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Flavor physics and fine-tuning in theory space

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Recently a new class of composite Higgs models have been developed which give rise to naturally light Higgs bosons without supersymmetry. Based on the chiral symmetries of "theory space," involving replicated gauge groups and appropriate gauge symmetry breaking patterns, these models allow the scale of the underlying strong dynamics giving rise to the composite particles to be as large as of order 10 TeV, without any fine tuning to prevent large corrections to Higgs boson mass(es) of order 100 GeV. In this paper we show that the size of flavor violating interactions arising generically from underlying flavor dynamics constrains the scale of the Higgs boson compositeness to be greater than of order 75 TeV, implying that significant fine-tuning is required. Without fine-tuning, the low-energy structure of the composite Higgs model alone is not sufficient to eliminate potential problems with flavor-changing neutral currents or excessive CP violation; solving those problems requires additional information or assumptions about the symmetries of the underlying flavor or strong dynamics. We also consider the weaker, but more model-independent, bounds which arise from limits on weak isospin violation.

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I. INTRODUCTION

Recently a new class [1,2] of composite Higgs models [3] has been developed which give rise to naturally light Higgs bosons without supersymmetry. Inspired by discretized versions of higher-dimensional gauge theory [4,5], these models are based on the chiral symmetries of "theory space" [4]. The models involve replicated gauge groups and corresponding gauge symmetry breaking patterns. They allow the scale (Λ) of the underlying strong dynamics giving rise to the composite particles to be as large as 10 TeV, without causing large corrections to the Higgs boson mass(es) of order 100 GeV.

Various possibilities exist for the underlying physics (the "high-energy completion") which gives rise to the chiral-symmetry breaking pattern required, and produces the "pion" which becomes the composite Higgs boson. However, regardless of the precise nature of the underlying strongly interacting physics, there must be flavor dynamics at a scale of order Λ or greater that gives rise to the different Yukawa couplings of the Higgs boson to ordinary fermions. As in extended technicolor theories [6,7], if this flavor dynamics arises from gauge-interactions it will generically cause flavor-changing neutral currents [7].

In this paper we review and update the lower bound on Λ arising from the experimental constraints on extra contributions to the neutral meson mass differences [8]. We find that in composite Higgs models the size of flavor-violating interactions arising from the high-energy theory constrains the scale Λ to be greater than of order 75 TeV. We then consider the "theory space" models, argue why this flavor bound applies to such models, and review the upper limit on Λ of order 10 TeV necessary to avoid fine-tuning [1,2]. Raising the scale Λ to 75 TeV to be consistent with the flavor bounds mentioned above, then, necessitates fine tuning of order 2%. We compare these bounds to those arising from limits on the amount of CP violation and isospin violation in the composite Higgs theory.

The implication of our findings is that the low-energy structure of the composite Higgs model alone is not sufficient to eliminate potential problems with flavor-changing neutral current or excessive CP violation; solving those problems requires additional information or assumptions about the symmetries of the underlying strong dynamics.[1]

II. FLAVOR AND COMPOSITE HIGGS BOSONS[2]

We begin by considering what the observed masses of the ordinary fermions imply about the underlying flavor physics. Providing the different masses of the fermions requires flavor physics [analogous to extended-technicolor interactions

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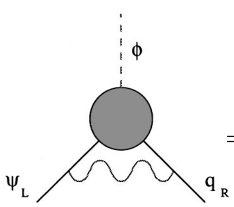
[1]See also [9], which emphasizes that the properties of the underlying strong dynamics may affect the details of the low-energy phenomenology.

[2]This section reviews and updates material from [8].

(ETC) [6,7]] which couples the left-handed quark doublets ψ_L and right-handed singlets q_R to the strongly interacting constituents of the composite Higgs doublet. At low energies, these interactions produce the quark Yukawa couplings.

