Mass spectra of doubly heavy $\Omega_{QQ'}$ baryons

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We evaluate the masses of baryons composed of two heavy quarks and a strange quark, taking into account spin-dependent splittings in the framework of the potential model with the K^{2}O potential [V. V. Kiselev, A. E. Kovalsky, and A. I. Onishchenko, Phys. Rev. D **64**, 054009 (2001)] motivated by QCD with a three-loop β function for the effective charge consistent with both the perturbative limit at short distances and a linear confinement term at long distances between the quarks. The factorization of dynamics is assumed and explored in the nonrelativistic Schrödinger equation for motion in a system of two heavy quarks constituting the doubly heavy diquark and the strange quark interaction with the diquark. The limits of the approach, its justification, and uncertainties are discussed. Excited quasistable states are classified by the quantum numbers of a heavy diquark composed of heavy quarks of the same flavor.

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I. INTRODUCTION

The long-lived doubly heavy baryons open a new battlefield in the study of heavy quark dynamics in various aspects such as the interplay of strong and weak interactions in decays as well as confinement (see a recent review on this subject in [2]). Indeed, there are some features of these baryons in contrast to the mixed flavor meson B_c composed of two heavy quarks, for which one can explore the technique of nonrelativistic QCD (NRQCD) [3] in the form of potential NRQCD (PNRQCD) [4] or velocity-counting NRQCD (VNRQCD) [5–7], while the description of doubly heavy baryons involves interaction with light or strange quarks constituting the baryon together with the doubly heavy diquark. Therefore, one should combine the above NRQCD approaches with the heavy quark effective theory (HQET) [8] constructed in order to take into account the interactions of a local heavy spinor field with the soft light quarks and gluons. Thus, doubly heavy baryons give the possibility to test these methods in different conditions specific for the baryons QQq or QQs.

An experimental opportunity for such studies could be quite real at Run II of Tevatron. Indeed, the B_c meson was observed by the CDF Collaboration at FNAL [9] under a lower statistics of Run I. Its spectroscopic characteristics and decays were theoretically described and predicted in the framework of QCD sum rules [10], potential models [11,12], and operator product expansion [13]. Those predictions served to isolate some reasonable constraints preferable for the detection of a B_c signal such as the intervals of mass, production rate, lifetime, and branching ratios of various decay modes with quite a high efficiency. The strategy in the study of doubly heavy baryons is the same as that for B_c [14]. First, one estimates the spectroscopic characteristics in the framework of potential models [15,16] and sum rules of QCD or NRQCD [17] that resulted in definite predictions of mass spectra and wave functions, which determine the normalization of cross sections in terms of a soft binding factor [18]. Second, corresponding yields in hadronic collisions and e^+e^- annihilation can be obtained in perturbative theory (fourth order in α_s for the production of two pairs of heavy quarks in QCD), so that the rates are large enough to expect a positive result in the experimental search for doubly heavy baryons at Run II [18]. Third, the potential models and operator product expansion were used to calculate both exclusive and inclusive branching ratios [19,20], respectively, as well as the lifetimes [20,21]. A new field of interest for the investigations is radiative, electromagnetic, or hadronic transitions between the quasistable states in the families of baryons with two heavy quarks. A first step in the study of this problem was recently done in [22], wherein some preliminary results were obtained on the electromagnetic transitions between the levels of Ξ_{bc}.

In the present paper, we analyze the basic spectroscopic characteristics for the families of doubly heavy strange baryons $\Omega_{QQ'}=(QQ's)$ in the framework of the potential model and compare the results with the calculations in NRQCD sum rules and lattice simulations for ground states.

