PHYSICAL REVIEW D **66**, 036005 (2002)

Radiative corrections to Kaluza-Klein masses

Hsin-Chia Cheng*
Enrico Fermi Institute, The University of Chicago, Chicago, Illinois 60637

Konstantin T. Matchev†
Department of Physics, University of Florida, Gainesville, Florida 32611

Martin Schmaltz‡
Physics Department, Boston University, Boston, Massachusetts 02215

(Received 1 May 2002; published 22 August 2002)

Extra-dimensional theories contain a number of almost degenerate states at each Kaluza-Klein level. If extra dimensional momentum is at least approximately conserved then the phenomenology of such nearly degenerate states depends crucially on the mass splittings between KK modes. We calculate the complete one-loop radiative corrections to KK masses in general 5 and 6 dimensional theories. We apply our formulas to the example of universal extra dimensions and show that the radiative corrections are essential to any meaningful study of the phenomenology. Our calculations demonstrate that Feynman diagrams with loops wrapping the extra dimensions are well-defined and cutoff independent even though higher dimensional theories are not renormalizable.

DOI: 10.1103/PhysRevD.66.036005 PACS number(s): 11.10.Gh, 04.50.+h, 11.10.Kk, 11.25.Mj

I. INTRODUCTION

Radiative corrections are known to play an important role for precision measurements, but are generally not expected to radically change the nature of high energy "discovery" processes like the production and decay of new particles in collider experiments.

In this paper we point out that this expectation can be completely wrong with respect to the collider physics of some extra-dimensional models. Radiative corrections are crucial for determining the decays of Kaluza-Klein (KK) excitations. This is because at the tree level KK masses are quantized, and all momentum preserving decays are exactly at threshold. Radiative corrections then become the dominant effect in determining which decay channels are open.

Consider for example the simplest case of a massless field propagating in a single circular extra dimension with radius R. This theory is equivalently described by a four dimensional theory with a tower of states with tree level masses $m_n = n/R$. The integer n corresponds to the quantized momentum p_5 in the compact dimension and becomes a quantum number (KK number) under a $U(1)$ symmetry in the $4d$ description. The tree level dispersion relation of a 5D-massless particle is fixed by Lorentz invariance of the tree level Lagrangian

$$E^2 = \vec{p}^{\,2} + p_5^2 = \vec{p}^{\,2} + m_n^2, \tag{1}$$

where $\vec{p}$ is the momentum in the usual three spatial directions. Ignoring branes and orbifold fixed points, KK number is a good quantum number and is preserved in all interactions and decays. We see from Eq. (1) that at the tree level the KK modes of level $n > 1$ are exactly at threshold for decaying to lower level KK modes. For example, in 5D QED with massless photons and electrons the reaction

$$e^{(2)} \rightarrow e^{(1)} + \gamma^{(1)} \tag{2}$$

is exactly marginal at the tree level. It is straightforward to include electroweak symmetry breaking masses. This gives no mass shift to the photon and its KK modes and generates masses $\sqrt{m_n^2 + m_e^2}$ for the electrons at KK level n. Including these shifts one finds that the reaction (2) is just barely forbidden by phase space, and one concludes that all electron KK modes are stable. However, using realistic values $m_e \sim$ MeV and $R^{-1} \sim$ TeV, the difference between the total masses on both sides of Eq. (2) normalized to the KK mode masses is only of order $m_e^2/m_n^2 \sim 10^{-12}$. Clearly, this minuscule mass splitting is completely irrelevant if there are radiative corrections to Eq. (1) which would start at order $\alpha \sim 10^{-2}$. This is reminiscent of the case of W-ino-LSP in supersymmetric models where the tiny tree level W-ino mass splitting is overwhelmed by the radiative corrections [1].

We now show that there are indeed radiative corrections to the KK masses. The dispersion relation (1) follows from local 5D Lorentz invariance of the tree level Lagrangian. However, 5D-Lorentz invariance is broken by the compactification. This breaking is non-local and cannot be seen in the renormalized couplings of the local 5D Lagrangian, but it contributes to the $4d$ masses of KK modes because of their delocalized wave functions in the fifth dimension. More explicitly, the leading mass corrections δm_n^2 to Eq. (1) come from loop diagrams with internal propagators which wrap around the compactified dimension. The sign and n dependence of these corrections determines which decay channels are open and which KK modes are stable. For the example of

*Electronic address: hcheng@theory.uchicago.edu
†Electronic address: matchev@phys.ufl.edu
‡Electronic address: schmaltz@bu.edu

5D QED, we find radiative corrections at order α as anticipated; they render the reaction (2) allowed with phase space of order $\alpha R^{-1}\sim 10$ GeV.

In this paper we compute mass corrections at one loop for a general theory with fields of spin 0, $\frac{1}{2}$ and 1. Our results are finite and well defined. At first sight, this might seem surprising since the 5D theory is not renormalizable. However, the 5D Lorentz violating corrections to KK mode masses involve propagation over finite distances (around the extra dimension) and are exponentially suppressed for momenta which are large compared to the compactification scale. Thus our results are UV finite and do not depend on the choice of regulator as long as it is 5D Lorentz invariant and sufficiently local.

Applying these results to the standard model requires introducing an additional complication. Obtaining chiral fermions in 4D from a 5D theory is only possible with additional breaking of 5D Lorentz invariance. Two frequently discussed choices are introducing chiral fermions on branes or imposing orbifold boundary conditions on fermions in the bulk. We focus on the latter because we wish to minimize the breaking of 5D Lorentz invariance. The resulting model in which all the standard model fields live in the bulk of an orbifold is known as ''universal extra dimensions'' [2]. We consider the orbifolds S^1/Z_2 and T^2/Z_2.

Both orbifolds have fixed points which break extra-dimensional translation invariance, and we expect new interactions localized on the fixed points. Clearly, the presence of such localized interactions violates 5D momentum conservation, and KK number is no longer preserved. However, a discrete subgroup remains unbroken. In the S^1/Z_2 case, this is ''KK parity,'' a parity flip of the extra dimension. In the 4D description KK parity is a Z_2 symmetry under which only KK modes with odd KK number are charged. The symmetry implies that the lightest KK particle at level 1 (the LKP) is stable. Note that KK parity and the LKP play an analogous role to R parity and the LSP in supersymmetry.

In the presence of orbifold boundaries higher level KK modes can decay to lower level KK modes via KK number violating interactions. These decays compete with KK number preserving decays, and it becomes a phenomenologically important question which channels dominate. The answer can be understood very simply. Since the KK number violating interactions exist only on the boundaries they turn into volume suppressed couplings between KK modes. This implies that even though KK number violating decays have larger phase space they are more strongly suppressed because they are proportional to the square of smaller coupling constants. Therefore, the question of which momentum preserving decays are allowed by phase space remains phenomenologically important also in theories on orbifolds.

In addition to giving rise to new interactions, the boundary terms also include 5D Lorentz violating kinetic terms which contribute to the masses of KK modes and are important in determining decay patterns. In Ref. [3] it was shown that the coefficients of boundary terms receive logarithmically divergent contributions at one loop. Thus it is not only possible to include boundary terms in orbifold theories, it is inconsistent not to include them. The coefficients of these terms correspond to new parameters of the theory. They contain incalculable contributions from unknown physics at the cutoff as well as contributions from loops in the low-energy theory which we compute in this paper.

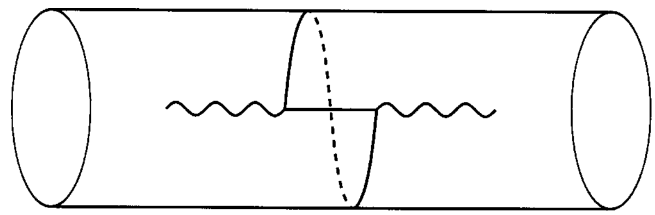

FIG. 1. Lorentz violating loop.

This paper is structured as follows. In the next section we compute radiative corrections to masses of KK modes for an arbitrary theory with scalars, fermions, and gauge fields in a 5D theory on a circle. In Sec. III we discuss the additional complications which arise for orbifolds where all fields propagate in the bulk and compute the renormalization of boundary couplings. In Sec. IV we apply the results of the previous sections to the standard model in ''universal extra dimensions'' and determine the complete one-loop corrected spectrum. Section V contains our conclusions. Details of our calculations can be found in the Appendixes.

II. BULK CORRECTIONS FROM COMPACTIFICATION

To begin, we discuss the simplest higher dimensional theory: an extra dimension compactified on a circle S^1 with radius R $(x_5+2\pi R\sim x_5)$. We assume that 5D Lorentz invariance is respected by the short-distance physics, and is only broken by the compactification. The momentum in the 5th dimension, which is quantized in units of $1/R$, becomes a mass for the KK modes after compactification. If 5-dimensional Lorentz invariance were exact, the KK mode masses coming from the 5th dimensional momentum would not receive corrections. For example, the kinetic term of a scalar field living in 5 dimensions is

$$\mathcal{L}\supset Z\partial_\mu\phi\partial^\mu\phi - Z_5\partial_5\phi\partial_5\phi,\quad \mu=0,1,2,3. \tag{3}$$

Both Z and Z_5 receive divergent quantum corrections. However, if 5-dimensional Lorentz invariance were exact, these contributions would be equal, so that the masses of the KK modes coming from the $(\partial_5\phi)^2$ term would stay uncorrected. More generally, exact Lorentz invariance would imply that the energy is only a function of $|\vec{p}|^2+p_5^2$, and hence $E^2=|\vec{p}|^2+p_5^2+m^2$ does not receive p_5-dependent corrections. All KK mode masses would be given by $p_5^2+m^2$ with the same p_5 dependence, and the only correction would be due to renormalization of the zero mode mass m.

However, 5-dimensional Lorentz invariance is broken at long distances by the compactification, so in general the masses of the KK modes do receive radiative corrections. Feynman diagrams are sensitive to the Lorentz symmetry breaking if they have an internal loop which winds around the circle of the compactified dimension, as shown in Fig. 1, so that it can tell that this direction is different from the others. This is a non-local effect as the size of the loop can not be shrunk to zero. Such non-local loop diagrams are well-defined and finite, even though the higher-dimensional theory is non-renormalizable.

We can isolate the finite 5D Lorentz violating corrections from the divergent 5D Lorentz invariant corrections by employing a very simple subtraction prescription: from every loop diagram in the compactified theory we subtract the corresponding diagram of the uncompactified theory. The UV divergences are canceled because the two theories are identical at short distances, but the KK mass corrections are unaltered because the subtraction is 5D Lorentz invariant.