To estimate the sizes of various effects of the underlying physics, we rely on dimensional analysis [10]. As noted by Georgi [11], a theory with light scalar particles belonging to a single symmetry-group representation depends on two parameters: Λ, the scale of the underlying physics, and f (the analog of f_π in QCD), which measures the amplitude for producing the scalar particles from the vacuum. Our estimates of the sizes of the low-energy effects of the underlying physics will depend on the ratio $\kappa \equiv \Lambda/f$, which determines the sizes of coupling constants in the low-energy theory. Naive dimensional analysis corresponds to $\kappa = 4\pi$ [10].

Assuming that these new flavor interactions are gauge interactions with gauge coupling g and gauge boson mass M, dimensional analysis [10] allows us to estimate that the size of the resulting Yukawa coupling is [3] of order $(g^2/M^2)(\Lambda^2/\kappa)$, i.e.



$$\Rightarrow \frac{g^2}{M^2}\frac{\Lambda^2}{\kappa}\overline{q}_R \phi \psi_L . \tag{1}$$

In order to give rise to a quark mass m_q, the Yukawa coupling must be equal to

$$\frac{\sqrt{2} m_q}{v} \tag{2}$$

where $v \approx 246$ GeV. This implies

$$\Lambda \approx \frac{M}{g}\sqrt{\sqrt{2}\kappa \frac{m_q}{v}}. \tag{3}$$

Thus, if we set a lower limit on M/g from low-energy flavor physics, Eq. (3) will give a lower bound on Λ.

The high-energy flavor physics responsible for the generation of the Yukawa couplings *must* distinguish between different flavors so as to give rise to the different masses of the corresponding fermions. In addition, the flavor physics will give rise to flavor-specific couplings among ordinary fermions [6,7]. These will generically give rise to flavor-changing neutral currents (as previously noted in [7] for the case of ETC theories) that affect kaon, D-meson, and B-meson physics.

Consider the interactions responsible for the c-quark mass. Through Cabibbo mixing, these interactions must couple to the u-quark as well. Neglecting mixing with the top-quark, this will generally give rise to the interactions

$$\begin{aligned}\mathcal{L}_{eff} = &-(\cos\theta_L^c \sin\theta_L^c)^2 \frac{g^2}{M^2}(\overline{c}_L\gamma^\mu u_L)(\overline{c}_L\gamma_\mu u_L)\\ &-(\cos\theta_R^c \sin\theta_R^c)^2 \frac{g^2}{M^2}(\overline{c}_R\gamma^\mu u_R)(\overline{c}_R\gamma_\mu u_R)\\ &-2\cos\theta_L^c \sin\theta_L^c \cos\theta_R^c \sin\theta_R^c \frac{g^2}{M^2}(\overline{c}_L\gamma^\mu u_L)\\ &\times(\overline{c}_R\gamma_\mu u_R),\end{aligned} \tag{4}$$

where the coupling g and mass M are of the same order as those in the interactions which ultimately give rise to the c-quark Yukawa coupling in Eq. (1), and the angles θ_L^c and θ_R^c represent the relation between the gauge eigenstates and the mass eigenstates. The operators in Eq. (4) will clearly affect neutral D-meson physics. Similarly, the interactions responsible for other quarks' masses will give rise to operators that contribute to mixing and decays of the corresponding mesons.

The color-singlet products of currents in Eq. (4) will contribute directly to D-meson mixing. In the vacuum-insertion approximation, the purely left-handed or right-handed current-current operators yield

$$\left(\frac{M}{g}\right)_{\mathrm{LL,RR}} \gtrsim f_D\left(\frac{2 m_D B_D}{3\Delta m_D}\right)^{1/2}\cos\theta_{L,R}^c \sin\theta_{L,R}^c \approx 225 \text{ TeV}, \tag{5}$$

where we have used the limit on the neutral D-meson mass difference, $\Delta m_D \lesssim 4.6\times 10^{-11}$ MeV [12], and $f_D\sqrt{B_D} = 0.2$ GeV [13], $\theta_{L,R}^c \approx \theta_C$. The bound on the scale of the underlying strongly interacting dynamics follows from Eq. (3):

$$\Lambda \gtrsim 21\,\text{TeV}\sqrt{\kappa\left(\frac{m_c}{1.5\,\text{GeV}}\right)}, \tag{6}$$

so that $\Lambda \gtrsim 75$ TeV for $\kappa \approx 4\pi$.