A general approach of potential models to calculate the masses of baryons containing two heavy quarks was discussed in detail in [15,16]. There are two clear physical arguments in this problem, which differ in the kinds of interquark forces in the baryon. The first motivation in the form of description is pair interactions, while the second is the stringlike presentation based on the Wilson loop for three static sources, where one source interacts with the other two sources through a string connected to the diquark. For doubly heavy baryons, these two approaches result in quite certain predictions, which can be clearly distinguished in the splitting between the excitations as described by Gershtein *et al.* in [16]. The peculiarity of a doubly heavy diquark is its small size, which can be used in order to construct an effective approximation in the form of combined NRQCD and

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HQET by implementing the hierarchy of the following scales: the size of the QQ'-diquark antitriplet color subsystem, $r_{QQ'}$, which is about the inverse of both the characteristic momentum transfer between the heavy quarks $|\boldsymbol{k}|$ and the relative momentum of heavy quark motion $|\boldsymbol{p}|\sim|\boldsymbol{k}|\sim m_{Q,Q'}\cdot v$ with a small nonrelativistic velocity $v\ll 1$, the confinement scale $\Lambda_{\rm QCD}$ for the dynamics of light or strange quarks, and the heavy quark masses $m_{Q,Q'}$, so that

$$r_{QQ'}\cdot\Lambda_{\rm QCD}\ll 1,\quad \Lambda_{\rm QCD}\ll m_Q.$$

Under such conditions, we can factorize the doubly heavy diquark as a local source of colored field in the interactions with the light quark, while the dynamics in the diquark can be reliably described in terms of nonrelativistic spinors, i.e., NRQCD, PNRQCD, or VNRQCD. In the present paper, we explore the quark-diquark factorization of baryon wave functions in the framework of the nonrelativistic potential model and use the QCD-motivated potential combining the known perturbative calculations for the static potential at short distances [23,24] and the linear confining term at large distances in the way similar to the Buchmüler-Tye method [25] extended to the three-loop β function for the effective charge in the static potential [1]. Such calculations differ from the approaches of PNRQCD and VNRQCD, since the PNRQCD deals with the static potential of heavy sources fixed at a distance r not including the static energy of ultrasoft fields such as the contributions from the quark-gluon string or sea, while the VNRQCD describes the nonrelativistic Coulomb system in the limit of negligible contributions by ultrasoft fields valid at $m_{Q,Q'}\cdot v^2\gg\Lambda_{\rm QCD}$, which is broken for the heavy-heavy systems composed of the bottom and charmed quarks under study. The static potential $\mathrm{K^2O}$ [1] is consistent with the high-virtuality normalization of the coupling constant in QCD as well as with the slope of Regge trajectories, which define two scale parameters of the model. The heavy quarkonia spectra [1] and leptonic constants [26] are described with quite good accuracy in this approach, which results in the fixed values of heavy quark masses implemented in the potential model with the nonrelativistic quarks. The connection of these quantities with the pole and current masses of heavy quarks was discussed in [1].

We adjust the nonrelativistic Schrödinger equation for the description of light and strange quark dynamics. So, we determine the constituent mass of light and strange quarks with minimal binding energy of quarks in the static field of the colored source in the framework of a presentation with the constituent mass formed as a piece of ultrasoft energy in the linear confining term of the potential or the string (see Sec. II B). Then we test the estimated mass values of light and strange quarks in the potential model by calculating the masses of bound states with a single heavy quark, i.e., the charmed and beauty mesons. We discuss uncertainties and apply the procedure for the doubly heavy baryons.

If the heavy quarks composing the antitriplet-color diquark have the same flavor, $Q=Q'$, we have to take into account the Pauli principle for the identical fermions. Then we find that for the symmetric, spatial parity P-even wave functions of the diquark, $\Psi_d(\mathbf{r})$ (the orbital momentum equals $L_d=2n$, where $n=0,1,2,\dots$), the spin wave function of the diquark should be symmetric too, and the summed spin of quarks equals $S=1$, while for the antisymmetric, odd functions, $\Psi_d(\mathbf{r})$ (i.e., $L_d=2n+1$), we have $S=0$.

