fig. 1 along the internal fermion line. Diagram (a) is obtained from the tadpole self-energy, the result is the same for any neutrino flavor, and one has to sum over all the fermions f present in the background. Diagrams (b) and (c) correspond to cutting the Z-and W-exchange self-energy diagrams, consequently the background fermion necessarily has the same flavor as the test neutrino. In this way, $\mathcal{M}^l_f$ can be written as

$$\mathcal{M}^l_f = \mathcal{M}_a + \mathcal{M}_b\delta_{f\nu_l} + \mathcal{M}_c\delta_{fl}, \tag{3.13}$$

with $\delta_{ll}=\delta_{\nu_l\nu_l}=1$, $\delta_{fl}=0$ for $f\neq l$, and $\delta_{f\nu_l}=0$ for $f\neq\nu_l$. Here

$$\mathcal{M}_a = -\frac{1}{4}\overline{u}_{\nu_l}(k,s')\Gamma^Z_{f\alpha}u_{\nu_l}(k,s')\Delta^{\alpha\beta}_Z(0)\overline{u_f}(p,s)\times\Gamma^Z_{f\beta}u_f(p,s),$$

TABLE I. Effective potential for a neutrino propagating through a medium. The $+$ ($-$) sign refers to neutrinos (antineutrinos).

Neutrino	Background particle	V_{eff}
ν_e,ν_μ,ν_τ	p	$\pm\frac{G_F}{\sqrt{2}}(1-4\sin^2\theta_W)(N_p-N_{\bar{p}})$
ν_e,ν_μ,ν_τ	n	$\mp\frac{G_F}{\sqrt{2}}(N_n-N_{\bar{n}})$
ν_e	e	$\pm\frac{G_F}{\sqrt{2}}(1+4\sin^2\theta_W)(N_e-N_{\bar{e}})$ $+\frac{2\sqrt{2}G_F\omega m_e^2}{3M_W^2}\left[N_e\left\langle\frac{1}{E_e}\right\rangle+N_{\bar{e}}\left\langle\frac{1}{E_{\bar{e}}}\right\rangle-\frac{4}{m_e^2}(N_e\langle E_e\rangle+N_{\bar{e}}\langle E_{\bar{e}}\rangle)\right]$
ν_e	ν_e	$\pm\frac{4G_F}{\sqrt{2}}(N_{\nu_e}-N_{\bar{\nu}_e})-\frac{8\sqrt{2}G_F\omega}{3M_Z^2}[N_{\nu_e}\langle E_{\nu_e}\rangle+N_{\bar{\nu}_e}\langle E_{\bar{\nu}_e}\rangle]$
ν_μ,ν_τ	e	$\pm\frac{G_F}{\sqrt{2}}(-1+4\sin^2\theta_W)(N_e-N_{\bar{e}})$
ν_μ,ν_τ	ν_e	$\pm\frac{2G_F}{\sqrt{2}}(N_{\nu_e}-N_{\bar{\nu}_e})$

$$\mathcal{M}_b=\frac{1}{4}\overline{u}_{\nu_l}(k,s')\Gamma^Z_{f\alpha}u_f(p,s)\Delta_Z^{\alpha\beta}(k-p)\overline{u}_f(p,s)\times\Gamma^Z_{f\beta}u(k,s'),\tag{3.14}$$

$$\mathcal{M}_c=\frac{1}{4}\overline{u}_{\nu_l}(k,s')\Gamma^W_{\alpha}u_f(p,s)\Delta_W^{\alpha\beta}(k-p)\overline{u}_f(p,s)\times\Gamma^W_{\beta}u_{\nu_l}(k,s').$$

Since we are interested in contributions to V of order g^2/M_W^4 we expand the gauge propagator in power of M_B^{-2} up to the second order

$$\frac{g_{\alpha\beta}}{q^2-M_B^2}\approx-\frac{g_{\alpha\beta}}{M_B^2}\left(1+\frac{q^2}{M_B^2}\right).\tag{3.15}$$

Using this expansion and neglecting quantities of order m_f^2/M_B^2, we find

$$\mathcal{M}_a\approx 2\sqrt{2}G_F X_f k\cdot p,$$
$$\mathcal{M}_b\approx\sqrt{2}G_F\left[k\cdot p-\frac{2(p\cdot k)^2}{M_Z^2}\right],\tag{3.16}$$
$$\mathcal{M}_c\approx\sqrt{2}G_F\left[k\cdot p-\frac{2(p\cdot k)^2}{M_W^2}\right].$$

where $q=p-k$ and $G_F/\sqrt{2}\equiv g^2/8M_W^2$ is the Fermi coupling constant, and the factors X_f are given in Eqs. (3.7), (3.8), and (3.9) for leptons, neutrinos, and nucleons, respectively.