The $\Delta C = 2$, LR product of color-singlet currents gives a weaker bound than Eq. (6), but the LR product of color-octet currents,

$$\begin{aligned}\mathcal{L}_{eff} = &-2\cos\theta_L^c \sin\theta_L^c \cos\theta_R^c \sin\theta_R^c\\ &\times \frac{g^2}{M^2}(\overline{c}_L\gamma^\mu T^a u_L)(\overline{c}_R\gamma_\mu T^a u_R),\end{aligned} \tag{7}$$

where T^a are the generators of $SU(3)_C$, gives a stronger bound:

$$\left(\frac{M}{g}\right)_{\mathrm{LR}} \gtrsim \frac{4 f_D}{3(m_c+m_u)}\left(\frac{m_D^3 B_D'}{\Delta m_D}\right)^{1/2} \times (2\cos\theta_L^c \sin\theta_L^c \cos\theta_R^c \sin\theta_R^c)^{1/2} \tag{8}$$

$$\approx 590 \text{ TeV}\left(\frac{1.5 \text{ GeV}}{m_c}\right), \tag{9}$$

corresponding to

$$\Lambda \gtrsim 53 \ \text{TeV} \sqrt{\kappa\left(\frac{1.5 \ \text{GeV}}{m_c}\right)}. \tag{10}$$

There are also contributions to K-meson mixing from the color-singlet and color-octet products of currents analogous to those in Eqs. (4) and (7). The lower bound on Λ derived from the measured value of the $K_L K_S$ mass difference [8]

$$\Lambda \gtrsim 6.8 \ \text{TeV} \sqrt{\kappa\left(\frac{m_s}{200 \ \text{MeV}}\right)} \tag{11}$$

is weaker than Eq. (6) because the s-quark is lighter than the c-quark, while the d-s and u-c mixings are expected to be of comparable size [8]. However, in the absence of additional superweak interactions to give rise to CP-violation in K-mixing (ε), the flavor interactions responsible for the s-quark Yukawa couplings must violate CP at some level. In this case the bounds on the scale Λ are much stronger. Recalling that

$$\mathrm{Re}\, \varepsilon \approx \frac{\mathrm{Im}\, \mathrm{M}_{12}}{2\, \Delta M} \lesssim 1.65\times 10^{-3}, \tag{12}$$

and assuming that there are phases of order 1 in the $\Delta S = 2$ operators analogous to those shown in Eq. (4), we find the bound

$$\Lambda \gtrsim 120 \ \text{TeV} \sqrt{\kappa\left(\frac{m_s}{200 \ \text{MeV}}\right)}. \tag{13}$$

III. COMPOSITE HIGGS BOSONS FROM THEORY

A set of "theory space" composite Higgs models [1,2] is illustrated in Fig. 1, using "moose" or "quiver" notation [14]. In this diagram, each site except (1,1) represents a gauged $SU(3)$ group, while the links represent nonlinear sigma fields transforming as $(3,\overline{3})$'s under the adjacent groups:

$$U_{ij} \to W_{ij} U_{ij} W^{\dagger}_{i\, j+1}, \quad V_{ij} \to W_{ij} V_{ij} W^{\dagger}_{i+1\ \ j}. \tag{14}$$

The "toroidal" geometry of theory space implies that the indices i,j are periodic mod N. At the site (1,1), only the $SU(2)\times U(1)$ subgroup of an $SU(3)$ global symmetry is gauged. The kinetic energy terms in the Lagrangian then read

$$\mathcal{L}_{kin} = -\sum_{ij} \frac{1}{2g_{ij}^2} \mathrm{Tr}\, F_{ij}^2 + \frac{f^2}{4} \sum_{ij} \mathrm{Tr}|D^\mu U_{ij}|^2 + \frac{f^2}{4} \sum_{ij} \mathrm{Tr}|D^\mu V_{ij}|^2, \tag{15}$$

where g_{ij} are the gauge couplings and f is the "pion-decay constant" of the chiral symmetry breaking dynamics. For simplicity, in what follows we will assume that the gauge couplings $g_{ij} = g$ are the same for every site except for (1,1). The rules of naive dimensional analysis [10] then imply that the scale Λ of the underlying high-energy dynamics which gives rise to this theory is bounded by of order $4\pi f$.