Under factorization of the diquark and strange quark dynamics, we accept the following notations for the classification of levels in the system of QQs baryons: the summed spin of two heavy quarks, their orbital momentum, and principal quantum number are denoted by the letters S, L, and n_d, respectively, so that the ground state of the diquark, for instance, is marked as $n_d^{2S+1}L=1^3S$, while the motion of the strange quark is labeled by the letters $n_s\,l$. Such notations are based on the presentation that these quantum numbers are approximately conserved if we neglect the multipole interactions in QCD [27] with the emission of soft gluons, which can be absorbed by the associated strange quark in the baryon or involved in the scattering with the emission of a kaon, for example $g+s\to q+K$ with $q=u,d$. In this expansion, there are the following suppression factors mentioned above: the diquark size with respect to the scale of confinement, and the relative momentum of heavy quarks with respect to the heavy quark mass, which leads to suppression of both the chromoelectric and chromomagnetic dipole transitions, respectively. Moreover, in the doubly heavy baryons with identical flavors of two heavy quarks, some transitions between the levels or mixing operators have a double suppression, since for the corresponding operators one could need the properties providing the change of a summed heavy quark spin $\Delta S=1$ together with the change of their orbital momentum $\Delta L=2m+1$, $m\in\mathbb{Z}$. Suppose we expect that the excited $2\,^1P$ level of diquarks bb and cc would be quasistable under the transition to the ground level $1\,^3S$. This picture cannot be valid for the bc diquark, since both values of the summed heavy quark spin are admissible, and the spin changing operators are not removed and can result in a significant mixing of diquark states labeled by the summed spin. So, we restrict our consideration of the Ω_{bc} mass spectrum by the spin-dependent splitting of the ground state, while for the doubly heavy baryons Ω_{cc} and Ω_{bb} we present complete spectra of families.

The next point is the following evident condition for the applicability of diquark dynamics factorization. We calculate the size of the diquark subsystem in order to demonstrate the reliability of the calculations. We will see that the low-lying levels have quite small sizes, especially in the case of the bb diquark, while the higher excitations are large with respect to the distance to the strange quark, so that the approximation of the local colored source for such diquark levels results in less reliable estimates of baryon masses.

The paper is organized as follows. In Sec. II, we describe the static potential explored in the paper and calculate the relevant constituent masses of light and strange quarks. We test the accepted approximations in estimates of masses for mesons with a single heavy quark. Then we introduce the spin-dependent forces. In Sec. III, we present numerical results. Our conclusions are given in Sec. IV.

II. NONRELATIVISTIC POTENTIAL MODEL

Under the factorization of dynamics inside the doubly heavy diquark and in the system composed of the strange quark and diquark, we use the nonrelativistic Schrödinger equation in order to solve the corresponding two-body problems, which yields spin-independent wave functions and levels. We use the three-loop improved static potential in the Schrödinger equations with the heavy quark masses adjusted to the observed data on the bottomonium and charmonium. The procedure for the determination of constituent masses of light and strange quarks with the given potential is described and tested with the data on the heavy-light mesons. Then we introduce the spin-dependent forces treated as perturbations.

A. The static potential

In QCD, the static potential is defined in a manifestly gauge invariant way by means of the vacuum expectation value of a Wilson loop [28],

$$V(r) = -\lim_{T\to\infty}\frac{1}{iT}\ln\langle\mathcal{W}_\Gamma\rangle,$$

$$\mathcal{W}_\Gamma = \widetilde{\mathrm{tr}}\,\mathcal{P}\exp\left(ig\oint_\Gamma dx_\mu A^\mu\right). \tag{1}$$

Here, Γ is taken as a rectangular loop with time extension T and spatial extension r. The gauge fields A_μ are path-ordered along the loop, while the color trace is normalized according to $\widetilde{\mathrm{tr}}(\cdots) = \mathrm{tr}(\cdots)/\mathrm{tr}\,\mathbb{1}$. This definition corresponds to the calculation of the effective action for the case of two external sources fixed at a distance r during an infinitely long time period T, so that the time ordering coincides with the path ordering. Moreover, the contribution into the effective action by the path parts, where the charges have been separated to the finite distance during a finite time, can be neglected in comparison with the infinitely growing term of $V(r)\cdot T$.