To make this more explicit, first note that momenta in the compact dimension are discrete. Therefore the five-dimensional phase space integral

$$\int \frac{d^5k}{(2\pi)^5}\cdots \tag{4}$$

becomes

$$\frac{1}{2\pi R}\sum_{k_5}\int \frac{d^4k}{(2\pi)^4}\cdots \tag{5}$$

for compact dimensions.

Our subtraction prescription is to simply subtract Eq. (4) from Eq. (5) for each diagram. To better understand the physical meaning of this prescription and to explicitly demonstrate that Eqs. (4) and (5) contain the same divergence we rewrite the KK sum using the Poisson resummation identity

$$\frac{1}{2\pi R}\sum_{m=-\infty}^{\infty} F(m/R)=\sum_{n=-\infty}^{\infty} f(2\pi R n), \tag{6}$$

where $f(x)$ and $F(k)$ are related by Fourier transformation

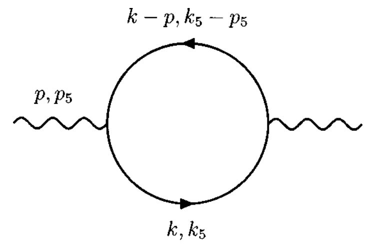

FIG. 2. Vacuum polarization diagram.

$$f(x)=\mathcal{F}^{-1}\{F(k)\}=\int_{-\infty}^{\infty}\frac{dk}{2\pi}e^{-ikx}F(k). \tag{7}$$

The resummation formula turns a sum over KK numbers m (or KK momenta $k_5=m/R$) into a sum over winding numbers n (or position space windings $n2\pi R$ around the fifth dimension). Note that the $n=0$ term in the sum is identical to the phase space integral of an uncompactified extra dimension

$$f(0)=\int_{-\infty}^{\infty}\frac{dk_5}{2\pi}F(k_5)=\int \frac{d^5k}{(2\pi)^5}\cdots. \tag{8}$$

Thus our subtraction prescription simply amounts to leaving out the divergent $n=0$ term in the re-summed expression for each Feynman diagram. The remaining terms in the sum (with $n\neq 0$) correspond to particle loops with net winding n around the compactified dimension.[1] They are all finite and so is their sum.

To illustrate the calculation, we consider the relatively simple example of QED in 4+1 dimensions with one spatial dimension compactified on a circle. We will calculate the correction to the masses of KK photons due to the electron loop. The one loop vacuum polarization (Fig. 2) is given by

$$\begin{aligned} i\Pi_{\mu\nu}&=-e^2\sum_{k_5}\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}\left[\gamma_\mu\frac{1}{\not k+i\gamma_5k_5}\gamma_\nu\frac{1}{(\not k-\not p)+i\gamma_5(k_5-p_5)}\right]\\ &=-4e^2\sum_{k_5}\int\frac{d^4k}{(2\pi)^4}\frac{k_\mu(k_\nu-p_\nu)+k_\nu(k_\mu-p_\mu)-g_{\mu\nu}k(k-p)+g_{\mu\nu}k_5(k_5-p_5)}{(k^2-k_5^2)[(k-p)^2-(k_5-p_5)^2]} \end{aligned} \tag{9}$$

where p,k are 4-momenta, $k_5=m/R$ with $m=$integers, and the volume factor $1/(2\pi R)$ has been absorbed into the gauge coupling $e^2=e_5^2/(2\pi R)$.

As usual we use Feynman parametrization to combine the denominators,

$$i\Pi_{\mu\nu}=-4e^2\int_0^1 d\alpha\sum_{k_5}\int\frac{d^4k}{(2\pi)^4}\frac{N_{\mu\nu}}{[k^2-k_5'^2+\alpha(1-\alpha)(p^2-p_5^2)]^2} \tag{10}$$

[1]More precisely, they correspond to diagrams in which the internal propagators form a non-contractible loop around the extra dimension. The parameter n is the winding number of the internal loop. The diagrams with a contractible loop are 5D Lorentz invariant and get subtracted.

where

$$N_{\mu\nu}=2k_\mu k_\nu+g_{\mu\nu}(-k^2+\alpha(1-\alpha)(p^2-p_5^2)+(2\alpha-1)p_5k_5'+k_5'^2)-2\alpha(1-\alpha)p_\mu p_\nu, \quad (11)$$

and $k_5'=k_5-\alpha p_5$. To calculate the correction to the masses of the KK modes, we concentrate on the terms proportional to $g_{\mu\nu}$,

$$\Pi_{\mu\nu}=g_{\mu\nu}\Pi_1-p_\mu p_\nu\Pi_2. \quad (12)$$

We can set $p^2=p_5^2$ in the leading order approximation. Replacing $k_\mu k_\nu$ by $g_{\mu\nu}k^2/4$, and performing the Wick rotation, we have

$$\Pi_1=-4e^2\int_0^1 d\alpha\sum_{k_5}\int\frac{d^4k_E}{(2\pi)^4}\frac{\frac{1}{2}k_E^2+(2\alpha-1)p_5k_5'+k_5'^2}{(k_E^2+k_5'^2)^2}. \quad (13)$$

It is convenient to rescale k_E,k_5,k_5',p_5 by $1/R$ so that they become dimensionless numbers and k_5,p_5 run over integers. Using the formula

$$\frac{1}{A^r}=\frac{1}{(r-1)!}\int_0^\infty d\ell\,\ell^{r-1}e^{-A\ell}, \quad (14)$$

we obtain

$$\Pi_1=-\frac{4e^2}{R^2}\int_0^1 d\alpha\sum_{k_5}\int\frac{d^4k_E}{(2\pi)^4}\int_0^\infty d\ell\,\ell\times\left[\frac{1}{2}k_E^2+(2\alpha-1)p_5k_5'+k_5'^2\right]e^{-(k_E^2+k_5'^2)\ell}. \quad (15)$$

Next, we perform the d^4k_E integral

$$\Pi_1=-\frac{4e^2}{16\pi^2R^2}\int_0^1 d\alpha\int_0^\infty d\ell\,\ell\times\sum_{k_5}\left[\frac{1}{\ell^3}+\frac{(2\alpha-1)p_5k_5'}{\ell^2}+\frac{k_5'^2}{\ell^2}\right]e^{-k_5'^2\ell}$$
$$=-\frac{e^2}{4\pi^2R^2}\int_0^1 d\alpha\int_0^\infty dt\times\sum_{k_5}\left[1+\frac{(2\alpha-1)p_5k_5'}{t}+\frac{k_5'^2}{t}\right]e^{-k_5'^2/t}, \quad (16)$$

where $t=1/\ell$. Now we use the Poisson resummation formula, Eq. (6), to turn the sum over k_5 into a sum over winding numbers. The inverse Fourier transformations needed are

$$\mathcal{F}^{-1}\{e^{-k_5^2/t}\}=\sqrt{\frac{t}{4\pi}}e^{-x^2t/4}$$
$$\mathcal{F}^{-1}\{k_5e^{-k_5^2/t}\}=-i\frac{xt}{2}\sqrt{\frac{t}{4\pi}}e^{-x^2t/4}$$
$$\mathcal{F}^{-1}\{k_5^2e^{-k_5^2/t}\}=\left(-\frac{x^2t^2}{4}+\frac{t}{2}\right)\sqrt{\frac{t}{4\pi}}e^{-x^2t/4} \quad (17)$$
$$\mathcal{F}^{-1}\{F(k_5'=k_5-\alpha p_5)\}=f(x)e^{-i\alpha xp_5}.$$

The result is

$$\Pi_1=-\frac{e^2}{2\pi R^2}\sum_{x=2\pi n}\int_0^1 d\alpha e^{-i\alpha xp_5}\int_0^\infty dt\sqrt{\frac{t}{4\pi}}\times e^{-x^2t/4}\left[\frac{3}{2}-i\left(\alpha-\frac{1}{2}\right)xp_5-\frac{x^2t}{4}\right]$$
$$=-\frac{e^2}{2\pi R^2}\sum_{x=2\pi n}\int_0^1 d\alpha e^{-i\alpha xp_5}\times\left[\frac{3}{2}\frac{2}{|x|^3}-i\left(\alpha-\frac{1}{2}\right)xp_5\frac{2}{|x|^3}-\frac{x^2}{4}\frac{12}{|x|^5}\right]$$
$$=-\frac{e^2}{2\pi R^2}\sum_{n=-\infty}^{\infty}\int_0^1 d\alpha e^{-i\alpha 2\pi np_5}\times(-i(2\alpha-1)2\pi np_5)\frac{1}{|2\pi n|^3}. \quad (18)$$

For the zero mode ($p_5=0$), we have $\Pi_1=0$, i.e., there is no correction to the mass as expected by gauge invariance. For nonzero KK modes, the correction to their masses is obtained by dropping the divergent $n=0$ term as discussed above

$$\delta m_{KK}^2=-\frac{e^2}{2\pi R^2}\sum_{n\neq0}\frac{2}{|2\pi n|^3}=-\frac{e^2}{4\pi^4R^2}\sum_{n=1}^{\infty}\frac{1}{n^3}=-\frac{e^2\zeta(3)}{4\pi^4R^2}, \quad (19)$$

which is finite and independent of the KK level.

It is straightforward to follow the same procedure to calculate the corrections in a more general theory which contains non-Abelian gauge fields, fermions, and scalars. In our calculation, we assumed that the zero mode masses are much smaller than the compactification scale so that we can ignore them in the calculations. With the possible exception of the Higgs boson and the top quark, this is also the case of interest for applications to the standard model. (For nonvanishing zero mode mass $m_0\ll1/R$, there will be KK level dependent corrections suppressed by m_0^2/p_5^2.) The one-loop contributions from various diagrams are listed in Appendix A and we summarize the results here.

The correction to the KK mode masses for the gauge field is given by

$$\delta m_{V_{KK}}^2 = \frac{g^2\zeta(3)}{16\pi^4 R^2}\left(3C(G) + \sum_{\text{real scalars}} T(r_s) - 4\sum_{\text{fermions}} T(r_f)\right), \tag{20}$$

where $C(G)\delta_{ab} = f_{acd} f_{bcd} [= N \text{ for } SU(N)]$, and $T(r)\delta_{AB} = \mathrm{tr}(T^A T^B)$ is the Dynkin index of the representation r, normalized to be 1/2 for the fundamental representation of $SU(N)$. The sum over scalars is over the real components and needs to be multiplied by 2 for a complex scalar. Note that for a supersymmetric theory the correction vanishes as it has to because the KK gauge bosons are BPS states. As in the case of QED5, the zero mode mass is not corrected as dictated by gauge invariance.

