

Black hole, string ball, and p -brane production at hadronic supercolliders

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In models of large extra dimensions, the string and Planck scales become accessible at future colliders. When the energy scale is above the string scale or Planck scale a number of interesting phenomena occur, namely, production of stringy states, p -branes, string balls, black hole, etc. In this work, we systematically study the production cross sections of black holes, string balls, and p -branes at hadronic supercolliders. We also discuss their signatures. At the energy scale between the string scale M_s and M_s/g_s^2 , where g_s is the string coupling, the production is dominated by string balls, while beyond M_s/g_s^2 it is dominated by black holes. The production of a p -brane is only comparable to black holes when the p -brane wraps entirely on small extra dimensions. Rough estimates on the sensitivity reaches on the fundamental Planck scale M_D are also obtained, based on the number of raw events.

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I. INTRODUCTION

The standard model (SM) of particle physics, though it can fit most of the present data, leaves a few fundamental problems unsolved, one of which is the gauge hierarchy problem. Since the second revolution of string theories, a crop of models with extra dimensions have been proposed to solve various theoretical problems. In an attractive model of large extra dimensions or TeV quantum gravity (the ADD model) [1], the fundamental Planck scale can be as low as a few TeV. This is made possible by localizing the SM particles on a brane (using the idea of D-branes in type I or II string theory), while gravity is free to propagate in all dimensions. The observed Planck scale ($\sim 10^{19}$ GeV) is then a derived quantity. Extensive phenomenology studies have been carried out in recent years. Signatures for the ADD model can be divided into two categories: sub-Planckian and trans-Planckian. The former is the one that was studied extensively, while the latter just recently received more attention, especially black hole production in hadronic collisions.

A black hole (BH) has been illusive for decades, as we cannot directly measure any properties of it, not to mention the production of black holes in any terrestrial experiments. This is due to the fact that in order to produce black holes in collider experiments one needs a center-of-mass energy above the Planck scale ($M_{\text{Pl}} \sim 10^{19}$ GeV), which is obviously inaccessible at the moment.

In models of large extra dimensions, the properties of black holes are modified and interesting signatures emerge [2–4]. The fact that the fundamental Planck scale is as low as TeV also opens up an interesting possibility of producing a large number of black holes at collider experiments (e.g. LHC) [5,6]. Reference [4] showed that a BH localized on a brane will radiate mainly in the brane, instead of radiating into the Kaluza-Klein states of gravitons of the bulk. In this case, the BH so produced will decay mainly into the SM particles, which can then be detected in the detector. This

opportunity has enabled investigation of the properties of BH at terrestrial collider experiments. There have been a number of such studies [5–18] at hadronic colliders. A typical signature of the BH decay is a high multiplicity, isothermal event, very much like a spherical “fireball.” On the other hand, BH production has also been studied in cosmic ray experiments [19–26]. The ultrahigh energy cosmic rays (UHECR) serve as a very energetic beam hitting on the atmosphere as the target. The primary ingredient of UHECR is probably protons or light nuclei, or could be photons or even neutrinos. If it is made up of protons, it could produce a large number of BH’s up at the top of the atmosphere, producing a giant air shower. If it is neutrinos, it could produce very interesting horizontal air showers [20–24], or black holes within a km-sized neutrino telescope [25,26].

An important quantity of a BH is its entropy S_{BH} . To fulfill the thermodynamical description, a BH requires a large entropy of order of 25 [9]. Such an entropy requirement implies that the BH mass must be at least five times the fundamental Planck scale [8,9]. This mass requirement makes the BH production not as large as previously calculated in a number of works [5,6,17], first pointed out in Ref. [8]. In addition, the signature of large multiplicity decay of a BH can only happen when the entropy is large. Even taking into account this mass requirement, the event rate is still large enough for detection. On the other hand, there were arguments from Voloshin that the cross section should be multiplied by an exponential factor $\sim \exp(-S_{\text{BH}}/n+1)$ [27] (we shall get back to this point later in Sec. III). However, this suppression factor becomes too severe for the production rate to be interesting, contrary to the conclusion of Ref. [17], because of the large entropy requirement. There have been continuous theoretical efforts to calculate the production and decay of BH’s in particle collisions [28–36].

Other interesting trans-Planckian phenomena include string balls [37], p -branes [38–40], and TeV string behaviors [41,42]. Dimopoulos and Emparan [37] pointed out that when a BH reaches a minimum mass, it transitions into a state of highly excited and jagged strings—a string ball (SB). The transition point is at

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$$M_{\text{BH}}^{\text{min}} = \frac{M_s}{g_s^2}, \quad (1)$$

where M_s is the string scale and g_s is the string coupling. Naively, SB's are stringy progenitors of BH's. The BH correspondence principle states that properties of a BH with a mass $M_{\text{BH}} = M_s/g_s^2$ match those of a string ball with $M_{\text{SB}} = M_s/g_s^2$. We can then equate the production cross sections of SB and BH at the transition point. In fact, we shall use this argument to write down the cross section for the SB at the transition point. The existence of string balls could be argued from the string point of view. When the energy of the scattering reaches the string scale, the scattering of particles is no longer described by point-particle scattering but replaced by string-string scattering. As the energy goes further up, the strings become highly excited, jagged and entangled string states, and become like a string ball. When the energy reaches the transition point, it turns into a BH. Previously, in the discussion of a BH, we mentioned a large entropy requirement on the BH in order for the object to be a BH. Such a large mass requirement makes the production cross section smaller than previously thought. Here in the case of SB's, the mass requirement is substantially lower, thus the production rate is significantly higher. Hence, an SB is more interesting in the experimental point of view if it decays with a distinct signature. Dimopoulos and Emparan [37] argued that the decay of a SB is similar to that of a BH, i.e., a high multiplicity decay into the SM particles, though in some intermediate stages the SB decays more likely into bulk particles.

Another interesting trans-Planckian object is the p -brane. A BH can be considered a 0-brane. In particle collisions, if one considers BH production, one should also consider p -brane production. In fact, the properties of p -branes reduce to those of a BH in the limit $p \rightarrow 0$. In extra dimension models, in which there are large extra dimensions and small extra dimensions of the size of the Planck length, let a p -brane wrap on r small and $p-r$ large dimensions. It was found [38] that the production of p -branes is comparable to BH's only when $r=p$, i.e., the p -brane wraps entirely on the small dimensions only. If $r < p$, the production of p -branes would be suppressed by powers of (M_*/M_{Pl}) , where M_* is the fundamental scale of the $4+n$ dimensions. Therefore, here we only consider the case in which $r=p$. The decay of p -branes is not well understood. One interesting possibility is cascade into branes of lower dimensions until they reach the dimension of zero. Whether the zero brane is stable depends on the model. Another possibility is the decay into brane and bulk particles, thus experimentally the decay can be observed. Or it can be a combination of cascade into lower-dimensional branes and direct decays.

