Quark-squark alignment reexamined

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We reexamine the possibility that the solution to the supersymmetric flavor problem is related to small mixing angles in gaugino couplings induced by approximate horizontal Abelian symmetries. We prove that, for a large class of models, there is a single viable structure for the down quark mass matrix with four holomorphic zeros. Consequently, we are able to obtain both lower and upper bounds on the supersymmetric mixing angles and predict the contributions to various flavor changing neutral current processes. We find that the most likely signals for alignment are Δm_D close to the present bound, significant CP violation in D^0-$\overline{D^0}$ mixing, and shifts of the order of a few percent in various CP asymmetries in B^0 and B_s decays. In contrast, the modifications to radiative B decays, to ε'/ε and to $K\to\pi\nu\overline{\nu}$ decays are small. We further investigate a new class of alignment models, where supersymmetric contributions to flavor changing processes are suppressed by both alignment and RGE-induced degeneracy.

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I. INTRODUCTION

Quark-squark alignment (QSA) is a mechanism that suppresses supersymmetric contributions to flavor changing neutral current processes via small mixing angles in flavor changing gaugino couplings [1,2]. The alignment could be precise enough that the models are viable without requiring any squark degeneracy. Alignment occurs naturally in all models with Abelian horizontal symmetries that induce the observed hierarchy in the Yukawa couplings. However, to achieve small enough mixing angles in the gaugino couplings that are relevant to Δm_K and ε_K, one has to carefully choose the symmetry and the charge assignments.

Existing models of alignment use holomorphic zeros in the down quark mass matrix to achieve small enough mixing angles between the first two generations. In Sec. II we reexamine the allowed structures for this mass matrix. We prove that there is a single structure (that is, a unique set of holomorphic zeros) that gives phenomenologically viable mixing angles. The unique structure of M^d gives this framework a strong predictive power: We are able to derive both lower and upper bounds on the parametric suppression of the supersymmetric mixing angles.

The most interesting prediction of models of quark-squark alignment is that the mass difference in the neutral D system, Δm_D, should be close to the experimental bound. (A more refined version of this statement is given in Sec. III.) Furthermore, D^0-$\overline{D^0}$ mixing could be CP violating. While recent analyses suggest that the standard model contribution to Δm_D could also be large [3], CP violation in the mixing will provide unambiguous evidence for new physics. Recently, there has been much progress in the search for D^0-$\overline{D^0}$ mixing. No signal has been found, and the bounds on the mixing parameters have improved. In Sec. III we examine the implications of these improved bounds on the viability of QSA models. It is important here that the experimental results on D^0-$\overline{D^0}$ mixing are analyzed allowing for CP violation. We thus use the results of Ref. [4] where the impact of weak (and strong) phases on the interpretation of the experimental bounds was taken into account.

The framework of alignment has a strong predictive power also for the mixing angles related to B^0-$\overline{B^0}$ mixing, B_s-$\overline{B_s}$ mixing and $b\to X\gamma$ decays. We analyze these predictions in Sec. IV. The implications for K physics—ε'/ε and $K\to\pi\nu\overline{\nu}$ decays—are discussed in Sec. V.

Another basic assumption made in the literature is that the *only* restriction on the soft supersymmetry breaking terms comes from the selection rules related to the small breaking of the horizontal symmetry. In particular, it was assumed that there is no degeneracy among squark masses, that is, $\Delta m^2/m^2=\mathcal{O}(1)$. This assumption may, however, be questioned. It is perhaps more plausible that this situation holds at a high energy scale, where the soft supersymmetry breaking terms are induced. But then, renormalization group evolution (RGE) would give a universal contribution to squark masses and lead to some degree of degeneracy. In Sec. VI we examine various aspects of "high energy alignment:" we estimate the size of the effect and its consequences for the constraints on mixing angles and for model building.

Future prospects for finding evidence for the alignment mechanism or for excluding it are discussed in Sec. VII.

II. SUPERSYMMETRIC MIXING ANGLES

The size of supersymmetric flavor violation depends on the overall scale of the soft supersymmetry breaking terms, on mass degeneracies between sfermion generations, and on the mixing angles in gaugino couplings. Within the framework of alignment, mixing angles play a significant role. For most of our purposes here, we can make the approximation that the mixing between $\tilde{q}_L$, the superpartners of the left-handed quarks, and $\tilde{q}_R$, the superpartners of the right-handed quarks, is small. Then there are four relevant 3×3 mixing matrices in the quark-squark sector, which we denote by K_L^d, K_R^d, K_L^u and K_R^u.

Consider, for example, the matrix elements $(K_L^d)_{ij}$ which parametrize the $\tilde{g}-(d_L)_i-(\tilde{d}_L)_j$ couplings. Given the down quark mass matrix in the interaction basis, M^d, we define the diagonalizing matrices, V_L^d and V_R^d, according to

$$V_L^d M^d V_R^{d\dagger} = \mathrm{diag}(m_d, m_s, m_b). \tag{1}$$

Given the the mass-squared matrix for the $\tilde{d}_L$ squarks, $\tilde{M}_{LL}^{2d}$, we can obtain the diagonalizing matrix $\tilde{V}_L^d$:

$$\tilde{V}_L^d \tilde{M}_{LL}^{2d} \tilde{V}_L^{d\dagger} = \mathrm{diag}(m_{\tilde{d}_1}^2, m_{\tilde{d}_2}^2, m_{\tilde{d}_3}^2). \tag{2}$$

Then we have

$$K_L^d = V_L^d \tilde{V}_L^{d\dagger}. \tag{3}$$

In this section we derive predictions for the flavor changing elements of the K_M^q matrices in the framework of alignment.

A. The down quark mass matrix

If the only suppression of supersymmetric flavor violation is related to alignment, then the constraints from K^0-$\overline{K^0}$ mixing (Δm_K and ε_K) require that the relevant supersymmetric mixing angles are much smaller than the corresponding CKM angle:

$$|(K_L^d)_{12}|, |(K_R^d)_{12}| \ll |V_{us}| = \lambda. \tag{4}$$

In models where alignment is induced by an Abelian horizontal symmetry, such a situation can be achieved by having holomorphic zeros in the down quark mass matrix.

We would like to argue that, for a large class of alignment models based on Abelian horizontal symmetries, there is a unique structure for the down quark mass matrix which is consistent with Eq. (4) and with the known values of the quark flavor parameters (masses and mixing angles):[1]

$$M^d \sim \begin{pmatrix} m_d & 0 & m_b V_{ub} \\ 0 & m_s & m_b V_{cb} \\ 0 & 0 & m_b \end{pmatrix}. \tag{5}$$

(The "$\sim$" sign here and below means that there is an arbitrary coefficient of order one, which we do not write explicitly, in each entry.) We will now prove this statement and spell out our assumptions along the way.

In order that Eq. (4) is satisfied, we must have $|(V_L^d)_{12}|, |(V_R^d)_{12}| \ll \lambda$. These matrix elements can be expressed in terms of the entries of M^d [2,7]. We define

$$y_{i1}^d = \frac{M_{i1}^d}{\sqrt{|M_{22}^d|^2 + |M_{33}^d|^2}},$$

$$y_{i2}^d = \frac{M_{i2}^d M_{33}^d - M_{i3}^d M_{32}^d}{|M_{22}^d|^2 + |M_{33}^d|^2},$$

$$y_{i3}^d = \frac{M_{i3}^d M_{33}^{d*} + M_{i2}^d M_{32}^{d*}}{|M_{22}^d|^2 + |M_{33}^d|^2}. \tag{6}$$

Then the relevant contributions to the matrix elements are given as follows:

$$(V_L^d)_{12} = \frac{y_{12}^d}{y_{22}^d} + \frac{y_{11}^d y_{21}^{d*}}{|y_{22}^d|^2},$$

$$(V_R^d)_{12} = \frac{y_{21}^{d*}}{y_{22}^{d*}} + \frac{y_{11}^{d*} y_{12}^d}{y_{22}^{d*}} - \frac{y_{31}^{d*} y_{23}^{d*}}{y_{22}^{d*}}. \tag{7}$$

To sufficiently suppress these mixing angles while providing acceptable values for the down quark masses, the following conditions are necessary [8]:

$$M_{12}^d = 0;$$

$$M_{21}^d = 0;$$

$$M_{31}^d = 0 \quad \text{or} \quad M_{23}^d = M_{32}^d = 0;$$

$$M_{32}^d = 0 \quad \text{or} \quad M_{13}^d = 0.$$

But not all the ways to satisfy these conditions can be realized in models of Abelian horizontal symmetries. In particular, we will now prove that in a large class of models we can have neither $M_{13}^d = 0$ nor $M_{23}^d = 0$.