Once the previous results are substituted in Eq. (3.11), the different contributions to V can be expressed in terms of the following integrals:

$$\int\frac{d^4p}{(2\pi)^3}p^\mu\delta(p^2-m^2)\,\eta_f(p\cdot u)=Au^\mu,\tag{3.17}$$

$$\int\frac{d^4p}{(2\pi)^3}p^\mu p^\nu\delta(p^2-m^2)\,\eta_f(p\cdot u)=Bu^\mu u^\nu+Cg^{\mu\nu}.$$

The scalar quantities A, B, and C are easily evaluated in the rest frame of the medium; the results are

$$A=\frac{1}{2}(N_f-N_{\bar{f}}),$$

$$B=\frac{1}{6}\left[m_f^2\left(\left\langle\frac{1}{E_f}\right\rangle N_f+\left\langle\frac{1}{E_{\bar{f}}}\right\rangle N_{\bar{f}}\right)-(\langle E_f\rangle N_f+\langle E_{\bar{f}}\rangle N_{\bar{f}})\right],\tag{3.18}$$

$$C=-\frac{1}{6}\left[m_f^2\left(\left\langle\frac{1}{E_f}\right\rangle N_f+\left\langle\frac{1}{E_{\bar{f}}}\right\rangle N_{\bar{f}}\right)-4(\langle E_f\rangle N_f+\langle E_{\bar{f}}\rangle N_{\bar{f}})\right].$$

In these equations N_f ($N_{\bar{f}}$) represents the density of fermions (antifermions) in the background

$$N_{f,\bar{f}}=g_f\int\frac{d^3p}{(2\pi)^3}\frac{1}{e^{\beta(E_f\mp\mu_f)}+1},\qquad(3.19)$$

and $\langle E^j_{f,\bar{f}}\rangle$ $(j=1,-1)$ denotes the statistical averaged of E_f and $1/E_f$

$$\langle E^j_{f,\bar{f}}\rangle=\frac{g_f}{N_{f,\bar{f}}}\int\frac{d^3p}{(2\pi)^3}E^j_f\frac{1}{e^{\beta(E_f\mp\mu_f)}+1}.\qquad(3.20)$$

Here, $E_f=\sqrt{p^2+m_f^2}$ and g_f is the number of spin degrees of freedom ($g_\nu=1$ for chiral neutrinos and $g_f=2$ for the electron and nucleons). Collecting these results it is straightforward to write down the effective potential for a neutrino of a given flavor.

The contributions to the effective potential for the various neutrino flavors and background particles are listed in Table I. They agree with the results given in the literature [6–8].

IV. IMAGINARY PART

The discontinuity of the self-energy is related to the damping rate γ that determines the imaginary part of the dispersion relation [see Eq. (2.11)], additionally the damping rate can be interpreted as the rate at which the single-particle distribution function approaches the equilibrium form [11]. The former interpretation follows if one considers a particle distribution that is slightly out of equilibrium, hence one has

$$\frac{\partial f}{\partial t}=-f\Gamma_a+(1+\sigma f)\Gamma_c,\qquad(4.1)$$

where Γ_a and Γ_c are the absorption and creation rates of the given particle, respectively. The parameter σ distinguishes bosons ($\sigma=1$) and fermions ($\sigma=-1$). The previous equation has for general solution

$$f(\omega_r,t)=\frac{\Gamma_c}{\Gamma_a-\sigma\Gamma_c}+C(\omega_r)e^{(\Gamma_a-\sigma\Gamma_c)t},\qquad(4.2)$$

where $C(\omega_r)$ is an arbitrary function that does not depend on time. Creation and absorption rates are related by the Kubo-Martin-Schwinger (KMS) relation

$$\Gamma_a(\omega)=e^{\omega/T}\Gamma_c(\omega).\qquad(4.3)$$

Consequently Eq. (4.2) can be written as

$$f(\omega_r,t)=\frac{1}{e^{\omega_r/T}-\sigma}+C(\omega_r)e^{-2\gamma t},\qquad(4.4)$$

where $\gamma=(\Gamma_a-\sigma\Gamma_c)/2$ is defined as the damping rate. Therefore γ can be interpreted as the inverse time scale it takes for a thermal distribution to reach equilibrium. The sign of the damping rate must necessarily be positive for stable systems. Additionally, the form of the dispersion relation implies for a normal mode to propagate that γ is small compared to ω_r. For neutrinos this condition is satisfied in a

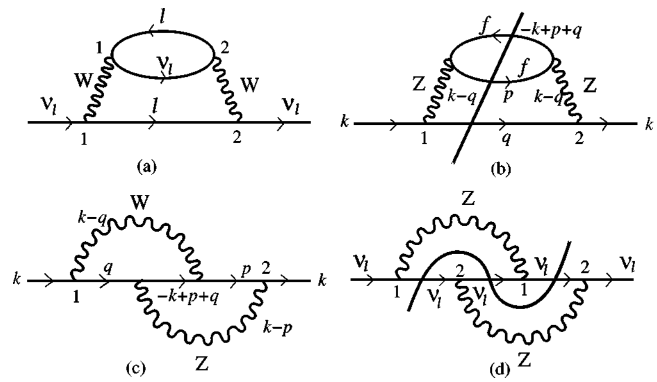

FIG. 3. Two-loop contributions to the self-energy of a neutrino in a thermal background of charged leptons, nucleons, and neutrinos.

normal matter, such as the core of the Sun. However, in a CP-symmetric medium the leading contributions to the real part vanish. Thus the first nonvanishing contributions are of order G_F/M_W^2; under these circumstances γ can become of the same order.