In this work, we study the production rates of the BH's, SB's, and p -branes in hadronic collisions, with emphasis on the LHC and the VLHC. The organization is as follows. In the next section, we briefly describe the relation between the fundamental Planck scale and the string scale. In Sec. III, we describe the production of BH's, SB's, and p -branes. In Sec. IV, we show our numerical results for the LHC and VLHC. We discuss the decays in Sec. V and conclude in Sec. VI.

II. PLANCK AND STRING SCALES

First let us address more clearly the configuration of the space-time. Let there be n total extra dimensions with m small extra dimensions and $n-m$ large extra dimensions. When we say small extra dimensions, we mean the size is of order of $1/M_*$, the fundamental Planck scale. The observed 4D Planck scale M_{Pl} is then a derived quantity given by [1]

$$M_{\text{Pl}}^2 = M_*^{2+n} V_m V_{n-m}, \quad (2)$$

where V_m and V_{n-m} are the volumes of the extra m and $n-m$ dimensions, respectively, given by

$$V_m = L_m^m \equiv \left(\frac{l_m}{M_*} \right)^m, \quad V_{n-m} = L_{n-m}^{n-m} \equiv \left(\frac{l_{n-m}}{M_*} \right)^{n-m}, \quad (3)$$

where we have expressed the lengths L_m, L_{n-m} in units of Planckian length $1/M_*$.¹ Suppose the small extra dimension has the size of $L_m \sim 1/M_*$, i.e., $l_m \sim 1$, then

$$M_{\text{Pl}}^2 = M_*^2 (l_{n-m})^{n-m}. \quad (4)$$

The fundamental Planck scale M_* is lowered to the TeV range if the size L_{n-m} is taken to be very large, of order $O(\text{mm})$.

The relation of the observed Planck scale to the string scale M_s is given by [43]

$$M_{\text{Pl}}^2 \sim \frac{M_s^{2+n}}{g_s^2} V_m V_{n-m}, \quad (5)$$

where we again take the small extra dimensions of size $L_m \sim 1/M_*$. From Eqs. (2) and (5) we can relate the string scale with the fundamental Planck scale as

$$M_*^{2+n} \sim \frac{M_s^{2+n}}{g_s^2}, \quad (6)$$

where we take the proportional constant of order $O(1)$, which depends on different compactification configurations. We shall also use the more conventional definition of the fundamental Planck scale M_D related to M_* by

$$M_D^{n+2} = \frac{(2\pi)^n}{8\pi G_{4+n}} = \frac{(2\pi)^n}{8\pi} M_*^{n+2}, \quad (7)$$

where G_{4+n} is the gravitational constant in $D=4+n$ dimensions (used in the Einstein equation: $\mathcal{R}_{AB} - \frac{1}{2}g_{AB}\mathcal{R} = -8\pi G_{4+n}T_{AB}$). Then M_D is related to the string scale M_s as

$$M_D^{n+2} = K \frac{M_s^{n+2}}{g_s^2}, \quad (8)$$

¹In the case of toroidal compactification, the length $L_i = 2\pi R_i$ ($i = m, n-m$) where R_i is the radius of the torus.

for a constant K of order $O(1)$. In the next section we shall simply use $K=1$ for discussions and cross-section calculations.

III. PRODUCTION

A. Black holes

A black hole is characterized by its Schwarzschild radius R_{BH} , which depends on the mass M_{BH} of the BH. A simplified picture for BH production is as follows. When the colliding partons have a center-of-mass (c.o.m.) energy above some thresholds of order of the Planck mass and the *impact parameter* less than the Schwarzschild radius R_{BH} , a BH is formed and almost at rest in the c.o.m. frame. The BH so produced will decay thermally (regardless of the incoming particles) and thus isotropically in that frame.

This possibility was first investigated for the LHC in Refs. [5,6]. In Refs. [5–7], black hole production in hadronic collisions is calculated in $2 \rightarrow 1$ subprocesses: $ij \rightarrow \text{BH}$, where i, j are incoming partons. The black hole so produced is either at rest or traveling along the beam pipe such that its decay products (of high multiplicity) have a zero net transverse momentum (p_T). Giddings and Thomas [5] and Dimopoulos and Landsberg [6] demonstrated that a BH so produced will decay with a high multiplicity.

In Ref. [8], we pointed out the “ $ij \rightarrow \text{BH} + \text{others}$ ” subprocesses, such that the BH is produced with a large p_T before it decays. The “ $ij \rightarrow \text{BH} + \text{others}$ ” subprocesses can be formed when the c.o.m. energy of the colliding particles is larger than the BH mass; the excess energy will be radiated as other SM particles.² In such subprocesses, the “others” are just the ordinary SM particles and usually of much lower multiplicity than the decay products of the BH. Therefore, the signature is as follows: on one side of the event there are particles of high multiplicity (from the decay of the BH), the total p_T of which is balanced by a much lower number of particles on the other side. Such a signature is very clean and should have very few backgrounds.

The next natural question to ask is how large the event rate is. Collider phenomenology is only possible if the event rate is large enough, especially if we want to study BH properties. It was first pointed out in Ref. [6] that the BH production rate is so enormous that the LHC is in fact a BH factory. However, this has not taken into account the entropy factor of the BH. It was shown in Ref. [8] that with a large entropy requirement ($S_{\text{BH}} \gtrsim 25$) the BH production rate decreases substantially, but still affords enough events for detection. For example, the production cross section is as high as 1 pb

²Another viewpoint on BH formation is as follows. The BH is formed when the two partons approach each other in a very short distance ($< R_{\text{BH}}$), and everything, including the “extra partons,” will be hidden behind the horizon. Thus, the entire energy is contained within the BH, which is considered a quantum state rather than a particle. In this picture, there is no transverse momentum given to the BH, unless by the initial-state radiation of the incoming partons. This was already studied by a Monte Carlo approach in Ref. [6].

for $M_D = 1.5$ TeV and $n=4$ with $M_{\text{BH}}^{\text{min}} = 5M_D$ at the LHC. This implies 10^5 events with an integrated luminosity of 100 fb^{-1} . Of course, if we relaxed this entropy constraint, the production cross sections would be increased tremendously, but the cross sections have to be interpreted with care because of the presence of large string effects in this regime.