We consider models with Abelian symmetries of the type $U(1)_1 \times U(1)_2 \times \cdots \times U(1)_n$. Each $U(1)_i$ subgroup is broken by a small parameter ϵ_i. It is convenient to express all ϵ_i's as powers of $\lambda, \epsilon_i \sim \lambda^{n_i}$ ($n_i > 0$). We emphasize that there is no loss of generality in doing so. Each matter supermultiplet Φ carries horizontal charges $H_i(\Phi), i = 1, \ldots, n$. Here Φ stands for any of the quark doublet superfields Q_i, the singlet anti-up superfields $\overline{u_i}$, the singlet anti-down superfields $\overline{d_i}$ and the Higgs superfields ϕ_u and ϕ_d. We use the freedom that comes from the $U(1)_Y \times U(1)_B \times U(1)_{\mathrm{PQ}}$ symmetry of the Yukawa sector to set $H_i(Q_3) = H_i(\phi_u) = H_i(\phi_d) = 0$ without loss of generality. It is also convenient to define an effective charge of a field, $H(\Phi) = \Sigma_i n_i H_i(\Phi)$. Then the selection rules for the entries in $M^q (q = d, u)$ are as follows:

(i) If, for all $i, H_i(Q_j) + H_i(\overline{q_k}) \geq 0$ then

$$M_{jk}^q = \langle \phi_q \rangle \lambda^{H(Q_j) + H(\overline{q_k})}.$$

(ii) If, for some $i, H_i(Q_j) + H_i(\overline{q_k}) < 0$ then $M_{jk}^q = 0$.

We assume that $m_t / \langle \phi_u \rangle = \mathcal{O}(1)$, namely it is not parametrically suppressed.[2] Then we must have $H_i(\overline{u_3}) + H_i(Q_3) = 0$ for all i. We also must have $M_{33}^d \simeq m_b$ which

[1]For related studies, see [5,6].

[2]The alignment model of Ref. [9] takes m_t to be parametrically suppressed and therefore is not subject to our analysis. Similarly, neither the mass matrix structures nor the phenomenological consequences proposed in Refs. [10,11] are possible in our framework.

means that $H_i(\overline{d_3})+H_i(Q_3)\geq 0$ for all i. These two conditions together imply that $H_i(\overline{d_3})\geq H_i(\overline{u_3})$. Then it is simple to see that if $M^d_{i3}=0$, we necessarily have also $M^u_{i3}=0$. But if $M^d_{23}=M^u_{23}=0$ we would obtain $|V_{cb}|\ll\lambda^2$. We conclude that we must not have $M^d_{23}=0$ and that, therefore, we must have $M^d_{31}=0$. But if $M^d_{31}=M^d_{13}=M^u_{13}=0$ we would obtain $|V_{td}|\ll\lambda^3$. We conclude that we must not have $M^d_{13}=0$ and that, therefore, we must have $M^d_{32}=0$. This completes the proof to our statement that the only viable down quark mass matrix within our framework and assumptions is that of Eq. (5).

B. The supersymmetric mixing angles

In the framework of alignment one assumes that there are no fine-tuned relations between $\mathcal{O}(1)$ coefficients. This means that we can use Eq. (3) to estimate $(K^d_L)_{ij}$:

$$(K^d_L)_{ij}\sim\max[(V^d_L)_{ij},(\tilde{V}^d_L)_{ji}]. \tag{8}$$

The uniqueness of the mass matrix M^d of Eq. (5) implies that the parametric suppression of all entries of the diagonalizing matrices $V^d_{L,R}$ is known within our framework:

$$V^d_L\sim\begin{pmatrix}1 & \lambda^5 & \lambda^3\\ \lambda^5 & 1 & \lambda^2\\ \lambda^3 & \lambda^2 & 1\end{pmatrix},\quad V^d_R\sim\begin{pmatrix}1 & \lambda^7 & \lambda^7\\ \lambda^7 & 1 & \lambda^4\\ \lambda^7 & \lambda^4 & 1\end{pmatrix}. \tag{9}$$

From Eq. (8) we conclude that the values of the various entries in V^d_L and V^d_R given in Eq. (9) constitute lower bounds on the corresponding entries in, respectively, K^d_L and K^d_R. In other words, the parametric suppression of $(K^d_M)_{ij}$ is at most as strong as that of $(V^d_M)_{ij}$ in Eq. (9).

We would now like to estimate the diagonalizing matrices for the squark mass-squared matrices. The selection rules for the diagonal block are simple:

(i) For the LL block, $(\tilde{M}^2_{LL})_{jk}\sim\tilde{m}^2_Q\lambda^{\sum_{i=1}^n n_i|H_i(Q_j)-H_i(Q_k)|}$ (for both down and up squarks).

(ii) For the RR block of the down sector, $(\tilde{M}^{2d}_{RR})_{jk}\sim\tilde{m}^2_D\lambda^{\sum_{i=1}^n n_i|H_i(\overline{d_j})-H_i(\overline{d_k})|}$.

(iii) For the RR block of the up sector, $(\tilde{M}^{2u}_{RR})_{jk}\sim\tilde{m}^2_U\lambda^{\sum_{i=1}^n n_i|H_i(\overline{u_j})-H_i(\overline{u_k})|}$.

(We here allow for the possibility that the typical mass-squared scale is different for each of the three sectors. In most cases we will assume that there is a single mass scale that characterizes all soft supersymmetry breaking terms and denote this scale by $\tilde{m}$.) The interesting point here is that one can find upper bounds on the off-diagonal elements of the diagonalizing matrices in terms of the quark flavor parameters, that is, the CKM angles and the quark masses. The latter can be written in terms of the effective charges:

$$|V_{ij}|\sim\lambda^{|H(Q_i)-H(Q_j)|},$$
$$m_{d_i}/m_{d_j}\sim\lambda^{H(Q_i)+H(\overline{d_i})-H(Q_j)-H(\overline{d_j})},$$
$$m_{u_i}/m_{u_j}\sim\lambda^{H(Q_i)+H(\overline{u_i})-H(Q_j)-H(\overline{u_j})}. \tag{10}$$

Then we get the following bounds (here $i\leq j$):

$$|(\tilde{V}^q_L)_{ij}|\lesssim|V_{ij}|,$$
$$|(\tilde{V}^q_R)_{ij}|\lesssim\frac{1}{|V_{ij}|}\frac{m_{q_i}}{m_{q_j}}. \tag{11}$$

There are cases in which one can derive an upper bound on $|(\tilde{V}^q_M)_{ij}|$ that is stronger than those in Eq. (11). These are the cases when a related entry in the down quark mass matrix is a holomorphic zero. For example, since $M^d_{31}=0$, the upper bound on $|(\tilde{V}^d_R)_{13}|$ in Eq. (11), $|(\tilde{V}^d_R)_{13}|\lesssim(m_d/m_b)/|V_{ub}|\sim\lambda$, is never saturated and a stronger bound holds. We now derive this bound. Our starting point is the application of the selection rule to this specific case,

$$|(\tilde{V}^d_R)_{13}|\sim\lambda^{\sum_{i=1}^n n_i|H_i(\overline{d_1})-H_i(\overline{d_3})|}. \tag{12}$$

The source of the upper bound in Eq. (11) is the inequality

$$\sum_{i=1}^n n_i|H_i(\overline{d_1})-H_i(\overline{d_3})|\geq\sum_{i=1}^n n_i[H_i(\overline{d_1})-H_i(\overline{d_3})]. \tag{13}$$

For the upper bound in Eq. (11) to be saturated, Eq. (13) should become an equality. That would imply that $H_i(\overline{d_1})-H_i(\overline{d_3})\geq 0$ for all i. As we mentioned before, we must have $M^d_{33}\simeq m_b$ which means that $H_i(\overline{d_3})+H_i(Q_3)\geq 0$ for all i. The combination of the two requirements gives $H_i(\overline{d_1})+H_i(Q_3)\geq 0$ for all i. But then $M^d_{31}\neq 0$, in contradiction to Eq. (5). The minimal extra suppression of $|(\tilde{V}^d_R)_{13}|$ compared to the upper bound in Eq. (11) is by two powers of the largest among the small parameters ϵ_i, that is,

$$|(\tilde{V}^d_R)_{13}|\lesssim\frac{m_d}{|V_{ub}|m_b}\epsilon^2_{\max},\quad \epsilon_{\max}\equiv\max_i(\epsilon_i). \tag{14}$$

In particular, if $\epsilon_i\lesssim\lambda$ for all i, then $|(\tilde{V}^d_R)_{13}|\lesssim\lambda^3$. Together with Eq. (9), we obtain

$$\lambda^7\lesssim|(K^d_R)_{13}|\lesssim\lambda^3. \tag{15}$$

Similar considerations apply to other supersymmetric mixing angles. Within the up sector, the structure of the mass matrix is less restricted. The only strict requirements are that the eigenvalues of M^u would be (m_u,m_c,m_t) and that, given that the Cabibbo mixing is not induced by the diagonaliza-

TABLE I. Bounds on supersymmetric mixing angles in models of alignment. The estimates in powers of $\lambda\sim 0.2$ refer to our evaluation of the quark mass ratios in powers of λ and to $\epsilon_{\max}\equiv\max_i(\epsilon_i)\lesssim\lambda$. (‡) In viable models these mixing angles are set to be smaller than the formal upper bounds so that the phenomenological bounds on the products $(K_L^q)_{12}(K_R^q)_{12}$ $(q=u,d)$ are satisfied.