To obtain the neutrino damping rate we have to evaluate Σ_{12} and then use Eqs. (2.2), (2.5), and (2.16). As explained further ahead the one loop contributions to Σ_{12} cancel. The diagrams that contribute to Σ_{12} at the two loop level are depicted in Fig. 3; there are also diagrams similar to those in (c) with the W and Z lines interchanged. According to the Feynman rules on the real time formulation of the TFT, the contributions of diagrams (a) and (b) can be written as

$$\begin{aligned}-i\Sigma_{12}(k)=&-\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\Gamma_1^\mu[iS_{12}(q)]\Gamma_2^\nu\,\mathrm{Tr}\\&\times\{\Gamma_1^\alpha[iS_{12}(p)]\Gamma_2^\beta[iS_{21}(p+q-k)]\}\\&\times i[\Delta^A_{\mu\alpha}(k-q)]_{11}i[\Delta^A_{\nu\beta}(k-q)]_{22}.\end{aligned}\qquad(4.5)$$

Similarly, for diagrams (c) and (d) we have

$$\begin{aligned}-i\Sigma_{12}(k)=&\int\frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\Gamma_1^\mu[iS_{12}(q)]\Gamma_2^\beta\\&\times[iS_{21}(q+p-k)]\Gamma_1^\alpha[iS_{12}(p)]\Gamma_2^\nu\\&\times i[\Delta^A_{\mu\alpha}(k-q)]_{11}i[\Delta^B_{\nu\beta}(k-p)]_{22}.\end{aligned}\qquad(4.6)$$

In the above expressions $A,B=W,Z$ and $\Gamma_2^\alpha=-\Gamma_1^\alpha$, with Γ_1^α representing any of the vertex Γ_α^W or $\Gamma_{\alpha f}^Z$ given in the previous section. The S_{12} and S_{21} components of the propagator matrix of the fermion are given by [13]

$$\begin{aligned}S_{12}(p)&=2\pi i\,\delta(p^2-m^2)[\eta_f(p)-\theta(-p\cdot u)](\not p+m),\\S_{21}(p)&=2\pi i\,\delta(p^2-m^2)[\eta_f(p)-\theta(p\cdot u)](\not p+m).\end{aligned}\qquad(4.7)$$

In principle, the internal vertices should be added over the thermal indices ($a=1,2$). However, since temperature is small as compared to the gauge boson masses, the matrices of the W and Z bosons' propagators are diagonal with

$[\Delta^A_{\alpha\beta}(k)]_{11} = -[\Delta^A_{\alpha\beta}(k)]^*_{22} = \Delta^A_{\alpha\beta}$, where $\Delta^A_{\alpha\beta}$ is the vacuum propagator given in Eq. (3.3). This explains the cancellation of the g^2 contribution to the neutrino damping rate. The one loop contribution to Σ_{12} is given by diagrams similar to those in Fig. 1 with iS_{12} and $i[\Delta^A_{\alpha\beta}(k)]_{12}$ replacing the internal fermionic and bosonic lines; however, the bosonic propagator is diagonal, hence Σ_{12} cancel at this order.

According to Eq. (2.16) γ is directly proportional to $b_i(\omega,\kappa)$, that is given by

$$b_i(\omega,\kappa) = \frac{1}{4}\mathrm{Tr}\{\not{\chi}\,\mathrm{Im}\Sigma\}, \tag{4.8}$$

with $\not{\chi}$ defined in Eq. (3.2).

Taking into account the previous results it is demonstrated after a lengthy calculation that the neutrino damping rate can be expressed in the form

$$\gamma = -i\frac{\epsilon(k\cdot u)}{2in_F}\left\{C^{WW} + C^{ZZ} - \sum_{AB=Z,W} D^{AB}\right\}, \tag{4.9}$$

where C^{AA} is obtained from Eq. (4.5):

$$C^{AA} = \frac{1}{4}\int \frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4Q}{(2\pi)^4}\{\mathrm{Tr}\{\not{\chi}\Gamma^\mu_1(\not{q}+m_1)\Gamma^\nu_2\}\mathrm{Tr}\{\Gamma^\alpha_1(\not{p}+m_2)\Gamma^\beta_2(\not{Q}+m_3)\}i[\Delta^A_{\mu\alpha}(k-q)]_{11}i[\Delta^A_{\nu\beta}(k-q)]_{22}\}$$
$$\times(2\pi)^4\delta^{(4)}[Q-(q+p-k)]\delta(q^2-m_1^2)\delta(p^2-m_2^2)\delta(Q^2-m_3^2)[\eta_F(q_x)-\theta(-q\cdot u)][\eta_F(p_y)-\theta(-p\cdot u)]$$
$$\times[\eta_F(Q_z)-\theta(Q\cdot u)], \tag{4.10}$$

and D^{AB} is obtained from Eq. (4.6):

$$D^{AB} = \frac{1}{4}\int \frac{d^4q}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\frac{d^4Q}{(2\pi)^4}\{\mathrm{Tr}\{\not{\chi}\Gamma^\mu_1(\not{q}+m_1)\Gamma^\nu_2(\not{Q}+m_2)\Gamma^\alpha_1(\not{p}+m_3)\Gamma^\beta_2\}i[\Delta^A_{\mu\alpha}(k-q)]_{11}i[\Delta^B_{\nu\beta}(k-p)]_{22}\}$$
$$\times(2\pi)^4\delta^{(4)}[Q-(q+p-k)]\delta(q^2-m_1^2)\delta(p^2-m_2^2)\delta(Q^2-m_3^2)[\eta_F(q_x)-\theta(-q\cdot u)][\eta_F(p_y)-\theta(-p\cdot u)]$$
$$\times[\eta_F(Q_z)-\theta(Q\cdot u)]. \tag{4.11}$$