Generally, in the momentum space one rewrites the above definition of the QCD potential of static quarks in terms of the relevant quantity α_V representing a so-called V scheme of the QCD coupling constant as follows:

$$V(\boldsymbol{q}^2) = -C_F\frac{4\pi\alpha_V(\boldsymbol{q}^2)}{\boldsymbol{q}^2}. \tag{2}$$

After the introduction of $\mathfrak{a} = \alpha/4\pi$, the β function is actually defined by

$$\frac{d\mathfrak{a}(\mu^2)}{d\ln\mu^2} = \beta(\mathfrak{a}) = -\sum_{n=0}^{\infty}\beta_n\cdot\mathfrak{a}^{n+2}(\mu^2), \tag{3}$$

so that in the perturbative limit at high $\boldsymbol{q}^2$ the coefficients were calculated up to three-loop order in the V scheme by two-loop matching of the static potential with $\alpha_s^{\overline{\mathrm{MS}}}$ [23,24]. Then $\beta_{0,1}^V = \beta_{0,1}^{\overline{\mathrm{MS}}}$ and $\beta_2^V = \beta_2^{\overline{\mathrm{MS}}} - a_1\beta_1^{\overline{\mathrm{MS}}} + (a_2 - a_1^2)\beta_0^{\overline{\mathrm{MS}}}$, where the coefficient a_i corresponds to the short-distance expansion

$$\alpha_V(\boldsymbol{q}^2) = \alpha_{\overline{\mathrm{MS}}}(\mu^2)\sum_{n=0}^{2}\tilde{a}_n(\mu^2/\boldsymbol{q}^2)\left(\frac{\alpha_{\overline{\mathrm{MS}}}(\mu^2)}{4\pi}\right)^n$$
$$= \alpha_{\overline{\mathrm{MS}}}(\boldsymbol{q}^2)\sum_{n=0}^{2}a_n\left(\frac{\alpha_{\overline{\mathrm{MS}}}(\boldsymbol{q}^2)}{4\pi}\right)^n. \tag{4}$$

Note that expansion (4) cannot be straightforwardly extended to higher orders of perturbative QCD because of infrared problems that result in nonanalytic terms in the three-loop perturbative potential, as was first discussed by Appelquist, Dine, and Muzinich [28].

Buchmüller and Tye proposed a procedure for the reconstruction of the β function in the whole region of charge variation by the known limits of asymptotic freedom to a given order in α_s and the confinement regime. Generalizing their method, the β_{PT} function found in the framework of asymptotic perturbative theory (PT) to three loops is transformed to the β function of effective charge as follows:

$$\frac{1}{\beta_{\mathrm{PT}}(\mathfrak{a})} = -\frac{1}{\beta_0\mathfrak{a}^2} + \frac{\beta_1 + \left(\beta_2^{\mathrm{V}} - \frac{\beta_1^2}{\beta_0}\right)\mathfrak{a}}{\beta_0^2\mathfrak{a}} \Rightarrow$$

$$\frac{1}{\beta(\mathfrak{a})} = -\frac{1}{\beta_0\mathfrak{a}^2\left(1 - \exp\left[-\frac{1}{\beta_0\mathfrak{a}}\right]\right)} + \frac{\beta_1 + \left(\beta_2^V - \frac{\beta_1^2}{\beta_0}\right)\mathfrak{a}}{\beta_0^2\mathfrak{a}}\exp\left[-\frac{l^2\mathfrak{a}^2}{2}\right], \tag{5}$$

where the exponential factor in the second term contributes to the next-to-next-to-leading order at $\mathfrak{a}\to 0$. This function has the essential peculiarity at $\mathfrak{a}\to 0$, so that the expansion is the asymptotic series in $\mathfrak{a}$. At $\mathfrak{a}\to\infty$, the β function tends to the confinement limit