Mixing angle	Lower bound	Upper bound
$(K_L^d)_{12}$	$V_{ub}V_{cb}(\sim\lambda^5)$	$V_{us}\epsilon_{\max}^2(\sim\lambda^3)^{\ddagger}$
$(K_R^d)_{12}$	$\frac{m_d}{m_s}V_{ub}V_{cb}(\sim\lambda^7)$	$\frac{m_d}{m_s V_{us}}\epsilon_{\max}^2(\sim\lambda^3)^{\ddagger}$
$(K_L^d)_{13}$	$V_{ub}(\sim\lambda^3)$	$V_{ub}(\sim\lambda^3)$
$(K_R^d)_{13}$	$\frac{m_d}{m_b}V_{ub}(\sim\lambda^7)$	$\frac{m_d}{m_b V_{ub}}\epsilon_{\max}^2(\sim\lambda^3)$
$(K_L^d)_{23}$	$V_{cb}(\sim\lambda^2)$	$V_{cb}(\sim\lambda^2)$
$(K_R^d)_{23}$	$\frac{m_s}{m_b}V_{cb}(\sim\lambda^4)$	$\frac{m_s}{m_b V_{cb}}\epsilon_{\max}^2(\sim\lambda^2)$
$(K_L^u)_{12}$	$V_{us}\sim\lambda$	$V_{us}(\sim\lambda)$
$(K_R^u)_{12}$	$\frac{m_u}{m_c}V_{us}(\sim\lambda^4)$	$\frac{m_u}{m_c\lvert V_{us}\rvert}(\sim\lambda^2)^{\ddagger}$

tion of M^d, we should have $|(V_L^u)_{12}|=|V_{us}|$. (In addition, we must have $|(V_L^u)_{13}|\lesssim|V_{ub}|$ and $|(V_L^u)_{23}|\lesssim|V_{cb}|$.) These requirements are enough to find constraints on the $|(K_M^u)_{12}|$ mixing angles. The bounds on various mixing angles in our framework of alignment are given in Table I.

III. *D* PHYSICS

The most promising way to find evidence for quark-squark alignment is through CP violation in D^0-$\overline{D^0}$ mixing. The best way to exclude a large class of alignment models is by improving the bounds on D^0-$\overline{D^0}$ mixing. The most important quantity here is the dispersive part of the D^0-$\overline{D^0}$ mixing amplitude, M_{12}^D. To constrain the supersymmetric flavor parameters, we need to find the phenomenological bounds on this transition amplitude. The analysis is not straightforward because the possible presence of strong phases and of weak phases in the relevant decay processes complicates the relation between M_{12}^D and the experimentally measured parameters. A careful analysis was performed in Ref. [4] with the result[3]

$$|M_{12}^D|\leqslant 6.2\times 10^{-11}\ \text{MeV}(95\%\ \text{C.L.}). \tag{16}$$

In the next subsection we interpret this bound in the framework of supersymmetric models with quark-squark alignment.

A. Mixing angle constraints without squark degeneracy

Supersymmetric box diagrams with intermediate gauginos and squarks contribute to neutral meson mixing. It is our purpose in this subsection to estimate the supersymmetric contribution to M_{12}^D in the framework of quark-squark alignment models and to compare it to the experimental bound (16).

The size of the contribution depends on the masses of the intermediate particles and on the mixing angles in the gaugino couplings to quarks and squarks. The interest in D^0-$\overline{D^0}$ mixing lies in the fact that alignment models predict the value of one relevant mixing angle:

$$|(K_L^u)_{12}|\simeq\lambda. \tag{17}$$

Here $\lambda\equiv|V_{us}|=0.22$ is the Wolfenstein parameter. The mixing angle $(K_L^u)_{12}$ gives the coupling of the gluino (or a neutralino) to a left-handed up quark and a "left-handed" charm squark. Then one can calculate the contribution to M_{12}^D in terms of the three relevant masses, $m_{\tilde g},\tilde m_2$ and $\tilde m_1$ (where the latter are, respectively, the masses of $\tilde c_L$ and $\tilde u_L$).

One often calculates the supersymmetric contributions to neutral meson mixing in the mass insertion approximation (MIA). This is equivalent to Taylor expanding around a common squark mass $\tilde m_q$ and keeping only the leading term in $\Delta\tilde m_{21}^2/\tilde m_q^2$, where

$$\tilde m_q=\frac{1}{2}(\tilde m_2+\tilde m_1),$$

$$\Delta\tilde m_{21}^2=(\tilde m_2^2-\tilde m_1^2). \tag{18}$$

[The particular choice of $\tilde m_q$ in Eq. (18) is explained in Ref. [12].] It is convenient to define the following dimensionless quantity:

$$(\delta_{LL}^u)_{12}\equiv\frac{(V_L^u\tilde M_{LL}^{2u}V_L^{u\dagger})_{12}}{\tilde m_q^2}\sim(K_L^u)_{12}\frac{\Delta\tilde m_{21}^2}{\tilde m_q^2}. \tag{19}$$

In the second equation we assumed that the terms related to $(K_L^u)_{13}(K_L^u)_{23}$ can be neglected and that, furthermore, the diagonal matrix elements, $(K_L^u)_{ii}$, are not parametrically suppressed. These assumptions are always valid in our framework of Abelian horizontal symmetries. The leading contribution in the MIA depends on $m_{\tilde g},\tilde m_q$ and $(\delta_{LL}^u)_{12}$. The MIA result for the contributions to M_{12}^D involving $\tilde c_L$ and $\tilde u_L$ is given by [13]

$$M_{12}^D=\frac{\alpha_s^2 m_D B_D f_D^2\eta_D}{\tilde m_q^2}\left[\frac{11}{108}\tilde f_6(m_{\tilde g}^2/\tilde m_q^2)+\frac{1}{27}\frac{m_{\tilde g}^2}{\tilde m_q^2}f_6(m_{\tilde g}^2/\tilde m_q^2)\right]\times[(\delta_{LL}^u)_{12}]^2, \tag{20}$$

where

$$f_6(x)=\frac{6(1+3x)\ln x+x^3-9x^2-9x+17}{6(1-x)^5},$$

[3] In the literature, the effects of weak and strong phases on the interpretation of searches for D^0-$\overline{D^0}$ mixing are often ignored. Consequently, a stronger bound is often quoted. See [4] for details.