The Schwarzschild radius R_{BH} of a BH of mass M_{BH} in $4+n$ dimensions is given by [44]

$$\begin{aligned} R_{\text{BH}} &= \frac{1}{M_D} \left(\frac{M_{\text{BH}}}{M_D} \right)^{1/(n+1)} \left(\frac{2^n \pi^{(n-3)/2} \Gamma\left(\frac{n+3}{2}\right)}{n+2} \right)^{1/(n+1)} \\ &= \frac{1}{M_D} \left(\frac{M_{\text{BH}}}{M_D} \right)^{1/(n+1)} f(n), \end{aligned} \quad (9)$$

where $f(n)$ is introduced for convenience and M_D is the fundamental Planck scale in the model of large extra dimensions already defined in Eq. (7). The radius R_{BH} is much smaller than the size of the extra dimensions. BH production is expected when the colliding partons with a center-of-mass energy $\sqrt{s} \gtrsim M_{\text{BH}}$ pass within a distance less than R_{BH} . A black hole of mass M_{BH} is formed and the rest of the energy, if there is any, is radiated as ordinary SM particles. This semiclassical argument calls for a geometric approximation for the cross section for producing a BH of mass M_{BH} as

$$\sigma(M_{\text{BH}}^2) \approx \pi R_{\text{BH}}^2. \quad (10)$$

In the $2 \rightarrow 1$ subprocess, the c.o.m. energy of the colliding partons is just the same as the mass of the BH, i.e., $\sqrt{s} = M_{\text{BH}}$, which implies a subprocess cross section

$$\hat{\sigma}(\hat{s}) = \int d\left(\frac{M_{\text{BH}}^2}{\hat{s}}\right) \pi R_{\text{BH}}^2 \delta(1 - M_{\text{BH}}^2/\hat{s}) = \pi R_{\text{BH}}^2. \quad (11)$$

On the other hand, for the $2 \rightarrow k$ ($k \geq 2$) subprocesses the subprocess cross section is

$$\hat{\sigma}(\hat{s}) = \int_{(M_{\text{BH}}^2)_{\text{min}}/\hat{s}}^1 d\left(\frac{M_{\text{BH}}^2}{\hat{s}}\right) \pi R_{\text{BH}}^2. \quad (12)$$

Another important quantity that characterizes a BH is its entropy given by [44]

$$\begin{aligned} S_{\text{BH}} &= \frac{4\pi}{n+2} \left(\frac{M_{\text{BH}}}{M_D} \right)^{(n+2)/(n+1)} \\ &\quad \times \left(\frac{2^n \pi^{(n-3)/2} \Gamma\left(\frac{n+3}{2}\right)}{n+2} \right)^{1/(n+1)}. \end{aligned} \quad (13)$$

The variation of S_{BH} versus the ratio M_{BH}/M_D is shown in Fig. 1. To ensure the validity of the above classical description of a BH [9], the entropy must be sufficiently large, of order 25 or so. From the figure we can see that when $M_{\text{BH}}/M_D \gtrsim 5$, the entropy $S_{\text{BH}} \gtrsim 25$. Therefore, to avoid getting into the nonperturbative regime of the BH and to ensure

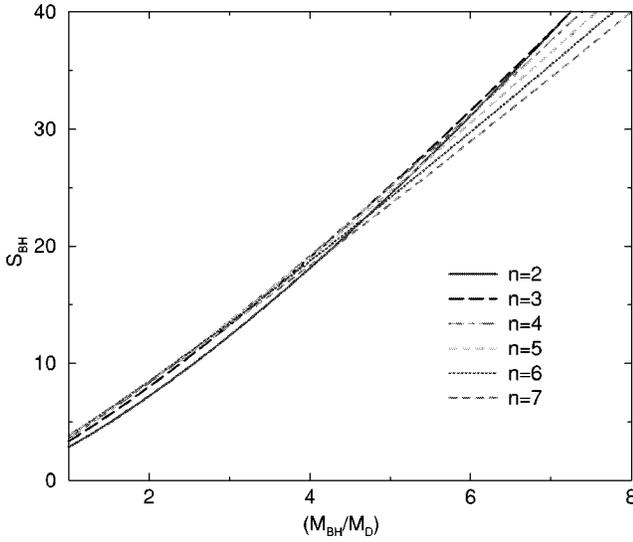


FIG. 1. The entropy S_{BH} of a black hole vs the ratio (M_{BH}/M_D) in $4+n$ dimensions.

the validity of the semiclassical formula, we restrict the mass of the BH to be $M_{\text{BH}} \geq y M_D$, where $y \equiv M_{\text{BH}}^{\text{min}}/M_D$ is of order 5.

Voloshin [27] pointed out that the semiclassical argument for the BH production cross section is not given by the geometrical cross-section area, but, instead, suppressed by an exponential factor,

$$\exp\left(-\frac{S_{\text{BH}}}{n+1}\right). \quad (14)$$

There are, however, counter arguments [9,37] that the simple geometric formula should be valid.³ In Ref. [8], we have considered both forms of cross sections: the naive πR_{BH}^2 and the πR_{BH}^2 multiplied with the exponential factor of Eq. (14). The suppression factor renders the cross section too small for detection, because the exponential factor contains the entropy S_{BH} , which has to be sufficient large (e.g., ≥ 25) to define a black hole. The suppression is more than two orders of magnitude and, therefore, we shall not be concerned with this suppression factor anymore. This seems to contradict the results of Ref. [17], in which Rizzo concluded that even when Voloshin's suppression factor is included, the cross section is still large enough, because he did not impose a large entropy requirement on the validity of the BH. If he had done so, he would also have gotten a very large suppression. Nevertheless, careful interpretation is needed if the BH has only a small entropy.

³The controversy of using the exponential suppression factor seems to be resolved by now. The naive geometric cross section is correct, as pointed out or even derived by various authors in Refs. [28–37]. In particular, Ref. [32] explicitly pointed out a logical error by Voloshin and derived the geometric cross-section formula.

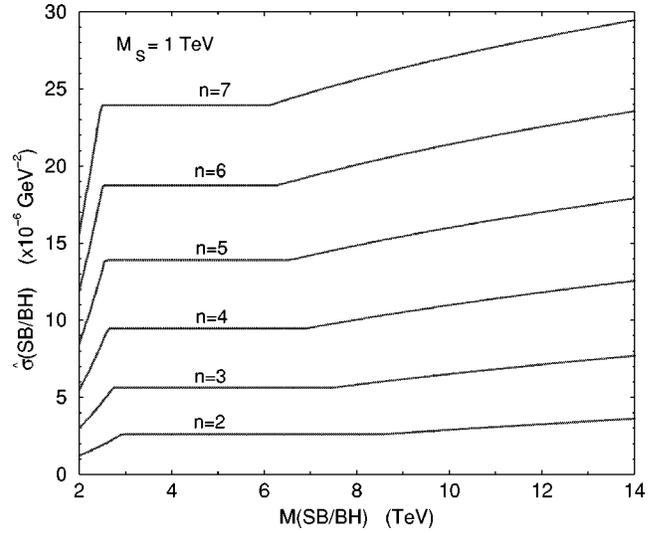


FIG. 2. The subprocess cross section $\hat{\sigma}(\text{SB/BH})$ for string ball or black hole vs the mass of SB or BH. Here we have used a string scale $M_s = 1$ TeV, and we require the SB-BH correspondence point at $M_s/g_s^2 = 5M_D$, where M_D is related to M_s by $M_s = M_D g_s^{2/(n+2)}$.