$$\frac{d\alpha_V(\boldsymbol{q}^2)}{d\ln\boldsymbol{q}^2} \to -\alpha_V(\boldsymbol{q}^2), \tag{6}$$

which gives the confinement asymptotics for the static potential at long distances r as usually represented by the linear potential (see discussion in Ref. [29])

$$V^{\mathrm{conf}}(r) = k\cdot r. \tag{7}$$

The construction of Eq. (5) is based on the idea of removing the pole from the coupling constant at a finite energy, but in contrast to the ''analytic'' approach developed in [30] and modified in [31], the ''smoothing'' of peculiarity occurs in the logarithmic derivative of charge related with the β function, but in the expression for the charge itself.[1] Indeed, in the one-loop perturbation theory we have

[1]In [31], the analytic approach is extended to the β function, too.

$$\frac{d\ln \mathfrak{a}}{d\ln \mu^2} = -\beta_0 \mathfrak{a} = -\frac{1}{\ln\frac{\mu^2}{\Lambda^2}},$$

because of

$$\mathfrak{a} = \frac{1}{\beta_0 \ln\frac{\mu^2}{\Lambda^2}},$$

and the pole can be canceled in the logarithmic derivative itself, so that

$$\frac{d\ln \mathfrak{a}}{d\ln \mu^2} \Rightarrow -\beta_0 \mathfrak{a}\left(1-\frac{\Lambda^2}{\mu^2}\right) \approx -\beta_0 \mathfrak{a}\left(1-\exp\left[-\frac{1}{\beta_0 \mathfrak{a}}\right]\right).$$

As we see in the perturbative limit $\mathfrak{a}\to 0$, the deviation in the β function is exponentially small, and the usual solution for the running coupling constant is valid.

Equation (5) can be integrated out, so that implicit representation of effective charge can be inverted by the iteration procedure, so that well-approximated solution has the form

$$\mathfrak{a}(\mu^2) = \frac{1}{\beta_0 \ln\left(1+\eta(\mu^2)\frac{\mu^2}{\Lambda^2}\right)}, \tag{8}$$

where $\eta(\mu^2)$ is expressed through the coefficients of the perturbative β function and parameter l in Eq. (5), which is related to the slope of Regge trajectories α'_P and the integration constant, the scale Λ [1]. Thus, the dimensionless parameter l determines the theoretically unknown ratio of the perturbative scale in the QCD coupling constant Λ to the confinement scale involved by the Regge slope.

The slope of Regge trajectories, determining the linear part of the potential, is assumed equal to $\alpha'_P = 1.04\ \mathrm{GeV}^{-2}$, so that in Eq. (7) we put the parameter $k = 1/2\pi\alpha'_P$. We also use the measured value of the QCD coupling constant [32] and pose

$$\alpha_s^{\overline{\mathrm{MS}}}(m_Z^2) = 0.123$$

as the basic input of the potential. The transformation into the configuration space was done numerically in [1], so that the potential is presented in the form of a file in the notebook format of the MATHEMATICA system.

The analysis of quark masses and mass spectra of heavy quarkonia results in the following values ascribed to the potential approach [1]:

$$m_c^V = 1.468\ \ \mathrm{GeV}, \quad m_b^V = 4.873\ \ \mathrm{GeV}. \tag{9}$$

Thus, the spectroscopic characteristics of systems composed of nonrelativistic heavy quarks are determined in the approach with the static potential described above.

B. Constituent masses of light and strange quarks

A fast moving light or strange quark interacting with a static source of a gluon field nonperturbatively emits virtual quarks and gluons forming a quark-gluon sea or a string. Such a cloud of confined virtual fields with the valence quark is not a local object, which of course has some internal excitations. However, these excitations correspond to unstable hybrid states related with additional $q\bar{q}$ pair or glueball degrees of freedom in the hadron. We do not consider such exotics in the present paper. Then, the fast-moving light quark surrounded by the virtual soft and ultrasoft fields with no internal excitations can be described as a whole object with an effective mass, which we call a constituent mass μ_q. Since we postulate the nonrelativistic Schrödinger equation, we fix the dispersion law of the constituent quark: $E_q = \boldsymbol{p}^2/2\mu_q$. Further, we present a consistent determination of μ_q, when the static potential entering the Schrödinger equation is fixed.