For later convenience an extra integration over the momentum Q has been introduced. In what follows we shall see that the neutrino damping rate can be expressed in terms of amplitudes squared and weighted with the statistical factors that account for the various physical processes. To derive these results we notice first that fermion propagators in Eqs. (4.5) and (4.6) are either type 12 or 21. According to Eq. (4.7) the propagator $S_{12}(p)$ contains a delta function $\delta(p^2-m^2)$ and a factor $(\not{p}+m)$. The delta function put the fermion on their mass shell mass, in other words self-energy diagrams in Fig. 3 are cut along all the internal fermion lines. Whereas the second factor is concerned, insertion of the fermion projectors

$$\not{p}+m = \sum_s u(p,s)\overline{u}(p,s),$$
$$\not{p}-m = \sum_s v(p,s)\overline{v}(p,s), \tag{4.12}$$

allow us to rewrite the resulting expressions in terms of amplitudes for the physical processes arising from the cuts. The bosonic lines are not cut because they do not include thermal distributions for $T \ll M_W$.

For definitiveness let us consider diagram (b) in Fig. 3 and also that the fermion in the internal loop is a proton ($f = P$). When the diagram is cut as shown in the figure, we obtain a series of physical processes for the neutrinos ν_1 and ν_2 and protons P_1 and P_2. Of these particles one of the neutrinos (ν^*) is considered a test particle, all others are thermalized. According to the notation in Eq. (4.6), the momentum and chemical potentials are assigned as $(\nu_1:\, k,\mu_{\nu_1})$, $(\nu_2:\, q,\mu_{\nu_2})$, $(P_1:\, Q,\mu_{P_1})$, and $(P_2:\, p,\mu_{P_2})$. With momentum and charge conservation conserved depending on the process, e.g., for $\nu_1 P_1 \rightarrow \nu_2 P_2$:

$$k+Q = q+p, \tag{4.13}$$

$$\mu_{\nu_1} + \mu_{P_1} = \mu_{\nu_1} + \mu_{P_1}.$$

The processes obtained from the mentioned cut rules include the two neutrinos and the two protons distributed into the initial and final states in all possible ways. Hence in general we expect to obtain 16 different processes; this is explicitly displayed in Eq. (4.16). The resulting expression comes out with the appropriated thermal distribution; for this we have to rewrite the thermal contributions that appear in Eq. (4.10) utilizing the following identities:

$$\eta_f(x_k)-\theta(\pm k\cdot u)=\mp\epsilon(k\cdot u)n_f(\mp x_k),$$

$$n_f(x_k)=e^{-x_k}n_f(-x_k)=e^{-x_k}[1-n_f(x_k)],$$

$$\frac{1}{n_f(x_Q)}n_f(x_k)n_f(x_q)n_f(x_p)=n_f(x_k)n_f(x_q)n_f(x_p)+e^{-x_k-x_q+x_p}[1-n_f(x_k)]\times[1-n_f(x_q)]n_f(x_p), \tag{4.14}$$

where

$$x_k=\beta(k\cdot u-\mu_{\nu_1}),\quad x_q=\beta(q\cdot u-\mu_{\nu_2}),$$
$$x_p=\beta(p\cdot u-\mu_{P_1}),\quad x_Q=\beta(Q\cdot u-\mu_{P_2}). \tag{4.15}$$

Taking into account these considerations and performing an integration over the timelike components of the momentum integration, it is possible to cast the contribution to γ arising from the proton loop in diagram (b) of Fig. 3 into the following form:

$$\gamma^P=\frac{1}{2\kappa}\int\frac{d^3q}{2E_q(2\pi)^3}\frac{d^3p}{2E_p(2\pi)^3}\frac{d^3Q}{2E_Q(2\pi)^3}(2\pi)^4\Big[\delta^{(4)}(k+Q-q-p)|\mathcal{M}(P\nu\leftrightarrow P\nu)|^2$$
$$\times\{[1-n_\nu(E_q)][1-n_P(E_p)]n_P(E_Q)+n_\nu(E_q)n_P(E_p)[1-n_P(E_Q)]\}+\delta^{(4)}(k+Q+q-p)|\mathcal{M}(P\nu\bar{\nu}\leftrightarrow P)|^2$$
$$\times\{n_{\bar{\nu}}(E_q)n_P(E_Q)[1-n_P(E_p)]+[1-n_{\bar{\nu}}(E_q)][1-n_P(E_Q)]n_P(E_p)\}+\delta^{(4)}(k+Q+p-q)|\mathcal{M}(P\bar{P}\nu\leftrightarrow\nu)|^2$$
$$\times\{[1-n_\nu(E_q)]n_{\bar{P}}(E_p)n_P(E_Q)+n_\nu(E_q)[1-n_{\bar{P}}(E_p)][1-n_P(E_Q)]\}+\delta^{(4)}(k+Q+p+q)|\mathcal{M}(P\bar{P}\nu\bar{\nu}\leftrightarrow 0)|^2$$
$$\times\{n_{\bar{\nu}}(E_q)n_{\bar{P}}(E_p)n_P(E_Q)+[1-n_{\bar{\nu}}(E_q)][1-n_{\bar{P}}(E_p)][1-n_P(E_Q)]\}+\delta^{(4)}(k-Q-q-p)|\mathcal{M}(\nu\leftrightarrow P\bar{P}\nu)|^2$$
$$\times\{[1-n_\nu(E_q)][1-n_P(E_p)][1-n_{\bar{P}}(E_Q)]+n_\nu(E_q)n_P(E_p)n_{\bar{P}}(E_Q)\}+\delta^{(4)}(k+q-Q-p)\frac{1}{2}|\mathcal{M}(\nu\bar{\nu}\leftrightarrow P\bar{P})|^2$$
$$\times\{n_{\bar{\nu}}(E_q)[1-n_P(E_p)][1-n_{\bar{P}}(E_Q)]+[1-n_{\bar{\nu}}(E_q)]n_P(E_p)n_{\bar{P}}(E_Q)\}+\delta^{(4)}(k+p-Q-q)|\mathcal{M}(\bar{P}\nu\leftrightarrow\bar{P}\nu)|^2$$
$$\times\{[1-n_\nu(E_q)]n_{\bar{P}}(E_p)[1-n_{\bar{P}}(E_Q)]+n_\nu(E_q)[1-n_{\bar{P}}(E_p)]n_{\bar{P}}(E_Q)\}+\delta^{(4)}(k+p+q-Q)|\mathcal{M}(\bar{P}\nu\bar{\nu}\leftrightarrow\bar{P})|^2$$
$$\times\{n_{\bar{\nu}}(E_q)n_{\bar{P}}(E_p)[1-n_{\bar{P}}(E_Q)]+[1-n_{\bar{\nu}}(E_q)][1-n_{\bar{P}}(E_p)]n_{\bar{P}}(E_Q)\}\Big]. \tag{4.16}$$

Regardless of its length the interpretation of this equation is quite simple. The first two terms are interpreted as the absorption and emission of a neutrino via the dispersion $\nu P\rightarrow\nu P$ with statistical weight $(1-n_\nu)(1-n_P)n_P$ and $n_\nu n_P(1-n_P)$, respectively. As expected a n_f factor appears for each background fermions in the initial state, whereas fermions in the final state contribute with a Pauli blocking term $1-n_f$. As already discussed for femions the absorption and emission decay rates must be added [11]. The scattering amplitude for both processes are the same because they are related by CPT inversion. Similarly the third and fourth contributions represent neutrino annihilation and creation via the $P\nu\bar{\nu}\rightarrow P$ and $P\rightarrow P\nu\bar{\nu}$, respectively; they include, as expected, the statistical factors $n_{\bar{\nu}}n_P(1-n_P)$ and $n_P(1-n_{\bar{\nu}})(1-n_P)$. The same reasoning applies to the remaining terms.

Taking into account the δ-function constrains some of the quoted processes are not allowed. In what follows we focus on conditions with temperatures $m_f\approx T\ll M_W$ where, for example, the composition of the primeval plasma is dominated by (anti-) neutrinos, (anti-) electrons, nucleon, and photons. We recall [see the discussion below Eq. (4.4)] that for fermions the contributions to γ of decay and absorption add together. Hence the statistical factors appearing in the previous equation can be simplified. For example, the absorption $\nu^*P\rightarrow\nu P$ and decay $\nu P\rightarrow\nu^*P$, where ν^* is the test particle, add according to

$$[1-n_\nu(E_q)][1-n_P(E_p)]n_P(E_Q)+n_\nu(E_q)n_P(E_p)[1-n_P(E_Q)]=n_P(E_p)[1-n_P(E_Q)]+n_\nu(E_q)[n_P(E_p)-n_P(E_Q)]. \tag{4.17}$$

However, we can drop out the last term in the right-hand side of the equation because its contribution vanishes when substituted into Eq. (4.16).

With all these results we finally find that the neutrino damping rate can be expressed as

$$\gamma=\pm\frac{(2\pi)^4}{2\omega}\left\{\frac{1}{2}\int\frac{d^3p}{(2\pi)^3 2E_p}\frac{d^3q}{(2\pi)^3 2E_q}\frac{d^3Q}{(2\pi)^3 2E_Q}\times|\mathcal{M}(\nu e\leftrightarrow\nu e)|^2\delta^2(k+p-q-Q)\times[1-n_e(E_Q)]n_e(E_p)+\text{all the possible processes}\right\},\tag{4.18}$$

where the $+$ $(-)$ sign stands for test neutrinos (antineutrinos). For all possible processes we mean all the kinematically allowed processes obtained by cutting all the fermionic internal lines of Feynman diagrams shown in Fig. 3. The corresponding processes and their cross sections are listed below in Eq. (4.22).