B. String balls

Dimopoulos and Emparan [37] pointed out that when a BH reaches a minimum mass, it transitions into a state of highly excited and jagged strings, dubbed a string ball. They are the stringy progenitors of BH's and share some properties of BH's, such as large production cross sections at hadronic supercolliders and similar signatures when they decay. They made an important observation [37] that the minimum mass $M_{\text{BH}}^{\text{min}}$ (the transition point) above which a BH can be treated general-relativistically is M_s/g_s^2 , where M_s and g_s are the string scale and the string coupling, respectively. Below this transition point, the configuration is dominated by string balls. Since the mass of a string ball is lower than a BH, the corresponding production cross section is larger than that of a BH. Thus, at the LHC, string ball production may be more important.

According to the BH correspondence principle, the properties of a BH with a mass $M_{\text{BH}} = M_s/g_s^2$ match those of a string ball with a mass $M_{\text{SB}} = M_s/g_s^2$. Therefore, the production cross section of a string ball or a BH should be smoothly joined at $M_{\text{BH}} = M_s/g_s^2$, i.e.,

$$\sigma(\text{SB})|_{M_{\text{SB}}=M_s/g_s^2} = \sigma(\text{BH})|_{M_{\text{BH}}=M_s/g_s^2}.$$

The production cross section for string balls with mass between the string scale M_s and M_s/g_s grows with s until M_s/g_s , beyond which, due to unitarity, it should stay constant. Therefore, we can use the BH cross section and match to the string ball cross section at the transition point M_s/g_s^2 . This string ball cross section then stays constant between M_s/g_s and M_s/g_s^2 . Then below M_s/g_s the string ball cross section grows like M_{SB}^2/M_s^4 .

The cross sections for the SB or BH are given by

$$\hat{\sigma}(\text{SB/BH}) = \begin{cases} \frac{\pi}{M_D^2} \left(\frac{M_{\text{BH}}}{M_D} \right)^{2(n+1)} [f(n)]^2, & \frac{M_s}{g_s} \leq M_{\text{BH}} \\ \frac{\pi}{M_D^2} \left(\frac{M_s/g_s^2}{M_D} \right)^{2(n+1)} [f(n)]^2 = \frac{\pi}{M_s^2} [f(n)]^2, & \frac{M_s}{g_s} \leq M_{\text{SB}} \leq \frac{M_s}{g_s^2} \\ \frac{\pi g_s^2 M_{\text{SB}}^2}{M_s^4} [f(n)]^2, & M_s \leq M_{\text{SB}} \leq \frac{M_s}{g_s}, \end{cases} \quad (15)$$

in which we have set $M_D^{n+2} = M_s^{n+2}/g_s^2$ as in Eq. (8) with $K=1$. A graphical presentation of these cross sections is shown in Fig. 2 for $n=2-7$.

In the next section, when we calculate the production cross sections for BH's and SB's, we use the above equation, together with Eq. (8) with $K=1$. The production then depends on the following parameters: M_s , g_s , n , and M_D . The M_D can be determined by Eq. (8). We also require that at the BH-SB transition point, $M_{\text{BH}}^{\text{min}} = M_s/g_s^2$, the mass of the BH is already at $5M_D$ (this ensures that the BH has a sufficiently large entropy ~ 25 [9]). Therefore, the production cross sections depend on M_s and n only. We shall present the results in terms of M_D and n for easy comparison with existing literature.

C. p -branes

A black hole can be considered a zero-brane. In principle, higher-dimensional objects, e.g., p -branes ($p\text{B}$), can also be formed in particle collisions, in particular when there exist small extra dimensions of the size $\sim 1/M_*$ in addition to the large ones of the size $\gg 1/M_*$. It was pointed out by Ahn *et al.* [38] that the production cross section of a p -brane completely wrapped on the small extra dimensions is larger than that of a spherically symmetric black hole. A similar situation is true in cosmic ray experiments [39,40].

Consider an uncharged and static p -brane with a mass $M_{p\text{B}}$ in $(4+n)$ -dimensional space-time (m small Planckian size and $n-m$ large size extra dimensions such that $n \geq p$). Suppose the p -brane wraps on r ($\leq m$) small extra dimensions and on $p-r$ ($\leq n-m$) large extra dimensions. Then the "radius" of the p -brane is

$$R_{p\text{B}} = \frac{1}{\sqrt{\pi} M_*} \gamma(n,p) V_{p\text{B}}^{-1/(1+n-p)} \left(\frac{M_{p\text{B}}}{M_*} \right)^{1/(1+n-p)}, \quad (16)$$

where $V_{p\text{B}}$ is the volume wrapped by the p -brane in units of the Planckian length. Recall from Eq. (2), $M_{\text{Pl}}^2 = M_*^2 l_{n-m}^{n-m} l_m^m$, where $l_{n-m} \equiv L_{n-m} M_*$ and $l_m \equiv L_m M_*$ are the lengths of the size of the large and small extra dimensions in units of Planckian length ($\sim 1/M_*$). Then $V_{p\text{B}}$ is given by

$$V_{p\text{B}} = l_{n-m}^{p-r} l_m^r \approx \left(\frac{M_{\text{Pl}}}{M_*} \right)^{2(p-r)/(n-m)}, \quad (17)$$

where we have taken $l_m \equiv L_m M_* \sim 1$. The function $\gamma(n,p)$ is given by

$$\gamma(n,p) = \left[8 \Gamma \left(\frac{3+n-p}{2} \right) \sqrt{\frac{1+p}{(n+2)(2+n-p)}} \right]^{1/(1+n-p)}. \quad (18)$$

The $R_{p\text{B}}$ reduces to the R_{BH} in the limit $p=0$.

The production cross section of a p -brane is similar to that of BH's, based on a naive geometric argument [38]. When the partons collide with a center-of-mass energy \sqrt{s} larger than the fundamental Planck scale and an impact parameter less than the size of the p -brane, a p -brane of mass $M_{p\text{B}} \leq \sqrt{s}$ can be formed. That is,

$$\hat{\sigma}(M_{p\text{B}}) = \pi R_{p\text{B}}^2. \quad (19)$$

Therefore, the production cross section for a p -brane is the same as BH's in the limit $p=0$ (i.e., a BH can be considered a 0-brane). In $2 \rightarrow 1$ and $2 \rightarrow k$ ($k \geq 2$) processes, the parton-level cross sections are given by similar expressions in Eqs. (11) and (12), respectively.