The constructive procedure is the following. Since the constituent mass is a part of the energy in the string confining the quarks, and this confining energy is represented in the static potential by the term linearly growing with the quark separation, we argue that this part of the energy should be subtracted from the potential. So, we will solve the Schrödinger equation for the light quark with the constituent mass μ_q and a static source with the potential $V(r)-\mu_0$, where μ_0 is the energy subtraction. We expect that the parameters μ_q and μ_0 are very close to each other, of course. A difference between them should be suppressed, since it is determined by such systematic reasons as the nonlocality of the constituent quark as well as its dispersion relation. At fixed μ_0 we can investigate the dependence of the binding energy of the constituent quark on the parameter μ_q. So, we numerically solve the Schrödinger equation

$$\left[\frac{\boldsymbol{p}^2}{2\mu_q} + V(r)\right]\Psi(r) = [\bar{\Lambda}(\mu_q) + \mu_0 - \mu_q]\Psi(r), \tag{10}$$

for the ground state. Thus, the binding energy is given by $\bar{\Lambda}(\mu_q)$, and it is shown in Fig. 1.

We see that the binding energy $\bar{\Lambda}$ has an optimal value corresponding to its minimum. We ascribe *the position of minimum* $\mu_q = \mu_q^\star = 0.37$ GeV as *the valid constituent mass* of the light quark attributed to the Schrödinger equation with the given static potential. The difference between the normalization point μ_0 and the constituent mass $\mu_q^\star$ can be extracted from the experimental data on the spin-averaged ground-state masses of heavy-light mesons by comparison with the theoretical expectation

$$M_Q(1S) = m_Q + \bar{\Lambda}(\mu_q^\star)|_{\mu_0=\mu_q^\star} + \frac{\langle \boldsymbol{p}^2\rangle}{2\mu_Q} + \delta\mu, \tag{11}$$

where we put $\delta\mu = \mu_q^\star - \mu_0$ and use the subleading approximation including the kinetic energy of the heavy quark with a finite mass m_Q, whereas the average $\langle \boldsymbol{p}^2\rangle$ can be estimated numerically with the wave functions extracted from the Schrödinger equation. Remember that $\langle \boldsymbol{p}^2\rangle = 2\mu_q^\star T$, where T

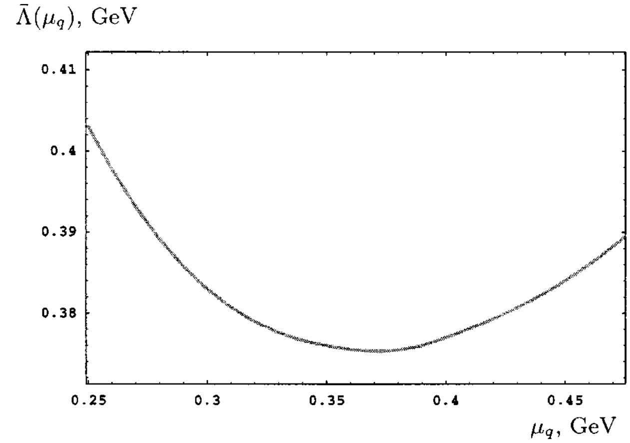

FIG. 1. The binding energy of light quark in the potential field of static source obtained from the solution of Eq. (10) vs the constituent mass μ_q. The normalization $\mu_0 = 0.37$ GeV corresponds to the minimum of binding energy.

