Local chirality of low-lying Dirac eigenmodes and the instanton liquid model

I. Horváth,[1] S. J. Dong,[1] T. Draper,[1] N. Isgur,[3] F. X. Lee,[2,3] K. F. Liu,[1] J. McCune,[4] H. B. Thacker,[4] and J. B. Zhang[5]
[1]*University of Kentucky, Lexington, Kentucky 40506*
[2]*Center for Nuclear Studies and Department of Physics, George Washington University, Washington, D.C. 20052*
[3]*Jefferson Lab, 12000 Jefferson Avenue, Newport News, Virginia 23606*
[4]*Department of Physics, University of Virginia, Charlottesville, Virginia 22901*
[5]*CSSM and Department of Physics and Mathematical Physics, University of Adelaide, Adelaide SA 5005, Australia*
(Received 15 January 2002; published 14 August 2002)

The reasons for using low-lying Dirac eigenmodes to probe the local structure of topological charge fluctuations in QCD are discussed, and it is pointed out that the qualitative double-peaked behavior of the local chiral orientation probability distribution in these modes is necessary, but not sufficient, for dominance of instantonlike fluctuations. The results with the overlap Dirac operator in Wilson gauge backgrounds at lattice spacings ranging from $a \approx 0.04$ fm to $a \approx 0.12$ fm are reported, and it is found that the size and density of local structures responsible for double peaking of the distribution are in disagreement with the assumptions of the instanton liquid model. More generally, our results suggest that vacuum fluctuations of topological charge are not effectively dominated by locally quantized (integer-valued) lumps in QCD.

DOI: 10.1103/PhysRevD.66.034501 PACS number(s): 11.15.Ha

I. INTRODUCTION

It has been recently suggested [1] that questions about the dynamical nature of topological charge fluctuations in the QCD vacuum are worth reexamining. The notion that gauge field topology is relevant for understanding QCD started with the discovery of instantons [2], and their subsequent use as a basis for, among other things, the qualitative resolution of the $U_A(1)$ problem [3], and the discovery of the Θ dependence of QCD physics [4]. These successes were associated with the use of semiclassical methods and a concrete *local* picture of the vacuum, the instanton gas picture [5], characterized by formation of well-separated (anti-)self-dual lumps of quantized topological charge. However, it was soon realized that such a vacuum does not lead to confinement, thus putting the relevance of instantons immediately into question. Reflecting upon this situation, Witten has argued [6] that large quantum fluctuations associated with the confining vacuum naturally generate large fluctuations of topological charge, and lead to qualitative effects similar to those usually ascribed to instantons. In fact, he has conjectured that instantons are not important dynamically, arguing that the semiclassical picture is invalidated by large quantum corrections, and suggesting that topological charge fluctuates in a more or less continuous manner.

Nevertheless, QCD instantons remained rather popular as a frequently preferred (and analytically accessible) way of thinking about vacuum topology. The instanton solution was used as a basis for developing a rather successful semiclassically motivated phenomenology, the instanton liquid model (ILM) [7], where correlations among instantons were introduced to suppress the infrared divergences present in the instanton gas. Even though it is termed a ''liquid,'' the corresponding vacuum is quite dilute with (anti-)instantons of radius $\rho \approx \frac{1}{3}$ fm and density $n \approx 1\ \mathrm{fm}^{-4}$ occupying a small fraction of space-time volume and retaining their identity. This setup allows for an interesting mechanism of spontaneous chiral symmetry breaking [8], in which the 't Hooft ''would-be'' zero modes associated with individual (anti-)instantons mix, and supply the finite density of near-zero modes required by the Banks-Casher relation [9]. While the ILM is a phenomenological model, this elegant mixing picture invokes the impression that instantons play an important *microscopic* dynamical role in the QCD vacuum. A primary aim of our investigation is to examine whether the microscopic relevance of the ILM picture can be justified.

The support for the foundations of the ILM has frequently been drawn from lattice QCD simulations, using equilibrated gauge configurations as typical representatives in the path integral (see, e.g., Ref. [10]). Indeed, it is quite reasonable to expect that lattice QCD will eventually provide us with detailed answers about the nature of topological charge fluctuations. However, finding a clean and satisfactory way to infer this information from lattice QCD has proven to be a nontrivial issue due to the fact that lattice gauge fields are found to be very rough at the scale of the lattice spacing. The apparent necessity to eliminate the short-distance fluctuations in some way resulted in manipulating the gauge fields in various cooling or smoothing procedures. While useful for estimating the global topological charge, these techniques are significantly biased as to the local structure of the vacuum they reveal, and there is always an inherent subjective element present.

The idea of using fermions to study global topology is relatively old [11], but was not exploited very much due to both computational demands and problems with chiral symmetry. However, the suggestion to use low-lying fermionic modes to extract the information about *local* fluctuations of topological charge is recent [1] (see also Ref. [12]). In light of the above remarks, the preference for using fermions in this respect is quite reasonable, but there is also a good physical motivation for doing so. In particular, local fluctuations of topological charge are of interest because they are believed to cause light quarks to generate the η' mass [3,6] and the chiral condensate [8], for example. It is thus physically very natural to look for the imprint of these fluctuations

in the low-lying Dirac eigenmodes which dominate the propagation of light quarks.

The concrete proposal of Ref. [1] is to investigate the behavior of local chirality in the low-lying modes. This approach uses the fact that the instantonlike gauge fluctuations would leave a specific imprint in the individual modes, thus offering an opportunity to examine the consistency of the instanton picture. More precisely, it was argued that if there are extended regions of (anti-)self-duality in the gauge background, then the probability distribution of the chiral orientation parameter X (X distribution) over the regions of strong field should exhibit double-peaked behavior in the vicinity of extremal chiralities. The initial study of the X distribution in Ref. [1] indicated a flat behavior and thus a *qualitative* discrepancy with the instanton picture. However, later studies on finer lattices and/or with chirally symmetric fermionic actions [13,14] revealed a visible degree of double peaking. This situation precludes any definite conclusions on purely qualitative grounds since the double-peaked structure is *necessary but not sufficient* for instanton dominance. It is a main purpose of this work to offer a more *quantitative* point of view.

In the first part of the paper (Sec. II), we explain in detail why we choose to rely on the information encoded in low-lying Dirac modes, rather than using the conventional approaches for probing the gauge field topology on the lattice. We then concentrate on clarifying the possible implications of the X distribution and argue that a double-peaked structure is not uniquely associated with self-duality. For example, if the gauge fields form approximately quantized[1] isolated lumps of topological charge, then strong double peaking is expected as well, even when such lumps are not self-dual. Also, some chiral peaking in the low eigenmodes may occur even for nonquantized topological charge excitations through some as-yet-unknown mechanism. The observed behavior of the X distribution is thus not necessarily associated with an instanton-dominated vacuum. We suggest that the tentative conclusions of such nature be supplemented by quantitative comparison of QCD X distributions to those for the existing ILM ensembles. The material in Sec. II extends and complements the general discussion of Ref. [1].

In the second part of this paper (Sec. III), we present new results for X distributions from eigenmodes of the overlap operator in Wilson gauge backgrounds at lattice spacings ranging from $a \approx 0.04$ fm to $a \approx 0.12$ fm. In an attempt to verify whether the observed behavior can be ascribed to gauge structures with the parameters of the ILM, we determine the size and density of fermionic structures that contribute to the peaks of the X distribution. This is motivated by the fact that for instantonlike excitations, there is a direct correspondence between gauge and fermionic structure. Our consistency check leads to significant disagreement with the ILM. In fact, the lattice spacing dependence of the average radius (defined without reference to the 't Hooft zero-mode profile, which does not fit reasonably) indicates a finite continuum limit with value $R \approx 0.15$ fm, while the density of structures at $a \approx 0.04$ fm is $n \approx 50\ \mathrm{fm}^{-4}$, and we cannot exclude the possibility that it diverges in the continuum limit. We then point out that this not only disagrees with the ILM, but also suggests that the bulk of topological charge in QCD is not locally quantized in approximately integer units. We give several arguments supporting this interpretation. Needless to say, if confirmed, this would have profound implications for the possible microscopic explanation of spontaneous chiral symmetry breaking. Finally, we give several arguments that our data are not contaminated by lattice artifacts usually referred to as ''dislocations,'' and support this conclusion by presenting the data for the Iwasaki gauge action. A preliminary version of this work is presented in compact form in Ref. [15].

II. COOLING OR SMOOTHING VERSUS FERMIONS

The structure of equilibrium lattice Monte Carlo configurations provides a unique window for examining the nature of gauge fluctuations in the QCD vacuum. It also represents a distinctive way of thinking about how low-energy phenomena in QCD arise in terms of fundamental degrees of freedom. For example, as we will examine in some detail below, in the ILM mechanism of spontaneous chiral symmetry breaking, the condensate arises due to mixing of 't Hooft's ''would-be'' zero modes associated with (anti-)self-dual lumps, carrying approximately quantized topological charge. This line of reasoning inherently assumes that this happens at the ''configuration level'' with the vacuum dynamically generating the gauge potentials with these properties. If this is the case, then the picture has a fundamental microscopic meaning. Otherwise, it represents only a phenomenological modeling.

Unfortunately, examining lattice gauge fields directly leads to ambiguous results, mainly due to the fact that fields are typically rough even at the scale of a single lattice spacing. While the situation will improve with the approach to the continuum limit, this is probably not so when comparing the behavior at a fixed physical distance. It is thus perhaps inevitable that some sort of filtering procedure be used to interpret the local structure of lattice gauge fields. In this section, we examine various ways of approaching this issue.

Before proceeding to discuss this in more detail, it is useful to fix the language that will be used in what follows. In particular, we will use the term ''*isolated lump*'' or simply ''*lump*'' to denote a local structure in the gauge field that can be enclosed by a hypersurface on which the field is approximately pure gauge. Consequently, the lump contains approximately integer-valued topological charge. A tendency towards local ''lumpy structure'' is one of the inherent properties of the ILM picture which distinguishes it from the situation when the gauge field fluctuates very inhomogeneously, forming peaks of topological charge density, but without any tendency for such quantization.