A similar calculation yields the correction to the mass of the zero mode of A_5. We find

$$\delta m_{A_5^0}^2 = 3\,\delta m_{V_{KK}}^2, \tag{21}$$

which is in agreement with earlier calculations [4]. Note that the KK modes of A_5 are "eaten" and become longitudinal components of the KK gauge fields.

For fermions, we find

$$\delta m_{f_{KK}} = 0. \tag{22}$$

Fine tuning is required for a scalar to be light, as its (Lorentz invariant) mass receives power divergent corrections no matter whether the extra dimension is compact or not. We are interested in the difference between the corrections to the KK modes and the zero mode, assuming that the zero mode mass has been fine tuned to be smaller than the compactification scale. In calculating the potentially 5D Lorentz violating contributions from loops with nonzero winding number, we find that the lowest order corrections to the squared masses of the zero mode and KK modes are the same,

$$\delta m_{S_{KK}}^2 = \delta m_{S_0}^2. \tag{23}$$

Therefore, they can be absorbed into the (infinitely renormalized) zero mode mass, and the n-th KK mode mass is simply given by

$$m_{S_n}^2 = \frac{n^2}{R^2} + m_0^2 \tag{24}$$

with no corrections at the lowest order.

The radiative corrections due to 5-dimensional Lorentz violation are long-distance effects, so they are saturated by the contributions from the lowest lying KK modes, where the effects of accelerated running of couplings are not very important yet. Here we confine ourselves to one-loop order in the calculation of the KK masses. The evolution of the couplings contributes to the KK mass corrections starting at two-loop order. They are to be included in a higher-loop calculation as well as other two-loop or higher effects which cannot be absorbed into the running couplings. Also, in the above calculations we have ignored graviton loops [5]. Their effects on KK mass splittings are negligible as they are suppressed by powers of $M_{Pl}R$.

III. ORBIFOLD COMPACTIFICATIONS

In the previous section we considered the simplest compactification on a circle (the generalization to a torus in more extra dimensions is discussed in Appendix A). However, a higher dimensional fermion has 4 or more components. Its four dimensional zero mode consists of both left-handed and right-handed fermions when compactified on a torus, and the resulting four dimensional theory is vector-like. To obtain chiral fermions in four dimensions, we need more complicated compactifications. One possibility is to compactify the extra dimensions on an orbifold. In this section, we consider the simplest example, an S^1/Z_2 orbifold, where Z_2 is the reflection symmetry $x_5 \to -x_5$. In addition to their indirect transformation via their x_5 dependence, fields can be even or odd under this Z_2 symmetry. A consistent assignment is to have A_μ, $\mu = 0,1,2,3$ even, and A_5 odd for the gauge field, and ψ_L even (odd), ψ_R odd (even) for the fermions. The scalars can be either even or odd. From a field theory point of view, the orbifold is simply a line segment of length $L = \pi R$ with boundary points (orbifold fixed points) at $x_5 = 0, \pi R$. Even (odd) fields have Neumann (Dirichlet) boundary conditions, $\partial_5 \phi = 0$ $(\phi = 0)$ at $x_5 = 0, \pi R$.

The KK decomposition for even and odd fields is given by

$$\Phi_+(x,x_5) = \frac{1}{\sqrt{\pi R}}\phi_+^{(0)}(x) + \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} \cos\frac{n x_5}{R}\,\phi_+^{(n)}(x),$$
$$\Phi_-(x,x_5) = \sqrt{\frac{2}{\pi R}}\sum_{n=1}^{\infty} \sin\frac{n x_5}{R}\,\phi_-^{(n)}(x). \tag{25}$$

The zero mode of the odd field is projected out by the orbifold Z_2 symmetry (or Dirichlet boundary conditions). For a fermion ψ, only ψ_L (or ψ_R) has a zero mode, hence we obtain a chiral fermion in the four dimensional theory. Similarly, the A_5 zero mode is projected out and there is no massless adjoint scalar from the extra component of the gauge field.

The orbifold introduces additional breaking of higher dimensional Lorentz invariance which leads to further corrections to KK mode masses. The orbifold fixed points break translational symmetry in the x_5 direction, therefore momentum in the x_5 direction (KK number) is no longer conserved, and we expect mixing among KK modes. However, a translation by πR in the x_5 direction remains a symmetry of the orbifold. We can see from Eq. (25) that under this transformation the even number ($n=$even) KK modes are invariant while the odd number ($n=$odd) KK modes change sign. Therefore, KK parity $(-1)^{KK}$ (not the Z_2 in S^1/Z_2) is still a good symmetry. Note that KK parity is a flip of the line segment about its center at $x_5=\pi R/2$ combined with the Z_2 transformation which flips the sign of all odd fields.

Because 5D Lorentz and translation invariance are broken at the orbifold boundaries, radiative corrections generate additional Lagrangian terms which are localized at the boundaries and do not respect 5D Lorentz symmetry. The boundary terms contribute to masses and mixing of KK modes. To calculate them, we follow the work by Georgi, Grant, and Hailu [3]. Fields on the S^1/Z_2 orbifold can be written as

$$\phi(x,x_5)=\frac{1}{2}(\Phi(x,x_5)\pm\Phi(x,-x_5)),$$
$$\psi(x,x_5)=\frac{1}{2}(\Psi(x,x_5)\pm\gamma_5\Psi(x,-x_5)), \tag{26}$$

where Φ,Ψ are unconstrained 5-dimensional boson and fermion fields, and the upper (lower) sign, $+(-)$, corresponds to ϕ,ψ_R being even (odd) under $x_5\to -x_5$. The propagators such as

$$S(x-x',x_5-x_5')=\langle\psi(x,x_5)\overline{\psi}(x',x_5')\rangle \tag{27}$$

can be expressed in terms of unconstrained fields (26). The results are

$$S(p,p_5,p_5')=\frac{i}{2}\left\{\frac{\delta_{p_5,p_5'}}{\not p+i\gamma_5 p_5}\mp\frac{\delta_{-p_5,p_5'}}{\not p+i\gamma_5 p_5}\gamma_5\right\} \tag{28}$$

for the fermion,

$$D_{\mu\nu}(p,p_5,p_5')=\frac{-ig_{\mu\nu}}{2}\left\{\frac{\delta_{p_5,p_5'}+\delta_{-p_5,p_5'}}{p^2-p_5^2}\right\},$$
$$D_{55}(p,p_5,p_5')=\frac{-ig_{55}}{2}\left\{\frac{\delta_{p_5,p_5'}-\delta_{-p_5,p_5'}}{p^2-p_5^2}\right\}, \tag{29}$$

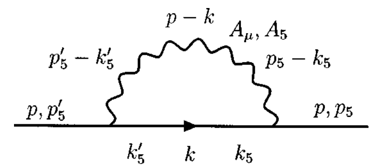

FIG. 3. Electron self-energy diagram.

for the gauge field (in the Feynman–'t Hooft gauge), and

$$D(p,p_5,p_5')=\frac{i}{2}\left\{\frac{\delta_{p_5,p_5'}\pm\delta_{-p_5,p_5'}}{p^2-p_5^2}\right\} \tag{30}$$

for the scalar boson. p_5 and p_5' are the outgoing and incoming fifth dimensional momenta (KK numbers). They can be different because 5D momentum is not conserved.

We calculate the one-loop diagrams with these modified propagators. Consider, for example, the one-loop contribution to the electron self-energy in 5D QED (Fig. 3). Let us first focus on the summation over momenta in the fifth dimension. The summations are of the form

$$\sum_{k_5,k_5'}(\delta_{k_5,k_5'}+\delta_{-k_5,k_5'}\gamma_5)(\delta_{p_5-k_5,p_5'-k_5'}+\delta_{-(p_5-k_5),p_5'-k_5'})$$
$$=(\delta_{p_5,p_5'}+\delta_{-p_5,p_5'}\gamma_5)\sum_{k_5}+\sum_{k_5}(\delta_{2k_5,p_5+p_5'}+\delta_{2k_5,p_5-p_5'}\gamma_5). \tag{31}$$

Up to a factor of $\frac{1}{2}$, the term proportional to $\delta_{p_5,p_5'}+\delta_{-p_5,p_5'}\gamma_5$ reproduces the corresponding diagram in 5D QED on a circle, and we can simply recycle the result of the previous section. The relative factor of $\frac{1}{2}$ arises because the Z_2 orbifolding projects out half of the states of the theory on S^1. The second term gives rise to new contributions to the self-energy which violate 5D momentum by integer multiples of $2/R$. We will see shortly that these terms are log divergent. The corresponding counter terms are localized on the fixed points of the orbifold at $x_5=0$ and $x_5=\pi R$.

Denoting the "boundary" contribution to the self-energy by $\overline{\Sigma}(p;p_5,p_5')$ we have

$$-i\overline{\Sigma}(p;p_5,p_5')=-\frac{g^2}{4}\sum_{k_5}\int\frac{d^4k}{(2\pi)^4}\left[\frac{\gamma^\nu(\not k+i\gamma_5 k_5)\gamma^\mu g_{\mu\nu}-\gamma_5(\not k+i\gamma_5 k_5)\gamma_5}{(k^2-k_5^2)[(k-p)^2-(k_5-p_5)^2]}\right](\delta_{2k_5,p_5+p_5'}\pm\delta_{2k_5,p_5-p_5'}\gamma_5) \tag{32}$$

where the first term in the numerator comes from the 4-dimensional gauge field components and the second term comes from the 5th component of the gauge field. After Feynman parametrization and Wick rotation, this becomes

$$
\begin{aligned}
-i\overline{\Sigma}(p;p_5,p_5')&=\frac{ig^2}{4}\sum_{k_5}\int_0^1 d\alpha\int\frac{d^4k_E}{(2\pi)^4}\frac{(\alpha\not{p}+5i\gamma_5k_5)(\delta_{2k_5,p_5+p_5'}\pm\delta_{2k_5,p_5-p_5'}\gamma_5)}{[k_E^2-\alpha(1-\alpha)p^2+k_5^2-2\alpha k_5p_5-\alpha p_5^2]^2}\\
&\to\frac{ig^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\sum_{k_5}\left[\frac{1}{2}\not{p}+5i\gamma_5k_5\right](\delta_{2k_5,p_5+p_5'}\pm\delta_{2k_5,p_5-p_5'}\gamma_5)\\
&=\frac{ig^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\not{p}\frac{1\pm\gamma_5}{2}+5i\gamma_5p_5\frac{1\pm\gamma_5}{2}+5i\gamma_5p_5'\frac{1\mp\gamma_5}{2}\right]\quad\text{for even }R(p_5-p_5'). \qquad (33)
\end{aligned}
$$

The arrow in the second line indicates that we have only kept the leading logarithmic divergence. In the log, Λ represents the cutoff and μ is the renormalization scale. The equality in the final line holds only for $R(p_5-p_5')$ even, for odd differences we have $\overline{\Sigma}(p,p_5,p_5')=0$.