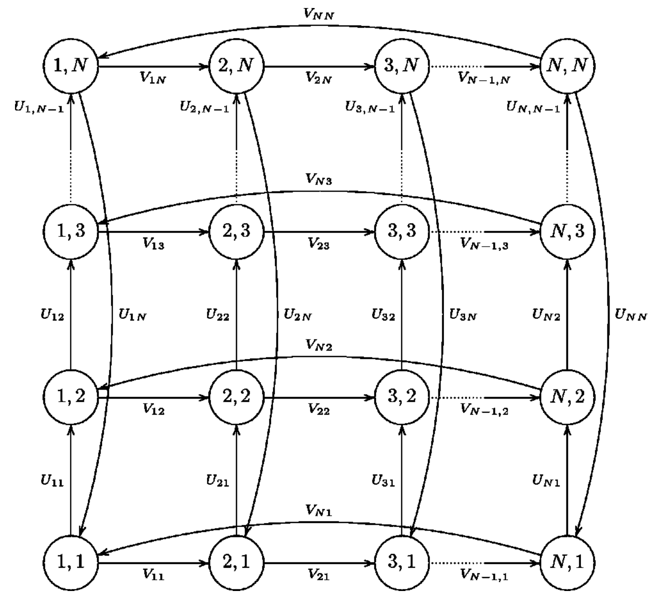


FIG. 1. A composite Higgs model based on an $N\times N$ toroidal lattice "theory space." $SU(3)$ gauge groups live at every site except (1,1), while the links represent nonlinear sigma fields transforming as $(3,\overline{3})$'s under the adjacent gauge symmetries. Only an $SU(2)\times U(1)$ subgroup of an $SU(3)$ global symmetry group is gauged at site (1,1). As described in the text, N^2-1 sets of Goldstone bosons are eaten, N^2-1 get mass from "plaquette operators" which explicitly break the chiral symmetries, and two sets remain in the very low-energy theory. This illustration comes from [9].

The $2N^2$ Goldstone bosons of the chiral symmetry breaking dynamics are incorporated into the sigma-model fields

$$U_{ij} = \exp 2i\pi_{u,ij}/f, \quad V_{ij} = \exp 2i\pi_{v,ij}/f. \tag{16}$$

The gauge symmetry breaking pattern implied is $SU(3)^{N^2-1}\times SU(2)\times U(1) \to SU(2)\times U(1)$, resulting in N^2-1 sets of "eaten" Goldstone bosons. The remaining N^2+1 sets of Goldstone bosons in the physical spectrum interact via the gauge interactions, which explicitly violate the chiral symmetries. However, because of the "topology" of theory space, the lowest-order interaction in the effective theory which breaks the chiral-symmetries in the same way as the gauge interactions only occurs at high order [1]. Therefore, the leading contribution to the masses of these remaining scalars from the low-energy gauge interactions is *finite*, and arises at $\mathcal{O}(g^4)$ from the Coleman-Weinberg potential [15].

An important ingredient in these models is a set of non-derivative chiral-symmetry breaking operators of the form of "plaquette" interactions[3]

[3]Because of the reduced symmetry at site (1,1), additional operators are present there which play an important role in the detailed phenomenology of the composite scalar particles [1,2].

$$\mathcal{L}_{pl} = \lambda f^4 \sum_{ij} \mathrm{Tr}(U_{ij} V_{i\,j+1} U^\dagger_{i+1\,j} V^\dagger_{ij}) + \text{H.c.}, \tag{17}$$

where (again, for simplicity) we have assumed that the dimensionless coupling constants λ are the same for every plaquette. Expanding these operators in terms of the Goldstone bosons fields, we find

$$\mathcal{L}_{pl} = -4\lambda f^2 \sum_{ij} \mathrm{Tr}(\pi_{u,ij} + \pi_{v,i\,j+1} - \pi_{u,i+1\,j} - \pi_{v,ij})^2 + \mathcal{O}(\pi^4) + \cdots . \tag{18}$$