A. Cooling

One popular way of addressing the problem of short-distance fluctuations is the cooling method [16]. We will be

[1]When we refer to local quantization of topological charge, we always imply quantization in units of ± 1.

rather generic when using this term, assuming just that it is a local minimization procedure for the gauge action with the initial state being the Monte Carlo generated QCD configuration. We will not be concerned with various implementations or variations on the original idea.

With the elementary step being local, it is naturally expected that if the original configuration can be assigned a global topology in some way, then it will be preserved to a large extent during such a procedure. One can thus turn the argument around and to *define the global topological charge* of the original configuration through field-theoretic definition, let us say, on the corresponding cooled counterpart. Such a definition may not be entirely satisfactory (the number of cooling sweeps is rather subjective and nonunique) but it can be made into a fairly well-motivated working scheme especially for the improved versions [17].

However, the situation is different if one is interested in the *local structure* of topological charge fluctuations. In the continuum, there are true local minima of the action for strictly self-dual or strictly anti-self-dual fields. The superpositions of not overwhelmingly overlapping instantons and anti-instantons, while not exact minima, correspond to plateaus (or shallow valleys) in the action profile. After a few cooling sweeps when the gauge field undergoes large changes, the local minimization can bring the configuration to the vicinity of such a plateau. Here the fields become naturally smooth, the field-theoretic definition of topological charge leads to values that cluster around integers, and the evolution in the configuration space slows down. Further cooling possibly leads the configuration into another plateau, and eventually into the global minimum with nonperturbative fields completely removed.[2] By the nature of the argument, in the vicinity of the plateau the configuration will necessarily resemble a (multi-)instanton–anti-instanton state regardless of the local properties of the original configuration. In other words, a possible observation of lumpy structure and local (anti-)self-duality in the cooled configuration is to be expected, and cannot be used as an independent, logically satisfactory input for making conclusions about local fluctuations of topological charge or about the dynamical importance of instantonlike gauge fluctuations in the QCD vacuum.

To illustrate this point, it is perhaps instructive to imagine that we are interested in studying the local structure of a sufficiently strongly coupled lattice QCD vacuum, where instantonlike fluctuations are not expected to play any role. Consequently, an unbiased approach should find no traces of them. However, cooling the corresponding equilibrium configurations will still lead to plateaus and locally lumpy, self-dual behavior. Can we use this as a basis to conclude that instantons play a significant role in the strongly coupled lattice QCD vacuum? Certainly not.

B. Smoothing

The inherent reason why cooling is a biased way of studying local properties of gauge fluctuations is that it uses the gauge action as a basis for the procedure. This is not easily curable by improvements. However, one can also attempt to eliminate the unphysical short-distance fluctuations by some sort of *smoothing* without reference to the action. Cooling itself is a smoothing procedure in the sense that it replaces the original configuration with a smoother one. For the discussion in this section, however, we will be more specific in that respect, and call smoothing any procedure that can be interpreted as an action-independent space-time averaging of the fields. The prototype for this would be, for example, an APE smearing [18] that was actually used in this context [19].

While smoothing is a less invasive approach than cooling, the subjective element in determining just how much smoothing is enough remains. This is troublesome because smoothing, by its nature, is also biased towards lumpy structure and can qualitatively change the local behavior. To see that, consider a very rough, inhomogeneous gauge field with various structures in it. Upon smoothing, the field varying over short distances will be quickly removed or reduced to a smooth four-dimensional ''bump'' if there is an underlying long-distance structure present. Also, the lower-dimensional structures, such as ridges, sheets, etc., will be eliminated soon by four-dimensional averaging. Thus, as the smoothing progresses, one naturally expects the stage when the configuration will be considerably lumpy with physical fields concentrated in the lumps. Further smoothing will cause the lumps to grow in size, to overlap, and eventually to reach a homogeneous situation with all physical fields removed. An unsettling question is where in this process one should stop and claim that the fields at the given stage represent the filtered local structure of the original configuration.

C. Fermions

Fortunately, there is a clean and meaningful strategy for approaching the above issues, namely to rely on the fermionic response to the corresponding gauge background. To explain that, it is easiest to first think in terms of a theory in the continuum where a typical configuration is also expected to have a lot of ultraviolet fluctuations. Using fermions is quite plausible since we know that, for sufficiently smooth gauge fields, fermions reflect global topology exactly [20] and for nondifferentiable gauge fields the index of the Dirac operator can actually serve as an extended definition of topological charge. It is thus natural to expect that the local structure can be inferred from fermionic response as well, e.g., by studying the divergence of the flavor-singlet axial-vector current.

The new idea which makes the fermionic approach attractive is to look for the imprints of topological charge fluctuations in the low-lying eigenmodes of the Dirac operator [1]. This is physically well-motivated, practical, and, at the same time, has the potential to naturally solve the problem of ultraviolet fluctuations without the subjective element involved. Indeed, the space-time structure of the low-lying modes is naturally smoother than that of the gauge fields themselves. This fact has two origins. First, the analogous Schrödinger-like eigenvalue problems typically yield smooth

[2]For improved cooling, it is in principle possible that the configuration remains indefinitely in the vicinity of the local self-dual minimum (with instanton–anti-instanton pairs annihilated).

stationary states even when the underlying potential has discontinuous jumps. Secondly, among the eigenstates, the infrared modes are expected to be least sensitive to the short-distance features on top of the long-distance structure in the potential.[3] At the same time, it is necessary to realize that the underlying physics we want to understand is the mechanism of how gauge fluctuations cause the light quarks to propagate in such a way as to form a quark condensate and to give the η' its mass. In other words, the goal is to understand the dynamics underlying the propagation of light quarks. This physics is encoded in the low-lying Dirac modes [1], and this is why we say that concentrating on these modes (''fermion filtering'') is physically well motivated.

One possible approach is to infer the information about topological charge fluctuations from the individual low-lying modes [1]. This method is indirect and is motivated by the instanton picture of the vacuum. In other words, the properties of instantonlike gauge fluctuations imply a specific qualitative behavior of the individual modes, thus giving an opportunity to check whether the observed structure is consistent with that scenario. The specific proposal of Ref. [1] is to study the probability distribution of the local chiral orientation parameter $X(n)$ defined by

$$\tan\left(\frac{\pi}{4}(1+X)\right)=\frac{|\psi_L|}{|\psi_R|}, \tag{1}$$

over the points n where the eigenmode $(\psi_n^+\psi_n)$ is large. Here $2\psi_L=(1-\gamma_5)\psi, 2\psi_R=(1+\gamma_5)\psi$ are the left and right components of the eigenmode ψ. $X(n)$ is a local angle in the $|\psi_L|$-$|\psi_R|$ plane rescaled so that $X(n)=-1$ for purely right-handed and $X(n)=+1$ for purely left-handed spinor ψ_n. For ψ_L, ψ_R generated in a random independent fashion, the local orientation parameter would be uniformly distributed between -1 and $+1$.

D. The implications of the *X* distribution

The usefulness of the X distribution was discussed in detail in Ref. [1], where it was argued that if the gauge background contains extended regions of strong (anti-)self-dual fields, then the X distribution should exhibit peaks near the extremal values. This is a consequence of the fact that in the eigenvalue problem for the Dirac operator, the (anti-)self-dual part of the gauge field enters as a potential term for the (right) left component of the eigenmode.

Here we would like to emphasize that this is not a unique way for the double-peaked behavior to arise. To illustrate that, consider subjecting the fermion to a background configuration that is lumpy in the sense that we have defined it here, i.e., consisting of relatively isolated lumps surrounded by approximately field-free regions. (Such configurations can be artificially prepared irrespective of whether or not they resemble typical equilibrium configurations of QCD.) The fields comprising the lumps have no definite duality properties, but yet they carry approximately quantized topological charge. A particular lump can thus be thought of as an individual object and, in the absence of all the other lumps, it would induce zero modes of the Dirac operator with chirality dictated by the index theorem. For simplicity, consider a situation with just two lumps with topological charges $Q=\pm 1$. The residual interaction will cause these ''would-be'' zero modes to mix and, to a first approximation, the true eigenmodes would be the two linear combinations of the strictly chiral ''would-be'' zero modes localized on the lumps. In the resulting *topological* near-zero modes, the X distribution will be strongly peaked at ± 1.

What we have described above is the same scenario that is a basis for the ILM mechanism of spontaneous chiral symmetry breaking [8]. The point is that self-duality is not essential for this argument since we did not need to invoke instantons at all. It is only the local quantization of topological charge that matters, and yet the peaked X distribution is expected, as well as spontaneous chiral symmetry breaking if such configurations turned out to be dynamically important.

We emphasize this point to show that the qualitative double-peaked behavior of the X distribution alone does not in itself provide a basis for the verification of the instanton picture. It is a necessary but certainly not a sufficient condition for such a conclusion. Apart from the above argument, there can possibly be other mechanisms for producing the peaked distribution (as well as spontaneous chiral symmetry breaking). In fact, we will argue later that there are reasons to believe that topological charge is actually not locally quantized in QCD. Nevertheless, the study of *quantitative* characteristics of the X distribution can still be very useful for distinguishing various scenarios as well as for other purposes [14].

E. Lattice fermions

When considering the lattice-regularized theory, we have the usual extra freedom in choosing the lattice action. According to the standard universality assumption, all local actions with appropriate symmetries and correct classical behavior should give consistent results sufficiently close to the continuum limit. However, comparing results from different discretizations at finite lattice spacing usually requires some care. When asking questions about the nature of fluctuations in the pure gauge vacuum, it is natural to fix a particular lattice gauge action and study the theory at different lattice spacings to be able to extrapolate to the continuum limit. Moreover, if we choose to examine the nature of these fluctuations using a lattice fermion, then it is essential that the fermionic action be kept fixed as well, even though it only plays the role of a probe.

