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Left-right symmetry in 5D and neutrino mass in TeV-scale gravity models

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We construct a left-right symmetric model based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in five dimensions where both the gauge bosons and fermions reside in all five dimensions. The orbifold boundary conditions are used not only to break the gauge symmetry down to $SU(2)_L \times U(1)_Y \times U(1)_{Y'}$ but also to ''project'' the right handed neutrino out of the zero mode part of the spectrum, providing a new way to understand the small neutrino masses without adding (singlet) bulk neutrinos. This formulation of the left-right model also has two new features: (i) it avoids most existing phenomenological bounds on the scale of the right handed W_R boson allowing for the possibility that the right handed gauge bosons could have masses under a TeV, and (ii) it predicts a stable lepton with mass of order of the inverse radius of the fifth dimension.

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I. INTRODUCTION

Left-right symmetric models of weak interactions were introduced to understand the origin of parity violation in weak interactions [1]. Several features that have made this class of models interesting are (i) the complete quark-lepton symmetry, (ii) a more physically meaningful formula for the electric charge than the standard model [2], (iii) a natural way to understand small neutrino masses via the seesaw mechanism [3], (iv) a solution to the strong CP problem [4], (v) the suppression of R-parity violating operators present in the minimal extension of the standard model due to local $B-L$ symmetry [5], and (vi) the phenomenology of the extra Z' boson [6]. Many recent string constructions have also led to left-right symmetric models below the string scale [7].

An important question for phenomenology of these models has been the scale of parity breaking. For a long time there has been interest in models where the masses of the right handed W_R and Z' are in the multi-TeV range. With the recent discovery of nonzero neutrino masses, the case for left-right models has become more compelling for two reasons: (i) the right handed neutrino, which is necessary to implement the seesaw mechanism, is an integral part of these models and (ii) the local $B-L$ symmetry which protects the right handed neutrino mass from being at the Planck scale is also part of the gauge symmetry. However, the present observations of neutrino oscillations, coupled with bounds on neutrino masses from tritium beta decay require that if the seesaw mechanism is to explain neutrino masses, then the scale of parity breaking must be very high (of order 10^{12} GeV or higher). This makes the effects associated with right handed gauge symmetry (such as the extra Z' boson, right handed current induced flavor changing effects, right handed current effects in weak decays, etc.) completely unobservable. An interesting question therefore is whether it is possible to have a low scale for $SU(2)_R$ symmetry and still have a natural understanding of small neutrino masses with a minimal particle content. With the recent revival of theories with low scale for new physics in the context of brane bulk picture of space time, this question becomes a very pertinent one. It is quite reasonable to ask whether a case can be made for the right handed scale being in the multi-TeV range close to the fundamental scale of nature and yet have a solution to the neutrino mass problem. Coupled with the fact that the brane-bulk models provide an attractive alternative to supersymmetry as a way to solve the gauge hierarchy problem [8], one would now have an additional reason to consider the multidimensional low scale models as the ultimate theory of nature below the string scale.

The most widely discussed scenarios of the brane-bulk type put the standard model fermions in the brane and gauge bosons either in the brane or the bulk. Either of these cases have the difficulty that they cannot accommodate the attractive seesaw mechanism for the neutrino masses. As a result, to solve the neutrino mass problem one adds two new ingredients: (i) extra singlet fermions [9] in the bulk to suppress the Dirac mass of the neutrino and (ii) a global symmetry such as $B-L$ to prevent dangerous higher dimensional operators of the form $(LH)^2/M_*$ (L,H are lepton and Higgs doublets of the standard model, respectively). On the one hand, the bulk neutrinos can cause drastic revisions of our understanding of such issues as big bang nucleosynthesis and supernova dynamics and on the other hand, the assumption of global $B-L$ symmetry contradicts the fact that string theories are not supposed to have any continuous global symmetries [10].

A natural question to ask is whether the above problems arise due to our insistence that the gauge group is the standard $SU(2)_L \times U(1)_Y$ all the way up to the string scale. We examine this question by introducing the left-right symmetric

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gauge structure in the low fundamental scale models and see whether it can provide a simpler way of understanding the small neutrino masses without introducing the bulk neutrinos and without assuming global $B-L$ symmetry. The additional advantage is that due to low $SU(2)_R$ scale, the right handed current induced effects could possibly be accessible to experiments providing new tests of low scale gravity models. In the present paper, we present such a model in the five-dimensional context.

Before proceeding to the presentation of our formulation of the five-dimensional left-right symmetry, we note that in a recent paper [11], Mimura and Nandi have constructed a brane model with the left-right symmetric structure in five-dimensional space time where the gauge bosons reside in all five dimensions and the fermions are confined to the $3+1$ dimensional brane. They showed that the orbifold projections can break the gauge symmetry down to $SU(2)_L\times U(1)_Y \times U(1)_{Y'}$. Understanding small neutrino masses in this formulation would require new ingredients since within the minimal Higgs sector adopted in Ref. [11], one expects a large ($\sim$ weak scale) Dirac mass for the neutrino.

In this paper we present a new formulation of left-right symmetry in the five dimensions, where we allow both gauge bosons and fermions to reside in five dimensions and use orbifold boundary conditions in such a way that they not only make the right handed charged W_R boson massive but also ''project out'' the right handed neutrino from the standard model brane, and so, out of the zero mode part of the spectrum. This assumption leads to a number of interesting consequences.

(i) The first implication of this new construction is that there is now no Dirac mass for the left handed neutrino; secondly there are no lower order nonrenormalizable operators that could give a large mass to neutrinos; the lowest order operator in five dimensions that contributes to neutrino masses is $D=10$; this allows us to have a solution to the neutrino mass problem even though the scale of right handed symmetry is only in the multi-TeV range. No (singlet) bulk neutrino need be invoked [9].

(ii) Another consequence of our orbifold breaking is that many of the conventional phenomenological constraints on the W_R mass do not apply; as, for example, the W_L-W_R box graph [12] that provided a stringent constraint on W_R in usual left-right models is now absent. Similarly, the collider constraints [13] as well as the low energy muon [14] and beta decay constraints [15] are now forbidden by the five-dimensional momentum conservation.

(iii) The model predicts the existence of a heavy stable particle (a lepton) whose mass is equal to the inverse radius of the fifth dimension. This can lead to interesting cosmological as well as phenomenological possibilities.

This paper is organized as follows: in Sec. II we discuss the orbifold breaking of the gauge symmetry as well as the boundary conditions satisfied by the matter fields of the theory; in Sec. III we discuss the masses and mixings of the gauge bosons; in Sec. IV we present the gauge boson fermion couplings; in Sec. V we discuss neutrino masses and the prediction of a heavy sterile neutrino; and in Sec. VI we present some phenomenological implications such as the $K^0-\overline{K}^0$ mixing, decay modes of the W_R, etc. In Sec. VII we speculate on further extensions of the model and inclusion of supersymmetry. There we also discuss how parity restoration occurs in the theory. In Sec. VIII we present a summary of our results and conclude.