This result can be understood (following [3]) as the renormalization of terms in the 5D Lagrangian which are localized at the boundaries of the orbifold. Fourier transforming to position space, we obtain

$$
\begin{aligned}
\delta\overline{\mathcal{L}}\supset\left(\frac{\delta(x_5)+\delta(x_5-L)}{2}\right)\frac{Lg^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\\
\times[\overline{\psi}_+ i\not{\partial}\psi_+ +5(\partial_5\overline{\psi}_-)\psi_+ +5\overline{\psi}_+(\partial_5\psi_-)],
\end{aligned}
\qquad (34)
$$

where L appears because of a change in normalization of fields in going from 4D to 5D; L combines with the 4D gauge coupling to give $Lg^2=g_5^2$. The delta functions are normalized to $\int_0^\epsilon\delta(x)dx=1$. We have been using Feynman gauge in the above calculation. For general 't Hooft ξ gauges, one can show that the coefficients in front of $i\not{\partial}$ and ∂_5 are given by $1+2(\xi-1)$ and $5+(\xi-1)$, respectively.

The logarithmically divergent result means that we should include counterterms localized at the boundaries to cancel the divergence. Our calculation only determined the running contribution between the cutoff Λ and μ, given initial values for the boundary terms at Λ. We implicitly assumed in our calculations that the boundary terms at the cutoff are small. If large boundary terms were present, they would mix KK modes of different levels and correspondingly shift their masses. Both effects would have to be taken into account in calculating the radiative corrections. The KK spectrum would then have a complicated dependence on the unknown boundary terms at the high scale. We continue to assume that there are no large boundary terms, and the logarithmic divergences can be absorbed into the cutoff Λ with Λ not too large. Note that this assumption is self-consistent because the boundary terms which are generated by radiative corrections are loop-suppressed.

The leading order correction to the mass of the n-th KK mode is obtained from Lagrangian terms which are quadratic in the n-th KK mode. Mass corrections due to the mixing among different KK modes are of higher order.

We expand the boundary terms (34) in terms of the KK modes and consider the modification of the kinetic terms for the n-th KK mode ($n\neq 0$),

$$
Z_{n+}\overline{\psi}_{n+}i\not{\partial}\psi_{n+}+\overline{\psi}_{n-}i\not{\partial}\psi_{n-}+Z_{n5}(\overline{\psi}_{n+}\partial_5\psi_{n-}-\overline{\psi}_{n-}\partial_5\psi_{n+}),
\qquad (35)
$$

where

$$
\begin{aligned}
Z_{n+}&=1+2[1+2(\xi-1)]\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2},\\
Z_{n5}&=1+2[5+(\xi-1)]\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}.
\end{aligned}
\qquad (36)
$$

Note that $Z_{n-}=1$ because ψ_{n-} vanishes on the boundary. After rescaling ψ_{n+} to canonical kinetic terms, the correction to the KK mode mass is given by

$$
\frac{\overline{\delta}m_n}{m_n}=\frac{Z_{n5}}{\sqrt{Z_{n+}}}-1=\frac{9}{4}\frac{g^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2},
\qquad (37)
$$

which is independent of the gauge parameter ξ. The correction is proportional to the n-th mode mass n/R, in contrast with the bulk contribution discussed in the previous section.

For a more general theory which contains non-Abelian gauge fields, fermions and scalars, the radiatively generated boundary terms from various diagrams are listed in Appendix B. In the following, we summarize the one-loop corrections to the KK mode masses. We always assume that the boundary terms are small, and can be treated as perturbations.

The corrections to the masses of KK modes for gauge bosons, fermions, Z_2 even scalars, and Z_2 odd scalars are given by

$$
\begin{aligned}
\overline{\delta}m_{V_n}^2=m_n^2\frac{g^2}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\\
\times\left[\frac{23}{3}C(G)-\frac{1}{3}\sum_{\text{real scalars}}(T(r)_{\text{even}}-T(r)_{\text{odd}})\right],
\end{aligned}
\qquad (38)
$$

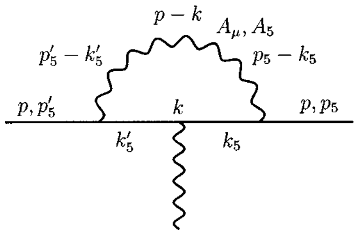

FIG. 4. One-loop diagram for the KK number violating vertex in the 5 dimensional QED.

$$\overline{\delta} m_{f_n} = m_n \frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2} \times\left[9C(r)g^2 - \sum_{\text{even scalars}} 3h_+^2 + \sum_{\text{odd scalars}} 3h_-^2\right], \tag{39}$$

$$\overline{\delta} m^2_{S_{+n}} = \overline{m}^2 + m_n^2 \frac{1}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2} \times\left[6g^2T(r) - \sum_{\text{even scalars}} \frac{\lambda_{++}}{2} + \sum_{\text{odd scalars}} \frac{\lambda_{+-}}{2}\right], \tag{40}$$

$$\overline{\delta} m^2_{S_{-n}} = m_n^2 \frac{1}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2} \times\left[9g^2T(r) + \sum_{\text{even scalars}} \frac{\lambda_{+-}}{2} - \sum_{\text{odd scalars}} \frac{\lambda_{--}}{2}\right], \tag{41}$$

where h and λ are Yukawa and quartic scalar couplings respectively. Their normalization is chosen to yield vertices with no numerical factors in the Feynman rules. The $\overline{m}^2$ in the expression for the even scalars contains a contribution $+2\overline{m}^2$ to the KK mode mass from a boundary mass term, minus a contribution $\overline{m}^2$ to the zero mode mass from the same boundary term. The relative factor of two between zero mode and KK modes comes from the normalization of the wave functions in Eq. (25).

The boundary terms also induce KK number violating couplings. Because KK parity is not broken, KK number can only be violated by even units in these couplings. Using the QED on S^1/Z_2 example, we can calculate the one-loop vertex diagram for the KK number violating coupling between the photon and the electron, Fig. 4. The result is simply to replace $\partial\!\!\!/$ in Eq. (34) by the covariant derivative $D\!\!\!\!/$. To obtain the couplings among the physical eigenstates, however, we have to take into account the kinetic and mass mixing effects on the external legs. A more detailed discussion is in Appendix C. The result can be related to the mass corrections from the boundary terms as both come from operators localized at the boundaries. For example, we find that the $\overline{\psi}_0\gamma^\mu T^a P_+\psi_0 A_{2\mu}$ coupling is given by

$$\frac{g}{\sqrt{2}}\left[\frac{\overline{\delta}(m^2_{A_2})}{m_2^2} - 2\frac{\overline{\delta}(m_{f_2})}{m_2}\right]. \tag{42}$$

On the other hand, couplings involving the zero mode gauge boson are governed by gauge invariance which implies that KK number violating interactions such as $\overline{\psi}_2\gamma^\mu T^a P_+\psi_0 A_{0\mu}$ vanish.

IV. THE STANDARD MODEL IN UNIVERSAL EXTRA DIMENSIONS

We now apply the results obtained in the previous two sections to the standard model in extra dimensions. The KK modes of standard model fields receive additional tree level mass contributions from electro-weak symmetry breaking which we have not taken into account in the calculations of the previous sections. Here, we include all these contributions but we ignore effects which involve both electro-weak symmetry breaking and radiative corrections. They are suppressed by both $g^2/16\pi^2$ and v^2/m_n^2 and are numerically negligible.

We consider the case in which all the standard model fields propagate in the same extra dimensions (universal extra dimensions) [2,6]. Theoretical motivations for considering such scenarios include electroweak symmetry breaking [6], the number of fermion generations [7], and proton stability [8]. Here we take a phenomenological approach and consider the simplest case of one universal extra dimension compactified on an S^1/Z_2 orbifold. The orbifold compactification is necessary to produce chiral fermions in four dimensions. In [2,9,10] it was shown that the current constraint on the compactification scale for one universal extra dimension is only about 300 GeV. Because of tree-level KK number conservation, KK states can only contribute to precision observables in loops, and direct searches for KK states require pair production. If the compactification scale is really so low, then KK states will be copiously produced at future colliders [11,12]. As we have argued in the Introduction, the radiative corrections have to be taken into account in any meaningful study of the phenomenology of these KK modes.

We assume the minimal field content of the standard model in one extra dimension. The fermions Q_i, u_i, d_i, L_i, e_i, $i=1,2,3$ are all 4-component fermions in 4+1 dimensions. [The upper case letters represent $SU(2)$ doublets and the lower case letters represent $SU(2)$ singlets.] Under the Z_2 orbifold symmetry, Q_L, u_R, d_R, L_L, e_R are even so that they have zero modes, which are identified with the standard model fermions. Fermions with opposite chirality $[Q_R, u_L, d_L, L_R, e_L]$ are odd and their zero modes are projected out. In order to allow Yukawa couplings the Higgs field must be even under the Z_2.

To obtain the corrections to the masses of the KK modes of the standard model fields we simply substitute into the formulas from the previous two chapters and include appropriate group theory and multiplicity factors. The bulk corrections are given by (bulk contributions in the S^1/Z_2 orbifold are half of those in the S^1 compactification)

$$\delta(m_{B_n}^2) = -\frac{39}{2}\frac{g'^2\zeta(3)}{16\pi^4}\left(\frac{1}{R}\right)^2,$$

$$\delta(m_{W_n}^2) = -\frac{5}{2}\frac{g_2^2\zeta(3)}{16\pi^4}\left(\frac{1}{R}\right)^2,$$

$$\delta(m_{g_n}^2) = -\frac{3}{2}\frac{g_3^2\zeta(3)}{16\pi^4}\left(\frac{1}{R}\right)^2, \tag{43}$$

$$\delta(m_{f_n}) = 0,$$

$$\delta(m_{H_n}^2) = 0,$$

where B_n are the KK modes of the $U(1)$ hypercharge gauge boson, W_n are the KK modes of the $SU(2)_W$ gauge bosons and g_n are the KK modes of the gluon.