To illustrate this point, consider some lattice Dirac operator $D(U)$ and the set of related operators $D_n(U)\equiv D(U_n)$, where U_n is the gauge configuration obtained from U by performing n smearing steps (e.g., APE steps) with other parameters of the smearing procedure fixed.[4] If $D(U)$ is an

[3]Low-lying modes remain sensitive to the *isolated* small structures, as they should.

[4]See Ref. [12] for example.

acceptable lattice Dirac operator, then $D_n(U)$ is acceptable as well, assuming that U_n exists. However, if we investigate the local properties in the equilibrium ensemble $\{U\}_{\rm eq}$ using D_n, then the answers we get will clearly depend on n. In particular, D_{1000} will probably indicate much larger structures in the gauge field than D_1. Nevertheless, one is allowed in principle to use D_{1000} as long as it is used consistently in *meaningful* extrapolations to the continuum limit, which might be difficult. However, one can hardly conclude anything meaningful about the behavior in the continuum limit by comparing the results from D_1 at one lattice spacing and from D_{1000} at a different lattice spacing.

Regarding the choice of the fermionic action, we would like to point out one particular aspect illustrated by comparing the two extremes represented by the Wilson-Dirac operator and the overlap operator based on it. The overlap operator has exact lattice chiral symmetry at finite lattice spacing, allowing for continuumlike theoretical analysis with respect to chiral symmetry and topology. On the other hand, it is nonultralocal and, due to extended gauge connections, effectively samples the gauge field potential over a nonzero physical distance.[5] This should not matter for the continuum limit, based on the expectation that the operator is local over equilibrium ensembles of Wilson pure gauge theory at sufficiently weak coupling. The Wilson-Dirac operator, on the other hand, has multilated chiral properties, and when using it in this context, one relies heavily on the assumption that this problem will go away in the continuum limit. At the same time, the Wilson-Dirac operator has a perfect resolution in the sense that there is no additional smoothness beyond the fact that we end up inspecting the infrared eigenmodes. It would be very interesting to know whether the two actions provide for consistent continuum extrapolations for quantities related to topology. In this work, we use an overlap operator which, even though expensive to implement, is very convenient for its theoretical advantages.

The above considerations also suggest that there may be a more general complementarity between the degree of chiral symmetry and the resolution for lattice fermionic actions. Indeed, if we start with a maximally ultralocal operator, we can try to improve its chiral properties by including couplings at larger distances and adding more complicated gauge connections, but by doing so we worsen the resolution. On the other hand, if one starts with the operator with exact lattice chiral symmetry, then this operator must be nonultralocal [22], and an attempt to improve its resolution by dropping couplings at large distances will result in deteriorated chiral properties.

III. LOW-LYING MODES OF THE OVERLAP OPERATOR

We have calculated and examined the low-lying modes of the overlap operator in Wilson gauge backgrounds over a wide range of lattice spacings. The massless overlap operator [23]

$$D=\rho\left[1+(D_W-\rho)\frac{1}{\sqrt{(D_W-\rho)+(D_W-\rho)}}\right] \qquad (2)$$

with D_W the Wilson-Dirac operator and $\rho=1.368$ ($\kappa=0.19$) is used throughout this paper. The rational approximation for the matrix sign function [24,25] was utilized to implement D and a small number of eigenmodes of $\gamma_5 D_W$ were projected out both to increase the accuracy of the approximation and to speed up the convergence.

We used the Ritz variational method [26] to obtain the low-lying eigenmodes of $H^2\equiv D^+D$. As a consequence of normality ($[D,D^+]=0$) and γ_5 Hermiticity ($D^+=\gamma_5 D\gamma_5$), the nonzero low-lying modes of H^2 are doubly degenerate and the eigenmodes of D can be constructed in the corresponding subspaces since $[H^2,D]=0$. Also, H^2 is proportional to the chirally nonsymmetric part of D, implying that $[\gamma_5,H^2]=0$. This allows for diagonalization in separate chiral sectors in different runs, thus speeding up the process and easing the memory requirements for large lattices. The typical accuracy of calculated eigenvalues (as measured by differences of complex-conjugate pairs) is about one part in 10^5 or better.

To avoid confusion, it is useful to fix the language that will be used when discussing the local properties of a given eigenmode ψ of D. We will refer to

$$d(n)\equiv\psi_n^+\psi_n, \quad c(n)\equiv\psi_n^+\gamma_5\psi_n \qquad (3)$$

as *density* and *chirality*, respectively, and to $X(n)$ of Eq. (1) as *chiral angle*. Since $d(n)$, $c(n)$, and $X(n)$ are identical for eigenmodes corresponding to complex conjugate nonzero eigenvalues ($\psi_{\lambda *}=\gamma_5\psi_\lambda$), we will treat $\psi_\lambda,\psi_{\lambda *}$ as a pair.

The parameters of the Wilson gauge ensembles used are listed in Table I. Configurations for the three finest lattices are separated by 20 000 sweeps. The quoted values of lattice spacings were obtained from the Sommer parameter using the interpolation formula given in Ref. [27]. Our finest lattice spacing is outside the interpolation range and we linearly extrapolate from $\beta=6.5$. Note that the physical volumes involved are such as to contain on average three to four (anti-)instantons if the ILM scenario is relevant, ensuring that the mixing of 't Hooft zero modes would take place. For all the configurations we have calculated the eigenfunctions for the zero modes and at least two pairs of near-zero modes.

TABLE I. Ensembles of Wilson gauge configurations.

β	a (fm)	V	No. configs.
5.85	0.123	$10^3\times 20$	12
6.00	0.093	14^4	12
6.20	0.068	20^4	8
6.55	0.042	32^4	5

[5]Because of this ''chiral smoothing,'' the features smaller than some physical threshold will not be resolved at finite lattice spacing. The results of Ref. [21] are a manifestation of this fact.

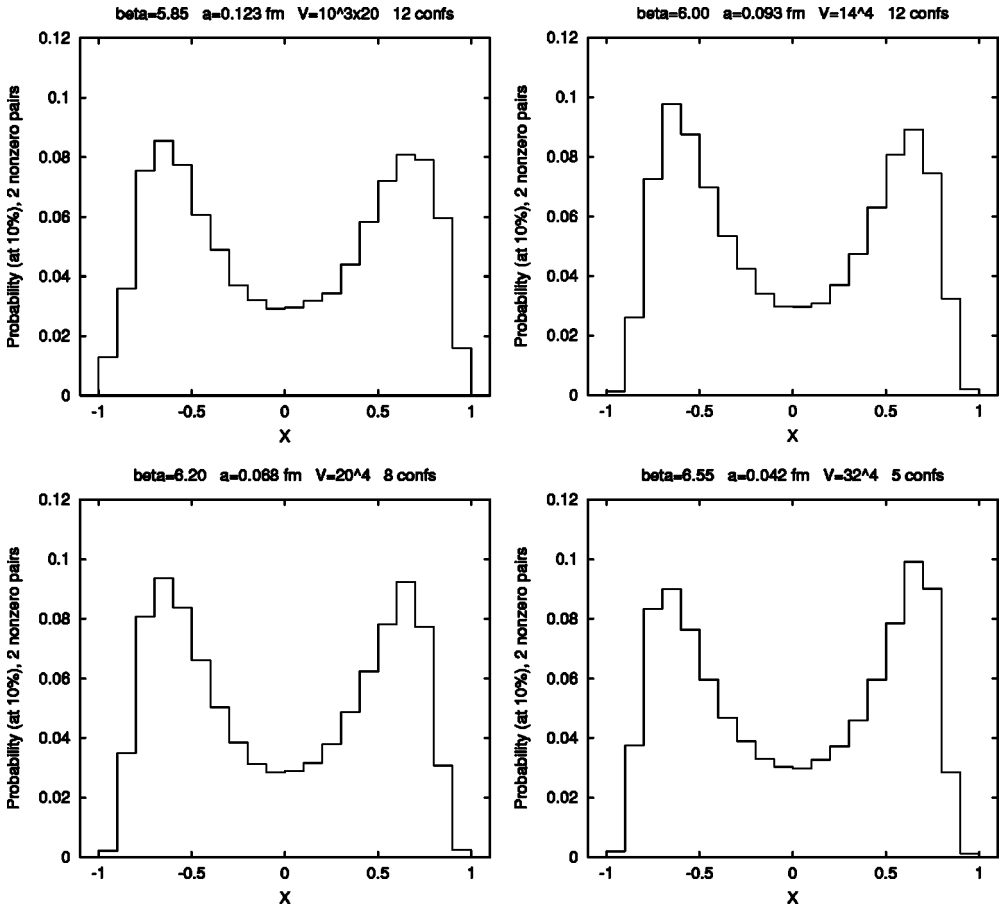

FIG. 1. X distribution for four Wilson gauge ensembles considered.

A. Results for the X distribution

To make meaningful comparisons for the behavior of the X distribution at different lattice spacings and different lattice sizes, it is necessary to fix the fraction f of the points that are examined on each lattice. In fact, it would be more appropriate to always speak of X_f distributions. For a given low-lying mode we order the lattice sites by the magnitude of density and consider the top fV points, where V is the lattice volume. It is assumed that the underlying gauge field is strongest in the regions so selected [1]. For most of the results discussed here, we fixed $f=0.1$. This is mostly motivated by the fact that we intend to relate our findings to the ILM. Diakonov and Petrov [7] use the packing fraction $\frac{1}{8}$ in their theoretical arguments. On the other hand, if one naively calculates the packing fraction as $f=nV_\rho$, where n is the density in fm^4 and V_ρ the volume of a four-dimensional sphere of radius ρ, then one obtains approximately $\frac{1}{20}$ if ILM values are used. We thus view $f=\frac{1}{10}$ as a reasonable compromise. The qualitative and even quantitative conclusions that we will make do not depend on the precise value from the range quoted above as long as it is fixed.