II. FERMIONS, HIGGS BOSONS, AND ORBIFOLD BREAKING OF $SU(2)_R$

We now discuss the detailed particle content of the model and the orbifold conditions to implement symmetry breaking. We denote the gauge symmetry in five dimensions as $SU(2)_1\times SU(2)_2\times U(1)_{B-L}$, with the later identification of subscripts as $1\to L$ and $2\to R$. The gauge bosons are denoted by $W^{\pm,3}_{1,M}$, $W^{\pm,3}_{2,M}$, and B_M, with $M=\mu,5$ (μ denotes the $3+1$ Minkowski indices and 5 corresponds to the compactified fifth dimension). We choose four sets of quark doublets denoted by Q_1, Q_1', Q_2, and Q_2' and similarly four sets of lepton doublets denoted by ψ_1, ψ_1', ψ_2, and ψ_2'. The subscripts (1,2) next to fermions represent that they transform as doublets under the corresponding $SU(2)$ group. Notice that all fermions contain both left and right handed components, and five-dimensional interactions are originally vectorlike. Nevertheless, the nontrivial boundary conditions introduced by orbifolding can break both the gauge symmetry [16,17] and the vectorlike nature of fermions. We will use this feature in what follows.

Let us now proceed with the orbifold compactification of the fifth dimension. We compactify the fifth coordinate (denoted by y) on an orbifold $S_1/Z_2\times Z_2'$, where under the first Z_2: $y\to -y$; and under the second Z_2': $y'\to -y'$; where $y'=y+\pi R/2$. The orbifold is then an interval $[0,\pi R/2]$. Since both gauge bosons and fermions in our model propagate in all five dimensions, we have to specify their transformation properties under the $Z_2\times Z_2'$. This is equivalent to defining the boundary conditions that those fields should satisfy.

As far as the gauge fields go, we choose the same boundary conditions as in Ref. [11]. Defining the gauge boson matrix for both the $SU(2)$'s as

$$W=\begin{pmatrix} W_3 & \sqrt{2}W^+ \\ \sqrt{2}W^- & -W_3 \end{pmatrix} \tag{1}$$

we can write the $Z_2\times Z_2'$ transformation properties of $W_{1,2}$ as

$$\begin{aligned} W_\mu(x^\mu,y)&\to W_\mu(x^\mu,-y)=PW_\mu(x^\mu,y)P^{-1},\\ W_5(x^\mu,y)&\to W_5(x^\mu,-y)=-PW_5(x^\mu,y)P^{-1},\\ W_\mu(x^\mu,y')&\to W_\mu(x^\mu,-y')=P'W_\mu(x^\mu,y')P'^{-1};\\ W_5(x^\mu,y')&\to W_5(x^\mu,-y')=-P'W_5(x^\mu,y)P'^{-1}; \end{aligned} \tag{2}$$

where P and P' are two by two diagonal matrices that we chose as (i) $P = P'=\mathrm{diag}(1,1)$ for the $SU(2)_1$ gauge bosons; and (ii) $P=\mathrm{diag}(1,1)$ and $P'=\mathrm{diag}(1,-1)$ for those of $SU(2)_2$. The B boson, on the other hand, has a single transformation property under both Z_2 and Z_2' projections, which is $B_\mu(x^\mu,y)\rightarrow B_\mu(x^\mu,-y)$ and $B_5(x^\mu,y)\rightarrow -B_5(x^\mu,-y)$, and same under Z_2'.

Under these transformations the gauge fields develop explicit fifth-dimensional parities identified as the $Z_2\times Z_2'$ quantum numbers (this is to be distinguished from the usual space time parity). With above transformation rules we find the following parity assignments:

$$\begin{aligned} &W^{3,\pm}_{1,\mu}(+,+); \quad B_\mu(+,+); \quad W^3_{2,\mu}(+,+);\\ &W^{\pm}_{2,\mu}(+,-); \quad W^{3,\pm}_{1,5}(-,-); \quad B_5(-,-);\\ &W^3_{2,5}(-,-); \quad W^{\pm}_{2,5}(-,+). \end{aligned} \tag{3}$$

To be more explicit about the meaning of these parities, note that a minus sign indicates that such an odd field vanishes at the fixed point associated to the corresponding Z_2 or Z_2' transformation. Thus from Eq. (3) we see that at $y=\pi R/2$ the $SU(2)_2$ gauge symmetry has been broken down to its diagonal $U(1)_{I_{3,2}}$ subgroup, while the other groups remain unbroken. Thus at this fixed point the remaining symmetry can be identified as $SU(2)_1\times U(1)_Y\times U(1)_{Y'}$. That is the standard model (SM) symmetry with an extra $U(1)_{Y'}$ factor generated by the orthogonal generator to the hypercharge: $\frac{1}{2}Y'\equiv\sqrt{\frac{2}{5}}I_{3,2}-3/\sqrt{10}\frac{1}{2}(B-L)$. At the other boundary ($y=0$) the whole gauge symmetry remains intact. Due to the breaking of the symmetry at one of the boundaries, the effective four-dimensional theory will be invariant only under $SU(2)_1\times U(1)_Y\times U(1)_{Y'}$ symmetry

Turning now to the fermions, the general transformation rules under $Z_2\times Z_2'$ of any of our doublet fermion representations have the form

$$\begin{aligned} &\Psi(x^\mu,y)\rightarrow\Psi(x^\mu,-y)=\pm\gamma_5 P\Psi(x^\mu,y),\\ &\Psi(x^\mu,y')\rightarrow\Psi(x^\mu,-y')=\pm\gamma_5 P'\Psi(x^\mu,y'), \end{aligned} \tag{4}$$

where the sign controls which chiral component of Ψ is being projected out of the fixed points by the action of γ_5. In the last equation P and P' act on the group space and are given by the very same matrices used in Eq. (2). In conclusion, left and right components of the same fermion will hold opposite parities. Up and down components of any doublet representations under $SU(2)_1$ will hold the same parity assignments. For $SU(2)_2$ doublets the situation is as follows. Parity assignments under Z_2 for the up and down fields will be the same; however, those associated to Z_2' will be opposite to each other, due to the nontrivial election of P' acting on this group sector.