The damping rate can be written in terms of the thermal average of the cross section. For the dispersion $\nu f\leftrightarrow\nu f$, the differential cross section is given by

$$d\sigma_f=\frac{1}{V_{\rm rel}2\omega 2E_p}|\mathcal{M}(\nu f\rightarrow\nu f)|^2(2\pi)^4\delta^2\times(k+p-q-Q)\frac{d^3q}{(2\pi)^3 2E_q}\frac{d^3Q}{(2\pi)^3 2E_Q},\tag{4.19}$$

where $V_{\rm rel}$ is the relative velocity between the neutrino and the background fermion; as we are considering massless neutrinos we simply have $V_{\rm rel}=1$.

Thus γ reads

$$\gamma=\pm\sum_f a_f\int\frac{d^3p}{(2\pi)^3}\langle d\sigma_f\rangle n_f(E_p)+\cdots,\tag{4.20}$$

where $a_f=1/2$ for $f=e,n,p$ and $a_f=1/4$ for $f=\nu$ and

$$\langle d\sigma_f\rangle=\frac{(2\pi)^4}{2\omega 2E_p v_{\rm rel}}\int\frac{d^3q}{(2\pi)^3 2E_q}\frac{d^3Q}{(2\pi)^3 2E_Q}\times|\mathcal{M}|^2\delta^2(k+p-q-Q)[1-n_f(E_Q)]\tag{4.21}$$

is the cross section thermally averaged by the Pauli blocking term $[1-n_f(E_Q)]$. In Eq. (4.18) the ellipsis represents the other possible process; in each case the corresponding statistical factors in Eqs. (4.18) and (4.19) and the dispersion amplitude are replaced by the pertinent factors. We recall that according to Eq. (2.19) γ is directly related to the imaginary part of the index of refraction. Hence Eq. (4.20) can be identified with the optical theorem.

In what follows we shall apply the previously obtained results to explicitly evaluate the neutrino damping rate in a background composed of neutrinos, electrons, protons, and neutrons. We suppose that the Pauli blocking term can be neglected, in addition, we consider temperatures $m_f\approx T\ll M_W$, hence in the thermal averages we can assume that $q^2\ll M_W^2$. First let us consider the cross section for the relevant processes. It is common to quote the cross sections, assuming ultrarelativistic neutrinos and neglecting the fermion masses. However, for conditions as those of the early universe, temperature and consequently the average neutrino energy can be comparable to the nucleon masses, and sometimes to the lepton masses. Hence, keeping fermion masses, the various neutrino cross sections can be calculated as

$$\tilde\sigma(\nu_e e\leftrightarrow\nu_e e)=16\delta^2+12\delta+3-(40\delta^2+30\delta+6)\frac{m_e^2}{s}+(12\delta^2+8\delta+1)\frac{3m_e^4}{s^2}-(16\delta^2+2\delta)\frac{m_e^6}{s^3}+4\delta^2\frac{m_e^8}{s^4},\tag{4.22}$$

$$\tilde\sigma(\nu_{\mu,\tau} e\leftrightarrow\nu_{\mu,\tau} e)=16\delta^2-12\delta+3-(26\delta^2-15\delta+3)\frac{2m_e^2}{s}+(12\delta^2-8\delta+1)\frac{3m_e^4}{s^2}-(8\delta^2-3\delta)\frac{2m_e^6}{s^3}+4\delta^2\frac{m_e^8}{s^4},$$

$$\tilde\sigma(\nu_e\overline{e}\leftrightarrow\nu_e\overline{e})=16\delta^2+4\delta+1-(40\delta^2+10\delta+1)\frac{m_e^2}{s}+(12\delta^2-4\delta)\frac{3m_e^4}{s^2}-(16\delta^2+10\delta+1)\frac{m_e^6}{s^3}+(2\delta+1)^2\frac{m_e^8}{s^4},$$

$$\tilde\sigma(\nu_{\mu,\tau}\overline{e}\leftrightarrow\nu_{\mu,\tau}\overline{e})=16\delta^2-4\delta+1-(40\delta^2-16\delta+1)\frac{m_e^2}{s}+(12\delta^2-4\delta)\frac{3m_e^4}{s^2}-(16\delta^2-12\delta+1)\frac{m_e^6}{s^3}+(-2\delta+1)^2\frac{m_e^8}{s^4},$$

$$\tilde\sigma(\nu_i\nu_i\leftrightarrow\nu_i\nu_i)=12,$$

$$\tilde\sigma(\nu_i\overline{\nu_i}\leftrightarrow\nu_i\overline{\nu_i})=8,$$

$$\tilde\sigma(\nu_i\nu_j\leftrightarrow\nu_i\nu_j)=6,$$

$$\tilde\sigma(\nu_i\overline{\nu_j}\leftrightarrow\nu_i\overline{\nu_j})=2,$$

TABLE II. Coefficients A_f and B_f appearing in Eq. (4.24) for various processes between neutrinos and the quoted background particles. Here $\delta=\sin^2\theta_W$.