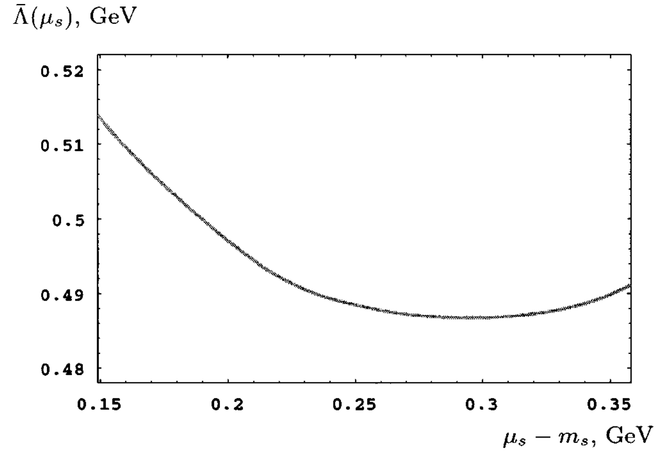

FIG. 2. The binding energy of a strange quark in the potential field of a static source obtained from the solution of the Schrödinger equation vs the constituent mass μ_s. The normalization $\mu_0^s = 0.29$ GeV corresponds to the minimum of binding energy.

is the kinetic energy, which is phenomenologically independent of the quark flavor, since the bound states are posed in the intermediate region with the change of Coulomb regime to the linear confinement, so that the potential is close to the logarithmic form, in which the kinetic energy is flavor-independent [33].

The experimental data determine the spin-averaged masses in accordance with

$$M_Q(1S) = \frac{3M_V + M_P}{4},$$

where $M_{V,P}$ are the masses of vector and pseudoscalar states. Then, we find that Eq. (11) is consistent with the experimental data if

$$\delta\mu = 35 \ \ \text{MeV}, \tag{12}$$

and we see that $\delta\mu \ll \mu_q^\star$ as expected.

We remark that the same result on $\delta\mu$ can be reproduced by the numerical solution of the relevant Schrödinger equation for the two-body problem with the corresponding kinetic terms for the heavy and light quarks, so that the difference between the values of $\delta\mu$ in these two approaches occurs less than 5 MeV, which points to the reliability of this result.

The determination of the constituent mass of a strange quark is very similar, but slightly more complicated. Indeed, the strange quark has a valuable current mass depending on the normalization point. For definiteness we fix this mass at a low virtuality relevant to the dynamics of bound states, so that we put $m_s = 0.24$ GeV, which is consistent with the extraction of the strange quark mass from the QCD sum rules at the scale 1 GeV [34]. Then we solve Eq. (10) with the substitutions $\mu_q \to \mu_s = m_s + \mu$ and $\mu_0 \to \mu_0^s$, where μ should be close to μ_0^s, and add the following perturbation:

$$\delta V_0 = k \cdot (\langle r \rangle - \langle r_0 \rangle)\, \theta\left(\frac{\mu_s}{k} - \langle r \rangle\right), \tag{13}$$

where k is the coefficient in the linear term of the potential, the average size $\langle r \rangle$ is calculated under the wave functions dependent on the scale μ_s, so that $\langle r \rangle = [\int \Psi^\dagger(r)\Psi(r) r^2 d^3 r]^{1/2}$, and this perturbation has quite a clear origin. Indeed, the length of string having weight μ_s equals μ_s/k, and we ascribe this weight to the constituent mass. However, in the case of a quark possessing a valuable current mass, the hadron can have the size $\langle r \rangle$, which is less than the attributed length of string μ_s/k determining the constituent mass. Then, we have to subtract a part of the fake energy that is given by the excess of length in order to be consistent in the presentation. The value of $\langle r_0 \rangle$ is determined at a normalization point μ_0^s given by the minimum of binding energy for such a constituent strange quark.

The dependence of the binding energy in the field of the static potential for the strange quark versus the constituent mass is presented in Fig. 2.