Using these rules we demand that the various fermion doublets get the following $Z_2\times Z_2'$ quantum numbers, for quarks:

$$\begin{aligned} &Q_{1,L}\equiv\begin{pmatrix} u_{1L}(+,+)\\ d_{1L}(+,+)\end{pmatrix}, \quad Q'_{1,L}\equiv\begin{pmatrix} u'_{1L}(+,-)\\ d'_{1L}(+,-)\end{pmatrix},\\ &Q_{1,R}\equiv\begin{pmatrix} u_{1R}(-,-)\\ d_{1R}(-,-)\end{pmatrix}, \quad Q'_{1,R}\equiv\begin{pmatrix} u'_{1R}(-,+)\\ d'_{1R}(-,+)\end{pmatrix},\\ &Q_{2,L}\equiv\begin{pmatrix} u_{2L}(-,-)\\ d_{2L}(-,+)\end{pmatrix}, \quad Q'_{2,L}\equiv\begin{pmatrix} u'_{2L}(-,+)\\ d'_{2L}(-,-)\end{pmatrix},\\ &Q_{2,R}\equiv\begin{pmatrix} u_{2R}(+,+)\\ d_{2R}(+,-)\end{pmatrix}, \quad Q'_{2,R}\equiv\begin{pmatrix} u'_{2R}(+,-)\\ d'_{2R}(+,+)\end{pmatrix}, \end{aligned} \tag{5}$$

and for leptons:

$$\begin{aligned} &\psi_{1,L}\equiv\begin{pmatrix} \nu_{1L}(+,+)\\ e_{1L}(+,+)\end{pmatrix}, \quad \psi'_{1,L}\equiv\begin{pmatrix} \nu'_{1L}(-,+)\\ e'_{1L}(-,+)\end{pmatrix},\\ &\psi_{1,R}\equiv\begin{pmatrix} \nu_{1R}(-,-)\\ e_{1R}(-,-)\end{pmatrix}, \quad \psi'_{1,R}\equiv\begin{pmatrix} \nu'_{1R}(+,-)\\ e'_{1R}(+,-)\end{pmatrix},\\ &\psi_{2,L}\equiv\begin{pmatrix} \nu_{2L}(-,+)\\ e_{2L}(-,-)\end{pmatrix}, \quad \psi'_{2,L}\equiv\begin{pmatrix} \nu'_{2L}(+,+)\\ e'_{2L}(+,-)\end{pmatrix},\\ &\psi_{2,R}\equiv\begin{pmatrix} \nu_{2R}(+,-)\\ e_{2R}(+,+)\end{pmatrix}, \quad \psi'_{2,R}\equiv\begin{pmatrix} \nu'_{2R}(-,-)\\ e'_{2R}(-,+)\end{pmatrix}. \end{aligned} \tag{6}$$

Let us note the mode expansion of a generic field, $\varphi(x^\mu,y)$ with given $Z_2\times Z_2'$ quantum numbers (z_1,z_2):

$$\varphi^{(z_1,z_2)}(x^\mu,y)=\sum_n^\infty \xi_n^{(z_1,z_2)}(y)\varphi_n(x^\mu), \tag{7}$$

for $z_1,z_2=\pm$ and where the Fourier parity eigenfunctions, $\xi^{(z_1,z_2)}$, properly normalized on the interval $[0,\pi R/2]$, are given by

$$\begin{aligned} &\xi_n^{(+,+)}=\frac{2\eta_n}{\sqrt{\pi R}}\cos\frac{(2n)y}{R},\\ &\xi_n^{(+,-)}=\frac{2}{\sqrt{\pi R}}\cos\frac{(2n-1)y}{R},\\ &\xi_n^{(-,+)}=\frac{2}{\sqrt{\pi R}}\sin\frac{(2n-1)y}{R},\\ &\xi_n^{(-,-)}=\frac{2}{\sqrt{\pi R}}\sin\frac{(2n)y}{R}, \end{aligned} \tag{8}$$

where η_n is $1/\sqrt{2}$ for $n=0$ and 1 for $n>0$. Notice from here that only fields with $Z_2\times Z_2'$ quantum numbers $(+,+)$ have zero modes. For all others $n>0$. Thus we conclude that indeed, after orbifolding the gauge group is $SU(2)_1\times U(1)_Y\times U(1)_{Y'}$ and the zero mode fermion content is the same as the standard model plus an additional neutrino per family which is a sterile neutrino since it does not couple with the W

and Z bosons of the standard model. This is one of the first predictions of the model. This prediction is purely a consequence of putting the fermions in the bulk and therefore different from Ref. [11]. As we show later on in this paper, the sterile neutrino can acquire a large mass and will therefore decouple from the low energy spectrum. From now on we identify the fermion fields having zero modes with the self-explaining standard notation: Q,L,u_R,d_R,e_R, and ν_s. Moreover, from here on, we will refer to $SU(2)_1$ as to $SU(2)_L$, and consequently to $SU(2)_2$ as $SU(2)_R$, which are perhaps more meaningful to us.

Let us now discuss the Higgs sector of the model. We need Higgs bosons to break the remaining $U(1)_{Y'}$ and the standard model gauge group, as well as to give mass to the fermions. We choose the minimal set required for the purpose, i.e., a bidoublet $\phi(2,2,0)$ and doublets $\chi_L(2,1,-1)$ and $\chi_R(1,2,-1)$. We assign the following $Z_2\times Z_2'$ quantum numbers to the various components of the Higgs bosons:

$$\phi\equiv\begin{pmatrix} \phi_u^0(+,+) & \phi_d^+(+,-) \\ \phi_u^-(+,+) & \phi_d^0(+,-) \end{pmatrix}, \quad \chi_L\equiv\begin{pmatrix} \chi_L^0(-,+) \\ \chi_L^-(-,+) \end{pmatrix};$$

$$\chi_R\equiv\begin{pmatrix} \chi_R^0(+,+) \\ \chi_R^-(+,-) \end{pmatrix}. \tag{9}$$

They are consistent with the generic $Z_2\times Z_2'$ transformation rules for scalar fields. For instance, $\chi\to\pm P\chi$ and $\chi\to\pm P'\chi$, respectively. We see from these assignments that the only fields that have zero modes are (ϕ_u^0,ϕ_u^-) and χ_R^0. The former doublet acts like the standard model doublet. We will assign vacuum expectation values (vev's) to $\langle\phi_u^0\rangle=v_{wk}$ and $\langle\chi_R^0\rangle=v_R$. The first vev breaks the standard model symmetry whereas the second vev breaks the $U(1)_{Y'}$ symmetry, or equivalently, it breaks the group $SU(2)_R\times U(1)_{B-L}$ down to $U(1)_Y$, as in the four-dimensional theories, though in our case there is no $W_R^\pm$ zero mode since $SU(2)_R$ is already broken by the orbifold anyway.

III. MASSES AND MIXINGS

We now turn our attention to the spontaneous symmetry breaking induced by $\langle\phi_u\rangle$ and $\langle\chi_R\rangle$. Let us first note that only zero mode fields have vevs and therefore they actually induce 5D mass terms. One can then simplify the analysis of masses and mixings just by looking directly at these 5D terms. In the effective 4D theory, the mass induced by the Higgs vacuum will generate a global shifting of the well-known Kaluza-Klein (KK) masses, such that the actual mass of each level will be given as

$$m_n^2=m_0^2+m_{n,KK}^2, \tag{10}$$

where m_0 stands for the 5D Higgs induced mass. Here $m_{n,KK}$ is the usual KK mass contribution, which is given by integer multiples of the inverse compactification scale R^{-1}. The values it takes only depend on the parity of the corresponding field, so one gets

$$m_{n,KK}R=\begin{cases} 2n & \text{for } (+,+)\text{and } (-,-), \\ 2n-1 & \text{for}(+,-)\text{and } (-,+). \end{cases} \tag{11}$$

Similarly as far as mixings go, if they exist among any two fields at the zero mode level, they will also be global, i.e., they will be independent of the KK number, and there will be no mixings among fields with different KK numbers. For instance, on a given excited level of SM fields there will be the very same mixings as the ones produced at the zero mode level. Our model therefore predicts an exact KK replication of the SM spectrum beyond the compactification scale. Obviously, there will also be towers associated with all other nonstandard fields that are present on the model.