These operators have the extraordinary feature that they give rise to masses to N^2-1 of the remaining scalars, but leave massless the two combinations

$$\pi_{u,ij} \equiv \frac{U}{N} \quad \pi_{v,ij} \equiv \frac{V}{N} \tag{19}$$

which are uniform in either the ''u'' or ''v'' directions. The factors of N arise so as to normalize the U and V fields correctly. Both the U and V fields contain $SU(2)\times U(1)$ doublet scalars ϕ_u and ϕ_v with the quantum numbers of the Higgs boson. The theory gives rise to two light (so far in this discussion, massless) composite Higgs bosons with nonderivative interaction of the form [1,2]

$$\mathcal{L}_{pl} \supset \frac{4\lambda}{N^2} \mathrm{Tr}(\phi_u \phi_u^\dagger - \phi_v \phi_v^\dagger)^2 + \frac{4\lambda}{N^2} (\phi_u^\dagger \phi_u - \phi_v^\dagger \phi_v)^2. \tag{20}$$

Additionally, a negative mass-squared for one or both Higgs bosons may be introduced either through a symmetry-breaking plaquette operator at the site (1,1) [1] or through the effect of coupling the Higgs bosons to the top-quark [2]. In either case, the resulting mass-squared of the Higgs bosons is of order

$$|m_h|^2 \simeq \frac{\lambda v^2}{N^2}. \tag{21}$$

The left- and right-handed quarks and leptons transform under the $SU(2)\times U(1)$ gauge interactions at the site (1,1) [1,2]. For the light fermions, Yukawa couplings between the fermions and the composite Higgs bosons are introduced. Such interactions violate the chiral symmetries protecting the Higgs bosons masses, but the size of the resulting corrections is small since $m_q \ll v$. This choice preserves a $[U(2)]^5$ flavor symmetry, broken only by the Yukawa couplings to the composite Higgs boson, suppressing flavor-changing neutral currents from the $SU(2)\times U(1)$ and $SU(3)^{N^2-1}$ gauge bosons. Because the light quarks obtain mass from Yukawa couplings to the composite scalars, the bounds on the compositeness scale derived in Sec. II apply to this model. As noted above, however, the light Higgs boson is ''delocalized'' in theory space, Eq. (19), and therefore has only an amplitude of order $1/N$ of being at site (1,1). Consequently, we would say that Λ must satisfy

$$\Lambda \gtrsim 21 \ \text{TeV} \sqrt{\kappa N \left(\frac{m_c}{1.5 \ \text{GeV}}\right)}, \tag{22}$$

and be at least of order $\sqrt{N}\times 75$ TeV for $\kappa = 4\pi$.

The top-quark presents a more difficult problem. In this case, no *direct* Yukawa coupling is introduced [1]. Instead, the top-quark is ''spread out'' in theory space: a family of massive $SU(3)$ vector fermions on the sites $(1,n_v)$ and $(n_u,1)$ is added (here $1 \leq n_{u,v} \leq N$), along with local interactions between the vector fermions at adjacent sites and the gauge-eigenstate top-quark [which has $SU(2)\times U(1)$ gauge interactions at site (1,1)] [1,2]. Upon diagonalizing the resulting mass matrix, the expected order-1 Yukawa coupling, y_t, of the Higgs boson to the top-quark is generated so long as the nearest neighbor couplings are of order 1 and the lightest vector-fermion mass m satisfies $m/f \simeq y_t$.

One might imagine that the bounds of Sec. II could be evaded in a different class of models in which the light fermions are also spread out in theory space, perhaps with ''families'' of $SU(3)$ vector fermions. Even in this case, however, the crucial flavor-violating couplings are still Yukawa couplings between an ordinary fermion at the site (1,1) and the appropriate component of a vector fermion at an adjacent site. The bounds described in Sec. II apply to these couplings and constrain the corresponding models.