Our results for the X distribution from the lowest two nonzero pairs of modes included for each configuration are shown in Fig. 1. The histograms are normalized so that the sum of the values in all bins adds up to unity. There are visible peaked maxima at $X\approx\pm 0.65$, but there appears to be very little change across the wide range of lattice spacings studied. To quantify this observation, we have calculated the average value of $|X|$ with the histograms serving as a probability profile, i.e.,

$$\langle |X| \rangle \equiv \sum_i p_{X_i} |X_i|, \tag{4}$$

where p_{X_i} is the value at the bin with center X_i. For truly peaked distribution, this should reflect the approximate position of the peak. The results are shown in Fig. 2 indicating a very flat behavior as a function of the lattice spacing. The underlying dynamics thus does not appear to generate more chiral peaking closer to the continuum limit, nor do the positions of the peaks move appreciably closer to ± 1.

As we have emphasized in Sec. II, the qualitative observation that the X distribution exhibits peaked maxima at nonzero X is not sufficient for concluding that the instanton picture of topological charge fluctuations in the QCD vacuum is correct. However, the study of quantitative characteristics of the distribution might still be very illuminating. We illustrate this by comparing in Fig. 3 the X distributions for the near-zero modes and for the exact zero modes. The latter is the same distribution that would be expected in the limit of a very dilute instanton gas (or a dilute gas of arbitrary topologically nontrivial lumps). It is clear that, even at our smallest lattice spacing, the X distribution of the near-zero modes

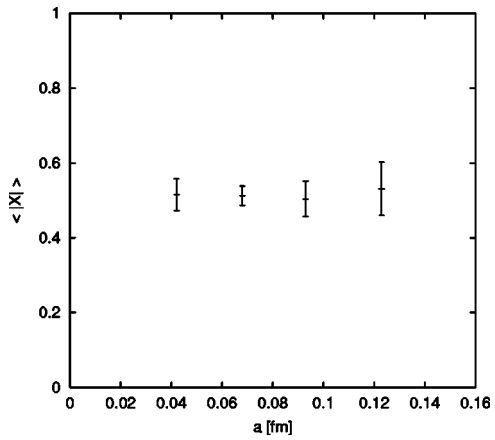

FIG. 2. Dependence of $\langle|X|\rangle$, Eq. (4), on the lattice spacing. The quoted uncertainty represents a rough estimate using the asymmetry $|p_{X_i}-p_{-X_i}|$ as a basis for the calculation.

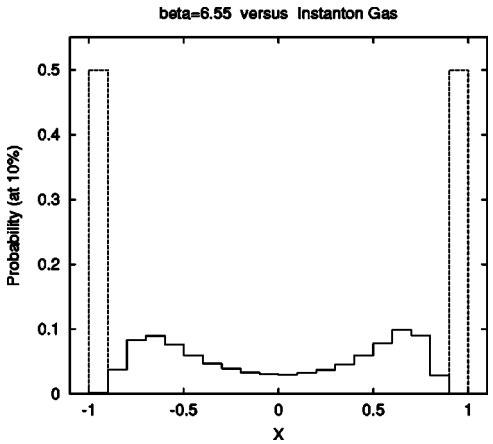

FIG. 3. Comparison of X distributions at $\beta=6.55$ for near-zero modes (solid line) and exact zero modes (dashed line) using the overlap Dirac operator.

is far less chirally peaked than that of the exact zero modes,[6] confirming the prevalent view that the dilute instanton gas picture is not realistic [7]. This also suggests that one reliable way to verify whether lattice QCD X distributions are consistent with the ILM prediction is a direct comparison to distributions from existing ILM ensembles.

B. Structures in the gauge field

Our next goal is to verify whether the observed X distribution can be attributed to the underlying local structures with properties similar to those assumed in the ILM. If the low-lying mode arises from mixing of 't Hooft "would-be" zero modes associated with ILM instantons, then this mode inherits the lumpy structure of the underlying gauge field, with lumps being of a prescribed size, shape, and abundance.

We identify such *possible* structures by finding the local maxima of density, $d(n)$, in the mode, and requiring that the profile of density around them resembles a four-dimensional peak at least in some average sense. To be more precise, we start by locating the maxima over the distance $\sqrt{3}$, i.e., finding the set

$$\mathcal{M}\equiv\{n:d(n)>d(m),|n-m|\leqslant\sqrt{3}\}. \tag{5}$$

For these candidates, specified by their position n, we then compute the functions

$$d_{n,\mu}(r)\equiv\langle d(m)\rangle_{\substack{|n-m|=r\\(n-m)\cdot\hat{\mu}>0}},$$

$$\mu=\pm1,\ldots,\pm4,\quad r>0 \tag{6}$$

representing the average of $d(m)$ over the spherical shell of radius r centered at n, restricted to the points for which $(n-m)$ has a component in the μ direction. The local maximum is retained only if $d_{n,\mu}(r)$ decays monotonically from origin over the distance $\sqrt{3}$ for all directions μ.

Note that the motivation for choosing $\sqrt{3}$ as a reference lattice distance in the above procedure is that it is a minimal distance for which the accidental occurrence of a structure can be statistically excluded for the largest lattice volumes we are working with. In other words, if we were to generate the density on sites of a 32^4 lattice using random numbers, no structures would typically be found using the above prescription. Conversely, if a structure is identified in a low-lying mode, we take it as a signal that a nontrivial fluctuation of the gauge field is causing its occurrence.

Finally, from the structures identified in this way, only those were selected that have a chance of contributing to the peaks of the X distribution. To make this final cut, we only retain structures for which the corresponding center n is among the 10% of points with highest density ($f=0.1$ was used to generate X distributions), and for which the chiral angle satisfies $|X(n)|\geqslant0.5$ as suggested by the position of the maxima in Fig. 1. The resulting density of structures as a function of lattice spacing is shown in Fig. 4, indicating a large disagreement with the ILM assumption at smaller lat-

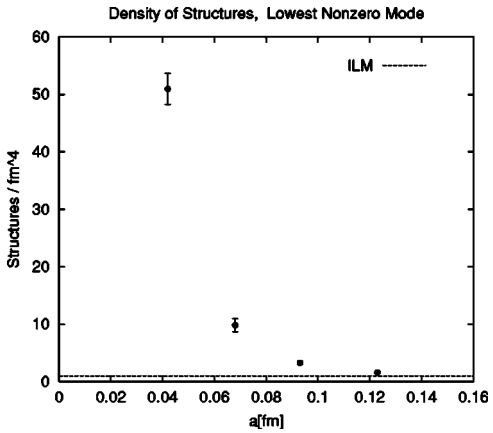

FIG. 4. Density of structures (in fm^{-4}) as a function of the lattice spacing.

[6]This is equally well reflected in the fact that we found typical values $\langle|X|\rangle\approx0.52$ (see Fig. 2), while at the same time $\langle|X|\rangle=1$ for distribution strictly peaked at $X=\pm1$.

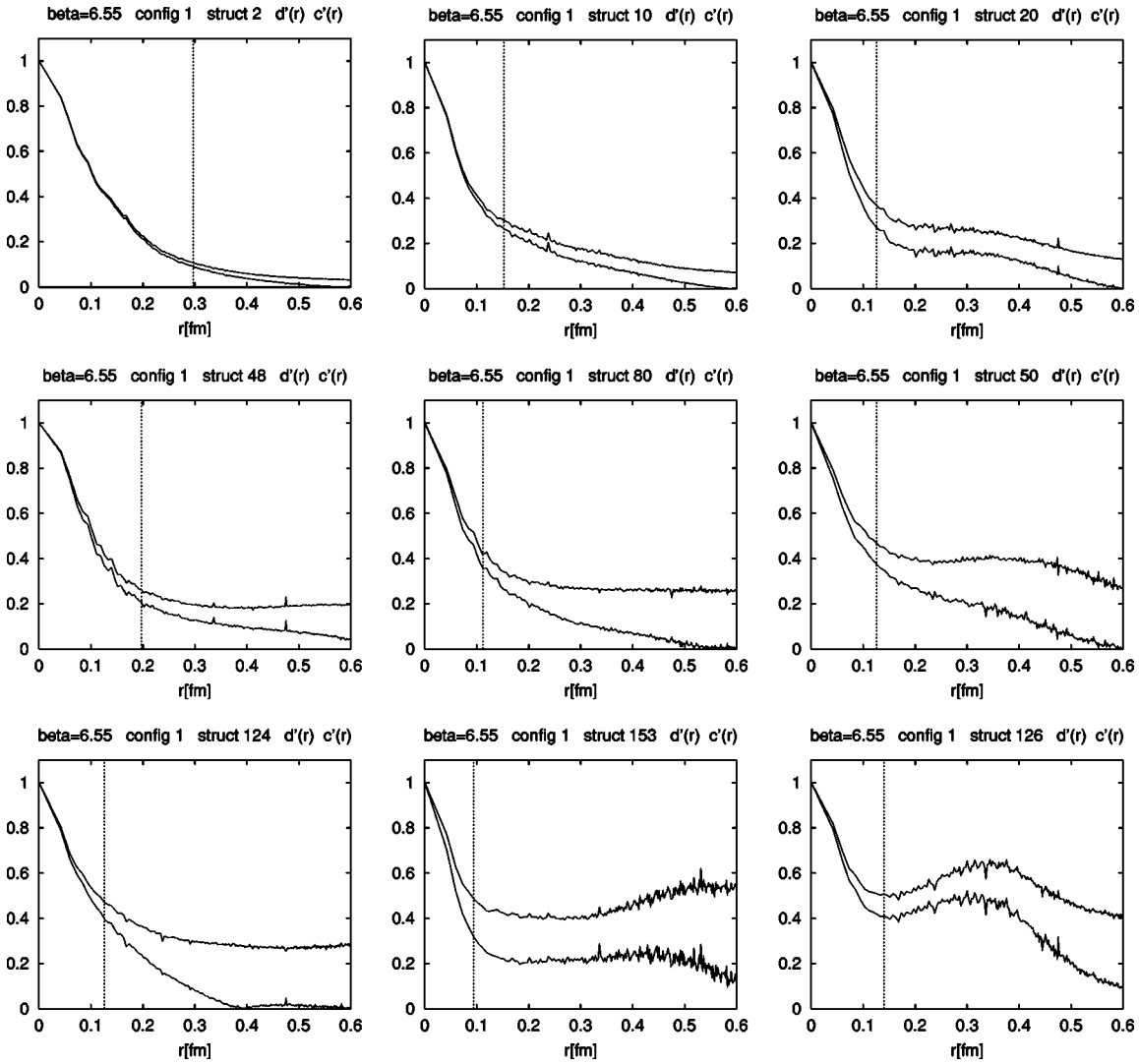

FIG. 5. The sample behavior of $d_n(r)/d_n(0)$ (upper curve) and $c_n(r)/c_n(0)$ (lower curve) for typical structures at $\beta=6.55$. The radius R_n of the corresponding region of coherent local chirality is shown as a vertical line. The figures are ordered so that the intensity of the peaks decreases vertically and the quality (degree of isolation) of peaks decreases horizontally.

tice spacings. We will return to the interpretation of this behavior later.