A. Charged gauge bosons

Let us now discuss with some detail the masses and mixings of the gauge sector (see also Ref. [11]). As only one neutral component of the ϕ field develops a vacuum, there is no left-right mixing between the charged gauge bosons. One then gets for the KK levels the masses

$$m_{n,W_L}^2=M_{W_L}^2+\left(\frac{2n}{R}\right)^2, \tag{12}$$

$$m_{n,W_R}^2=\frac{g_R^2}{2}(v_R^2+v_{wk}^2)+\left(\frac{2n-1}{R}\right)^2, \tag{13}$$

for $M_{W_L}^2=g_L^2v_{wk}^2/2$ being the mass of the standard W_L. Here $g_{L,R}$ represent the gauge coupling constants of left and right $SU(2)$ gauge groups, respectively. As W_R does not develop zero modes, its lower level mass has a nonzero contribution from the compactification scale, as it is clear from Eq. (13) for $n=1$.

B. Neutral gauge bosons

In order to simplify the analysis it is useful to define the mixing angles

$$\tan\alpha\equiv\frac{g_B}{g_R} \quad \text{and} \quad \tan\theta\equiv\frac{g_B\cos\alpha}{g_L}, \tag{14}$$

where g_B is the coupling constant of $U(1)_{B-L}$. Notice that $\tan\theta$ actually corresponds to the standard weak mixing angle, in terms of which the massless photon field is defined as

$$A(x,y)=\cos\theta B_Y(x,y)+\sin\theta W_L^3(x,y), \tag{15}$$

for $B_Y(x,y)=\cos\alpha\ B(x,y)+\sin\alpha W_R^3(x,y)$ is the standard hypercharge boson. A Lorentz index must be understood in these equations. As it is usual, the orthogonal boson to B_Y is called Z' and it is given by the combination $Z'(x,y)=\cos\alpha W_R^3(x,y)-\sin\alpha B(x,y)$, whereas the standard model neutral boson is the zero mode of the field $Z(x,y)=\cos\theta W_L^3(x,y)-\sin\theta B_Y(x,y)$.

By considering the mass terms induced by the VEVs, we notice that $\langle\chi_R\rangle$ only contributes to the mass of the Z' boson, whereas $\langle \phi_u \rangle$ generates masses for both Z' and the standard Z, and it also introduces a $Z-Z'$ mixing. The photon, as expected, remains massless. One can now write the $Z-Z'$ mass mixing matrix as

$$\begin{pmatrix} m_Z^2 & -m_Z^2 \sin\theta \cot\alpha \\ -m_Z^2 \sin\theta\cot\alpha & m_{Z'}^2 \end{pmatrix}. \tag{16}$$

In the last equation

$$m_Z^2 = \frac{M_{W_L}^2}{\cos\theta} \quad \text{and}$$

$$m_{Z'}^2 = \left(\frac{g_R^2 v_R^2}{2\cos^2\alpha}\right)\left[1+\left(\frac{v_{wk}}{v_R}\right)^2 \cos^2\alpha\right]. \tag{17}$$

In the limit when $v_R \gg v_{wk}$ the above mass matrix gives a mass correction to the Z boson,

$$M_Z^2 = m_Z^2 - \delta m_Z^2, \tag{18}$$

with $\delta m_Z^2 \approx (v_{wk}/v_R)^2 m_Z^2 \cos^4\alpha$. Correspondingly, one gets $M_{Z'}^2 = m_{Z'}^2 + \delta m_Z^2$. In the symmetric limit where $g_L = g_R$ the mass correction reads as

$$\frac{\delta m_Z^2}{m_Z^2} \approx \frac{\cos^2 2\theta}{\cos^4\theta}\left(\frac{v_{wk}}{v_R}\right)^2.$$

The $Z-Z'$ mixing is given by

$$\tan\beta \approx \left(\frac{v_{wk}}{v_R}\right)^2 \frac{\sin\alpha\cos^3\alpha}{\sin\theta} \overset{g_L=g_R}{\longrightarrow} \left(\frac{v_{wk}}{v_R}\right)^2 \frac{(\cos 2\theta)^{3/2}}{\cos^4\theta}. \tag{19}$$

KK modes of neutral gauge fields will have masses which follow the prescription in Eq. (10), that is

$$\begin{pmatrix} m_{n,A}^2 \\ m_{n,Z}^2 \\ m_{n,Z'}^2 \end{pmatrix} = \begin{pmatrix} 0 \\ M_Z^2 \\ M_{Z'}^2 \end{pmatrix} + \left(\frac{2n}{R}\right)^2. \tag{20}$$

C. Yukawa couplings and fermion masses

The most general Yukawa couplings one can write with our matter content, and which are invariant under both gauge and parity transformations, are

$$h_u \overline{Q}_1 \phi Q_2 + h_d \overline{Q}_1 \tilde{\phi} Q_2' + h_e \overline{\psi}_1 \tilde{\phi} \psi_2 + h_u' \overline{Q}_1' \phi Q_2' + h_d' \overline{Q}_1' \tilde{\phi} Q_2 + h_e' \overline{\psi}_1' \tilde{\phi} \psi_2' + \text{H.c.}, \tag{21}$$

where $\tilde{\phi} \equiv \tau_2 \phi^* \tau_2$ is the charge conjugate field of ϕ. The matrices $h_{u,d,e}$ and $h_{u,e}'$ are the 3×3 Yukawa coupling matrices in five dimensions. The above terms are invariant under the parity symmetry that interchanges the subscripts: $1 \leftrightarrow 2$ provided the Yukawa couplings satisfy the following constraints:

$$h_u = h_u^\dagger, \quad h_u' = h_u'^\dagger, \quad h_e = h_e^\dagger, \quad h_e' = h_e'^\dagger, \quad h_d = h_d'^\dagger. \tag{22}$$

This is the five-dimensional realization of the left-right symmetry. It is worth noticing that there are no trilinear couplings that involve χ doublet fields. Moreover, there are no couplings in the theory that may give rise to a Dirac mass for the neutrino. This is one of our most interesting results. From the above terms one can read out those couplings that give rise to the standard model fermion masses,

$$\mathcal{L} = h_u \overline{Q} \phi_u u_R + h_d \overline{Q} \tilde{\phi}_u d_R + h_e \overline{L} \tilde{\phi}_u e_R + \text{H.c.} \tag{23}$$

As it is obvious, generation mixings will come through the Yukawa couplings. Also important to note is the fact that while $h_{u,e}$ are Hermitian matrices, h_d is not; this fact has important implications for the nature of the quark mixings involving right handed quarks. To see this we first note that $h_{u,e}^{diag} = V_{u,e} h_{u,e} V_{u,e}^\dagger$ whereas $h_d^{diag} = V_d h_d U_d^\dagger$. Thus the $U_{\text{CKM}} = V_u^\dagger V_d$, whereas the corresponding right handed charged current mixing matrix for quarks is $U^R = V_u^\dagger U_d$. Thus unlike the case of standard left-right models, the left and right handed quark mixings are different from each other.