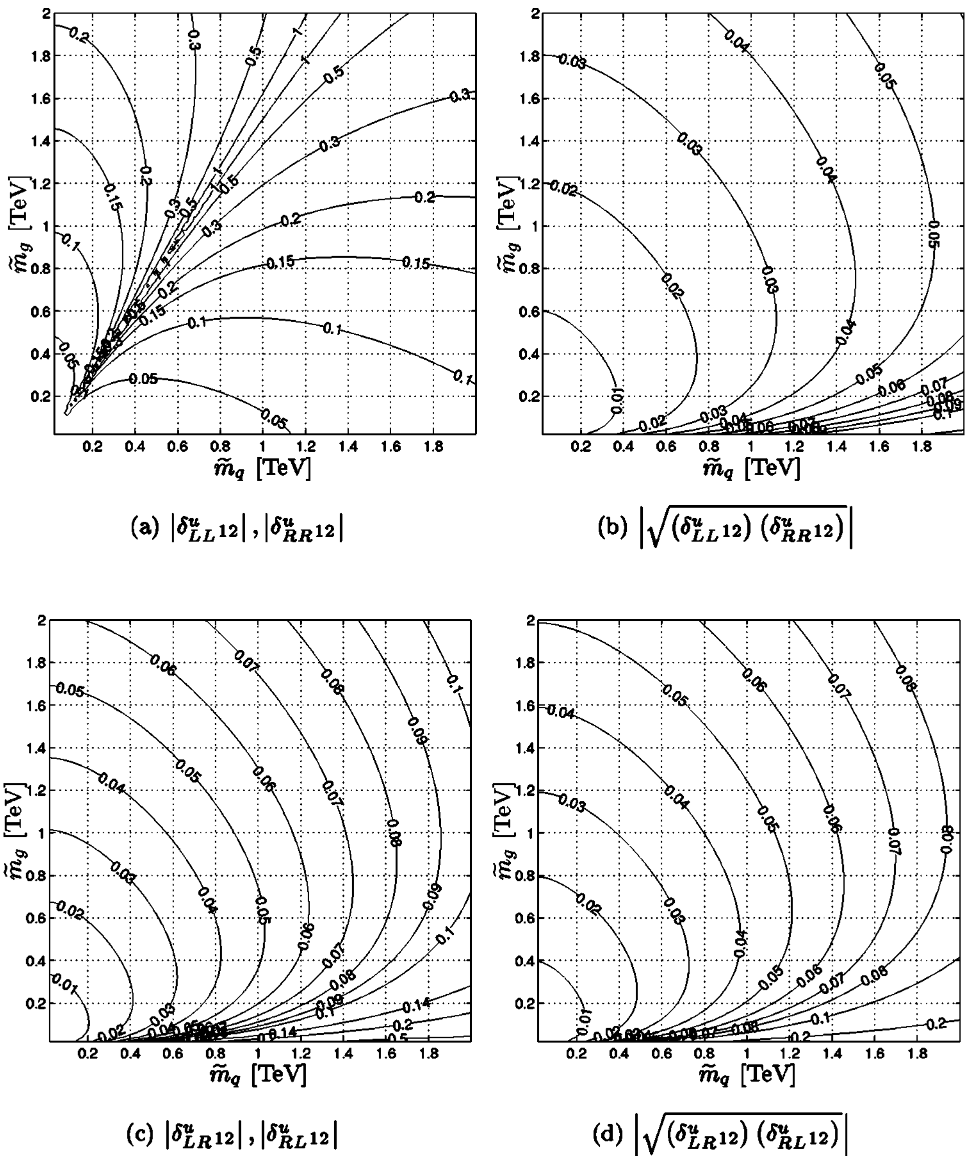

FIG. 1. Constraints on flavor changing mass insertions from D^0-$\overline{D^0}$ mixing as a function of the gluino mass $m_{\tilde{g}}$ and of the average squark mass $\tilde{m}_q$.

$$\tilde{f}_6(x)=\frac{6x(1+x)\ln x-x^3-9x^2+9x+1}{3(1-x)^5}. \tag{21}$$

Similarly, one can find the contributions that are porportional to $[(\delta^u_{RR})_{12}]^2,(\delta^u_{LL})_{12}(\delta^u_{RR})_{12},[(\delta^u_{LR})_{12}]^2,[(\delta^u_{RL})_{12}]^2$, and $(\delta^{\,u}_{LR})_{12}(\delta^{\,u}_{RL})_{12}$. [Generalizing Eq. (19), one defines $\delta^{\,q}_{MN} \equiv V^q_M \tilde{M}^{2q}_{MN} V^{q\dagger}_N/\tilde{m}^2$.] Requiring that each of these contributions separately is smaller than our bound (16) gives an upper bound on each of the $(\delta^{\,u}_{MN})_{12}$ combinations. These bounds are shown in Fig. 1. For example, with $m_{\tilde{g}}=\tilde{m}_q=1$ TeV, we find

$$(\delta^u_{LL})_{12} \lesssim 0.2. \tag{22}$$

Note that we do not take into account possible fine-tuned cancellations between the various contributions. Such cancellations would allow weaker bounds. While this option goes against the spirit of our work, where we try to explain small numbers by parametric suppression related to approximate symmetries and not by fine-tuning, one has to bear in mind that it is not impossible that the bounds are violated by a factor of a few and accidental cancellation does take place in Nature [14].

How should we interpret constraints that are calculated with the MIA within the framework of alignment? The answer is not simple for the following reason. Within models of alignment, the suppression of flavor changing $(\delta^{\,q}_{MN})_{ij}$ comes from the smallness of the mixing angles and not from squark degeneracy. Actually, in the spirit of alignment models, where all couplings that are not suppressed by the approximate horizontal symmetry are expected to be of $\mathcal{O}(1)$, one usually further assumes that there is no degeneracy among the relevant squarks, that is,

$$\frac{\Delta\tilde{m}^2_{21}}{\tilde{m}^2_q}=\mathcal{O}(1). \tag{23}$$

But the MIA is an expansion in $\Delta\tilde{m}^2/\tilde{m}^2$ (and not in δ). Therefore it is not necessarily a good approximation for alignment models. Reference [12] investigated the relation between the MIA and exact calculations within alignment models. The conclusion is that, in most of the parameter space, the MIA with the choice of $\tilde{m}_q$ as in Eq. (18) is a good approximation for the exact result. Thus, in the absence of any squark degeneracy, the constraints in Fig. 1 should be interpreted as an approximate upper bound on the mixing angle $|(K_L^u)_{12}|$. The approximation breaks only if there is a strong hierarchy between the two squark masses. If, on the other extreme, there is approximate degeneracy between the two squark masses, then the MIA constraint is (close to) exact but it applies to $|(K_L^u)_{12}|(\Delta\tilde{m}^2/\tilde{m}^2)$.

B. M_{12}^D with quark-squark alignment

In all models of alignment, Eq. (17) holds for the mixing angle. In the class of models considered in this section, Eq. (23) is assumed. In this class of models, the generic prediction is then that

$$(\delta_{LL}^u)_{12} \sim 0.2 \tag{24}$$

to be compared with the experimental bound of Eq. (22) or, more generally, with the constraints of Fig. 1(a). The regions of parameter space where the constraint on $(\delta_{LL}^u)_{12}$ is stronger than 0.2 are disfavored. The regions where the constraint is weaker are viable. We can make then the following three statements:

(i) Models of quark-squark alignment where $m_{\tilde{g}},\tilde{m}_q \gtrsim 1$ TeV are consistent with the experimental constraints from D^0-$\overline{D^0}$ mixing without any squark degeneracy.

(ii) Conversely, models where both of $m_{\tilde{g}}$ and $\tilde{m}_q$ are much lighter than 1 TeV are disfavored, unless there is some degeneracy between the first two generations of squarks.

(iii) There is a narrow region in the $m_{\tilde{g}},\tilde{m}_q$ plane where various contributions to M_{12}^D cancel against each other and the supersymmetric particles could be very light without violating the bound. While exact cancellation is unlikely, one should bear in mind that an accidental, approximate cancellation is possible and the TeV bound on the masses is not strict.

If supersymmetry is to solve the fine-tuning problem, supersymmetric masses should be $\lesssim$TeV. The conclusion of our discussion here is then that models without squark degeneracy require that $|M_{12}^D|$ is close to present experimental bounds.

IV. B PHYSICS

$B^0-\overline{B^0}$ mixing and rare B decays, such as the radiative $b\to s\gamma$, are an excellent probe of supersymmetry [15,16]. In this section we study the signatures of alignment in these processes.

A. B^0-$\overline{B^0}$ mixing

There are two important measurements that relate to B^0-$\overline{B^0}$ mixing. First, the mass difference between the neutral B mesons is given by [17]

$$\Delta m_B = (3.107\pm 0.112)\times 10^{-10}\ \ \text{MeV}. \tag{25}$$

Second, the CP asymmetry in $B\to\psi K$ decays is given by [18,19]

$$a_{\psi K} = 0.78\pm 0.08. \tag{26}$$

The supersymmetric contributions to B^0-$\overline{B^0}$ mixing can be calculated along the lines described in Sec. III A. The various contributions are proportional to $[(\delta^d_{LL})_{13}]^2$, $[(\delta^d_{RR})_{13}]^2$, $(\delta^d_{LL})_{13}(\delta^d_{RR})_{13}$, $[(\delta^d_{LR})_{13}]^2$, $[(\delta^d_{RL})_{13}]^2$ and $(\delta^d_{LR})_{13}(\delta^d_{RL})_{13}$. For each of these contributions, we find the value of the $(\delta^d_{MN})_{13}$ parameter that would saturate the experimental upper bound on $|M_{12}^B|$ from Eq. (25), $|M_{12}^B| \lesssim 1.7\times 10^{-10}$ MeV. The results of this analysis are shown in Fig. 2. For example, for $m_{\tilde{g}}=\tilde{m}_q=1$ TeV, we find that supersymmetric contributions would saturate Δm_B if at least one of the following conditions is satisfied:

$$(\delta^d_{LL})_{13} \sim 0.2,$$

$$\sqrt{(\delta^d_{LL})_{13}(\delta^d_{RR})_{13}} \sim 0.04. \tag{27}$$

We should now compare these results to the predictions given in Table I:

$$(K^d_{LL})_{13} \sim |V_{ub}| \sim 0.004,$$

$$\sqrt{(K^d_{LL})_{13}(K^d_{RR})_{13}} \lesssim \lambda\sqrt{m_d/m_b} \sim 0.01. \tag{28}$$

We obtain the following approximate range for the supersymmetric contribution to M_{12}^B:

$$\lambda^4 \lesssim \left|\frac{(M_{12}^B)^{\text{SUSY}}}{(M_{12}^B)^{\text{EXP}}}\right| \lesssim \lambda^2. \tag{29}$$

In particular, the supersymmetric contribution to the B^0-$\overline{B^0}$ mixing amplitude M_{12}^B, and hence to Δm_B and to $a_{\psi K}$, is at most a few percent.