In Eq. (16), we can see that the radius of a p -brane is suppressed by some powers of the volume $V_{p\text{B}}$ wrapped by the p -brane. It is then obvious that the production cross section is largest when $V_{p\text{B}}$ is minimal, in other words, the p -brane wraps entirely on the small extra dimensions only, i.e., $r=p$. When $r=p$, $V_{p\text{B}}=1$. We can also compare the production cross section of p -branes with BH's. Assuming that their masses are the same and the production threshold M^{min} is the same, the ratio of cross sections is

$$\begin{aligned} R &\equiv \frac{\hat{\sigma}(M_{p\text{B}}=M)}{\hat{\sigma}(M_{\text{BH}}=M)} \\ &= \left(\frac{M_*}{M_{\text{Pl}}} \right)^{4(p-r)/[(n-m)(1+n-p)]} \\ &\quad \times \left(\frac{M}{M_*} \right)^{2p/[(1+n)(1+n-p)]} \left(\frac{\gamma(n,p)}{\gamma(n,0)} \right)^2. \end{aligned} \quad (20)$$

In the above equation, the most severe suppression factor is in the first set of parentheses on the right-hand side. Since we are considering physics of TeV M_* , the factor $(M_*/M_{\text{Pl}}) \sim 10^{-16} - 10^{-15}$. Thus, the only meaningful production of a

TABLE I. The ratio $R \equiv \hat{\sigma}(M_{pB}=M)/\hat{\sigma}(M_{BH}=M)$ of Eq. (20) for various n and p with $n-m \geq 2$. We have used $M_D = 1.5$ TeV and $M_{BH} = M_{pB} = 5M_D$. We have assumed that the p -brane wraps entirely on small extra dimensions, i.e., $r=p$. In order to obtain the largest ratio R we have chosen $p=m$.

	$p=0$	$p=1$	$p=2$	$p=3$	$p=4$	$p=5$
$n=2$	1					
$n=3$	1	1.77				
$n=4$	1	1.41	2.46			
$n=5$	1	1.25	1.72	3.02		
$n=6$	1	1.17	1.42	1.94	3.46	
$n=7$	1	1.12	1.27	1.54	2.10	3.78

p -brane occurs for $r=p$, and then their production is comparable. In Table I, we show this ratio for various values of n and p .

IV. PRODUCTION AT THE LHC AND VLHC

The production of BH's and SB's depends on M_s, n, M_D, g_s , but they are related by Eq. (8). Since we also require the transition point (M_s/g_s^2) at $5M_D$, we can therefore solve for M_s and g_s for a given pair of M_D and n . We present the results in terms of M_D and n . The minimum mass requirement for the SB is set at $2M_s$. The production of a p -brane also depends on m and r . For an interesting level of event rates, r has to be equal to p , i.e., the p -brane wraps entirely on small (of Planck length) extra dimensions. So after setting all parameters, we are ready to present our numerical results.

In Fig. 3, we show the total production cross sections for BH's, SB's, and p -branes, including the $2 \rightarrow 1$ and $2 \rightarrow 2$ sub-

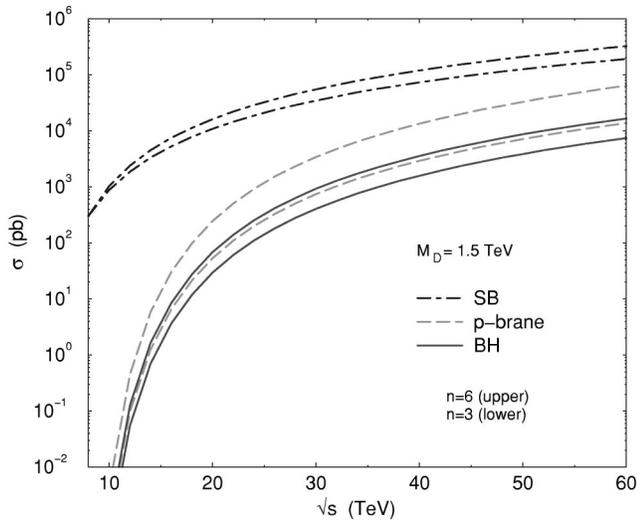


FIG. 3. Total production cross section σ including $2 \rightarrow 1$ and $2 \rightarrow 2$ processes for black hole (BH), string ball (SB), and p -brane (pB) at pp collisions vs \sqrt{s} for $n=3$ and 6 . Here we have used a fundamental scale $M_D = 1.5$ TeV. The minimum mass on the BH and p -brane is $M_{BH}^{\min}, M_{pB}^{\min} = 5M_D$, while that on SB is $M_{SB}^{\min} = 2M_s$. $M_s = 1.0$ and 1.2 TeV for $n=3$ and 6 , respectively, in our scheme.

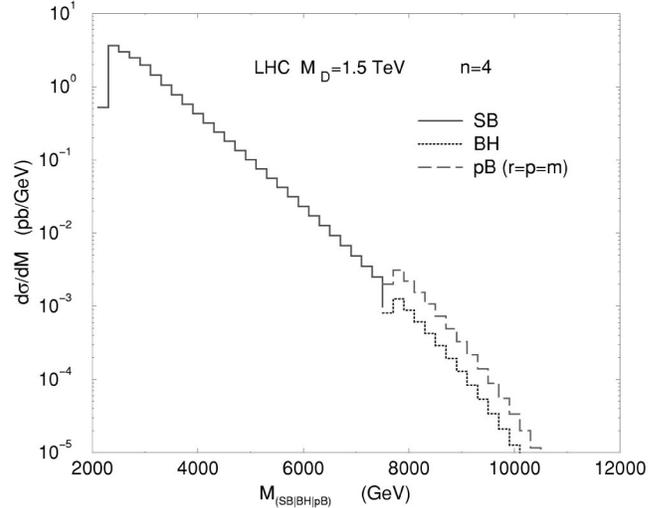


FIG. 4. Differential cross section $d\sigma/dM$ vs the mass M of black hole (BH), string ball (SB), or p -brane (pB) at the LHC. Here we have used a fundamental scale $M_D = 1.5$ TeV and $n=4$. The minimum mass on the BH and p -brane is $M_{BH}^{\min}, M_{pB}^{\min} = 5M_D$, while that on SB is $M_{SB}^{\min} = 2M_s$. $M_s = 1.1$ TeV for $n=4$.

processes (when computing the $2 \rightarrow 2$ subprocess we require a p_T cut of 500 GeV to prevent double counting). Typically, the $2 \rightarrow 2$ subprocess contributes at a level of less than 10%. For the BH, SB, and p -brane, we show the results for $n=3$ and $n=6$. The results for $n=4,5$ lie in between. Since we require $M_{BH}^{\min}, M_{pB}^{\min} = 5M_D$, their production is only sizable when \sqrt{s} reaches about 10 TeV, unlike the SB, which only requires $M_{SB}^{\min} = 2M_s$. The p -brane cross section is about a few times larger than the BH, as we have chosen $r=p=m=n-2$. String ball production is, on average, two orders of magnitude larger than that of a BH in the energy range between 20 and 60 TeV. Below 20 TeV (e.g., at the LHC), the SB cross section is at least three orders of magnitude larger than the BH.