Neutrino	Background fermion f	A_f	B_f
ν_e	e	$16\delta^2+12\delta+3$	$-(40\delta^2+30\delta+6)$
$\nu_{\mu,\tau}$	e	$16\delta^2-12\delta+3$	$-(52\delta^2-30\delta+6)$
ν_e	e^-	$16\delta^2+4\delta+1$	$-(40\delta^2+10\delta+1)$
$\nu_{\mu,\tau}$	e^-	$16\delta^2-4\delta+1$	$-(40\delta^2-16\delta+1)$
ν_i	ν_i	12	0
ν_i	$\nu_j,\ i\neq j$	6	0
ν_i	$\overline{\nu_i}$	8	0
ν_i	$\overline{\nu_j},\ i\neq j$	2	0
ν_i	n	3	-6
ν_i	$\overline{n}$	2	-2
ν_i	p	$16\delta^2-12\delta+1$	$-(40\delta^2-30\delta+6)$
ν_i	$\overline{P}$	$16\delta^2-4\delta+1$	$-(40\delta^2-10\delta+1)$
$\nu_e\overline{\nu_e}\leftrightarrow e\overline{e}$	annihilation process	$16\delta^2+8\delta+2$	-6
$\nu_{\mu,\tau}\overline{\nu}_{\mu,\tau}\leftrightarrow e\overline{e}$	annihilation process	$16\delta^2-8\delta+2$	-1

$$\tilde{\sigma}(\nu_i n\leftrightarrow\nu_i n)=3-\frac{6m_n^2}{s}+\frac{3m_n^4}{s^2},$$

$$\tilde{\sigma}(\nu_i\overline{n}\leftrightarrow\nu_i\overline{n})=2-\frac{2m_n^2}{s}-\frac{2m_n^6}{s^3},$$

$$\tilde{\sigma}(\nu_i p\leftrightarrow\nu_i p)=(16\delta^2-12\delta+1)-(40\delta^2-30\delta+6)\frac{m_p^2}{s}+(36\delta^2-24\delta+1)\frac{m_p^4}{s^2}-(16\delta^2-6\delta)\frac{m_p^6}{s^3}+(4\delta^2-2\delta)\frac{m_p^8}{s^4},$$

$$\tilde{\sigma}(\nu_i\overline{p}\leftrightarrow\nu_i\overline{p})=(16\delta^2-4\delta+1)-(40\delta^2-10\delta+1)\frac{m_p^2}{s}+(36\delta^2-12\delta)\frac{m_p^4}{s^2}-(16\delta^2-10\delta+1)\frac{m_p^6}{s^3}+(4\delta^2-2\delta)\frac{m_p^8}{s^4},$$

$$\tilde{\sigma}(\nu_e\overline{\nu}_e\leftrightarrow e\overline{e})=\left(1-\frac{4m_e^2}{s}\right)^{1/2}\left[16\delta^2+8\delta+2+\left(8\delta^2+4\delta-\frac{1}{2}\right)\frac{4m_e^2}{s}\right],$$

$$\tilde{\sigma}(\nu_{\mu,\tau}\overline{\nu}_{\mu,\tau}\leftrightarrow e\overline{e})=\left(1-\frac{4m_e^2}{s}\right)^{1/2}\left[16\delta^4-8\delta+2+\left(16\delta^2-8\delta+\frac{3}{2}\right)\frac{2m_e^2}{s}\right],$$

where $\tilde{\sigma}=\sigma/\sigma_0$ with $\sigma_0=(G_F^2/12\pi)s$, $i=e,\mu,\tau$, $s=(k+p)^2$ is the Mandelstam variable, and $\delta=\sin^2\theta_W\approx0.229$. These results reduce, in the zero fermion limit, to those found in Enqvist, Kainulainen, and Thomson [9] and Langacker and Liu [10].

Once the cross sections are inserted in Eq. (4.20) the thermal averages should be evaluated according to the constraints of the problem. If we consider temperatures well below the nucleon mass, then the proton and neutron contributions will be suppressed by the Boltzmann factor, and their contribution neglected. On the other hand if $T\approx m_f$ the complete average of the cross sections in Eq. (4.22) should be considered. In what follows we consider the situation in which $m_f<T$ and we retain terms of order m_f^2/s in Eq. (4.22). This leads us to consider thermal averages that contain integrals of the following type

$$\int\frac{d^4p}{(2\pi)^3}(k\cdot p)(k\cdot u)\,\theta(k\cdot u)\,\delta(p^2-m_f^2)n_f(p). \tag{4.23}$$

These integrals are easily evaluated in terms of the thermal average of the fermion density and energy, utilizing Eqs. (3.17)–(3.20). The results can be collected in a general formula that gives the contributions to the neutrino damping rate arising from various background particles (to first order in m_f):

$$\gamma_{\nu_i}=\frac{G_F^2}{12\pi}\sum_f \frac{a_f N_f}{g_f}\{2\omega A_f\langle E_f\rangle + m_f^2(A_f+B_f)\}, \tag{4.24}$$

where ω is the neutrino energy, the summation in f is taken over fermions in the background, and the corresponding factors A_f and B_f are summarized in Table II. In this equation N_f represents the density of fermions and antifermions in the medium [Eq. (3.19)] and the statistical averages of E_f is defined in Eq. (3.20).