B. B_s-$\overline{B_s}$ mixing

Within the standard model, the ratio between the mass differences in the B_s and B^0 systems, $\Delta m_{B_s}/\Delta m_B$, depends on the CKM elements [up to SU(3) breaking effects of order twenty percent],

$$\frac{\Delta m_{B_s}}{\Delta m_B} \sim \left|\frac{V_{ts}}{V_{td}}\right|^2 \sim \frac{1}{\lambda^2}. \tag{30}$$

Note that Δm_{B_s} has not been measured yet and only a lower bound exists [17],

$$\frac{\Delta m_{B_s}}{\Delta m_B} \gtrsim 30. \tag{31}$$

The prediction of alignment models can be read from Table I. The relevant ratios are

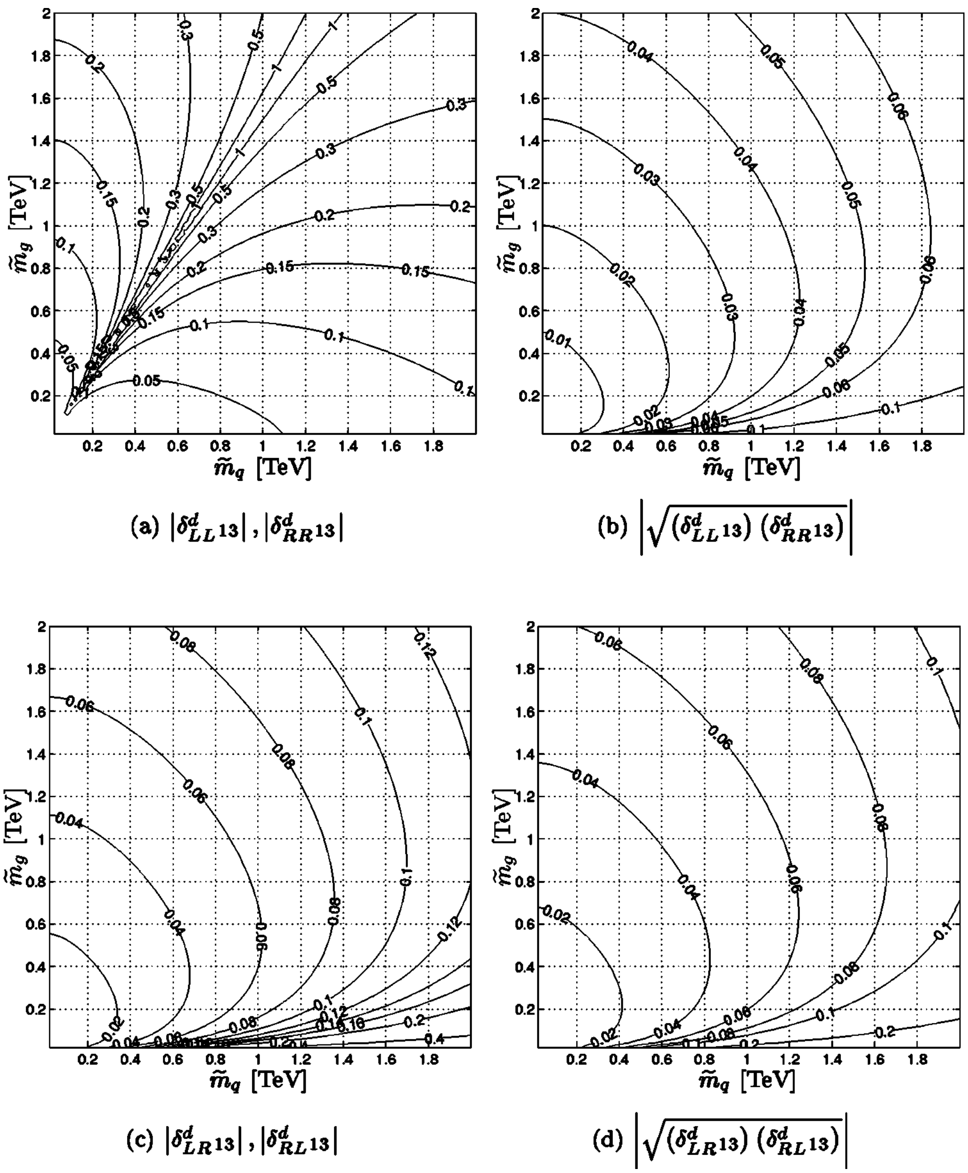

FIG. 2. Constraints on flavor changing mass insertions from B^0-$\overline{B^0}$ mixing as a function of the gluino mass $m_{\tilde{g}}$ and of the average squark mass $\tilde{m}_q$.

$$\frac{(K_L^d)_{23}^2}{(K_L^d)_{13}^2} \sim \frac{1}{\lambda^2},$$

$$\frac{\max[(K_L^d)_{23}(K_R^d)_{23}]}{\max[(K_L^d)_{13}(K_R^d)_{13}]} \sim \frac{1}{\lambda^2}. \tag{32}$$

Based on these results, we conclude that the supersymmetric contribution to B_s-$\overline{B_s}$ mixing is at most of order a few percent. Such an effect is too small to be clearly observed through a measurement of Δm_{B_s}. However, the standard model prediction for the CP asymmetries in B_s decay to $\psi\phi$ (or in any other $b \to c\bar{c}s$ process leading to a final CP eigenstate) is of order λ^2; these predictions can then be violated in a significant way.

C. $b \to X\gamma$

Within our framework, the structure of $\tilde{M}_{LR}^{2q}$ is similar to that of M^q: the same holomorphic zeros appear in both, and the same parametric suppression holds for the non-vanishing entries (though the coefficients of order one are, in general, different). Consequently, alignment models predict also the parametric suppression of the chirality-changing couplings, $(\delta^d_{MN})_{ij} \equiv (V_M^d \tilde{M}_{MN}^{2d} V_N^{d\dagger})_{ij}/\tilde{m}^2$ with $M \neq N$. These predictions are given in Table II.

These predictions imply that the supersymmetric contributions to $b \to X_s\gamma$ are small in our framework. For example, with $\tilde{m} \sim 500$ GeV, the prediction is $(\delta^d_{LR})_{23} \sim \lambda^5$ while the requirement, for the supersymmetric contribution to be significant, is $(\delta^d_{LR})_{23} \sim \lambda^2$. Thus the modificiation of the standard model prediction is of order 10^{-4}. Even for $\tilde{m}$ close to m_Z, the supersymmetric contribution is below the percent level. Similar conclusions hold for the $b \to X_d\gamma$ decay.

V. K PHYSICS

K physics have played an enormous role in shaping our thinking on supersymmetry breaking. The very idea of align-

TABLE II. Predictions for supersymmetric chirality-changing, flavor-changing mass insertions in models of alignment. The estimates in powers of $\lambda\sim 0.2$ refer to our evaluation of the quark mass ratios in powers of λ.

$(\delta^d_{MN})_{ij}$	Prediction
$(\delta^d_{LR})_{12}$	$\lambda^7 \ (m_b/\tilde{m})$
$(\delta^d_{RL})_{12}$	$\lambda^9 \ (m_b/\tilde{m})$
$(\delta^d_{LR})_{13}$	$\lambda^3 \ (m_b/\tilde{m})$
$(\delta^d_{RL})_{13}$	$\lambda^7 \ (m_b/\tilde{m})$
$(\delta^d_{LR})_{23}$	$\lambda^2 \ (m_b/\tilde{m})$
$(\delta^d_{RL})_{23}$	$\lambda^4 \ (m_b/\tilde{m})$

ment comes from the strong constraints on the soft supersymmetry breaking terms that follow from the smallness of K^0-$\overline{K^0}$ mixing. Future developments in K physics—particularly ε'/ε and $K\to\pi\nu\overline{\nu}$ decays—are likely to test in various ways the solutions that have been proposed to the supersymmetric flavor problem. As concerns ε'/ε, one may hope that future *theoretical* developments will allow us to tell whether indeed the standard model accounts for the measured value. As concerns the rare $K\to\pi\nu\overline{\nu}$ decays, in the future the measurement of the charged ($K^{\pm}$) mode might be improved and the neutral (K_L) mode might be measured, providing important information on supersymmetric flavor and CP violation. Whether deviations from the standard model are found or not, the results will help in testing alignment.

A. ε'/ε

Direct CP violation in $K\to\pi\pi$ decays has now been measured with high accuracy (for a review, see [20] and references therein):

$$\frac{\varepsilon'}{\varepsilon}=(1.72\pm 0.18)\times 10^{-3}. \tag{33}$$

For

$$\mathrm{Im}[(\delta^d_{LR})_{12}]\sim\lambda^7\left(\frac{\tilde{m}}{500\ \ \mathrm{GeV}}\right), \tag{34}$$

(and/or for a similar magnitude of $\mathrm{Im}[(\delta^d_{RL})_{12}]$), the supersymmetric contribution could saturate ε'/ε [21]. From Table II we learn that the predicted size is

$$(\delta^d_{LR})_{12}\sim\lambda^7\left(\frac{m_b}{\tilde{m}}\right). \tag{35}$$

We learn that models of alignment cannot explain a large deviation from the standard model prediction [8,22]. (See, however, Ref. [23] for a related model where the supersymmetric contribution is significant.) As mentioned above, experiments have determined ε'/ε rather accurately; the question of whether there is room (or even necessity) for a large supersymmetric contribution can only be answered if the theoretical determination of the relevant hadronic matrix elements improves in a significant way.