The "chiral partners" of the standard model fields, i.e., those that come in the same 5D representation, will have similar couplings, $\overline{Q}_{1R} \phi_u u_{2L} + \overline{Q}_{1R} \tilde{\phi}_u d_{2L} + \overline{\psi}_{1R} \tilde{\phi}_u e_{2L} + \text{H.c.}$, and so, they will get an equal contribution to its mass from the vacuum as that of its SM partner. The final mass spectrum will have the SM fermions with usual masses at the zero mode level. Extra degenerate pairs of (excited) massive fermion states will be present above the compactification scale. Thus, at each excited level, the spectrum of particles will duplicate the SM one. The masses of these Kaluza-Klein (KK) modes will be of the form

$$m_n^2 = m_{\text{SM}}^2 + \left(\frac{2n}{R}\right)^2. \tag{24}$$

There will also be some extra particles in the KK spectrum, those that get mass contributions from the Yukawa couplings: u_1' paired to u_2'; d_1' paired to d_2, and e_1' paired to e_2'. Such mass terms will shift the KK mass: $(2n-1)/R$. The neutral fermions ν_2 and ν_1' will all get Dirac masses: $(2n-1)/R$. Corresponding to the fields ν_1 and ν_2' we get KK modes with Dirac masses $2n/R$. At this point, only the standard neutrino $\nu_L \equiv \nu_{0,1L}$ and the sterile neutrino $\nu_s \equiv \nu_{0,2L}'$ remain massless, as we have already anticipated. These fields, however, may acquire masses from nonrenormalizable operators as we will discuss later on.

D. Scalar sector

As for the scalar fields, at the zero mode level there will be two massive neutral scalars. The standard Higgs, $H^0 = \phi_0$, with a mass of order v_{wk}, as usual, and a heavier field, χ_{0R}, with a mass $\sim v_R$. Clearly, they will come accompanied with their own tower of excited modes, with masses shifted as indicated in Eq. (24). What is perhaps more interesting to notice is that there will be no KK modes associated to ϕ_u^- nor to χ_R^-, both the fields (so, both theirs towers) have been absorbed, for the $W_{L,R}$ gauge bosons get masses through the Higgs mechanism. ϕ_d, and χ_L towers, on the contrary, will be present with mass spectra following Eqs. (10) and (11), according to the parity of each field.

IV. GAUGE BOSON FERMION COUPLINGS

Gauge couplings on the theory under consideration are essentially five-dimensional. They are generically of the form:

$$\mathcal{L}=g_{5D}G_M(x,y)J^M(x,y), \tag{25}$$

where $J^M=\overline{\Psi}\gamma^M\Psi$ is the (vectorlike) five-dimensional fermion current coupled to a gauge boson G. Note that the effective four-dimensional gauge coupling constant, g, and the five-dimensional one, g_{5D}, are related through the simple scaling: $g=\xi_0^{(+,+)}g_{5D}$. Notice also that g_{5D} is a dimensionful quantity whose mass dimension balances the higher dimensionality of the fields, whereas g is dimensionless. This scaling has already been taken into account in all the previous analyses such that $g_{L,R,B}$ were taken as the actual four-dimensional couplings.

Now, in order to do the KK decomposition of the theory one should go to the unitary gauge where the KK modes of the gauge boson get well-defined masses by absorbing the modes associated to its own fifth Lorentz component. In such a gauge one takes $G_5=0$, which reduces the Lorentz index in Eq. (25) to μ. This gauge fixing does not preclude the existence of the effective four-dimensional gauge invariance associated to the zero modes. On such a gauge we get the general effective couplings among KK modes:

$$\mathcal{L}_{eff}\equiv\int dy\mathcal{L} = \sum_{mn} g[G^L_{mn,\mu}(x)J^L_{mn}{}^{\mu}(x)+G^R_{mn,\mu}(x)J^R_{mn}{}^{\mu}(x)]. \tag{26}$$

Here, the left and right handed fermion current are given in terms of the excited modes $J^{L,R\mu}_{mn}=\overline{\Psi}_{mL,R}\gamma^\mu\Psi_{nL,R}$, whereas $G^{L,R}_{mn,\mu}$ stands for

$$G^{L,R}_{mn,\mu}(x)\equiv\sqrt{\frac{\pi R}{2}}\int dy\,\xi^{L,R}_m(y)\xi^{L,R}_n(y)G_\mu(x,y). \tag{27}$$

The right hand side of the last equation can be expanded by introducing the KK expansion of G. This procedure indicates that only excited gauge boson modes that conserve the KK number at the vertex contribute to the above couplings. Indeed, the explicit expansion of Eq. (27) involves integrals of the type $\int\xi_n\xi_m\xi_l$, which generically give the result: $\delta_{l,m+n}+\delta_{m,n+l}+\delta_{n,l+m}$. This can also be seen as a consequence that translational invariance along the fifth dimension has not been explicitly broken in the theory. Particularly, at the zero mode level only the $G_{0,\mu}$ couples to the purely zero mode current, $J^{L,R\mu}_{00}$. This has the consequence that KK modes could only be produced by pairs at colliders. Following the above prescription, it would be enough to write down the couplings at the level of the fifth-dimensional theory to be able to read out those of the effective theory.

A. Charged currents

As there is no left right mixing between charged bosons, one can write the Lagrangian of the charged currents straightforwardly,

$$\mathcal{L}^{cc}=\frac{g_L}{\sqrt{2}}W^-_{L,\mu}J_1^{c,\mu}+\frac{g_R}{\sqrt{2}}W^-_{R,\mu}J_2^{c,\mu}+\text{H.c.} \tag{28}$$

For $J_1^{c,\mu}=J_{1L}^{c,\mu}+J_{1R}^{c,\mu}$, with

$$J_{1L}^{c,\mu}=\overline{u_L}\gamma^\mu d_L+\overline{\nu_L}\gamma^\mu e_L+\overline{u'_{1L}}\gamma^\mu d'_{1L}+\overline{\nu'_{1L}}\gamma^\mu e'_{1L} \tag{29}$$

that contains the standard charged current as its zero mode component. $J_{1R}^{c,\mu}$, on the other hand, is given by a similar expression with all fields changed by its chiral partners (that is by taking $u_L\to u_{1R}$, and so on), that gives $J_{1R}^{c,\mu}=\overline{u_{1R}}\gamma^\mu d_{1R}+\overline{\nu_{1R}}\gamma^\mu e_{1R}+\overline{u'_{1R}}\gamma^\mu d'_{1R}+\overline{\nu'_{1R}}\gamma^\mu e'_{1R}$. The latter does not contain any zero mode, thus it is absent on the low energy level. A CKM matrix acting on the family space should be understood. Also note that a normalization factor for the gauge coupling has been omitted for simplicity.