IV. FLAVOR AND FINE-TUNING IN THEORY SPACE

In order to understand the implications of the lower bound from flavor physics, we will examine an upper bound imposed by the wish to avoid fine-tuning in the Higgs boson masses. As noted before, the chiral symmetries of theory space imply that the leading contributions to the Higgs boson masses are *finite* contributions arising from the Coleman-Weinberg potential. The rules of power-counting are easily modified in this case [1,2] in order to estimate the size of these finite contributions to parameters in the low-energy theory. In particular, the size of these contributions is the same as that in a standard scalar Higgs model with a cutoff equal to the mass of the lowest appropriate resonance.

For example, gauge boson loop corrections to the Higgs boson masses are of order [1,2]

$$\delta m_H^2 \simeq \frac{e^2}{16\pi^2 \sin^2\theta_W}\left(\frac{gf}{N}\right)^2 \simeq \left(\frac{\alpha}{4\pi \sin^2\theta_W}\right)^2 \Lambda^2, \tag{23}$$

where the mass of the first vector resonance (of order gf/N) plays the role of the cutoff of the low-energy theory, and we have assumed that $g = \mathcal{O}(Ne/\sin\theta_W)$ in order to yield the appropriate low-energy weak coupling constant. Here, and in the rest of this paper, we take $\Lambda \simeq 4\pi f$ which corresponds to $\kappa = 4\pi$ above. The size of these finite corrections to the Higgs boson mass must be compared to the desired low-energy mass-squared given in Eq. (21). To avoid fine-tuning, we require that

$$\left|\frac{\delta m_H^2}{m_H^2}\right| \lesssim 1, \tag{24}$$

which yields

$$\Lambda \lesssim \left(\frac{4\pi \sin^2\theta_W}{\alpha}\right)\frac{\sqrt{\lambda}v}{N} \approx \frac{108 \ \ \mathrm{TeV}\sqrt{\lambda}}{N}. \tag{25}$$

If gauge-boson loop corrections were the only issue, the cutoff could be taken to be of order 100 TeV without any fine-tuning.

However, the most important corrections to the Higgs boson masses arise from the interactions added to give rise to the top-quark mass. The fermion loop Coleman-Weinberg contribution to the Higgs mass-squared is of order

$$|\delta m_H^2| \simeq \frac{N_c y_t^2 m^2}{16\pi^2} \approx \frac{N_c y_t^4}{(16\pi^2)^2}\Lambda^2, \tag{26}$$

where $N_c = 3$ accounts for color. In this case, the absence of fine-tuning ($\delta m_H^2/m_H^2 \lesssim 1$) implies

$$\Lambda \lesssim \frac{16\pi^2\sqrt{\lambda}v}{\sqrt{N_c}y_t^2 N} \approx \frac{22 \ \ \mathrm{TeV}\sqrt{\lambda}}{N}. \tag{27}$$

Comparing Eqs. (27) and (22) we see that for $N=2$, fine-tuning on the order of 1% is required if the bound from $\Delta C = 2$ mixing is to be satisfied. If the bound from CP violation (13) must also be satisfied, the fine-tuning required is of order .04%.

V. ISOSPIN VIOLATION

A crucial issue in all composite Higgs models is the size of weak-isospin violation [16,8,17,18]. Recall that the standard one-doublet Higgs model has an accidental custodial isospin symmetry [19], which naturally implies that the weak-interaction ρ-parameter is approximately 1. While all $SU(2)\times U(1)$ invariant operators made of a single scalar-doublet field that have dimension less than or equal to 4 automatically respect custodial symmetry, terms of higher dimension that arise from the underlying physics at scale Λ in general will not. Furthermore, the interaction given in Eq. (20) does not respect custodial symmetry. However, the effect of these interactions is to introduce custodial violation in the spectrum of Higgs boson masses and therefore only affects the weak interaction ρ parameter at one-loop.