B. $K\to\pi\nu\overline{\nu}$

The measurement of $\mathrm{BR}(K^+\to\pi^+\nu\overline{\nu})$ has been recently improved [24]:

$$\mathrm{BR}(K^+\to\pi^+\nu\overline{\nu})=(1.57^{+1.75}_{-0.82})\times 10^{-10}. \tag{36}$$

The supersymmetric contribution can saturate this rate if [25–28]

$$(\delta^d_{LL})_{12}\sim\lambda^2 \tag{37}$$

[or if $(\delta^d_{LL})_{13}(\delta^d_{LL})_{23}\sim\lambda^2$]. Examining Table I, we learn that the relevant flavor changing couplings are much smaller. We conclude that models of alignment cannot explain a large deviation from the standard model prediction [25]. This situation might actually be helpful in probing alignment: while it may be difficult to be convinced of new contributions at the level of a few percent from a direct comparison between $\Delta m_B/\Delta m_{B_s}$ or $a_{\psi K}$ and the standard model prediction, such deviations can be probed by a violation of the standard model correlations between these observables and the $K\to\pi\nu\overline{\nu}$ decay rates [29,30].

VI. ALIGNMENT AT HIGH SCALE

The starting point of most previously-studied models of alignment is the assumption that the flavor structure of the soft supersymmetry breaking terms is determined solely by the selection rules related to the approximate horizontal symmetry. When we consider, however, a high scale of supersymmetry breaking, renormalization group evolution (RGE) of squark masses may induce an approximate degeneracy at low scale. Our purpose in this section is to investigate this effect and describe the phenomenological consequences.

A. RGE-induced degeneracy

The RGE effects on the Yukawa matrices are small [31], so we need to consider only the soft supersymmetry breaking terms. For our purposes, it is sufficient to consider the one-loop RG equations in the limit where all Yukawa couplings are set to zero [32]:

$$\partial_t\tilde{m}_a=-\frac{1}{4\pi}b_a\alpha_a\tilde{m}_a,$$

$$\partial_t(\tilde{M}^2_{LL})_{ij}=\frac{\delta_{ij}}{4\pi}\left(\frac{16}{3}\alpha_3\tilde{m}_3^2+3\alpha_2\tilde{m}_2^2+\frac{1}{9}\alpha_1\tilde{m}_1^2\right)$$
$$-\frac{m^2_{3/2}}{16\pi^2}[(A^uA^{u\dagger})_{ij}+(A^dA^{d\dagger})_{ij}],$$

$$\partial_t(\tilde{M}^{2u}_{RR})_{ij}=\frac{\delta_{ij}}{4\pi}\left(\frac{16}{3}\alpha_3\tilde{m}_3^2+\frac{16}{9}\alpha_1\tilde{m}_1^2\right)-\frac{m_{3/2}^2}{8\pi^2}(A^{u\dagger}A^u)_{ij},$$

$$\partial_t(\tilde{M}^{2d}_{RR})_{ij}=\frac{\delta_{ij}}{4\pi}\left(\frac{16}{3}\alpha_3\tilde{m}_3^2+\frac{4}{9}\alpha_1\tilde{m}_1^2\right)-\frac{m_{3/2}^2}{8\pi^2}(A^{d\dagger}A^d)_{ij}, \tag{38}$$

$$\partial_t A^u_{ij}=\frac{1}{4\pi}\left(\frac{8}{3}\alpha_3+\frac{3}{2}\alpha_2+\frac{13}{18}\alpha_1\right)A^u_{ij},$$

$$\partial_t A^d_{ij}=\frac{1}{4\pi}\left(\frac{8}{3}\alpha_3+\frac{3}{2}\alpha_2+\frac{7}{18}\alpha_1\right)A^d_{ij},$$

where $t=2\ln(M_S/Q)$, M_S is the scale at which supersymmetry breaking is communicated to the MSSM, and $b_{1,2,3}=(11,1,-3)$. The A^q matrices are defined through $\tilde{M}^{2q}_{LR}=m_{3/2}A^q\langle\phi_q\rangle$. The important point to notice is that the squark mass-squared matrices, $\tilde{M}^{2q}_{MM}$, get large universal contributions that are proportional to the gauge couplings.

Let us take, for example, $M_S\approx M_{\rm GUT}$ and set $Q=m_Z$ ($t\simeq 67$). Then the weak scale parameters (unprimed) can be written in terms of the high scale parameters (primed) as follows [32]:

$$(\tilde{M}^2_{LL})_{ij}=(\tilde{M}^2_{LL})'_{ij}+7\delta_{ij}m'^2_{1/2}-m'^2_{3/2}[1.8(A^{u'}A^{u'\dagger})_{ij}+1.7(A^{d'}A^{d'\dagger})_{ij}],$$

$$(\tilde{M}^{2u}_{RR})_{ij}=(\tilde{M}^{2u}_{RR})'_{ij}+7\delta_{ij}m'^2_{1/2}-3.6m'^2_{3/2}(A^{u'\dagger}A^{u'})_{ij},$$

$$(\tilde{M}^{2d}_{RR})_{ij}=(\tilde{M}^{2d}_{RR})'_{ij}+7\delta_{ij}m'^2_{1/2}-3.4m'^2_{3/2}(A^{d'\dagger}A^{d'})_{ij}, \tag{39}$$

$$A^u_{ij}=3.7A^{u'}_{ij},$$

$$A^d_{ij}=3.6A^{d'}_{ij}.$$

Here $m'_{1/2}$ is the average gaugino mass at the GUT scale. Thus, RGE induces a universal contribution of order $\frac{7}{9}m_{\tilde{g}}^2$ to the weak-scale squark mass-squared matrices. We used here the fact that the RGE of gaugino masses yields $m_{\tilde{g}}\approx 3m'_{1/2}$.

We now make the crucial assumption that the structure of the soft supersymmetry breaking terms at M_S is solely determined by the horizontal symmetry. This assumption means that, at the high scale, the following order of magnitude relations hold:

$$\tilde{m}'^2_{3/2}\sim\tilde{m}'^2_{1/2}\sim(\tilde{M}^{2q}_{MM})'_{ii},$$

$$A^{u'}_{33}\sim 1,\ A^{d'}_{33}\sim m_b/\langle\phi_d\rangle. \tag{40}$$

But then, at low energy, squark masses acquire approximate degeneracy. The estimates of the supersymmetric mixing angles in Table I correspond in this case to the suppression of the high-scale δ_{ij} parameters. But the weak-scale δ_{ij} parameters are now suppressed not only by the small mixing angles but also by squark degeneracy. Explicitly, the low energy δ_{ij} parameters have the following RGE-induced suppression factors with respect to their high energy values:

$$(\delta^d_{LL})_{ij}\approx 0.25(\delta^d_{LL})'_{ij},\quad (ij)=(12),(13),(23),$$

$$(\delta^d_{RR})_{ij}\approx 0.15(\delta^d_{RR})'_{ij},\quad (ij)=(12),(13),(23), \tag{41}$$

$$(\delta^u_{MM})_{12}\approx 0.15(\delta^u_{MM})'_{12},\quad M=L,R.$$

In other words, if Eqs. (23) and (40) hold at the GUT scale, we have at the weak scale $\Delta\tilde{m}^2/\tilde{m}^2\approx 1/4$ (1/7) in the $\tilde{d}_L$ sector ($\tilde{d}_R$ sector and first two up squark generations). We would like to emphasize the following three points:

(i) The milder suppression of $(\delta^d_{LL})_{ij}$ depends on our assumption that the scale that characterizes the A terms is $m'_{3/2}$. If it is smaller, the degeneracy becomes as strong as in the other sectors. The degeneracy would be similarly enhanced if the A matrices were *exactly* proportional to the corresponding Yukawa matrices.

(ii) The results in Eq. (41) have been derived with $\tan\beta=\mathcal{O}(1)$. In the case that $\tan\beta\gg 1$, the suppression of $(\delta^d_{RR})_{23}$ becomes milder: $(\delta^d_{RR})_{23}\approx 0.5(\delta^d_{RR})'_{23}$ (for $\tan\beta\sim m_t/m_b$).

(iii) We used here, as an example, $M_S\approx M_{\rm GUT}$. Lower values of M_S correspond to weaker RGE effects and, therefore, to a milder suppression of the flavor changing effects. For $M_S\lesssim 10^9$ GeV, there is effectively no degeneracy and the phenomenology is the same as in the discussion in previous sections, where alignment is the only source of suppression of flavor changing couplings.