The next step is to quote some explicit results. Consider a CP-symmetric plasma composed of the three types of neutrinos ν_e, ν_μ, and ν_τ, electrons, and their corresponding antiparticles. Considering ν_e as a test particle, the contribution to the neutrino damping rate arising from the background neutrinos is given by

$$\gamma = 8.1\frac{G_F^2}{\pi^3}\omega T^4, \tag{4.25}$$

where ω is the neutrino energy. Whereas the contribution of the electron and positron background to the ν_e damping rate yields

$$\gamma = 0.39\frac{G_F^2}{\pi^3}\omega T^4 \rho\left(\frac{m_e}{T}\right)\left[\epsilon\left(\frac{m_e}{T}\right) - 0.6\frac{m_e^2}{\omega T}\right], \tag{4.26}$$

where the functions ρ and ϵ are defined by

$$\rho(\xi)=\int_0^\infty dx\,\frac{x^2}{e^{\sqrt{x^2+\xi^2}}+1},$$
$$\epsilon(\xi)=\frac{1}{\rho(\xi)}\int_0^\infty dx\,\frac{x^3}{e^{\sqrt{x^2+\xi^2}}+1}. \tag{4.27}$$

Similar expressions can be obtained for the other processes. These results reduce to those in Refs. [6,9,10] in the limit of zero electron mass.

V. CONCLUSIONS

In this paper we have systematically derived the neutrino damping rate in real-time thermal field theory. Starting from the discontinuity of the neutrino self-energy at the two loop level, we prove that the damping rate can be expressed as integrals over phase space of total cross sections, weighted with statistical factors that account for the possibility of particle absorption or emission from the medium.

Cutkosky rules are used to obtain the self-energy imaginary part at zero temperature. Weldon [11] and Kobes and Semenoff [12] (see also [19]) have studied the corresponding Cutkosky rules at finite temperature. In these references it is shown that for certain specific examples at one loop order the discontinuity of the self-energy can be expressed in terms of amplitudes squared and weighted with the statistical factors that account for the various physical processes. Here we prove that these results stand valid for the neutrino damping rate at the two loop order.

The complete results that account for all possible processes that contribute to γ appear in Eq. (4.16). Depending on the physical conditions some of these processes are forbidden. Specific results for conditions such as those of the early universe, where the primeval plasma is composed of (anti-)neutrinos, (anti-)electrons, and nucleons, were obtained. For those conditions the fermion masses are not always negligible; consequently we report a general formula [Eq. (4.24)] that includes mass correction to first order in $m_f^2/\langle s\rangle$; however, further improvements are easily obtained utilizing the values for the cross sections in Eq. (4.22). Our results, summarized in Eq. (4.24) and Table II, should be useful for the study of neutrino processes in the early universe, as well as in some astrophysical scenarios.

ACKNOWLEDGMENTS

This work has been partially supported by CIC-UMSNH, grant 8.10. and by CONACyT grants No. 32540-E and G32723-E.

[1] P. Langacker, J.P. Leveille, and J. Sheiman, Phys. Rev. D **27**, 1228 (1983).
[2] H.A. Bethe, Phys. Rev. Lett. **56**, 1305 (1986).
[3] L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).
[4] S.P. Mikheyev and A.Yu. Smirnov, Sov. J. Nucl. Phys. **42**, 913 (1985).
[5] J.N. Bahcall, M.C. Gonzalez-Garcia, and Carlos Pena-Garay, J. High Energy Phys. **08**, 014 (2001); J.N. Bahcall, *Neutrino Astrophysics* (Cambridge University, Cambridge, England, 1989).
[6] D. Nötzold and G. Raffelt, Nucl. Phys. **B307**, 924 (1988).
[7] K. Enqvist, K. Kainulainen, and J. Maalampi, Nucl. Phys. **B349**, 754 (1991).
[8] J.C. D'Olivo, J. Nieves, and M. Torres, Phys. Rev. D **46**, 1172 (1992).
[9] K. Enqvist, K. Kainulainen, and M. Thomson, Nucl. Phys. **B373**, 498 (1992).
[10] P. Langacker and J. Liu, Phys. Rev. D **46**, 4140 (1992).
[11] A. Weldon, Phys. Rev. D **28**, 2007 (1983).
[12] R. Kobes and G.W. Semenoff, Nucl. Phys. **B260**, 714 (1985).
[13] N.P. Landsman and CH.G. van Weert, Phys. Rep. **145**, 141 (1987).
[14] M. Le Bellac, *Thermal Field Theory* (Cambridge University Press, Cambridge, England, 1996).
[15] J.F. Nieves, Phys. Rev. D **42**, 4123 (1990).
[16] A. Weldon, Phys. Rev. D **26**, 2789 (1982); **40**, 2410 (1989).
[17] E. Fermi, *Nuclear Physics* (University of Chicago Press, Chicago, 1949), p. 201; M.L. Goldberger and K.M. Watson, *Collision Theory* (Wiley, New York, 1964), p. 768.
[18] J.C. D'Olivo and J.F. Nieves, Phys. Rev. D **52**, 2987 (1995).
[19] P. Jizba, Phys. Rev. D **57**, 3634 (1998).