C. Sizes from coherent regions of local chirality

To study the typical sizes of the structures, we first use the definition that is motivated only by the assumed lumpy behavior of the gauge field rather than the specific profile of an instanton. The mixing of ''would-be'' zero modes in the lumpy background would produce regions of coherent local chirality in the near-zero modes, concentrated around the maximum of the lump. We thus define the radius of the structure at point n as the radius of the largest hypersphere centered at n, containing points with the same sign of local chirality: namely,

$$R_n \equiv \max\{r: c(n)c(m)>0, |n-m|\leqslant r\}. \tag{7}$$

To assess the feasibility of this definition, we compute functions

$$d_n(r) \equiv \langle d(m)\rangle_{|n-m|=r}, \quad c_n(r) \equiv \langle c(m)\rangle_{|n-m|=r} \tag{8}$$

representing the average density and chirality over the spherical shell of radius r centered at n. In Fig. 5, we show the sample behavior of functions $d_n(r)/d_n(0)$ and $c_n(r)/c_n(0)$ for typical structures, and how this relates to the radius assigned by our definition (vertical line in Fig. 5). The examples of structures with high, medium, and low intensity (density) are put in the top, middle, and bottom row, respectively. Note that the level of ''background'' increases for lower intensities. Also, the structures with cleaner peaked behavior (typically more isolated ones) are put on the left and quality is decreasing to the right. The examples on the right represent the ''worst'' cases we have identified. From these examples it can be seen that the peaks of density and chirality are well contained within the determined radius.

Using the definition (7), we have studied the average sizes of structures as a function of the lattice spacing with results shown in Fig. 6. The straight line is a fit using the three

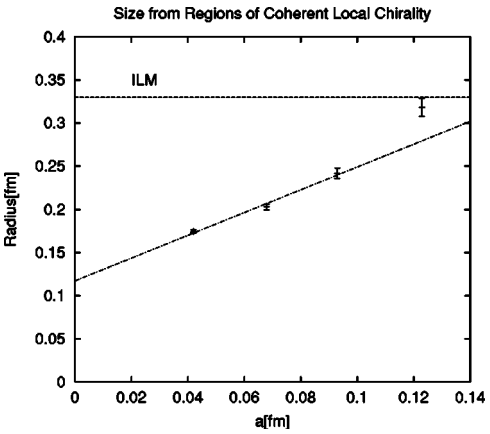

FIG. 6. Average radius $\langle R_n \rangle$ of structures from regions of coherent local chirality. The lowest nonzero mode was used for calculation. Data for the three smallest lattice spacings were used to obtain the fit. The horizontal line represents the radius of an ILM instanton.

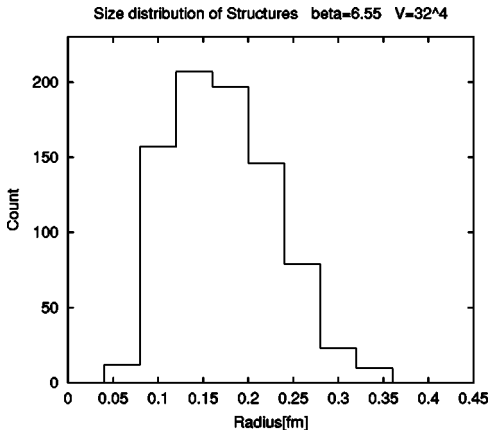

FIG. 7. The distribution of sizes R_n at $\beta=6.55$.

smallest lattice spacings considered. The average radius at $a \approx 0.12$ fm is close to the ILM value and consistent with a recent estimate at a similar lattice spacing [12]. However, it would be erroneous to conclude agreement with the ILM based on the data at single lattice spacing; the average value decreases significantly for finer lattices and the continuum-extrapolated estimate is in striking disagreement with the ILM. There appears to be a positive curvature in our data and a well-defined finite value (with a rough estimate of about 0.15 fm) in the continuum limit. The fact that our procedure leads to a finite size in physical units is significant because it characterizes the physical size of regions of coherent local chirality that are necessary to develop the large η' hairpin correlator required to solve the $U(1)$ problem [1]. We also emphasize that while our definition of size was motivated by assuming the lumpy structure of topological charge fluctuations, it makes very good sense even if the structures do not carry the quantized values of topological charge, and are not related to instantons. The distribution of sizes at $\beta=6.55$ is shown in Fig. 7.

D. Sizes from instanton-assumed profiles of density

If the near-zero mode of the Dirac operator is a mixture of ''would-be'' zero modes associated with instantons, then peaks of the wave function should resemble the profile of a 't Hooft zero mode. In particular, in the ideal case we should have [3]

$$\frac{d_n(r)}{d_n(0)} = \left(\frac{\rho_n^2}{\rho_n^2 + r^2} \right)^3 \equiv [y_n(r)]^3, \tag{9}$$

where ρ_n is the radius of the instanton located at n. Thus the function $r^2 y_n/(1-y_n)$ is a constant (ρ_n^2) independent of r for an instanton profile, and should be approximately constant for our structures if they represent the response to an instantonlike fluctuation. However, we find that this is not the case and the shapes of the peaks in our low eigenmodes do not resemble the instanton ansatz. To illustrate this, we display the situation for a typical structure in Fig. 8. Shown are also fits to the instanton profile over the distances 0.00–0.06 fm, 0.06–0.12 fm, and 0.12–0.18 fm. These fits are inconsistent with both the profile of the peak and with each

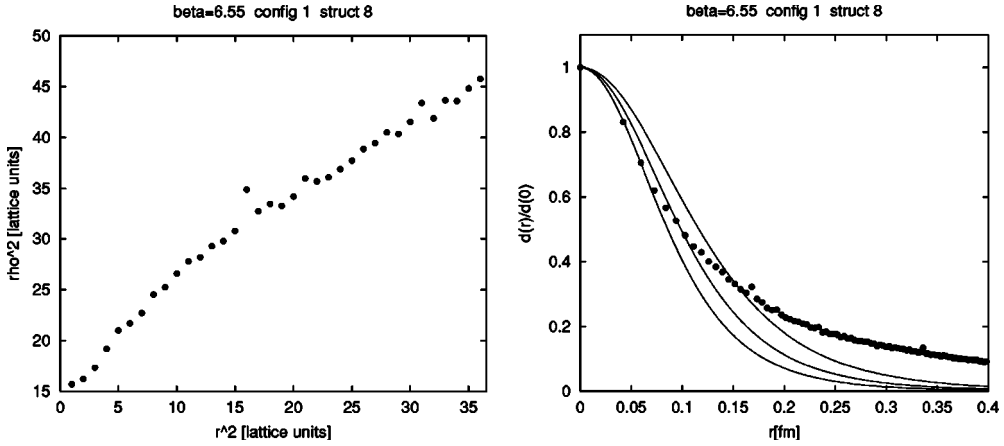

FIG. 8. Left: The function $r^2 y_n/(1-y_n)$ for a typical more intense structure. The data should be constant for an instanton profile. Right: The profile of the structure and attempted fits in the region 0.00–0.06 fm (leftmost curve), 0.06–0.12 fm (middle curve), and 0.12–0.18 fm (rightmost curve).

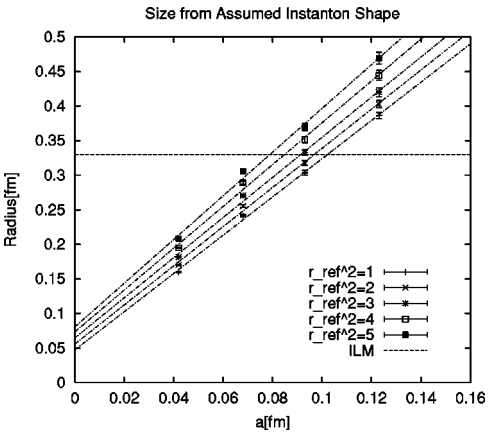

FIG. 9. Average radius $\langle \rho_n \rangle$ of structures determined from the assumed instanton shape of the peak at different reference distances. Lowest nonzero mode was used for calculation.

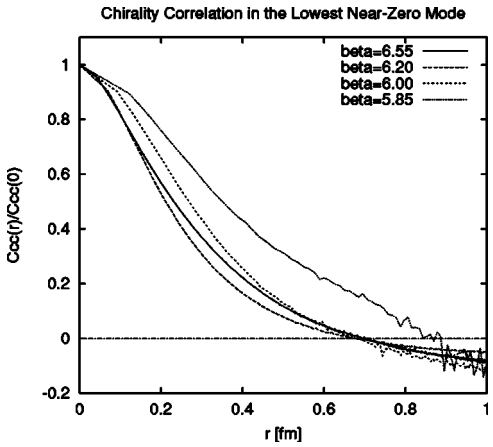

FIG. 10. Average $C_{cc}(r)/C_{cc}(0)$ from the lowest near-zero mode.

other. Note that the shapes of the peaks are already averaged over all the directions, and should exhibit a robust behavior.