B. Phenomenological consequences

The RGE-induced suppression of the flavor changing δ_{ij} parameters in the high scale models has important phenomenological consequences. Before we list the phenomenological implications of this class of models, let us point out that the predictions are here somewhat sharper. This is due to the fact that, given our assumption (40), we can estimate

$$x\equiv\frac{m_{\tilde{g}}^2}{\tilde{m}^2}\sim\frac{9}{7}. \tag{42}$$

Again, we used here as our example $M_S\approx M_{\rm GUT}$. This leaves essentially a single free parameter, say, $\tilde{m}$, in any given model in this class.

(i) D^0-$\overline{D^0}$ mixing: Eq. (24) is now replaced (for $M_S\approx M_{\rm GUT}$) with

$$(\delta^u_{LL})_{12}\sim 0.03, \tag{43}$$

to be compared with the constraints of Fig. 1 (along the curve $m_{\tilde{g}}/\tilde{m}\sim 1.1$). We can make the following statements:

(a) There is no region of parameter space that is disfavored by the experimental upper bound on $|M_{12}^D|$. In particular, the scale of squark and gluino masses could be as low as 300 GeV. This is true for a supersymmetry breaking scale as low as $M_S\sim 10^{14}$ GeV: for $M_S\gtrsim 10^{14}$ GeV our framework predicts $(\delta^u_{LL})_{12}\lesssim 0.05$ which, as can be seen in Fig. 1, is the upper bound for $\tilde{m}_q\sim 300$ GeV.

(b) For $\tilde{m}\sim 1$ TeV, the supersymmetric contributions to $|M_{12}^D|$ are a factor of $\mathcal{O}(50)$ below the experimental bound. Given the expected experimental sensitivity of future experiments, it will be impossible to exclude models of high-scale alignment based on non-observation of D^0-$\overline{D^0}$ mixing.

(c) For $\tilde{m}\sim 300$ GeV, the supersymmetric contributions to $|M_{12}^D|$ are a factor of $\mathcal{O}(3)$ below the experimental bound. It is then possible that D^0-$\overline{D^0}$ mixing will be observed in the future.

(ii) B^0-$\overline{B^0}$ mixing: Eq. (28) is now replaced with

$$(\delta^d_{LL})_{13}\sim 0.001,$$

$$\sqrt{(\delta^d_{LL})_{13}(\delta^d_{RR})_{13}}\lesssim 0.003, \tag{44}$$

to be compared with the constraints of Fig. 2. We can make the following statements:

(a) The supersymmetric contribution to B^0-$\overline{B^0}$ mixing is smaller by a factor of at least 10 compared to the low-scale models of similar squark and gluino masses. In particular, for $\tilde{m}\sim 1$ TeV, the modification to the standard model prediction for $a_{\psi K}$ is below the percent level.

(b) The fact that, in this class of alignment models, light [that is, $\mathcal{O}(300$ GeV$)$] squark masses are allowed, means that the maximal supersymmetric contributions could be comparable to the maximal low-scale model predictions. Indeed, with $\tilde{m}\sim 300$ GeV and large $\tan\beta$ [to give minimal suppression of $(\delta^d_{RR})_{13}$], the supersymmetric contribution could be of $\mathcal{O}(0.1)$ of M_{12}^B. This could lead to observable modifications of $a_{\psi K}$.

(iii) K^0-$\overline{K^0}$ mixing: the constraints from ε_K (assuming CP violating phases of order one in the supersymmetric mixing matrices) are given in Fig. 3. For example, with $m_{\tilde{g}}=\tilde{m}=1$ TeV, we obtain

$$(\delta^d_{LL})_{12}\lesssim 8\times 10^{-3},$$

$$\sqrt{(\delta^d_{LL})_{12}(\delta^d_{RR})_{12}}\lesssim 6\times 10^{-4}. \tag{45}$$

Given Eq. (41), these constraints can be translated into bounds on the supersymmetric mixing angles,

$$(K_L^d)_{12}\lesssim\lambda^2,$$

$$\sqrt{(K_L^d)_{12}(K_R^d)_{12}}\lesssim\lambda^3. \tag{46}$$

This is to be compared with the bounds $(K_L^d)_{12}\lesssim\lambda^3$ and $\sqrt{(K_L^d)_{12}(K_R^d)_{12}}\lesssim\lambda^5$ that apply in low scale models of alignment.

Models of alignment are constructed to satisfy the Δm_K and ε_K constraints. What we have just learned is that in models of GUT-scale alignment, the constraints on the mixing angles (46) are milder. The question then arises whether this situation has significant consequences for model building.

The most dramatic result would be if the "naive" alignment,

$$|(K_L^d)_{12}|\sim|V_{us}|\sim\lambda,\quad |(K_R^d)_{12}|\sim\frac{m_d}{m_s|V_{us}|}\sim\lambda, \tag{47}$$

were sufficient to satisfy the K^0-$\overline{K^0}$ constraints. If this were the case, then no holomorphic zeros would be required and the analysis of both model building and the phenomenological consequences of alignment would change considerably. What we learn from Eq. (45) is, however, that this is not the case. One could imagine that the parametric suppression gives $|(K_L^d)_{12}|\sim\lambda$ and that an accidental suppression of $\mathcal{O}(6)$ would make $(\delta^d_{LL})_{12}$ consistent with the bound (45). But then the second constraint would imply $(\delta^d_{RR})_{12}\lesssim 5\times 10^{-5}$, a factor of $\mathcal{O}(10^3)$ below the naive suppression. We conclude that the RGE-induced suppression in the GUT-scale models is not enough to allow viable models that employ no holomorphic zeros.

Under these circumstances, the milder constraints in Eq. (46) do not give a significant simplification for model building. In particular, relaxing the bound on $(K_L^d)_{12}$ from λ^3 in low-scale models to λ^2 in high scale models makes no difference at all. The point is that holomorphic zeros suppress $(K_L^d)_{12}$ compared to its naive value (47) by at least $\epsilon_{\max}^2$. Assuming, as we do in this work, that $\epsilon_{\max}\lesssim\lambda$, the consequences for model building of the λ^3 and λ^2 bounds are identical. On the other hand, the milder bound on $\sqrt{(K_L^d)_{12}(K_R^d)_{12}}$, λ^3 instead of λ^5, does allow horizontal charge assignments that would not be viable in low scale models.

We conclude that models of GUT scale alignment have phenomenological consequences that may be very different from low scale alignment. The difference in model building (in the framework of Abelian horizontal symmetries) is, however, of limited significance.

VII. CONCLUSIONS

We analyzed questions of model building and of phenomenological implications in the framework of quark-squark alignment. In models of alignment, three ingredients play a role in suppressing the supersymmetric contributions to flavor changing neutral currents:

(i) Approximate horizontal symmetries naturally suppress off-diagonal entries in both quark and squark mass matrices. This alignment of mass matrices induces small mixing angles in gaugino couplings.