Now we particularly look at the production rates at the LHC, operating at $\sqrt{s} = 14$ TeV with a nominal yearly luminosity of 100 fb^{-1} . The differential cross sections $d\sigma/dM$, where $M = M_{BH}, M_{SB}, M_{pB}$, are shown in Fig. 4, where we have shown the case of $n=4$ and $M_D = 1.5$ TeV. In our scheme, $M_s \approx 1.1$ TeV. The minimum SB mass starts at $2M_s \approx 2.2$ TeV, while the BH and p -brane start at $5M_D = 7.5$ TeV. The SB spectrum smoothly joined to the BH spectrum at the transition point $M_s/g_s^2 = 5M_D$. Similarly, the transverse momentum spectra for their production are shown in Fig. 5. Even at a very high $p_T \geq 1$ TeV, the cross section is still large enough for detection. We show the integrated cross sections for the LHC in Table II, including contributions from $2 \rightarrow 1$ and $2 \rightarrow 2$ processes (we imposed a p_T cut of 500 GeV in the $2 \rightarrow 2$ process).

Sensitivity information can be drawn from the table. The event rates for BH and p -brane production are negligible for $M_D = 2.5$ TeV and only moderate at $M_D = 2$ TeV. At $M_D = 2$ TeV, the number of BH events that can be produced in one year running (100 fb^{-1}) is about 120–340 for $n=3-7$ while the number for p -brane events is 210–1300.

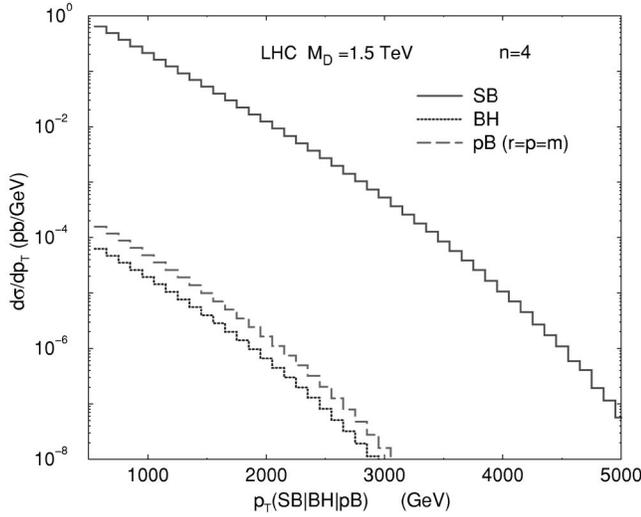


FIG. 5. Differential cross section $d\sigma/dp_T$ vs the transverse momentum p_T of black hole (BH), string ball (SB), or p -brane (pB) at the LHC. Here we have used a fundamental scale $M_D=1.5$ TeV and $n=4$. The minimum mass on the BH and p -brane is $M_{BH}^{\min}, M_{pB}^{\min}=5M_D$, while that on SB is $M_{SB}^{\min}=2M_s$. $M_s=1.1$ TeV for $n=4$.

Therefore, the sensitivity for a detectable signal rate for a BH and a p -brane is only around 2 TeV, if not much larger than 2 TeV. The SB event rate is much higher. Even at $M_D=3$ TeV, the cross section is of order of 30 pb. In Table II, we also show the $\sigma(SB)$ for $M_D=4-6$ TeV. Roughly, the sensitivity is around 6 TeV.

TABLE II. Total cross sections in pb for the production of BH, SB, and p -brane, for various values of n and M_D at the LHC. The minimum mass on the BH and p -brane is $M_{BH}^{\min}, M_{pB}^{\min}=5M_D$, while that on SB is $M_{SB}^{\min}=2M_s$.

	$n=3$	$n=5$	$n=7$
BH			
M_D (TeV)			
1.5	0.70	1.3	1.9
2.0	1.2×10^{-3}	2.2×10^{-3}	3.4×10^{-3}
2.5	1.3×10^{-8}	2.4×10^{-8}	3.6×10^{-8}
SB			
M_D (TeV)			
1.5	3300	4100	4900
2.0	590	670	760
2.5	130	130	140
3.0	33	29	28
4.0	2.4	1.5	1.1
5.0	0.16	0.060	0.033
6.0	0.0091	0.0015	0.00044
p -brane			
M_D (TeV)			
1.5	1.2	4.0	7.6
2.0	2.1×10^{-3}	6.9×10^{-3}	0.013
2.5	2.3×10^{-8}	7.3×10^{-8}	1.4×10^{-7}

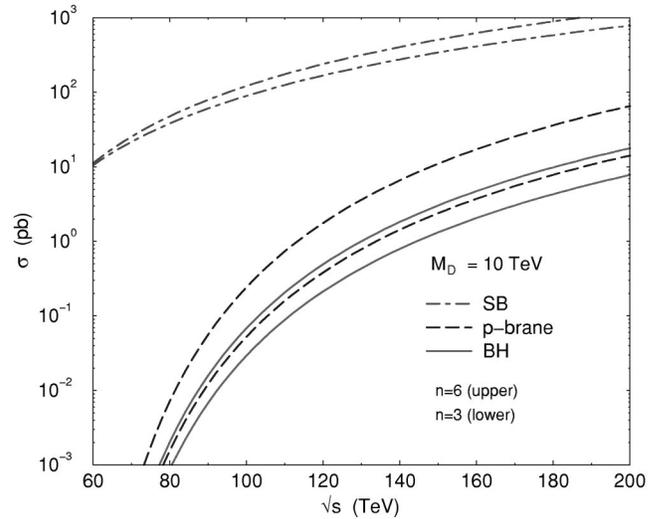


FIG. 6. Total production cross section σ including $2 \rightarrow 1$ and $2 \rightarrow 2$ processes for black hole (BH), string ball (SB), and p -brane (pB) at pp collisions vs \sqrt{s} from 60–200 TeV for $n=3$ and 6. Here we have used a fundamental scale $M_D=10$ TeV. The minimum mass on the BH and p -brane is $M_{BH}^{\min}, M_{pB}^{\min}=5M_D$, while that on SB is $M_{SB}^{\min}=2M_s$. $M_s=6.7$ and 7.9 TeV for $n=3$ and 6, respectively, in our scheme.

The VLHC (very large hadron collider) is another pp accelerator under discussion [45] in Snowmass 2001 [46]. The preliminary plan is to have an initial stage of about 40–60 TeV center-of-mass energy, and later an increase up to 200 TeV. The targeted luminosity is $(1-2) \times 10^{34} \text{ cm}^{-2} \text{ s}^{-1}$. In Fig. 6, we show the total production cross sections for BH's, SB's, and p -branes for $\sqrt{s}=60-200$ TeV and for $n=3$ and 6. The integrated cross sections for $\sqrt{s}=50, 100, 150$, and 200 TeV are shown in Table III. For a fixed M_D , the cross section obviously increases with \sqrt{s} . We choose to show the event rates for different values of M_D such that it roughly gives an idea about the sensitivity reach at each \sqrt{s} . We found that the sensitivity reaches for BH and p -brane production are roughly between 6 and 7 TeV for $\sqrt{s}=50$ TeV, 10 and 13 TeV for $\sqrt{s}=100$ TeV, 14 and 18 TeV for $\sqrt{s}=150$ TeV, and 20 and 25 TeV for $\sqrt{s}=200$ TeV. These estimates are rather crude based on the requirement that the number of raw events is $\geq 50-100$.