To see the inconsistency with the instanton profile on average, we choose a reference point $r_{\rm ref}$ (in lattice units) and assign a radius to each structure through

$$\rho_n(r_{\rm ref}) \equiv r_{\rm ref} \left(\frac{y_n(r_{\rm ref})}{1-y_n(r_{\rm ref})} \right)^{1/2}. \tag{10}$$

All of our structures are guaranteed to have a peaked behavior over the distance $r_{\rm ref}^2=3$ in lattice units, but the vast majority of them decays over much larger lattice distances. To treat all the structures on the same footing, we compute average radii from the above prescription for $r_{\rm ref}^2 = 1,2,\ldots,5$ and plot the resulting dependence on lattice spacing in Fig. 9. Wide vertical bands at every lattice spacing reflect the fact that our structures cannot be fit reasonably by the 't Hooft profile. Nevertheless, the tendency towards sizes substantially smaller than the ILM value in the continuum limit is still obvious, thus showing that the overall conclusion from Fig. 6 remains valid even for this definition of the radius.

We emphasize that contrary to the results in Fig. 6, we do not assign much physical significance to the precise behavior of data in Fig. 9. The main purpose of showing these results is to reveal the marked inconsistencies with the instanton picture of the vacuum.

E. Correlation functions of local chirality

We have also computed the chirality-chirality correlation function in the lowest Dirac nonzero mode ψ, namely

$$C_{cc}(r) \equiv \langle c(n)c(m) \rangle_{|n-m|=r} V^2. \tag{11}$$

This correlator has been studied in Ref. [12]. While the average information encoded in the correlator may be useful, it will not provide us with a detailed view of the space-time distribution of $c(n)$. The point is that if this $C_{cc}(r)$ has characteristic size ρ, one still cannot infer whether the typical space-time distribution of $c(n)$ in a configuration predominantly appears localized on spherical structures of radius ρ, or if it comes in one of the infinitely many other forms leading to the same average correlation.

The average $C_{cc}(r)/C_{cc}(0)$ from the lowest nonzero modes for our four ensembles is shown in Fig. 10. While larger statistics would be desirable, we have estimated the "size" of the correlators by evaluating $\langle r \rangle$ and $\langle r^2 \rangle^{1/2}$ with $C_{cc}(r)$ used as a probability distribution. In every case we have cut off the integration at the distance where the correlator first turns negative. The resulting values are collected in Table II. Note that values for $\langle r^2 \rangle^{1/2}$ are systematically higher than sizes obtained from coherent local chirality (cf. Fig. 6), which is to be expected for an inhomogeneous space-time distribution of fluctuations with various shapes. Nevertheless, the sizes of correlators we have obtained are still systematically lower than the ILM value for an instanton radius.

F. Interpretation of the results

The data in Figs. 4 and 6 represent rather interesting new results and we would now like to elaborate on their interpretation. We have started from the assumption that the vacuum has local properties identifiable with the ILM scenario, i.e., that it typically fluctuates in such a way as to form (anti-)self-dual lumps of approximately quantized topological charge. However, the identification of such presumed lumps resulted in quantitative characteristics that are in marked disagreement with the ILM on fine lattices. We observe many more structures of much smaller physical size (although this size remains *finite* even in the continuum limit). Given that the parameters of the ILM are rather tightly

TABLE II. Estimates of the size of the correlator $C_{cc}(r)$.

a (fm)	0.123	0.093	0.068	0.042
$\langle r^2 \rangle^{1/2}$ (fm)	0.33	0.24	0.22	0.24
$\langle r \rangle$ (fm)	0.26	0.19	0.17	0.19

fixed [7], the true microscopic picture as seen by the lattice fermion appears to be very different from that envisioned by the ILM.

Particularly puzzling from the instanton perspective is a large number of peaks that we observe in the lowest near-zero modes, implying that a propagating light quark feels many more ''kicks'' from the regions of strong gauge field than one would expect based on the ILM. Indeed, assuming that the underlying gauge structures represent elementary instanton tunneling events between classical vacua leads to clear contradictions. For example, from our results at $\beta = 6.55$ one would estimate that the gluon condensate should be of the order $\langle 0|G^2|0\rangle \approx 32\pi^2 n \approx 32\pi^2 50\ \mathrm{fm}^{-4} \approx 25\ \mathrm{GeV}^4$, where $32\pi^2$ is a contribution of a single instanton.[7] This is to be compared with the accepted value $\langle 0|G^2|0\rangle \approx 0.5-1.0\ \mathrm{GeV}^4$. A similar inconsistency arises from considering topological susceptibility under even weaker assumptions: Assuming that gauge structures are general lumps of quantized and approximately uncorrelated topological charge leads to the estimate of topological susceptibility $\langle Q^2\rangle/V \approx n \approx 50\ \mathrm{fm}^{-4} \approx 50(200\ \mathrm{MeV})^4$, i.e., about 50 times the accepted value. At the same time, the topological charges of our lattices (determined as a byproduct of the overlap calculation) are in very good agreement with the expected value of topological susceptibility in the pure gauge vacuum.[8]

The above considerations strongly suggest that the topological charge contained within the structure is probably not quantized. One is led to the same conclusion from consideration of radii of regions of coherent local chirality. Our data indicate the value $\langle R_n\rangle \approx 0.15$ fm in the continuum limit. However, the 't Hooft instanton formula for the density of quantized charges valid at short distances, i.e., $n(\rho) \propto \rho^6$, is frequently invoked as a basis for concluding that the occurrence of instantons of this size should be essentially negligible (certainly relative to $\rho \approx 0.3$ fm instantons for which the formula is still expected to be valid) [28,7]. We interpret this marked mismatch as a manifestation of the fact that the underlying assumption for the 't Hooft formula, namely the local quantization of topological charge, is probably not valid. Indeed, the suppression of small instantons is a consequence of their quantized topological charge: it is difficult to squeeze an entire unit of topological charge inside a small radius. However, without quantization, there is no reason to expect a suppression of smaller-scale fluctuations, which would simply contain proportionally less topological charge.

Finally, we would like to return to Fig. 4 and discuss the rapidly increasing density of the structures with decreasing lattice spacing. While the density at $\beta = 6.55$ is very large, the fraction of sites contained in the structures is still relatively small, as shown by the packing fractions plotted in Fig. 11. The peaks thus retain their identity, as can be seen from Fig. 5. However, let us elaborate for the moment on the hypothetical possibility that the density increases indefinitely, because this might suggest something unphysical even if topological charge is not locally quantized, as we argued above. Indeed, how should one interpret a diverging density of structures in the continuum limit, especially if their average size (cf. Fig. 6) remains finite? The point is that contrary to quantized topological charges, where the finite topological susceptibility forces the density to assume a finite physical value in the continuum limit, the peaks of nonquantized topological charge might well lose their identity closer to the continuum limit and their density might indeed be unphysical. (However, the regions of coherent local chirality identified with the help of these peaks and their sizes are still physical.) The picture that we have in mind is that of relatively isolated mountain peaks belonging to a larger mountain range and coalescing as the continuum limit is approached. Our algorithm to measure the size of regions of coherent local chirality would then sample the size of the mountain range rather than individual peaks. A more dynamical analogy is to imagine disturbing a calm surface of water. The regions where the water is above the original level (positive topological charge density) have various shapes and volumes, but on average have some typical size. Yet, within such regions there could be many local maxima at all length scales. While the sizes of individual maxima do not define the scale and may not be physically meaningful, the average radius measured relative to these maxima probes the typical finite size of these coherent regions. It is easy to imagine that in the continuum, the fluctuations of topological charge resemble violently disturbed surface of water. In such a hypothetical scenario, the diverging density of structures would actually be very natural.

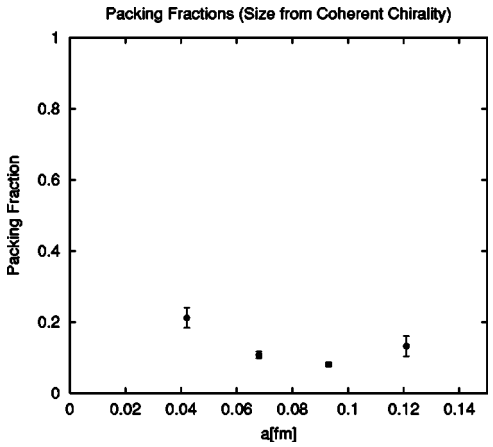

FIG. 11. Packing fractions. The sites contained in a given structure are bound by the largest hypersphere with the same sign of local chirality as in the center of the structure.

We emphasize that in this paper we do not attempt to put forward any specific low-energy scenario for the behavior of topological charge fluctuations in the continuum limit. The purpose of the discussion in the previous paragraph is to argue that as soon as one abandons the local quantization of topological charge, then the data in Fig. 4 do not represent anything unexpected.

[7]The estimates of this type are usually used to justify the ILM parameters [7].

[8]For example, from our small ensemble at $\beta = 6.55$ we estimate the topological susceptibility $\chi = 0.92 \pm 0.50\ \mathrm{fm}^{-4}$.