The embedding of $SU(2)\times U(1)$ in a global $SU(3)$ interaction is identical to the symmetry structure of the "Banks model," which is known to give rise to isospin violation [3]. This violation is most directly understood by expanding the kinetic energy terms in Eq. (15) to fourth-order in the pion fields. Keeping only the terms involving ϕ_u and ϕ_v, we find the isospin violating interactions

$$\mathcal{L}_{kin} \supset -\frac{1}{6Nf^2}[(\partial_\mu \phi_u^\dagger \phi_u)^2 - (\partial_\mu \phi_u^\dagger \phi_u)(\phi_u^\dagger \partial^\mu \phi_u) + (\phi_u^\dagger \partial^\mu \phi_u)^2] + u \leftrightarrow v. \tag{28}$$

Writing the vacuum expectation values of the Higgs fields as

$$\langle \phi_u \rangle = \begin{pmatrix} 0 \\ \dfrac{v\cos\beta}{\sqrt{2}} \end{pmatrix} \quad \langle \phi_v \rangle = \begin{pmatrix} 0 \\ \dfrac{v\sin\beta}{\sqrt{2}} \end{pmatrix}, \tag{29}$$

we find the contribution

$$\Delta\rho^\star = \alpha \Delta T = \frac{v^2}{4N^2 f^2}\left(1 - \frac{\sin^2 2\beta}{2}\right). \tag{30}$$

Current limits derived from precision electroweak observables [18] require that $\Delta T \lesssim 0.5$ at 95% confidence level for a Higgs mass less than 500 GeV. The bound in Eq. (30) implies that

$$\Lambda \simeq 4\pi f \gtrsim \frac{25 \ \ \mathrm{TeV}}{N}\left(1 - \frac{\sin^2 2\beta}{2}\right)^{1/2}. \tag{31}$$

Comparing this with Eq. (27), we see that the underlying strong dynamics cannot be at energies much less than 10 TeV, even if the high-energy theory contains approximate flavor and CP symmetries that nullify the limits of Eqs. (6) and (13).

VI. DISCUSSION

In this paper, we have shown that the size of flavor violating interactions arising generically from underlying flavor dynamics in composite Higgs models constrains the compositeness scale to be at least 75 TeV. This bound applies not only to the original composite Higgs models [3], but also to the recently developed "theory space" models [1,2]. For theory space models based on an $N\times N$ toroidal lattice, the lower limit is $\Lambda \gtrsim 75 \ \ \mathrm{TeV}\sqrt{N}$, so that the bound is 105 TeV for $N=2$. On the other hand, if fine-tuning of the Higgs mass is to be avoided in such models, $\Lambda \lesssim 22 \ \ \mathrm{TeV}\sqrt{\lambda}/N$; preventing flavor-changing neutral currents then leads to fine-tuning at the level of $10/N^3$%. We have also seen that the lower limits on Λ derived from considering weak isospin violation are somewhat weaker than those from FCNC, while those from CP-violation in the neutral kaon system are potentially much stronger.

It is also interesting to note how one might construct models that are not constrained by the bounds discussed in this paper. In order to produce the appropriate Yukawa couplings without potentially large effects in neutral-meson mixing, the underlying flavor or strong dynamics must incorporate additional structure. First, it may be possible to construct a theory in which the charm mass-eigenstates are eigenstates of the corresponding flavor gauge-interactions. In this case, no $\Delta C=2$ interactions arise at the scale relevant for producing the charm-quark Yukawa couplings. Since Cabibbo mixing exists, however, such interactions *will* necessarily arise at the scale relevant for strange-quark mass generation, yielding the result of Eq. (11). Second, the underlying strong dynamics could potentially be arranged to have a different scaling behavior, analogous to "walking technicolor" [20]. In this case one might have Yukawa couplings of order Λ/M rather than the square of that ratio. Or third, the underlying flavor dynamics could incorporate an approximate GIM symmetry [21,22]. Similarly, if the underlying dynamical theory incorporated an approximate CP symmetry, then the low-energy theory would not necessarily make the dangerously large contributions to ε discussed here.

In summary, we have seen that the low-energy structure of the composite Higgs model alone is not sufficient to eliminate potential problems with flavor-changing neutral current or excessive CP violation; solving those problems requires additional information or assumptions about the symmetries of the underlying strong dynamics.

Note added. After the completion of this manuscript, two minimal composite Higgs models have recently been proposed [23,24]. As noted by those authors, the constraints discussed in this paper are relevant to the new models as well.

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