The charged currents coupled to W_R are given by

$$J_{2R}^{c,\mu}=\overline{u_R}\gamma^\mu d_{2R}+\overline{u'_{2R}}\gamma^\mu d_R+\overline{\nu_{2R}}\gamma^\mu e_R+\overline{\nu'_{2R}}\gamma^\mu e'_{2R}. \tag{30}$$

That contains the couplings to the right handed SM fields. Note, however, that W_R does not couple u_R to d_R, but rather to the d_{2R} field, which belongs to the same representation. This has dramatic implications on the phenomenology as we shall mention below. Next, $J_{2L}^{c,\mu}$ is, again, just the chiral partner of $J_{2R}^{c,\mu}$, and it reads $J_{2L}^{c,\mu}=\overline{u_{2L}}\gamma^\mu d_{2L}+\overline{u'_{2L}}\gamma^\mu d'_{2L}+\overline{\nu_{2L}}\gamma^\mu e_{2L}+\overline{\nu_{sL}}\gamma^\mu e'_{2L}$. There is no zero mode component on these interactions, thus, any correction to SM processes due to this term will appear only through loops, and thus more suppressed than in the case of four-dimensional theories. Furthermore, in general the left and the right CKM matrices are not related (similar to the "nonmanifest" case of standard left-right models). These properties of our model are different from the model presented in Ref. [11].

B. Neutral currents

After performing the rotations introduced in the previous section (used to get gauge mass eigenstates) one gets the following neutral current interactions:

$$\mathcal{L}_{NC}=\left[eA_\mu Q+\left(\frac{g_L}{\cos\theta}\right)Z_\mu A_{NC} +\left(\frac{g_L}{\cos\theta}\right)Z'_\mu\ B_{NC}\right]J^{NC,\ \mu}, \tag{31}$$

where the neutral current has the contribution of all fermion representations in Eqs. (5) and (6):

$$J^{NC,\mu}=\sum_i \bar{Q}_i\gamma^\mu Q_i+\bar{\psi}_i\gamma^\mu\psi_i. \tag{32}$$

The zero mode components of this current are read out as

$$J^{NC,\mu}_{00,L}=\bar{Q}\gamma^\mu Q+\bar{L}\gamma^\mu L+\bar{\nu}_{sL}\gamma^\mu\nu_{sL},$$

$$J^{NC,\ \mu}_{00,R}=\bar{u}_R\gamma^\mu u_R+\bar{d}_R\gamma^\mu d_R+\bar{e}_R\gamma^\mu e_R, \tag{33}$$

that one identifies as the SM neutral current elements. The effective couplings, A_{NC} and B_{NC}, follow the general prescriptions:

$$A_{NC}=(\sin\theta\sin\beta\tan\alpha+\cos\beta)T_{3L}+\frac{\sin\theta\sin\beta}{\cos\alpha\sin\alpha}T_{3R}-(\sin\theta\sin\beta\tan\alpha+\cos\beta\sin^2\theta)Q$$

and

$$B_{NC}=(\sin\theta\cos\beta\tan\alpha-\sin\beta)T_{3L}+\frac{\sin\theta\cos\beta}{\cos\alpha\sin\alpha}T_{3R}-(\sin\theta\cos\beta\tan\alpha-\sin\beta\sin^2\theta)Q.$$

It is illustrative to notice that in the further limit when $v_R\rightarrow\infty$, that means that $\sin\beta\rightarrow 0$, one recovers the standard coupling of the Z boson from A_{NC}. In the case when one takes $g_L=g_R$, the above expressions reduce to

$$A_{NC}=\left(\cos\beta+\frac{\sin\beta\sin^2\theta}{(\cos 2\theta)^{1/2}}\right)T_{3L}+\frac{\sin\beta\cos^2\theta}{(\cos 2\theta)^{1/2}}T_{3R}-\sin^2\theta\left(\cos\beta+\frac{\sin\beta}{(\cos 2\theta)^{1/2}}\right)Q \tag{34}$$

and

$$B_{NC}=\left(\frac{\cos\beta\sin^2\theta}{(\cos 2\theta)^{1/2}}+\sin\beta\right)T_{3L}+\frac{\cos\beta\cos^2\theta}{(\cos 2\theta)^{1/2}}T_{3R}-\sin^2\theta\left(\frac{\cos\beta}{(\cos 2\theta)^{1/2}}-\sin\beta\right)Q. \tag{35}$$

V. NEUTRINO MASSES WITHOUT THE BULK SINGLET NEUTRINO

Let us now get back to the problem of neutrino masses. As we mentioned already, standard and sterile neutrinos do not get Dirac masses from Yukawa couplings [see Eq. (23)]. The physical reason is twofold. First, the zero mode right handed neutrino is missed in the theory. Second, the potential Yukawa terms, $\bar{\psi}_1\phi\psi_2$ or $\bar{\psi}_1\phi\psi'_2$, that may give rise to a large Dirac neutrino mass, and that one could expect from the matter content in Eq. (6), are not invariant under the parity symmetries, $Z_2\times Z'_2$. Thus the neutrino is massless at the renormalizable level of the effective four-dimensional theory. This is a completely new feature for this class of theories with left right symmetry and completely distinct from the four-dimensional left-right models. The neutrino mass does not arise in the present model from a see-saw mechanism, and thus there is not immediate constraint on the v_R scale from this sector. This allows for the possibility of a sufficiently small v_R so as to be accessible to the next generation collider experiments.

On the other hand, one common problem of theories with a low fundamental scale is a potentially dangerous large neutrino mass that comes from nonrenormalizable operators, such as $(LH)^2$. The question is then whether such a problem may be also present in our theory. As we now show, this is no longer a problem. Clearly such an operator, coming from its higher dimensional relative, $(\psi_1\phi)^2$, is forbidden by the conservation of $B-L$ on our model. Moreover, though the coupling $(\psi_1\chi_L)^2$ is perfectly possible, there is no neutrino mass induced from this operator due to the lack of a VEV for χ_L. Therefore the model is completely safe from dangerous operators that could give large neutrino masses.

To see how neutrino mass arises in this theory, let us note that there are three classes of nonrenormalizable operators of higher dimensions that remain invariant under all the symmetries of the theory and which contribute to neutrino masses.

(i) There are operators connecting the active left-handed neutrinos to themselves, i.e., $O_1\equiv\psi_1^T C_5\psi_1\phi\phi\chi_R\chi_R/M_*^5$, where $C_5\equiv\gamma^0\gamma^2\gamma^5$, where we have omitted the family index. Notice that it has dimension 10 on 5D. It generates, at the four-dimensional theory, the effective couplings [18]

$$\frac{h}{(M_*R)^2}\,\frac{(L\phi_u\chi_R)^2}{M_*^3}, \tag{36}$$

with h the dimensionless coupling. This operator induces a sufficiently small Majorana neutrino mass,

$$m_\nu=\frac{hv_{wk}^2v_R^2}{(M_*R)^2M_*^3}\approx h\cdot 1\ \ \text{eV}, \tag{37}$$

where the right-hand side has been estimated using $v_R\approx 1/R\approx 1$ TeV and $M_*\approx 100$ TeV. A soft hierarchy in the couplings (say $h\sim 0.01$) should provide the right spectrum on neutrino masses.