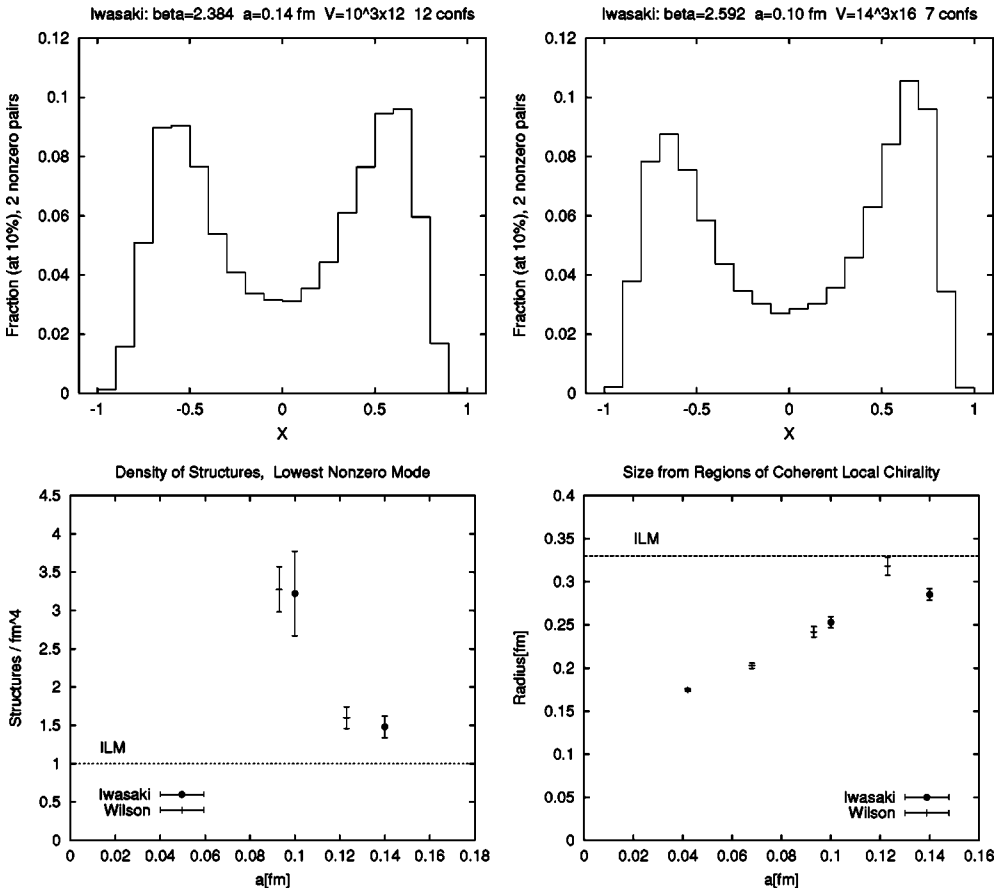

FIG. 12. Upper: X distributions for two Iwasaki gauge ensembles considered. Lower left: Density of structures for Iwasaki and Wilson actions. Lower right: Average size from coherent regions of local chirality for Iwasaki and Wilson actions.

G. The question of dislocations

In the lattice discussions of gauge field topology, one has to face the possible problem of ''dislocations.'' While this notion is frequently used as an unspecified synonym for ''lattice artifact,'' in the original discussion of Refs. [29,30] it has a rather well-defined meaning which we will adopt. In particular, for a given lattice gauge action and given topological charge operator, it is the local structure in the gauge field with action smaller than the continuum action of an instanton, and with unit topological charge.[9] In the loose sense, this is frequently pictured as a small instanton that is almost falling through the lattice and lives on one to two lattice spacings. Invoking entropy arguments, it was suggested in Ref. [30] that for the action–topological-charge combination where this happens, it is possible that dislocations dominate the topological-charge fluctuations, and can lead to unphysically large (possibly diverging) susceptibility. This raises the question of whether the large density of structures that we see is caused by unphysical dislocations.[10]

This question can be addressed at several levels. First of all, the arguments of Ref. [30] assume that the topological charge of the dislocation is concentrated on its core. Indeed, the idea that dislocations are important relies rather heavily on the picture of locally quantized topological charge. If the topological charge does not come in unit lumps, as we suggest here, then the issue simply does not arise.

Secondly, it is important to realize that dislocation is a concept assigned to the pair gauge-action–topological-charge-operator. While we work with the Wilson gauge action which is supposedly susceptible to the possibility of dislocations, we use chiral fermions to measure topological charge. It has frequently been noted (see, e.g., Ref. [7]) that fermions should not be sensitive to dislocations. This is indeed very plausible. As we have argued in Sec. II C, one naturally expects that infrared fermionic modes will be quite smooth and will not inherit the singular behavior of the underlying gauge fields.

Finally, there is no hint of behavior symptomatic of dislocations in our data. For example, even the smallest of our structures are extended objects in physical units and span at least four lattice spacings. Moreover, the estimates of topological susceptibility from topological charge measured by the overlap Dirac operator are in very good agreement with the accepted value of about 1 fm^{-4} (see footnote 8). If the underlying structures were dislocations (and hence carried topological charge close to unity), then this value should be

[9]We thank Tamás Kovács for clarifying this.

[10]We thank Thomas Schäfer for pointing this out.

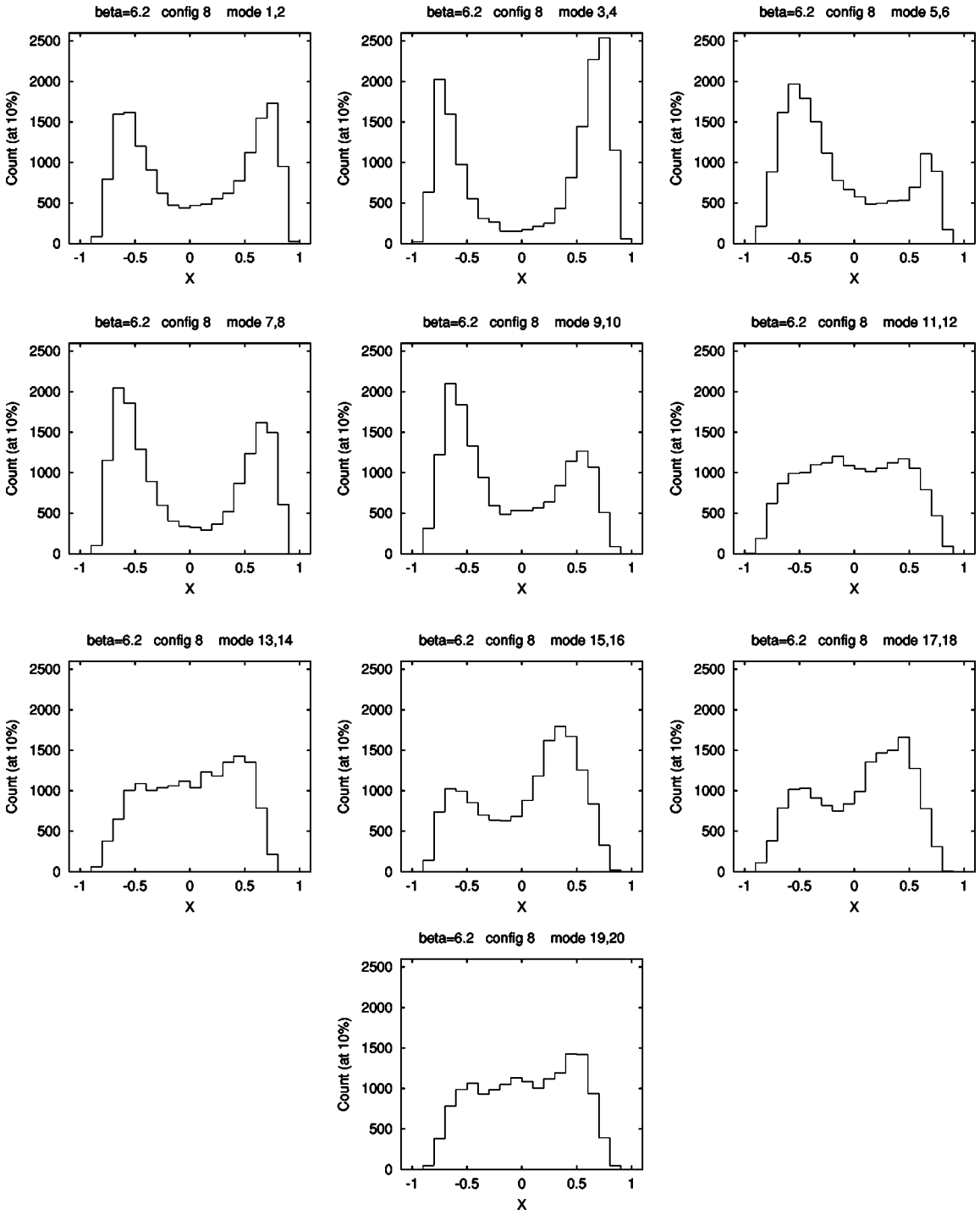

FIG. 13. The X distributions for the first 20 near-zero modes of configuration 8 at $\beta=6.2$.

substantially larger. This is one of the arguments that actually suggests that topological charge is not locally quantized, as we stressed in Sec. III F.

Nevertheless, to eliminate any concern that dislocations may be a problem, we have performed a comparative study with a renormalization-group improved gauge action. In this case, the possibility of dislocation dominance should be substantially reduced. In particular, we have employed the Iwasaki action [31], and studied overlap eigenmodes at two different lattice spacings. In the standard normalization for Iwasaki action (see Refs. [31,32]), we considered a $10^3\times 12$ lattice at $\beta=2.384$ and a $14^3\times 16$ lattice at $\beta=2.592$. This corresponds to lattice spacings $a=0.14$ and 0.10 fm, respectively, as determined from string tension by using the data of Ref. [32] and the two-loop beta function fit.

The summary of our results is shown in Fig. 12. The structure of the resulting X distributions is very similar to the structure for Wilson gauge ensembles, as can be seen by

comparing to Fig. 1. We thus applied identical procedures (with identical cuts) for finding the structures and determining their size as we did for Wilson gauge ensembles. Although the behavior of density for Wilson gauge action (see Fig. 4) might have suggested the possibility of dislocations, this is not the case because a similar pattern appears to be followed in the case of Iwasaki action as well. Similarly, the sizes of structures are systematically lower than the ILM values. It would obviously be interesting to investigate a larger window of lattice spacings with Iwasaki action, to be able to study the issues of continuum extrapolation. However, the purpose of this comparative study was to demonstrate that our conclusions are not sensitive to the choice of gauge action and that there is no problem of dislocations. Our results clearly indicate that this is indeed the case.

IV. CONCLUSIONS

Uncovering the local structure of topological charge fluctuations promises to have profound implications for our understanding of low-energy QCD. This expectation is born out of the fact that possible microscopic explanations for important phenomena such as spontaneous chiral symmetry breaking, the resolution of the $U_A(1)$ problem, and the Θ dependence of QCD are based on the picture of the vacuum wherein self-dual lumps of locally quantized topological charge (instantons) play a major role. It is consequently quite essential to determine whether such a picture (e.g., ILM) is indeed fundamental, or if there is in fact a different microscopic mechanism driving these important effects.