The boundary terms receive divergent contributions which require counterterms. The finite parts of these counterterms are undetermined and remain as free parameters of the theory.[2] Here we shall make the simplifying assumption that the boundary kinetic terms vanish at the cutoff scale Λ and compute their renormalization to the lower energy scale μ. The corrections from the boundary terms are then given by

$$\bar{\delta} m_{Q_n} = m_n\left(3\frac{g_3^2}{16\pi^2} + \frac{27}{16}\frac{g_2^2}{16\pi^2} + \frac{1}{16}\frac{g'^2}{16\pi^2}\right)\ln\frac{\Lambda^2}{\mu^2},$$

$$\bar{\delta} m_{u_n} = m_n\left(3\frac{g_3^2}{16\pi^2} + \frac{g'^2}{16\pi^2}\right)\ln\frac{\Lambda^2}{\mu^2},$$

$$\bar{\delta} m_{d_n} = m_n\left(3\frac{g_3^2}{16\pi^2} + \frac{1}{4}\frac{g'^2}{16\pi^2}\right)\ln\frac{\Lambda^2}{\mu^2},$$

$$\bar{\delta} m_{L_n} = m_n\left(\frac{27}{16}\frac{g_2^2}{16\pi^2} + \frac{9}{16}\frac{g'^2}{16\pi^2}\right)\ln\frac{\Lambda^2}{\mu^2},$$

$$\bar{\delta} m_{e_n} = m_n\frac{9}{4}\frac{g'^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}, \tag{44}$$

$$\bar{\delta}(m_{B_n}^2) = m_n^2\left(-\frac{1}{6}\right)\frac{g'^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2},$$

$$\bar{\delta}(m_{W_n}^2) = m_n^2\frac{15}{2}\frac{g_2^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2},$$

$$\bar{\delta}(m_{g_n}^2) = m_n^2\frac{23}{2}\frac{g_3^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2},$$

$$\bar{\delta}(m_{H_n}^2) = m_n^2\left(\frac{3}{2}g_2^2 + \frac{3}{4}g'^2 - \lambda_H\right)\frac{1}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2} + \bar{m}_H^2.$$

Here λ_H is the Higgs quartic coupling, $\mathcal{L} \supset -(\lambda_H/2)(H^\dagger H)^2$ ($m_h = \sqrt{\lambda_H}v, v = 246$ GeV), and $\bar{m}_H^2$ is the boundary mass term for the Higgs mode. The renormalization scale μ should be taken to be approximately the mass of the corresponding KK mode. In the above formulas, we have not included contributions from Yukawa couplings, which can be ignored except for the top quark Yukawa coupling. Including the top Yukawa coupling introduces no new corrections to the Higgs KK modes, but the KK modes of the third generation $SU(2)$ doublet quark Q_3 and the $SU(2)$ singlet t receive additional corrections,

$$\bar{\delta}_{h_t} m_{Q_{3n}} = m_n\left(-\frac{3}{4}\frac{h_t^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\right)$$

$$\bar{\delta}_{h_t} m_{t_n} = m_n\left(-\frac{3}{2}\frac{h_t^2}{16\pi^2}\ln\frac{\Lambda^2}{\mu^2}\right). \tag{45}$$

The corrected masses for most of the KK modes can simply be read off from Eqs. (43) and (44). However, for certain fields there are also non-negligible tree-level contributions from electro-weak symmetry breaking, which introduce mixings among the states. This effect is important for the "photon" and "Z," the two KK modes of the top quark, the Higgs boson KK modes, and to a lesser extent for the bottom and tau KK modes.

The mass eigenstates and eigenvalues of the KK "photons" and "Z's" are obtained by diagonalizing their mass squared matrix. In the B_n, W_n^3 basis it is

$$\begin{pmatrix} \frac{n^2}{R^2} + \hat{\delta} m_{B_n}^2 + \frac{1}{4}g'^2v^2 & \frac{1}{4}g' g_2 v^2 \\ \frac{1}{4}g' g_2 v^2 & \frac{n^2}{R^2} + \hat{\delta} m_{W_n}^2 + \frac{1}{4}g_2^2 v^2 \end{pmatrix}, \tag{46}$$

where $\hat{\delta}$ represents the total one-loop correction, including both bulk and boundary contributions. Note that the mixing angle is different from the zero mode Weinberg angle because of the corrections $\hat{\delta} m_{B_n}^2$ and $\hat{\delta} m_{W_n}^2$. Figure 5 shows the dependence of the mixing angle θ_n for the n-th KK level on (a) R^{-1} for fixed $\Lambda R = 20$; and (b) ΛR for fixed $R^{-1} = 300$ GeV. For large R^{-1} or ΛR, where the corrections become sizable, the neutral gauge boson eigenstates become approximately pure B_n and W_n^3.

[2]This is reminiscent of the case of low energy supersymmetry, where in the absence of an explicit theory of supersymmetry breaking we do not know the values of the soft masses at high scales. Nevertheless, we can compute their renormalization within a given visible sector model like the MSSM. Hence one can predict the superpartner masses only under specific assumptions about their values at the high scale.

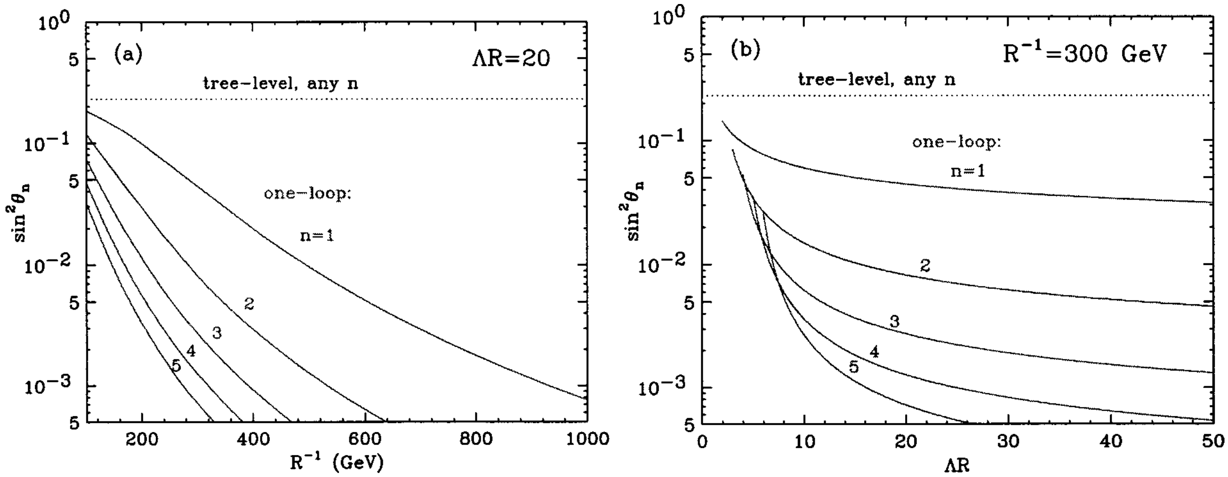

FIG. 5. Dependence of the "Weinberg" angle θ_n for the first few KK levels ($n=1,2,\ldots,5$) on (a) R^{-1} for fixed $\Lambda R=20$ and (b) ΛR for fixed $R^{-1}=300$ GeV.

Similarly, the eigenstates and eigenvalues of the KK fermions are obtained from the corresponding mass matrices. For example, the mass matrix for the top KK modes is

$$\begin{pmatrix} \frac{n}{R}+\hat{\delta}m_{T_n} & m_t \\ m_t & -\frac{n}{R}-\hat{\delta}m_{t_n} \end{pmatrix}, \tag{47}$$

where T_n and t_n represent $SU(2)$ doublet quarks and singlet quarks respectively.

Finally we discuss the KK modes of the Higgs field. The KK modes of W and Z acquire their masses by "eating" linear combinations of the fifth component of the gauge fields and the Higgs KK modes. The orthogonal combinations remain physical scalar particles. For $1/R \gg M_{W,Z}$, the longitudinal components of the KK gauge bosons mostly come from A_5, and the physical scalars are approximately the KK excitations of the Higgs field. There are 4 states at each KK level, $H_n^{\pm}, H_n^0, A_n^0$ (notice that $H_0^{\pm}$ and A_0^0 are just the usual Goldstone bosons in the SM). Their corrected masses are given by

$$\begin{aligned} m^2_{H_n^0} &\approx m_n^2+m_h^2+\hat{\delta}m^2_{H_n} \\ m^2_{H_n^{\pm}} &\approx m_n^2+M_W^2+\hat{\delta}m^2_{H_n} \\ m^2_{A_n^0} &\approx m_n^2+M_Z^2+\hat{\delta}m^2_{H_n}. \end{aligned} \tag{48}$$

In Fig. 6, we show a sample spectrum for the first KK excitations of all standard model fields, both at the tree level (a) and including the one-loop corrections (b). We have fixed $R^{-1}=500$ GeV, $\Lambda R=20$, $m_h=120$ GeV, $\overline{m}_H^2=0$ and assumed vanishing boundary terms at the cutoff scale Λ. We see that the KK "photon" receives the smallest corrections and is the lightest state at each KK level. Unbroken KK parity $(-1)^{KK}$ implies that the lightest KK particle (LKP) at level one is stable. Hence the "photon" LKP γ_1 provides an interesting dark matter candidate. The corrections to the masses of the other first level KK states are generally large enough that they will have prompt cascade decays down to γ_1.[3] Therefore KK production at colliders results in generic missing energy signatures, similar to supersymmetric models with stable neutralino LSP. Collider searches for this scenario appear to be rather challenging because of the KK mass degeneracy and will be discussed in a separate publication [13].

V. CONCLUSIONS

Loop corrections to the masses of Kaluza-Klein excitations can play an important role in the phenomenology of extra dimensional theories. This is because KK states of a given level are all nearly degenerate, so that small corrections can determine which states decay and which are stable.

In this paper we computed the corrections to the masses of the KK excitations of gauge fields, scalars and spin-$\frac{1}{2}$ fermions with arbitrary couplings in several extra-dimensional scenarios. Our results for one and two circular extra dimensions are presented in Sec. II and Appendix A. They are finite and cut-off independent as long as the cutoff is 5D Lorentz invariant and local. In Sec. III we extended our results to the case of orbifolds S^1/Z_2 and T^2/Z_2. We found divergences which introduce cut-off dependence. The corresponding counterterms can be seen to be localized at the fixed points of the orbifold. The same technique for calculating corrections to KK masses can be easily applied to more general compactifications, as long as the Lorentz invariance is preserved at short distance in the bulk.