V. DECAY SIGNATURES

A. Black holes

The main phase of the decay of a BH is via the Hawking evaporation. The evaporation rate is governed by its Hawking temperature, given by [44]

$$T_{BH} = \frac{n+1}{4\pi R_{BH}}, \quad (21)$$

which scales inversely with some powers of M_{BH} . The heavier the BH, the lower is the temperature. Thus, the evaporation rate is slower. The lifetime of the BH also scales inversely with the Hawking temperature as given by

TABLE III. Total cross sections in pb for the production of BH, SB, and p -brane, for various values of n and M_D in pp collisions with $\sqrt{s}=50,100,150,200$ TeV. The minimum mass on the BH and p -brane is $M_{\text{BH}}^{\text{min}}, M_{p\text{B}}^{\text{min}}=5M_D$, while that on SB is $M_{\text{SB}}^{\text{min}}=2M_s$.

	$n=3$	$n=5$	$n=7$
$\sqrt{s}=50$ TeV			
$M_D(\text{TeV})$		BH	
5.0	0.13	0.24	0.36
6.0	5.6×10^{-3}	0.010	0.016
7.0	1.3×10^{-4}	2.5×10^{-4}	3.7×10^{-4}
$M_D(\text{TeV})$		SB	
5.0	370	460	550
6.0	130	150	180
7.0	49	55	62
$M_D(\text{TeV})$		p -brane	
5.0	0.23	0.73	1.4
6.0	0.010	0.032	0.061
7.0	2.4×10^{-4}	7.6×10^{-4}	0.0014
$\sqrt{s}=100$ TeV			
$M_D(\text{TeV})$		BH	
8	0.49	0.91	1.4
10	0.029	0.055	0.082
13	2.2×10^{-4}	4.2×10^{-4}	6.3×10^{-4}
$M_D(\text{TeV})$		SB	
8	300	390	480
10	89	110	130
13	19	21	25
$M_D(\text{TeV})$		p -brane	
8	0.89	2.9	5.4
10	0.053	0.17	0.32
13	4.0×10^{-4}	0.0013	0.0024
$\sqrt{s}=150$ TeV			
$M_D(\text{TeV})$		BH	
10	1.3	2.4	3.6
14	0.033	0.061	0.092
18	5.6×10^{-4}	0.0011	0.0016
$M_D(\text{TeV})$		SB	
10	340	450	560
14	57	71	86
18	13	16	18
$M_D(\text{TeV})$		p -brane	
10	2.4	7.7	14
14	0.059	0.19	0.36
18	0.0010	0.0032	0.0061
$\sqrt{s}=200$ TeV			
$M_D(\text{TeV})$		BH	
10	7.7	14	21
15	0.23	0.43	0.64
20	0.0070	0.013	0.020
25	1.4×10^{-4}	2.5×10^{-4}	3.8×10^{-4}
$M_D(\text{TeV})$		SB	
10	780	1100	1400
15	100	130	160
20	21	26	31
25	5.7	6.6	7.6
$M_D(\text{TeV})$		p -brane	
10	14	46	86
15	0.42	1.3	2.5
20	0.013	0.040	0.076
25	2.4×10^{-4}	7.8×10^{-4}	0.0015

$$\tau \sim \frac{1}{M_D} \left(\frac{M_{\text{BH}}}{M_D} \right)^{(n+3)/(n+1)}. \quad (22)$$

From the above equation, it is obvious that the lifetime of a BH becomes much longer in models of large extra dimensions than in the usual 4D theory. However, the lifetime is still so short that it will decay once it is produced and no displaced vertex can be seen in the detector. For another viewpoint on the BH decay, please see Ref. [28].

An important observation is that the wavelength λ of the thermal spectrum corresponding to the Hawking temperature is larger than the size of the BH. This implies that the BH evaporates like a point source in s waves, therefore it decays equally into brane and bulk modes, and will not see the higher angular momentum states available in the extra dimensions. Since on the brane there are many more particles than in the bulk, the BH decays dominantly into brane modes, i.e., the SM particles in the setup. Furthermore, the BH evaporates “blindly” into all degrees of freedom. The ratio of the degrees of freedom for gauge bosons, quarks, and leptons is 29:72:18 (the Higgs boson is not included). Since the W and Z decay with a branching ratio of about 70% into quarks, and the gluon also gives rise to hadronic activities, the final ratio of hadronic to leptonic activities in the BH decay is about 5:1 [5].

Another important property of the BH decay is the large number of particles, in accord with the large entropy in Eq. (13), in the process of evaporation. It was shown [5,6] that the average multiplicity $\langle N \rangle$ in the decay of a BH is order of 10–30 for M_{BH} being a few times M_D for $n=2-6$. Since we are considering the BH that has an entropy of order 25 or more, it guarantees a high multiplicity BH decay. The BH decays more or less isotropically and each decay particle has an average energy of a few hundred GeV. Therefore, if the BH is at rest, the event is very much like a spherical event with many particles of hundreds of GeV pointing back to the interaction point (very much like a fireball). On the other hand, if the BH is produced in association with other SM particles (as in a $2 \rightarrow k$ subprocess), the BH decay will be a boosted spherical event on one side (a boosted fireball), the transverse momentum of which is balanced by a few particles on the other side [8]. Such spectacular events should have a negligible background.

B. String balls

Highly excited long strings emit massless quanta with a thermal spectrum at the *Hagedorn* temperature. (The Hagedorn temperature of an excited string matches the Hawking temperature of a BH at the corresponding point $M_{\text{BH}}^{\text{min}} \equiv M_s/g_s^2$.)

At $M_{\text{SB}} \leq M_s/g_s^2$, the wavelength λ corresponding to the thermal spectrum at the Hagedorn temperature is larger than R_{SB} . This argument is very similar to that of the BH, and so the string ball radiates like a point source and emits in s waves equally into brane and bulk modes. With many more particles (SM particles) on the brane than in the bulk, the SB radiates mainly into the SM particles.

When M_{SB} goes below M_s/g_s^2 , the SB has the tendency to puff up to a *random-walk size* as large as the λ of the emissions [37]. Therefore, it will see more of the higher angular momentum states available in the extra dimensions. Thus, it decays more into the bulk modes, but it is only temporary. When the SB decays further, it shrinks back to the string size and emits as a point source again [37]. Most of the time the SB decays into SM particles. On average, a SB decays into invisible quanta somewhat more often than a BH does.

High multiplicity decay of the BH should also apply to the SB, at least when the mass of the SB is close to the correspondence point [37]. Naively, we expect that if the SB mass decreases, the multiplicity will decrease. Thus, the signature of the SB is very similar to the BH, except that it may have lower multiplicity.