Expanding upon the ideas of Ref. [1], we have explained here in detail why studying low-lying Dirac eigenmodes provides a natural and reliable approach for exploring the local nature of topological charge fluctuations (at least as implied by the ILM). The study of the X distribution in low-lying modes has been designed as a tool for probing the local vacuum structure indirectly. While the available results [13,14] on the qualitative behavior of the X distribution could in principle be viewed as confirming the consistency of the instanton picture, we have pointed out here that the association with instantons is not unique. In other words, the double-peaked qualitative structure of X distribution is a necessary condition for instanton dominance, but not sufficient.

As a step towards resolving these issues, we have computed and analyzed low-lying modes of the overlap Dirac operator in Wilson gauge backgrounds over a wide range of lattice spacings ($a \approx 0.04-0.12$ fm). The double-peaked structure of the X distribution has been observed with maxima at $X \approx \pm 0.65$ (using a fraction $f=0.1$ of lattice points). The position of maxima shows very little change with lattice spacing without a visible tendency to move toward the extremal values of $X=\pm 1$, even though an exactly chiral lattice fermion is used. Our main initial aim in this study was to verify whether the characteristics of local structures contributing to the peaks of the X distribution are consistent with quantitative characteristics of the gauge features in the backgrounds proposed in the ILM. In other words, we attempted to verify whether low-lying modes can (at least approximately) be viewed as mixtures of 't Hooft "would-be" zero modes generated by gauge structures resembling approximately $\rho=\frac{1}{3}$ fm instantons of density $n=1$ fm^{-4}. To the contrary, we have found local characteristics indicating that the true nature of topological charge fluctuations is very different from that envisioned in the ILM.

We emphasize in this context that the underlying issue here is to determine whether one should view the ILM picture as a fundamental one, or as an effective description with phenomenological meaning only. For example, claims that the bulk of spontaneous chiral symmetry breaking is due to mixing of "would-be" zero modes associated with instantons have only well-defined microscopic meaning if the corresponding low-lying modes have the local structure predicted by this microscopic mechanism. To verify this proposition, we have no other choice but to examine these modes. This appears to be a well-defined problem with a well-defined answer. Our results indicate that the ILM scenario does not provide for accurate microscopic description of these modes and thus remains at the phenomenological level. It is this important distinction that we wish to stress in this paper.

While the detailed arguments are summarized in Sec. III F, our main conclusion can be verified in an independent manner. To make this point qualitatively, we have calculated 20 near-zero modes for configuration 8 from our $\beta=6.2$ ensemble.[11] This configuration has $Q=0$, and the physical volume is such that according to the ILM it should contain about three or four instantons and anti-instantons. Hence, the subspace spanned by 't Hooft would-be zero modes should have a dimension of that order. Consequently, it is the ILM prediction that chirally peaked X distributions should be observed for three or four near-zero modes (and possibly a few more) but the rest of the modes should resemble approximate free-field behavior with local chirality peaked around the origin. In Fig. 13, we plot the X distribution at $f=0.1$ for all 10 pairs of near-zero modes computed (the histogram is the same for both modes in a pair). Inspecting these results reveals that there are at least 14 modes with significant double-peaked structure and none of them is peaked at the origin. As a matter of fact, at $f=0.02$, *all* 20 calculated modes exhibit the double-peaked behavior and this most likely persists even for higher modes. While this fact can hardly be explained by the ILM, it is not very surprising in view of the results presented here. On this particular configuration, we have identified 32 local structures.[12]

Even though we have concentrated on the ILM in this paper, we believe that our results suggest a more general conclusion. In particular, as we have argued in Sec. III F, it is very difficult (if not impossible) to reconcile the topological susceptibility of the pure gauge vacuum and the assumption

[11] Needless to say, this was rather demanding on computer resources.

[12] Note that this observation also makes very unlikely the possibility that one could interpret several structures as forming a severely deformed quantized lump of topological charge which one could somehow still associate with the ILM instanton. In this case, the subspace of would-be zero modes should still be of size 3 or 4.

that local structures in fermionic near-zero modes are caused by underlying gauge excitations with locally quantized topological charge. This leads us to believe that the bulk of topological charge in the QCD vacuum is not locally quantized in integer units as suggested some time ago by Witten [6]. We will address this issue in detail in a forthcoming publication [33].

ACKNOWLEDGMENTS

This work has been supported in part by U.S. Department of Energy under Grants No. DE-FG05-84ER40154, No. DE-FG02-95ER40907, No. DE-FG02-97ER41027, and No. DE-AC05-84ER40150. We have benefited from discussions with Philippe de Forcrand and Tamás Kovács.

[1] I. Horváth, N. Isgur, J. McCune, and H. B. Thacker, Phys. Rev. D **65**, 014502 (2002).
[2] A. M. Polyakov, Phys. Lett. **59B**, 82 (1975); Nucl. Phys. **B121**, 429 (1977); A. A. Belavin, A. M. Polyakov, A. Schwartz, and Y. Tyupkin, Phys. Lett. **59B**, 85 (1975).
[3] G. 't Hooft, Phys. Rev. Lett. **37**, 8 (1976); Phys. Rev. D **14**, 3432 (1976).
[4] C. Callan, R. Dashen, and D. J. Gross, Phys. Lett. **63B**, 334 (1976); R. Jackiw and C. Rebbi, Phys. Rev. Lett. **37**, 172 (1976).
[5] C. G. Callan, R. Dashen, and D. J. Gross, Phys. Rev. D **17**, 2717 (1978).
[6] E. Witten, Nucl. Phys. **B149**, 285 (1979); **B256**, 269 (1979).
[7] E. V. Shuryak, Nucl. Phys. **B198**, 83 (1982); D. I. Diakonov and V. Y. Petrov, *ibid.* **B245**, 259 (1984); T. Schäfer and E. Shuryak, Rev. Mod. Phys. **70**, 323 (1998).
[8] D. I. Diakonov and V. Y. Petrov, Nucl. Phys. **B272**, 457 (1986).
[9] T. Banks and A. Casher, Nucl. Phys. **B169**, 125 (1980).
[10] M. I. Polikarpov and A. I. Veselov, Nucl. Phys. **B297**, 34 (1988); M. C. Chu, J. M. Grandy, S. Huang, and J. Negele, Phys. Rev. D **49**, 6039 (1994); C. Michael and P. S. Spencer, *ibid.* **52**, 4691 (1995).
[11] J. Smit and J. Vink, Nucl. Phys. **B284**, 234 (1987); **B298**, 557 (1988); M. Laursen, J. Smit, and J. Vink, *ibid.* **B343**, 522 (1990).
[12] T. DeGrand and A. Hasenfratz, Phys. Rev. D **64**, 034512 (2001).
[13] T. DeGrand and A. Hasenfratz, Phys. Rev. D **65**, 014503 (2002); I. Hip *et al.*, *ibid.* **65**, 014506 (2002); R. Edwards and H. Heller, *ibid.* **65**, 014505 (2002); T. Blum *et al.*, *ibid.* **65**, 014504 (2002).
[14] C. Gattringer *et al.*, Nucl. Phys. **B618**, 205 (2001); **B617**, 101 (2001).
[15] S. J. Dong *et al.*, Nucl. Phys. B (Proc. Suppl.) **106**, 563 (2001).
[16] B. Berg, Phys. Lett. **104B**, 475 (1981); J. Hoek, *ibid.* **166B**, 199 (1986); J. Hoek, M. Teper, and J. Waterhouse, Nucl. Phys. **B288**, 589 (1987).
[17] P. de Forcrand, M. G. Pérez, and I.-O. Stamatescu, Nucl. Phys. B (Proc. Suppl.) **47**, 777 (1996); M. G. Pérez, O. Philipsen, and I.-O. Stamatescu, Nucl. Phys. **B551**, 293 (1999).
[18] M. Falcioni, M. L. Paciello, G. Parisi, and B. Taglienti, Nucl. Phys. **B251**, 624 (1985); M. Albanese *et al.*, Phys. Lett. B **192**, 163 (1987).
[19] T. DeGrand, A. Hasenfratz, and T. G. Kovács, Nucl. Phys. **B520**, 301 (1998); A. Hasenfratz and C. Nieter, Phys. Lett. B **439**, 366 (1998).
[20] M. Atiyah and I. M. Singer, Ann. Math. **93**, 139 (1971).
[21] C. Gattringer *et al.*, Phys. Lett. B **522**, 194 (2001).
[22] I. Horváth, Phys. Rev. Lett. **81**, 4063 (1998); Phys. Rev. D **60**, 034510 (1999); I. Horváth, C. T. Balwe, and R. Mendris, Nucl. Phys. **B599**, 283 (2001).
[23] H. Neuberger, Phys. Lett. B **417**, 141 (1998); **427**, 353 (1998).
[24] R. G. Edwards, U. M. Heller, and R. Narayanan, Nucl. Phys. **B540**, 457 (1999).
[25] S. J. Dong, F. X. Lee, K. F. Liu, and J. B. Zhang, Phys. Rev. Lett. **85**, 5051 (2000).
[26] B. Bunk, K. Jansen, B. Jegerlehner, M. Lüscher, H. Simma, and R. Sommer, Nucl. Phys. B (Proc. Suppl.) **42**, 49 (1995); T. Kalkreuter and H. Simma, Comput. Phys. Commun. **93**, 33 (1996).
[27] M. Guagnelli, R. Sommer, and H. Wittig, Nucl. Phys. **B535**, 389 (1998).
[28] M. Teper, Nucl. Phys. B (Proc. Suppl.) **83-84**, 146 (2000).
[29] M. Lüscher, Nucl. Phys. **B200**, 61 (1982).
[30] D. Pugh and M. Teper, Phys. Lett. B **224**, 159 (1989).
[31] Y. Iwasaki, Nucl. Phys. **B258**, 141 (1985).
[32] M. Okamoto *et al.*, Phys. Rev. D **60**, 094510 (1999).
[33] I. Horváth *et al.*, hep-lat/0203027.