In Sec. IV we apply these results to the standard model in

[3]The first level graviton G_1 (or right-handed neutrino N_1 if the theory includes right handed neutrinos N_0) could also be the LKP. However, the decay lifetime of γ_1 to G_1 or N_1 would be comparable to cosmological scales. Therefore, G_1 and N_1 are irrelevant for collider phenomenology but may have interesting consequences for cosmology.

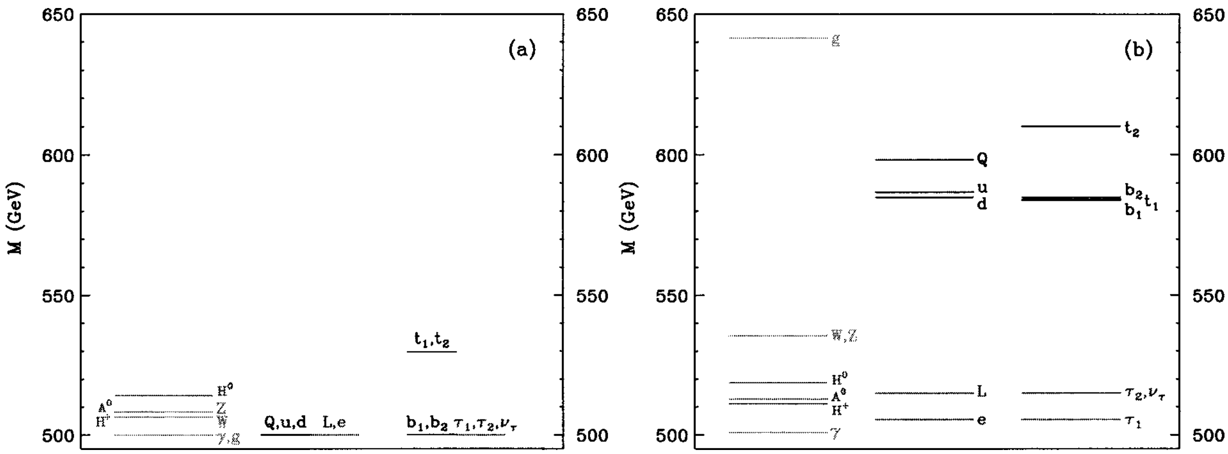

FIG. 6. The spectrum of the first KK level at (a) tree level and (b) one loop, for $R^{-1}=500$ GeV, $\Lambda R=20$, $m_h=120$ GeV, $\overline{m}_H^2=0$, and assuming vanishing boundary terms at the cut-off scale Λ.

extra dimensions and give explicit formulas for the corrected masses of all KK excitations. We hope that these results will be useful to practitioners of the phenomenology of universal extra dimensions and other models with standard model fields in the "bulk" (intriguing examples are [14,15]).

ACKNOWLEDGMENTS

We thank N. Arkani-Hamed, A. Cohen and B. Dobrescu for useful discussions. We also thank the Aspen Center for Physics for hospitality during the initial stage of this work. H.-C.C. is supported by the Department of Energy grant DE-FG02-90ER-40560. M.S. is supported in part by the Department of Energy under grant number DE-FG02-91ER-40676.

APPENDIX A: ONE-LOOP BULK CONTRIBUTIONS

In this appendix we list the one-loop corrections to KK masses from various diagrams with nonzero winding numbers.

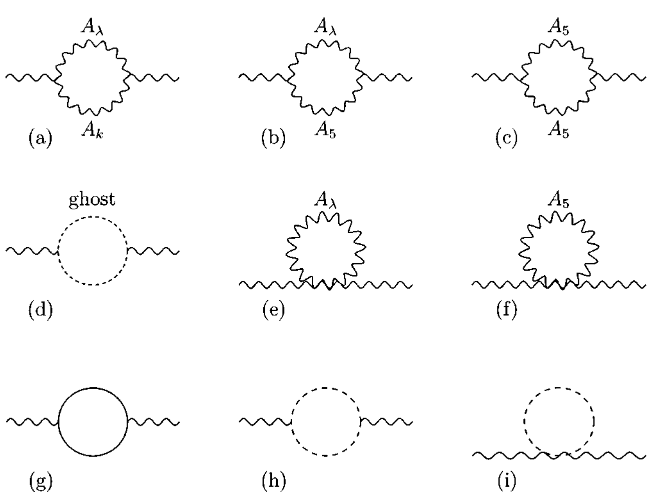

FIG. 7. One-loop diagrams for the gauge boson self-energy: (a) $A_\lambda - A_\kappa$ loop, (b) $A_\lambda - A_5$ loop, (c) $A_5 - A_5$ loop, (d) ghost loop, (e) A_λ loop, (f) A_5 loop, (g) fermion loop, (h) scalar-scalar loop, and (i) scalar loop.

We consider one extra dimension compactified on a circle S^1 with radius R. The various one-loop diagrams for the gauge boson self-energy are shown in Fig. 7. The contributions from nonzero winding numbers to the zero mode and nonzero modes in the Feynman–'t Hooft gauge are listed in Table I. After summing over all diagrams, we find that the total contribution to the zero mode is 0, and the contribution to nonzero modes is

$$\delta m^2_{V_{KK}} = \frac{g^2\zeta(3)}{16\pi^4 R^2}\left(3C(G) + \sum_{\text{real scalars}} T(r_s) - 4\sum_{\text{fermions}} T(r_f)\right). \tag{A1}$$

The one-loop contribution to the fermion self-energy is also obtained easily. For the example of QED,

$$\Sigma = -3e^2 \int d\alpha \sum_{k_5} \int \frac{d^4k_E}{(2\pi)^4} \times \frac{\alpha(\not{p} + i\gamma_5 p_5) + i\gamma_5 k_5'}{[k_E^2 + k_5'^2 - \alpha(1-\alpha)(p^2 - p_5^2)]^2}, \tag{A2}$$

TABLE I. The contributions from the diagrams in Figs. 7(a)–7(i). All these terms are multiplied $g^2\zeta(3)/16\pi^4R^2$. For the scalar loops in (h) and (i), the results are for each real component.

Diagram	Nonzero mode	Zero mode
(a)	$C(G)$	$-\frac{9}{2}C(G)$
(b)	$-2C(G)$	$C(G)$
(c)	0	$-C(G)$
(d)	0	$\frac{1}{2}C(G)$
(e)	$3C(G)$	$3C(G)$
(f)	$C(G)$	$C(G)$
(g)	$-4T(r_f)$	0
(h)	0	$-T(r_s)$
(i)	$T(r_s)$	$T(r_s)$

TABLE II. The contributions to a_1,a_2,a_3 from the diagrams in Figs. 7(a)–7(i) and Figs. 8(b′),8(c′). There is no contribution from fermions at one loop due to the cancellation between the Z_2 even and odd fermion components. For the scalar loops in (h) and (i), the upper (lower) sign is for the Z_2 even (odd) scalar, and the results are for each real component.

Diagram	a_1	a_2	a_3
(a)	$[\frac{19}{6}-(\xi-1)]C(G)$	$[\frac{11}{3}-(\xi-1)]C(G)$	$[\frac{9}{2}+\frac{9}{4}(\xi-1)]C(G)$
(b)	0	0	$[3+\frac{3}{4}(\xi-1)]C(G)$
(b′)	0	0	$\frac{3}{2}(\xi-1)C(G)$
(c)	$\frac{1}{3}C(G)$	$\frac{1}{3}C(G)$	$[-1+(\xi-1)]C(G)$
(c′)	0	0	$-2(\xi-1)C(G)$
(d)	$\frac{1}{6}C(G)$	$-\frac{1}{3}C(G)$	$-\frac{1}{2}C(G)$
(e)	$C(G)$	0	$[-3-\frac{3}{2}(\xi-1)]C(G)$
(f)	$C(G)$	0	$[1-(\xi-1)]C(G)$
(g)	0	0	0
(h)	$\mp\frac{1}{3}T(r_s)$	$\mp\frac{1}{3}T(r_s)$	$\pm T(r_s)$
(i)	0	0	$\mp T(r_s)$

where $k_5'=k_5-\alpha p_5$. The term proportional to k_5' vanishes after Poisson resummation. The remainder is a function of $\not{p}+i\gamma_5 p_5$ and therefore does not contribute to KK mode masses. Similar arguments apply to all other fermion self energy diagrams.

Scalar masses are not protected by symmetries, and they can receive power-divergent contributions. However, we can use the same method to isolate the finite contributions from loops with nonzero winding numbers. We find that these finite corrections are the same for zero mode and nonzero modes in the leading order for $m_0 \ll 1/R$. They are both given by

$$\frac{\zeta(3)}{16\pi^4R^2}\left(4g^2T(r)+\sum_{\text{real scalars}}\frac{\lambda}{2}-\sum_{\text{4-comp fermions}}4h_f^2\right) \tag{A3}$$

and can be absorbed into the overall mass term. At the lowest order, there is no relative correction between zero mode and nonzero mode.

One can also generalize to more extra dimensions. For example, we consider two extra dimensions compactified on a square torus with radius R for both dimensions. The result is very similar to the one extra dimension case, except that the factor

$$\zeta(3)=\sum_{n=1}^{\infty}\frac{1}{n^3}\approx 1.202 \tag{A4}$$

in the 5-dimensional formulas is replaced by

$$\begin{aligned}\frac{1}{\pi}\sum_{m,n\in Z}^{m^2+n^2\neq 0}\frac{1}{(m^2+n^2)^2} &= \frac{4}{\pi}\left(\zeta(4)+\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\frac{1}{(m^2+n^2)^2}\right)\\ &\equiv \frac{4}{\pi}[\zeta(4)+\Delta]\approx\frac{4}{\pi}\times 1.506,\end{aligned} \tag{A5}$$

and one has to include the A_6 loop, which contributes like a real adjoint scalar. There is also an extra adjoint scalar at each KK level coming from a linear combination of A_5 and A_6, which is not eaten by the KK gauge bosons. The correction to the KK mode masses of the gauge boson and the extra adjoint scalar are the same and are both given by

$$\delta m^2_{V_{KK}}(6D)=\frac{g^2[\zeta(4)+\Delta]}{4\pi^5R^2}\left(4C(G)+\sum_{\text{real scalars}}T(r_s)-\sum_{\text{4-comp fermions}}4T(r_f)\right). \tag{A6}$$

APPENDIX B: ONE-LOOP BOUNDARY CONTRIBUTIONS

In this appendix, we list the one-loop contributions to the boundary terms for gauge fields, fermions, and scalars for the S^1/Z_2 orbifold compactification. The results for the case of a two dimensional orbifold T^2/Z_2 are briefly discussed at the end.