(ii) The second class of operators connect ν to ν_s and have the form in lowest order $O_2\equiv\psi_1^T\tau_2\phi\chi_R\chi_R^T\tau_2C_5\psi'_2/M_*^{7/2}$. This operator after compactification has a magnitude

$$\simeq\frac{v_{wk}v_R^2}{M_*^2(M_*R)^{3/2}}\simeq 10\ \ \text{keV}.$$

(iii) The last class connects the left-handed neutrinos that transform under the $SU(2)_R$ group to themselves and have the form $O_3\equiv(\psi'_2\chi_R)^2/M_*^2$. They contribute to the ν_s-ν_s entry and have magnitude after compactification estimated to be $\simeq v_R^2/M_*^2R\simeq 1$–$10$ GeV. The full 6×6 $\nu-\nu_s$ mass matrix has a seesaw-like form and on diagonalization, leads to an effective mass for the light neutrino in the range of 0.1 eV or so.

One should note that, in the present model, there is no need to invoke extra bulk neutrinos living in a larger number of extra dimension [9] to get small neutrino masses. This is a major advantage of this model over the original models where a bulk neutrino was needed to get neutrino masses.

There should be, nevertheless, at least two more flat extra large dimensions to compensate for the gap between the fundamental and Planck scales. The size of such dimensions would be of order micrometers. Current bounds on M_* coming from bulk graviton effects [19] will apply. A single warped extra dimension [20], instead, could also provide a good explanation for the smallness of M_*. Of course, one could even bypass any large extra dimensions having a larger number of small $[r\sim(\mathrm{GeV})^{-1}]$ dimensions.

Sterile neutrino mass and cosmology

As noted in the previous subsection, there is a heavy mass ($\sim$1–10 GeV) sterile neutrino in this model due to the presence of the operator $\psi_2'^T C_5 \psi_2' \chi_R \chi_R$, that induces a Majorana neutrino mass,

$$m_{\nu_s}^2 = h_s \frac{v_R^2}{M_*^2 R}. \tag{38}$$

We do not enter into the detailed cosmological implication of such heavy sterile neutrinos except to note that they can annihilate via the exchange of Z' boson into lighter quarks and leptons. For a 1–10 GeV sterile neutrino annihilating via the exchange of 1 TeV Z', the typical temperature at which the ν_s goes out of equilibrium is around $T_* \sim M_{\nu_s}/15$. From this we estimate that the contribution of ν_s to the total energy density of the universe at the big bang nucleosynthesis (BBN) epoch, i.e., $T\simeq 1$ MeV is equivalent to about a tenth of a neutrino. Therefore this does not effect the usual helium synthesis scenarios of the standard big bang model.

VI. PHENOMENOLOGY

There is a series of dramatic implications on the phenomenology of the present model that makes it completely different from all previous left-right constructions.

There are no tree level contributions to muon decay from the new particles of the theory. This is due to the conservation of the KK number on the vertex, which forbids the mediation of the decay by any KK gauge bosons. That includes the W_R boson which itself is a KK mode (the lightest one in its tower) and all KK modes of W_L. Also, there are no new extra channels for the process since all lighter particles are only the ones in the standard model. Thus no good bounds on the masses of the new particles in the model can be obtained from here.

For similar reasons, W_R has no relevant contributions to neutrinoless double beta decay. At tree level, all external legs on the diagram are zero mode (SM) particles. This constrains the internal particles to be zero modes too. The process will still take place, but only as it is usually expected from the fact that neutrinos arise as Majorana particles in the model (see the previous section).

More generally, due to the conservation of the KK number, the excited modes can only contribute to SM processes through loops. Thus most of their contributions would be very suppressed, since all internal lines in the loop will get a heavy particle propagator on it.

Conservation of fifth momentum implies that the lightest mode with a nonzero KK number will be stable. This may, in principle, be either hadronic or leptonic depending on relative Yukawa couplings.

In four-dimensional versions of the model (see also Ref. [11]), the stringent bound on W_R mass usually comes from the contributions of W_R to the $K-\bar{K}$ mixing [12]. However, in the present case the situation changes considerably. The usual $W_L - W_R$ box graph that gives the largest contribution to this process in the standard left-right models with equal left and right quark mixings does not exist since the W_R coupling involves KK modes as can be seen from Eq. (30) and does not couple to the familiar charged current $\bar{u}_R \gamma^\mu d_R$ where u,d are both light quarks (or zero modes in this model). There would of course be a diagram similar to the standard model one, with W_R running on both internal boson lines instead of W_L. In this case the internal fermion lines will also be the KK modes of the fermions of the model. This will give a new nonzero contribution to the $K-\bar{K}$ mixing from right handed currents. Now, since all KK modes can run in the loop, their contribution should be summed up in the total amplitude. Notice that all internal lines on these last diagrams will correspond to KK exited modes of the same level. Thus they will come with a large suppression due to the heavy masses in the propagators. These contributions are suppressed compared to the left-handed ones by a factor of $(M_{W_L}/M_{W_R})^4$. The limits on the W_R mass from these considerations are therefore very weak.

In the four-dimensional left-right models, there are flavor changing neutral currents mediated by the second standard model Higgs doublet in the bidoublet. This in effect pushes the limit on the right-handed scale to 6–8 TeV [21]. In our model, however, since the second SM doublet has $(\pm)$ $Z_2 \times Z_2'$ symmetry, it couples only to KK modes to the SM fermions and therefore does not lead to such effects. Also constraints coming from processes such as $b\to s+\gamma$ [22] are also absent in this model.

Limits on R may come from collider physics from the production of the KK modes of the W_L boson [23] and are in the range of 400–800 GeV. The important point to note is that the W_R boson as well as all KK modes are produced in pairs both in e^+e^- as well as hadron colliders. The decay modes of the W_R are eN, $d\bar{U}$, etc. where N,U are the KK modes.

A limit on v_R comes from the $Z-Z'$ mixing. The analysis of this is as in the standard four-dimensional theories due to the universality of the mixing [11]. One gets $v_R \gtrsim 800$ GeV. This limit is likely to go up once LHC is turned on to the TeV range [24].

VII. COMMENTS AND OUTLOOK

A. Baryon nonconservation and six-dimensional extensions

In this model as in other five-dimensional brane-bulk models, one can write a baryon nonconserving operator $Q_1Q_1Q_1\psi_1/M_*^3$, which for low M_* leads to short lifetimes for the proton. There is also another baryon nonconserving operator of the form $Q_1Q_1Q_1\phi Q_2Q_2'Q_2'\chi_R\chi_R/M_*^{23/2}$. This leads to neutron–anti-neutron oscillation after compactification to four dimensions.

To avoid the problem of proton decay, one may proceed in one of the two following ways: (i) consider a fat brane with quarks located at a separate point from the leptons [25] or (ii) embed the model into a six-dimensional space time [26–28]. The second alternative is attractive from our point of view since it automatically brings in the right-handed neutrino into the picture and suggests a left-right symmetric model.