(ii) Supersymmetry requires that the Yukawa couplings

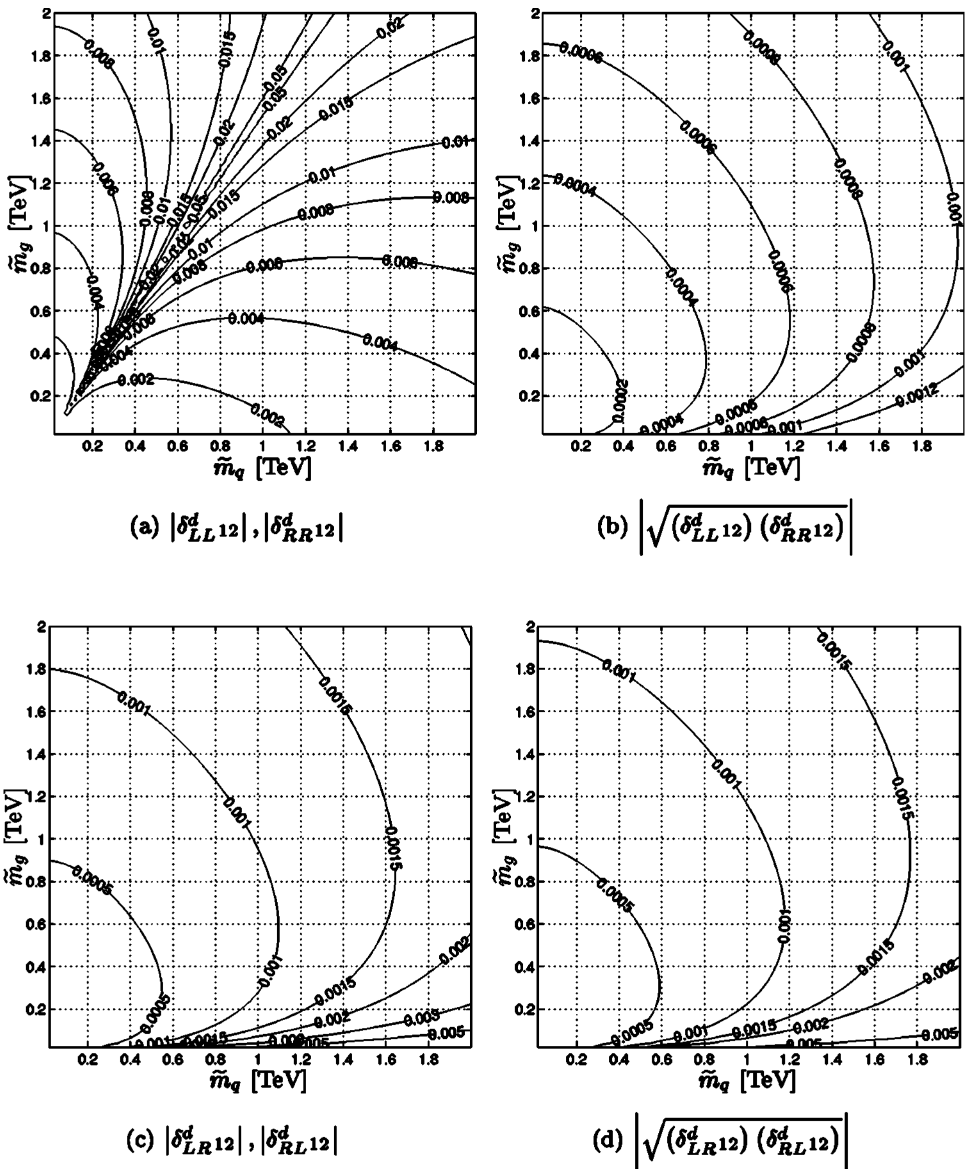

FIG. 3. Constraints on flavor changing mass insertions from K^0-$\overline{K^0}$ mixing as a function of the gluino mass $m_{\tilde{g}}$ and of the average squark mass $\tilde{m}_q$.

are holomorphic. In combination with the horizontal symmetries, zero textures may be required by holomorphy, opening up the possibility of a very precise alignment.

(iii) The running of the soft supersymmetry breaking terms may induce approximate degeneracy among squarks, even if there is no degeneracy in the high energy theory.

On the model-building side, we have made the following two main points:

(a) Under a few reasonable assumptions, there is a unique phenomenologically viable structure for the down quark mass matrix. In particular, four holomorphic zeros must appear, $M^d_{12}=M^d_{21}=M^d_{31}=M^d_{32}=0$.

(b) The possibility that a certain degree of degeneracy is induced by RGE somewhat relaxes the constraints on the required alignment. Still, "naive" alignment where, for example, the supersymmetric mixing angles for doublet quarks and squarks have the same parametric suppression as the corresponding CKM angles, is not viable. Consequently, the same holomorphic zeros must play a role and the complications of model building are not simplified.

On the phenomenological side, we would like to make the following points regarding the future prospects for discovering or excluding the idea of quark-squark alignment:

(a) Alignment models without squark degeneracy require that $|M^D_{12}|$ should be close to present experimental bounds. If the bounds on D^0-$\overline{D^0}$ mixing are improved by an order of magnitude, such models will be disfavored. Note that to improve the bound on M^D_{12} by an order of magnitude, it is not necessarily required to improve the bound on Δm_D by a similar factor. A mild experimental progress in constraining each of $x\equiv\Delta m_D/\Gamma, \phi_D$ (the relevant weak phase) and δ (the relevant strong phase) might give a significantly improved bound on $|M^D_{12}|$.

(b) The supersymmetric contribution to the B^0-$\overline{B^0}$ mixing amplitude M^B_{12} is at most a few percent of the experimental value. Experimentally, both Δm_B and $a_{\psi K_S}$ can be measured with an accuracy better than a few percent. The question of whether a deviation of order of a few percent from the standard model predictions can be convincingly signalled is related to the theoretical accuracy of the predictions. Given the hadronic uncertainties in the calculation of Δm_B, it will be impossible to have a convincing signal for this new contribution from the measurement of the mass difference. On the other hand, the hadronic uncertainties in the standard model relation $a_{\psi K} = \sin 2\beta$ are smaller than a percent. It is still an open question whether the value of 2β, constrained by other measurements, can be determined with the required accuracy.

(c) The supersymmetric contribution to the B_s-$\overline{B_s}$ mixing amplitude $M_{12}^{B_s}$ is at most a few percent of the experimental lower bound. Again, it would be difficult to have a convincing signal for this new contribution from the measurement of the mass difference Δm_{B_s}. On the other hand, the standard model predicts small $[\mathcal{O}(\lambda^2)]$ CP asymmetries in B_s decays to final CP eigenstates that involve the $b \to c\bar{c}s$ quark subprocess, so that the deviation can be significant.

(d) The supersymmetric contributions to $K \to \pi \nu \bar{\nu}$ decays are small. Thus the correlations between these decay rates and various observables related to B^0-$\overline{B^0}$ mixing, that are cleanly predicted by the standard model, may be violated.

We conclude that the observation of CP violation in D^0-$\overline{D^0}$ mixing and shifts of $\mathcal{O}(\lambda^2)$ from the standard model predictions for CP asymmetries in B^0 and B_s decays are the best possible clues for alignment. On the other hand, given the possibility of RGE-induced approximate degeneracy, it will be difficult to exclude the idea of alignment if no deviations from the standard model are observed. Stronger constraints on such deviations will simply translate into stronger lower bounds on the scale where alignment holds.

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[1] Y. Nir and N. Seiberg, Phys. Lett. B **309**, 337 (1993).
[2] M. Leurer, Y. Nir, and N. Seiberg, Nucl. Phys. **B420**, 468 (1994).
[3] A.F. Falk, Y. Grossman, Z. Ligeti, and A.A. Petrov, Phys. Rev. D **65**, 054034 (2002).
[4] G. Raz, hep-ph/0205113.
[5] M.S. Berger and K. Siyeon, Phys. Rev. D **64**, 053006 (2001).
[6] S.M. Barr, Phys. Rev. D **56**, 5761 (1997).
[7] L.J. Hall and A. Rasin, Phys. Lett. B **315**, 164 (1993).
[8] G. Eyal, A. Masiero, Y. Nir, and L. Silvestrini, J. High Energy Phys. **11**, 032 (1999).
[9] C.K. Chua and W.S. Hou, hep-ph/0110106.
[10] A. Arhrib, C.K. Chua, and W.S. Hou, Phys. Rev. D **65**, 017701 (2002).
[11] J.L. Diaz-Cruz, H.J. He, and C.P. Yuan, Phys. Lett. B **530**, 179 (2002).
[12] G. Raz, hep-ph/0205310.
[13] F. Gabbiani, E. Gabrielli, A. Masiero, and L. Silvestrini, Nucl. Phys. **B477**, 321 (1996).
[14] A. Arhrib, C.K. Chua, and W.S. Hou, Eur. Phys. J. C **21**, 567 (2001).
[15] S. Bertolini, F. Borzumati, A. Masiero, and G. Ridolfi, Nucl. Phys. **B353**, 591 (1991).
[16] S. Bertolini, F. Borzumati, and A. Masiero, Nucl. Phys. **B294**, 321 (1987); Phys. Rev. Lett. **59**, 180 (1987).
[17] Particle Data Group Collaboration, D.E. Groom *et al.*, Eur. Phys. J. C **15**, 1 (2000).
[18] BABAR Collaboration, B. Aubert *et al.*, Phys. Rev. Lett. **87**, 091801 (2001); hep-ex/0203007.
[19] Belle Collaboration, K. Abe *et al.*, Phys. Rev. Lett. **87**, 091802 (2001).
[20] Y. Nir, hep-ph/0109090.
[21] A. Masiero and H. Murayama, Phys. Rev. Lett. **83**, 907 (1999).
[22] M. Dine, E. Kramer, Y. Nir, and Y. Shadmi, Phys. Rev. D **63**, 116005 (2001).
[23] S.w. Baek, J.H. Jang, P. Ko, and J.H. Park, Phys. Rev. D **62**, 117701 (2000).
[24] E787 Collaboration, S. Adler *et al.*, Phys. Rev. Lett. **88**, 041803 (2002).
[25] Y. Nir and M.P. Worah, Phys. Lett. B **423**, 319 (1998).
[26] A.J. Buras, A. Romanino, and L. Silvestrini, Nucl. Phys. **B520**, 3 (1998).
[27] G. Colangelo and G. Isidori, J. High Energy Phys. **09**, 009 (1998).
[28] G. D'Ambrosio and G. Isidori, Phys. Lett. B **530**, 108 (2002).
[29] G. Buchalla and A.J. Buras, Phys. Lett. B **333**, 221 (1994); Phys. Rev. D **54**, 6782 (1996).
[30] S. Bergmann and G. Perez, J. High Energy Phys. **08**, 034 (2000).
[31] J.A. Aguilar-Saavedra and M. Masip, Phys. Rev. D **54**, 6903 (1996).
[32] D. Choudhury, F. Eberlein, A. Konig, J. Louis, and S. Pokorski, Phys. Lett. B **342**, 180 (1995).