The one-loop diagrams for the gauge boson self energy are shown in Fig. 7. We keep only logarithmically divergent contributions to the boundary terms. They can be written as

$$\bar{\Pi}_{\mu\nu}=\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left\{g_{\mu\nu}p^2a_1-p_\mu p_\nu a_2+g_{\mu\nu}\frac{p_5^2+p_5'^2}{2}a_3\right\}\quad\left(\text{for }p_5'=p_5+\frac{2n}{R}\right). \tag{B1}$$

In Table II, we list a_1,a_2,a_3 in the ξ gauge (using the gauge fixing of the 5 dimensional generalized Lorentz gauge condition). In this gauge, A_μ and A_5 do not decouple for $\xi\neq 1$, so there are additional divergent diagrams shown in Fig. 8. Adding all contributions together, we obtain

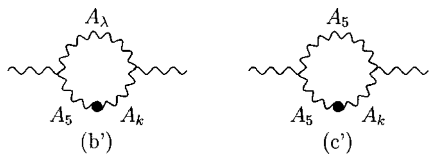

FIG. 8. Additional divergent contributions to the gauge boson self-energy in the ξ gauge where A_μ and A_5 do not decouple. (b′) $A_\lambda - A_5, A_\kappa$ loop; (c′) $A_5 - A_5, A_\kappa$ loop.

$$\bar{\Pi}_{\mu\nu} = \frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left\{(g_{\mu\nu}p^2 - p_\mu p_\nu)\left[\left(\frac{11}{3} - (\xi - 1)\right)C(G) - \frac{1}{3}\sum_{\text{real scalars}}(T(r)_{\text{even}} - T(r)_{\text{odd}})\right] + g_{\mu\nu}\frac{(p_5^2 + p_5'^2)}{2}(4 + (\xi - 1))C(G)\right\} \quad \left(\text{for } p_5' = p_5 + \frac{2n}{R}\right). \tag{B2}$$

The correction to the squared mass of the n-th mode KK gauge boson can be obtained from the term proportional to $g_{\mu\nu}$, by setting $p^2 = p_5^2 = p_5'^2 = m_n^2 = n^2/R^2$, and multiplying by the wave function normalization factor $(\sqrt{2})^2$,

$$\bar{\delta} m_{V_n}^2 = m_n^2 \frac{g^2}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\frac{23}{3}C(G) - \frac{1}{3}\sum_{\text{real scalars}}(T(r)_{\text{even}} - T(r)_{\text{odd}})\right]. \tag{B3}$$

One can see that the result is gauge independent.

The one-loop fermion self-energy diagrams are shown in Fig. 9. Keeping only the logarithmically divergent contributions, we can write

$$\bar{\Sigma} = \frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\not{p}\frac{1 \pm \gamma_5}{2}b_1 - \left(ip_5\frac{1 \pm \gamma_5}{2} - ip_5'\frac{1 \mp \gamma_5}{2}\right)b_2\right] \quad \left(\text{for } p_5' = p_5 + \frac{2n}{R}\right). \tag{B4}$$

The contributions to b_1, b_2 are listed in Table III. The correction to the fermion KK mode mass is given by

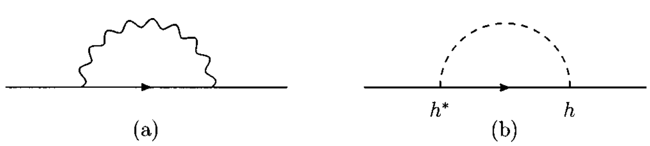

FIG. 9. One-loop diagrams for the fermion self-energy: (a) gauge boson loop; (b) scalar boson loop.

TABLE III. The contributions to b_1, b_2 from the diagrams in Figs. 9(a),9(b). $C(r)$ is defined $C(r)\delta_{ij} = \Sigma_a T_{ik}^a T_{kj}^a$ [$=(N^2-1)/(2N)$ for the fundamental representation of $SU(N)$ gauge group]. The upper (lower) sign in (b) is for Z_2 even (odd) scalars.

Diagram	b_1	b_2
(a)	$[-1-2(\xi-1)]g^2C(r)$	$[5+(\xi-1)]g^2C(r)$
(b)	$\mp h^2$	$\mp h^2$

$$\bar{\delta} m_{f_n} = m_n \frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[9C(r)g^2 - \sum_{\text{even scalars}} 3h_+^2 + \sum_{\text{odd scalars}} 3h_-^2\right]. \tag{B5}$$

A Z_2 even scalar can receive power-divergent contributions to both the bulk mass term and the boundary mass term. We need to fine tune these mass terms to have a light scalar. The boundary mass term causes mixing among KK modes and we need to re-diagonalize the mass matrix to find the eigenstates if it is large. The possibility of a light scalar arising because of cancellation between the bulk mass and the boundary mass may be interesting, but will not be considered here. Instead, we assume that both the bulk mass and the boundary mass are tuned to be much smaller than the compactification scale, so that we can treat the boundary mass term as a small perturbation and ignore the higher order mixing effects. The boundary mass term can be written as

$$\frac{L}{2}(\delta(x_5) + \delta(x_5 - L))\overline{m}^2\Phi^\dagger\Phi. \tag{B6}$$

Using the KK decomposition, Eq. (25), we find that the contribution to the zero mode is $\overline{m}^2$, while to the nonzero mode is $2\,\overline{m}^2$, due to the normalization factor $\sqrt{2}$ at the boundaries. Therefore, the nonzero KK modes receive a correction $\overline{m}^2$ relative to the zero mode from the boundary mass term (ignoring a weak scale dependence due to the wave function renormalization). We can also calculate the correction due to the boundary kinetic terms. The one-loop diagrams for the scalar self-energy are shown in Fig. 10. They can be written as

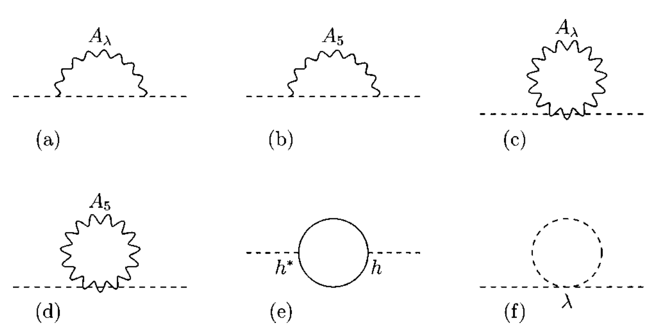

FIG. 10. One-loop diagrams for the scalar boson self-energy: (a) A_λ-scalar loop, (b) A_5-scalar loop, (c) A_λ loop, (d) A_5 loop, (e) fermion loop, and (f) scalar loop.

TABLE IV. The contributions to c_1, c_2 (in Feynman gauge) from the diagrams in Figs. 10(a)–10(f). The upper (lower) sign in (f) is for a Z_2 even (odd) scalar in the loop.

Diagram	c_1	c_2
(a)	$4g^2T(r)$	$2g^2T(r)$
(b)	0	$3g^2T(r)$
(c)	0	$-4g^2T(r)$
(d)	0	$g^2T(r)$
(e)	0	0
(f)	0	$\mp\frac{\lambda}{2}$

$$\frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left\{p^2c_1+\frac{p_5^2+p_5'^2}{2}c_2\right\}\quad\left(\text{for}\ \ p_5'=p_5+\frac{2n}{R}\right). \tag{B7}$$

The coefficients c_1, c_2 (in the Feynman gauge) are given in Table IV. Including the normalization factor $(\sqrt{2})^2$, we have the correction to the KK modes of an even scalar,

$$\overline{\delta}m^2_{S+_n}=\overline{m}^2+m_n^2\frac{1}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[6g^2T(r)-\sum_{\text{even scalars}}\frac{\lambda_{++}}{2}+\sum_{\text{odd scalars}}\frac{\lambda_{+-}}{2}\right], \tag{B8}$$

where the sum is over real components.

For an odd scalar, there is no boundary mass term. The correction comes only from boundary kinetic terms,

$$\frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}p_5p_5'd_1\quad\left(\text{for}\ p_5'=p_5+\frac{2n}{R}\right). \tag{B9}$$

The coefficients d_1 from one-loop diagrams are listed in Table V. The total correction to the KK modes of an odd scalar KK is

$$\overline{\delta}m^2_{S-_n}=m_n^2\frac{1}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[9g^2T(r)+\sum_{\text{even scalars}}\frac{\lambda_{+-}}{2}-\sum_{\text{odd scalars}}\frac{\lambda_{--}}{2}\right]. \tag{B10}$$

TABLE V. The contributions to d_1 (in Feynman gauge) from the diagrams in Figs. 10(a)–10(f). The upper (lower) sign in (f) is for a Z_2 even (odd) scalar in the loop.

Diagram	d_1
(a)	0
(b)	$5g^2T(r)$
(c)	$4g^2T(r)$
(d)	$-g^2T(r)$
(e)	0
(f)	$\pm\frac{\lambda}{2}$

Finally, we briefly describe the results for 2 extra dimensions compactified on a T^2/Z_2 orbifold, with a square torus T^2 of radius R for each side. The Z_2 is a 180° rotation in the x_5, x_6 plane, which flips the signs of both x_5 and x_6. The gauge components A_5, A_6 are odd under Z_2 while $A_\mu, \mu=0,1,2,3$ are even. There will be induced terms localized at the orbifold fixed points $(x_5,x_6)=(0,0),(0,\pi R),(\pi R,0),(\pi R,\pi R)$, which break 6-dimensional Lorentz invariance.