To see briefly the consequences of a six-dimensional embedding, note that the six-dimensional embedding comes at a minimal cost with the same fermion sector as in Eqs. (5) and (6). Our five-dimensional fermion representations can be straightforwardly written in terms of six-dimensional ones. Six-dimensional fermions are eight-component fields, but they may have a well-defined chirality through the operator $1\pm\Gamma^7$, with Γ^7 being the product of eight by eight Dirac matrices: $\Gamma^7=\Gamma^0\Gamma^1\cdots\Gamma^5$. After chiral projection, they reduce to four component spinors as required in the five-dimensional theory. The irreducible gauge and gravitational anomalies naturally cancel for the six-dimensional chirality assignments:

$$\mathcal{Q}_{1+},\ \ \mathcal{Q}_{1+}',\ \ \psi_{1+},\ \ \psi_{1+}',$$
$$\mathcal{Q}_{2-},\ \ \mathcal{Q}_{2-}',\ \ \psi_{2-},\ \ \psi_{2-}'. \tag{39}$$

One of the first implications of such an embedding is that all dangerous proton decay inducing operators are naturally forbidden [28]. This happens due to the extended Lorentz symmetry of the theory which is not broken in the Lagrangian, and which contains some discrete subgroups that only allow proton decay through operators of dimension 15 or higher. Notice that, up to a duplication of the spectrum, the particle content is the same considered in the analysis of Ref. [28]. Thus the details of the argument will follow the same lines.

Next, six-dimensional models have the potential of explaining the number of generations [26,27]. The argument is based on the fact that the cancellation of global $SU(2)$ anomalies imposes that [29]

$$N(2_+)-N(2_-)=0 \mod 6, \tag{40}$$

where $N(2_\pm)$ is the number of doublets with chirality $\pm$. This applies to both $SU(2)$ groups. The relation (40) is (nontrivially) satisfied only for at least three generations.

In the six-dimensional, version of our model, neutrino mass arises in an interesting manner. The simple neutrino mass operator involving the active neutrinos, i.e., O_1 alone discussed in Sec. V is now forbidden due to the $U(1)_{45}$ symmetry as is the operator O_2 involving only the ν_s. However the U_{45} symmetry allows the operator O_2 connecting ν and ν_s. Furthermore in six dimensions, O_2 has dimension eleven so that after compactification, it leads to the neutrino mass expression $m_\nu\sim v_{wk}v_R^2/M_*^2(M_*R)^3$. For $M_*\sim100$ TeV and $R^{-1}\sim1$ TeV, this gives an $m_\nu\sim1$ eV for a coupling strength multiplying these operators of order 0.1. Again, in this case, we do not need a bulk neutrino. The neutrino in this case is however a Dirac neutrino and will therefore not allow the neutrinoless double beta decay process.

We will pursue the details of the six-dimensional extension of the left-right model in a separate publication.

B. Supersymmetry

Another interesting alternative to extend the present model is to include supersymmetry at the 5D level. Notice that the above-described six-dimensional theory, however, cannot be trivially supersymmetrized, since the addition of new representations, as susy scalar and gauge partners, may destroy the properties that led to proton stability and the potential explanation to a number of generations.

The cost of introducing susy on our 5D model is minimal for the scalar content which now should be duplicated. The effective zero mode theory will be just the MSSM plus heavy sterile neutrinos. However, since one cannot write trilinear couplings in a $N=1$ 5D theory, supersymmetry has to be broken by the boundary conditions down to the effective $N=1$ 4D susy. This can be easily done at the fixed point $y=0$ without affecting the other properties of the model. As a consequence, all Yukawa couplings, Eq. (21), as well as the Higgs vevs, should now be localized on the border where susy is broken. The localization of these terms will change the profile of the wave function along the fifth dimension that will now try to match the presence of a pointlike source that comes as a localized mass term. This can also be understood in terms of the former Fourier expansion in Eq. (7) by noticing that a localized interaction term, as $\bar{Q}_1(x,y)\phi(x,y)Q_2(x,y)\delta(y)$, for instance, introduces a global mixing among all KK modes, and thus the basis has to be rotated to get the proper mass eigenstates. In such a case, most of the mentioned phenomenological properties associated to the universality of mixings and the conservation of the KK number fail, so their implications are likely to change.

C. Restoration of parity at high energies

In conventional left-right models, it is well known that as one goes to extreme high energies, i.e., when Q^2 in a process is much higher than the mass square of the W_R and Z' bosons, weak interaction processes involving left- and right-handed helicities become equal up to small corrections of order $m_{W_R}^2/Q^2$. Similarly, in the early universe when the temperature $T\gg m_{W_R}$, phase transition takes place leading to a symmetric phase of the theory and parity is fully restored. By the same token, as the Universe cools below the parity restoration scale (v_R scale), symmetry is broken and there are domains of even and odd parity separated by domain walls. The *a priori* possibility that such domain walls can cause extreme anisotropy in the universe always is a matter of concern for the cosmology of such models. A common way to deal with such issues is to "inflate" the domain walls away and keep the v_R scale above the reheating temperature so that the walls are not regenerated again [30].

For conceptual clarity let us explain how approximate parity restoration occurs in theories with orbifold compactification. For simplicity, consider the weak interaction contribution to the cross section for two scattering processes $\sigma_{e^-_L N}$ and $\sigma_{e^-_R N}$. For energies $E \leqslant R^{-1}$, $\sigma_{e^-_R N}=0$ whereas $\sigma_{e^-_L}$ is nonzero. As $E \geqslant R^{-1}$, $\sigma_{e^-_R N} \neq 0$ and also $\sigma_{e^-_N}$ receives an additional contribution both coming from the first KK excitation of fermions and gauge bosons. The additional contribution to $\sigma_{e^-_L N}$ and the value of $\sigma_{e^-_R N}$ are nearly equal apart from some propagator corrections. As $E \gg R^{-1}$, more and more states contribute to both processes and the zero mode contribution to $\sigma_{e^-_L N}$ which was nonzero in the beginning starts becoming a small part of the whole contribution and one has $\sigma_{e^-_N} \simeq \sigma_{e^-_R N}$. This is the sign of parity restoration in the class of models we are discussing. As $E \geqslant M_*$, then stringy contributions dominate and presumably both cross sections become equal obliterating the effect of the orbifold boundary conditions.

VIII. CONCLUSION

In this paper we have presented a five-dimensional left-right symmetric model where the gauge bosons as well as the fermions reside in the full five dimensions. The string scale in this model can be as low as 100 TeV and the radius of the fifth dimension of order of a $(\mathrm{TeV})^{-1}$. The gauge symmetry is partially broken by the orbifold boundary conditions. We show that one can consistently remove the right-handed neutrinos from the zero mode spectrum, which contains the standard model. The scale of parity breaking could therefore be under a TeV. The orbifold breaking enables this low scale to be compatible with all known low energy data. This model has a number of other features which are different from the standard four-dimensional left-right models. For instance, in our model, the W_R bosons are produced only in pairs. Therefore hadron colliders have no special advantage over the e^+e^- collider for testing this model. It predicts a stable lepton with mass R^{-1}. Furthermore, the structure of our model allows a new way to understand the small neutrino masses in low scale string models without introducing bulk neutrinos. Due to the completely new structure of this class of models compared to the standard four-dimensional left-right model, there will be many new phenomenological implications. We have briefly mentioned some of them. More details of these implications are under consideration.

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