C. p -branes

The decay of p -branes is not well understood, to some extent we do not even know whether it decays or is stable. Nevertheless, if it decays one possibility is the decay into lower-dimensional branes, thus leading to a cascade of branes. Therefore, they eventually decay to a number of 0-branes, i.e., BH-like objects. This is complicated by the fact that when the p -branes decay, their masses might not be high enough to become BH's. Therefore, the final 0-branes might be some excited string states or string balls. Whether the zero brane is stable or not depends on models. Another possibility is decay into brane and bulk particles, thus experimentally the decay can be observed. Or it can be a combination of cascade into lower-dimensional branes and direct decays. Since the size R_{pB} is much smaller than the size of the large extra dimensions, we expect p -branes to decay mainly into brane particles. However, the above is quite speculative.

VI. CONCLUSIONS

In this work, we have calculated and compared the production cross sections for black holes, string balls, and p -branes at hadronic supercolliders (LHC and VLHC). Provided that the fundamental Planck scale is of order of 1 to a few TeV, large numbers of BH, SB, and p -brane events should be observed at the LHC. At the VLHC (50–200 TeV), the events rates are enormous. We have also given rough estimates for the sensitivity reaches on the fundamental Planck scale M_D at various \sqrt{s} , based on the number of raw events. The sensitivity of BH and p -brane production is roughly 2 TeV at the LHC, 6–7 TeV for $\sqrt{s}=50$ TeV, 10–13 TeV for $\sqrt{s}=100$ TeV, 14–18 TeV for $\sqrt{s}=150$ TeV, and 20–25 TeV for $\sqrt{s}=200$ TeV.

Finally, we offer a few comments as follows.

(i) The production cross sections for BH estimated in this work are significantly smaller than others in the literature, because we have imposed a stringent entropy S_{BH} requirement on the BH. Such a requirement is necessary to make sure the object is a BH. Had this requirement relaxed, the

TABLE IV. Total cross sections in pb for BH and p -brane production at the LHC for various values of $y \equiv M_{\text{BH}}^{\text{min}}/M_D, M_{p\text{B}}^{\text{min}}/M_D$.

	BH			p -brane		
	$n=3$	$n=5$	$n=7$	$n=3$	$n=5$	$n=7$
$M_D = 1.5$ TeV						
$y=1$	5700	13000	22000	8100	26000	49000
$y=2$	580	1200	2000	910	2900	5600
$y=3$	75	150	230	120	400	760
$y=4$	8.5	16	25	15	47	89
$y=5$	0.70	1.3	1.9	1.2	4.0	7.6
$M_D = 2$ TeV						
$y=1$	1200	2900	4800	1700	5500	10000
$y=2$	72	150	250	110	360	690
$y=3$	4.1	8.4	13	6.8	22	42
$y=4$	0.13	0.26	0.39	0.23	0.73	1.4
$y=5$	0.0012	0.0022	0.0034	0.0021	0.0069	0.013
$M_D = 2.5$ TeV						
$y=1$	330	780	1300	460	1500	2800
$y=2$	10	22	36	16	52	98
$y=3$	0.19	0.39	0.62	0.32	1.0	1.9
$y=4$	6.9×10^{-4}	0.0013	0.0020	0.0018	0.0038	0.0072
$y=5$	1.3×10^{-8}	2.4×10^{-8}	3.6×10^{-8}	2.3×10^{-8}	7.3×10^{-8}	1.4×10^{-7}
$M_D = 3$ TeV						
$y=1$	100	250	420	140	460	880
$y=2$	1.5	3.2	5.2	2.3	7.5	14
$y=3$	0.0057	0.012	0.018	0.0093	0.030	0.057
$y=4$	1.8×10^{-7}	3.5×10^{-7}	5.4×10^{-7}	3.1×10^{-7}	9.9×10^{-7}	1.9×10^{-6}
$y=5$						

cross section would have increased substantially. For the purpose of comparing with others' results, we also show the cross sections for smaller values of $y \equiv M_{\text{BH}}^{\text{min}}/M_D, M_{p\text{B}}^{\text{min}}/M_D$ in Table IV. The cross sections listed for $y \leq 4$ should be interpreted with care, because the smaller the ratio $M_{\text{BH}}^{\text{min}}/M_D$ the stronger the string effect is and the classical description for BH may not be valid.

(ii) It was pointed out in Ref. [12] that a BH with an angular momentum J is likely to be formed in particle collisions when the incoming partons are collided at an impact parameter. In such a case, the radius of the BH decreases and thus the naive cross-section formula $\hat{\sigma} = \pi R_{\text{BH}}^2$ implies a smaller cross section for each angular momentum J . The higher the angular momentum, the larger is the suppression. Nevertheless, when all J (including $J=0$) are summed, the total cross section gives a factor of 2–3 enhancement to the case of nonspinning BH.

(iii) p -brane production is negligible if $r < p$, because of the large volume factor suppression. But when $r = p$ (the p -brane wraps entirely on the small extra dimensions of the size of the Planck length), the production cross section is sizable. Moreover, the cross section is a few times larger than the BH production for the case of $r = p = m$, where $m \leq n - 2$.

(iv) The production cross section for SB's is enormous because it does not suffer from a mass threshold as large as

for the BH. The minimum mass requirement is between M_s and M_s/g_s . We typically choose $2M_s$ as the starting point for the SB. Such a large event rate makes the tests for string ball properties and BH correspondence principle possible. As pointed out in Ref. [6], since only a very small fraction of the decay products of a BH has missing energies, the mass of the BH can be determined. Moreover, the energy spectrum of the decay products can be measured and fitted to the black-body radiation temperature. Thus, the Hawking radiation relationship between the mass and temperature of a BH can be tested. Here, similar to the BH, both the mass and the temperature of the SB can be determined by measuring the spectrum of the decay products. Thus, the relationship between the mass of the SB and the Hagedorn temperature can be tested.

(v) We have emphasized the importance and the advantages of using the $2 \rightarrow 2$ subprocess for production of BH's, SB's, and p -branes, which allows a substantial transverse momentum kick to the object, and at the same time produces an energetic high p_T parton, which provides a critical tag to the event.

(vi) At the LHC and VLHC, multiparton collisions and overlapping events may be likely to happen. A careful discrimination is therefore necessary, especially in the case in which the BH is produced at rest or is moving along the beam-pipe (i.e., in $2 \rightarrow 1$ subprocess). The $2 \rightarrow 2$ subprocess

affords an easier signature experimentally. The high p_T parton emerging as a jet, a lepton, jets, or leptons provides an easy tag.

(vii) In this study, we do not consider the difference in the decays of BH, SB, and p -brane. If we could distinguish the decay signatures of the BH and SB, we might be able to test the BH correspondence principle at the transition point. We can also test the decays of p -branes in more detail.

There just appears a short review article [47] on BH production at hadronic colliders and by ultrahigh energy cosmic neutrinos.

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