The KK states are labeled by a pair of KK numbers, (n_1,n_2), with (n_1,n_2) and $(-n_1,-n_2)$ identified. There are KK parities associated with each KK number. The results are similar to the 5-dimensional case on S^1/Z_2, except that we need to include the extra A_6 component, which contributes like an odd adjoint real scalar. We have

$$\overline{\delta}m^2_{V_{(n_1,n_2)}}=m^2_{(n_1,n_2)}\frac{g^2}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\times\left[8C(G)-\frac{1}{3}\sum_{\text{real scalars}}(T(r)_{\text{even}}-T(r)_{\text{odd}})\right], \tag{B11}$$

$$\overline{\delta}m_{f_{(n_1,n_2)}}=m_{(n_1,n_2)}\frac{1}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\times\left[12C(r)g^2-\sum_{\text{even scalars}}3h_+^2+\sum_{\text{odd scalars}}3h_-^2\right], \tag{B12}$$

$$\overline{\delta}m^2_{S+_{(n_1,n_2)}}=\overline{m}^2+m^2_{(n_1,n_2)}\frac{1}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\times\left[7g^2T(r)-\sum_{\text{even scalars}}\frac{\lambda_{++}}{2}+\sum_{\text{odd scalars}}\frac{\lambda_{+-}}{2}\right], \tag{B13}$$

$$\overline{\delta}m^2_{S-_{(n_1,n_2)}}=m^2_{(n_1,n_2)}\frac{1}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\times\left[8g^2T(r)+\sum_{\text{even scalars}}\frac{\lambda_{+-}}{2}-\sum_{\text{odd scalars}}\frac{\lambda_{--}}{2}\right]. \tag{B14}$$

In addition, there are also KK states corresponding to the linear combination of A_5 and A_6 which is not eaten by the KK gauge boson. These KK states are odd adjoint scalars. Their corrections are just like the odd adjoint scalars'

$$\overline{\delta}m^2_{P_{(n_1,n_2)}}=m^2_{(n_1,n_2)}\frac{g^2}{32\pi^2}\ln\frac{\Lambda^2}{\mu^2}\times\left[8C(G)+\sum_{\text{real scalars}}(T(r)_{\text{even}}-T(r)_{\text{odd}})\right]. \tag{B15}$$

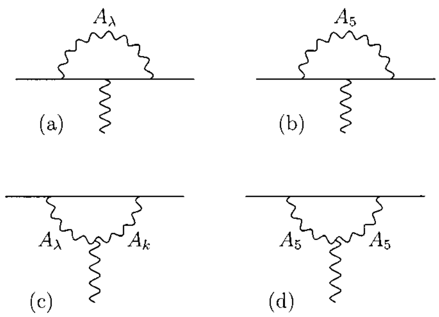

FIG. 11. One-loop diagrams for the fermion-gauge boson interaction: (a) A_λ-fermion-fermion loop, (b) A_5-fermion-fermion loop, (c) A_λ-A_κ-fermion loop, and (d) A_5-A_5-fermion loop,

APPENDIX C: KK NUMBER VIOLATING COUPLINGS

In this appendix we discuss the KK number violating couplings in an orbifold compactification. Using the example of one extra dimension compactified on S^1/Z_2, we consider the KK number violating couplings between the fermion and the gauge field. Figure 11 shows the one-loop vertex corrections for the fermion gauge interactions. The contributions to the KK number violating interaction are logarithmically divergent. They can be written as

$$\delta\mathcal{L}\supset -\frac{L}{2}(\delta(x_5)+\delta(x_5-L))f_1\frac{g^2}{64\pi^2}\times\ln\frac{\Lambda^2}{\mu^2}g\,\bar{\psi}\gamma^\mu T^a P_+\psi A^a_\mu, \tag{C1}$$

where $P_+=P_R$ or P_L is the projection on the Z_2 even fermions. The coefficients f_1 from the diagrams in Fig. 11 are listed in Table VI. Summing over them gives

$$f_1(\text{total})=C(r)[1+2(\xi-1)]+C(G)\left[2+\frac{1}{2}(\xi-1)\right]. \tag{C2}$$

To obtain the couplings among the physical mass eigenstates, we need to include the KK number violating mass and kinetic mixing effects on the external legs, since they are also one-loop effects. The (4-dimensional) kinetic mixing needs to be treated with some care. We illustrate this with a simple example of two real scalars, ϕ_p,ϕ_q, with masses $m_p<m_q$, and a small kinetic mixing proportional to ϵ:

TABLE VI. The contributions to f_1 from the diagrams in Figs. 11(a)–11(d).

Diagram	f_1
(a)	$[2C(r)-C(G)][1+(\xi-1)]$
(b)	$-C(r)+\frac{1}{2}C(G)$
(c)	$C(G)[3+\frac{3}{2}(\xi-1)]$
(d)	$-\frac{1}{2}C(G)$

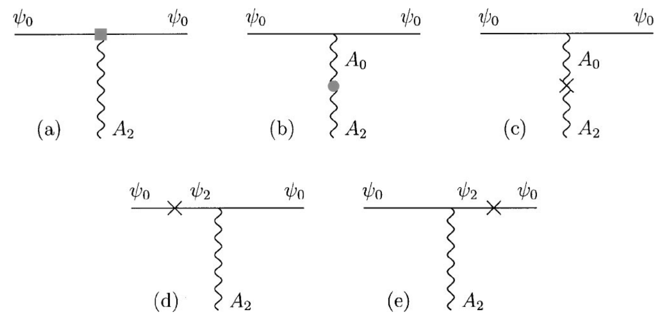

FIG. 12. The KK number violating coupling for $\overline{\psi}_0\gamma^\mu T^a P_+\psi_0 A^a_{2\mu}$. The dot represents the kinetic mixing and the cross represents the mass mixing. The contributions from various diagrams are $\sqrt{2}g(g^2/16\pi^2)\ln(\Lambda^2/\mu^2)\times$ (a) one-loop vertex: $\{C(r)[1+2(\xi-1)]+2C(G)[2+\frac{1}{2}(\xi-1)]\}$, (b) A_2(external)-A_0 kinetic mixing: $\{[\frac{11}{3}-(\xi-1)]C(G)-\frac{1}{3}\Sigma_{\text{real scalars}}(T(r_+)-T(r_-))\}$, (c) A_2-A_0 mass mixing: $[2+\frac{1}{2}(\xi-1)]C(G)$, and (d), (e) $\psi_0-\psi_2$ mass mixing: $\{-5+(\xi-1)C(r)\}\times 2$.

$$\mathcal{L}=\frac{1}{2}\partial_\mu\phi_p\partial^\mu\phi_p+\epsilon\partial_\mu\phi_p\partial^\mu\phi_q+\frac{1}{2}\partial_\mu\phi_q\partial^\mu\phi_q-\frac{1}{2}m_p^2\phi_p^2-\frac{1}{2}m_q^2\phi_q^2. \tag{C3}$$

We will only work in the leading order of ϵ. First, we redefine ϕ_p to absorb the mixing term,

$$\phi_p'=\phi_p+\epsilon\phi_q \qquad \phi_q'\approx\phi_q$$

or

$$\phi_p\approx\phi_p'-\epsilon\phi_q' \qquad \phi_q\approx\phi_q'. \tag{C4}$$

In terms of ϕ_p',ϕ_q', the mass terms become

$$-\frac{1}{2}m_p^2\phi_p'^2+\epsilon m_p^2\phi_p'\phi_q'-\frac{1}{2}m_q^2\phi_q'^2. \tag{C5}$$

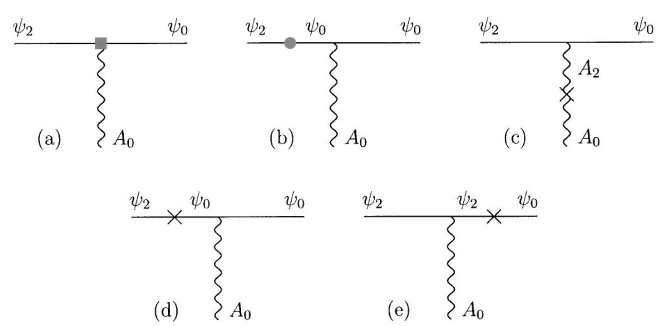

FIG. 13. The KK number violating coupling for $\overline{\psi}_2\gamma^\mu T^a P_+\psi_0 A^a_{0\mu}$. The dot represents kinetic mixing and the cross represents mass mixing. The contributions from various diagrams are $\sqrt{2}g(g^2/16\pi^2)\ln(\Lambda^2/\mu^2)\times$ (a) one-loop vertex: $\{C(r)[1+2(\xi-1)]+2C(G)[2+\frac{1}{2}(\xi-1)]\}$, (b) ψ_2(external)-ψ_0 kinetic mixing: $\{-[1+2(\xi-1)]\}$, (c) A_2-A_0 mass mixing: $\{-[2+\frac{1}{2}(\xi-1)]C(G)\}$, (d) ψ_2(external)-ψ_0 mass mixing: $[5+(\xi-1)]C(r)$, and (e) ψ_0(external)-ψ_2 mass mixing: $\{-[5+(\xi-1)]C(r)\}$.

Now we can diagonalize the mass matrix by a rotation between ϕ'_p and ϕ'_q. The physical eigenstates ϕ''_p and ϕ''_q are given approximately by

$$\phi''_p \approx \phi'_p + \frac{\epsilon m_p^2}{m_q^2 - m_p^2}\phi'_q \approx \phi_p + \frac{\epsilon m_q^2}{m_q^2 - m_p^2}\phi_q$$

$$\phi''_q \approx \phi'_q - \frac{\epsilon m_p^2}{m_q^2 - m_p^2}\phi'_p \approx \phi_q - \frac{\epsilon m_p^2}{m_q^2 - m_p^2}\phi_p. \tag{C6}$$

In particular, if one of them is massless, $m_p = 0$, the relation between the physical states and the original states is simply given by Eq. (C4).

As an example, we compute the coupling between the mass eigenstates of a second (or $2n$-th) KK mode gauge boson and two zero mode fermions. The contributions are shown in Fig. 12. Combining all contributions we obtain the $\overline{\psi}_0 - \psi_0 - A_2$ interaction vertex to be

$$(-i\gamma^\mu g T^a P_+)\sqrt{2}\frac{g^2}{64\pi^2}\ln\frac{\Lambda^2}{\mu^2}\left[\frac{23}{3}C(G) - \frac{1}{3}\sum_{\text{real scalars}}\left(T(r)_{\text{even}} - T(r)_{\text{odd}}\right) - 9C(r)\right]$$

$$= (-i\gamma^\mu g T^a P_+)\frac{\sqrt{2}}{2}\left[\frac{\overline{\delta}(m_{A_2}^2)}{m_2^2} - 2\frac{\overline{\delta}(m_{f_2})}{m_2}\right]. \tag{C7}$$

It is not too surprising that it is related to the mass corrections from the boundary terms. The $\sqrt{2}$ factor comes from the normalization of the KK mode at the boundaries.

One can also check the KK number violating couplings involving the zero mode gauge boson, e.g., $\overline{\psi}_2 \gamma^\mu T^a P_+ \psi_0 A^a_{0\mu}$ (Fig. 13). We find that they vanish as required by gauge invariance.[4]

[4] However, there can be higher dimensional operators such as $\overline{\psi}_2 \sigma^{\mu\nu} T^a P_+ \psi_0 F^a_{0\mu\nu}$.

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