Thus, in the potential approach we put the position of the minimum for the binding energy as the constituent mass of a strange quark, so that

$$\mu_s^\star = 0.53 \ \ \text{GeV}. \tag{14}$$

Following the same procedure as for the light quark we can use the experimental data on the heavy-strange mesons in order to extract the value of normalization point μ_0^s entering as the difference $\delta\mu_s = \mu_s^\star - \mu_0^s$ into the equation of ground-state mass, so that using Eq. (11) with $\langle \boldsymbol{p}^2 \rangle = 2T\mu_s^\star m_Q/(\mu_s^\star + m_Q)$, we find

$$\delta\mu_s = 15 \text{ MeV}, \tag{15}$$

while the solution of the Schrödinger equation taking into account the finite masses of heavy quarks in D_s and B_s mesons results in

$$\delta\mu_s = 10 \ \ \text{MeV}, \tag{16}$$

which we will use in our numerical estimates.

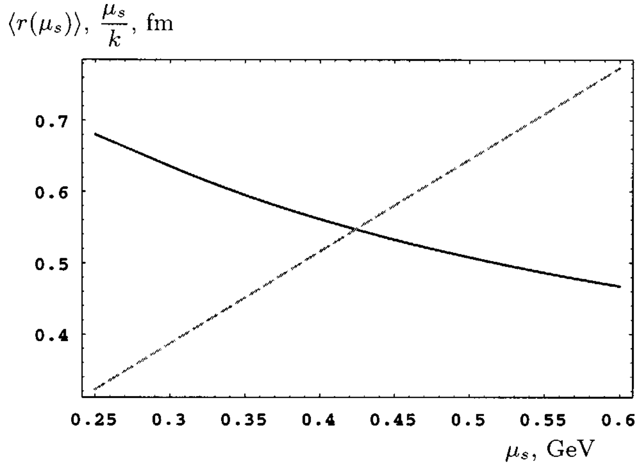

FIG. 3. The average size of a heavy-strange meson vs the constituent mass μ_s (the solid curve). The corresponding length of string with the weight μ_s is also shown by the dashed line.

The above values of $\delta\mu$ and $\delta\mu_s$ give the estimates of systematic accuracy of the potential model under consideration, so that the uncertainty of our calculations for the masses of bound states is about $\delta M = 40$ MeV.

The difference between the conditions of constituent mass formation in the heavy-light and heavy-strange mesons is demonstrated in Fig. 3, where we show the dependence of the average size of the meson on the constituent mass in comparison with the length of string with the weight of constituent mass. We see that if the constituent mass is less than 0.42 GeV, then we deal with the conditions in the heavy-light meson, while at μ_s greater than 0.42 GeV, the situation with the strange quark takes place, since at low virtualities in the bound state the running mass of the strange quark expanded in perturbative series over α_s has a significant contribution of a renormalon [35], reflecting the infrared singularity in α_s. Then, the running mass of the strange quark is greater than 0.42 GeV, and the correction (13) is justified. The renormalon contribution can be subtracted from the running mass, and this can be done by a redefinition of $\delta\mu_s$ in order to include the renormalon contribution in the value μ_0^s, as we have performed above, so that μ_0^s is rather small. Thus, the running mass of strange quark $m_s = 0.24$ represents the so-called subtracted running mass, which is correlated with the small value of μ_0^s, since we have rearranged the soft contribution between these two quantities.

Next, the above constituent masses of light and strange quarks are fixed in the procedure for the ground states of mesons with the static heavy quark, and we do not perform the same procedure for the excitations and use the fixed values in the predictions for the doubly heavy baryons, not only for the ground states, but also for excitations, too.

So, we numerically solve the following Schrödinger equations:

$$\left[\frac{\boldsymbol{p}^2}{2\mu_s^\star} + \frac{\boldsymbol{p}_{QQ'}{}^2}{2m} + V(r)\right] \quad \Psi_s(r) = \epsilon_s \Psi_s(r), \tag{17}$$

$$\left[\frac{\boldsymbol{p}_Q^2}{2m_Q} + \frac{\boldsymbol{p}_{Q'}{}^2}{2m_{Q'}} + \frac{1}{2}V(r)\right] \quad \Psi_{QQ'}(r) = \epsilon_{QQ'}\Psi_{QQ'}(r),$$

where the mass of the diquark is determined by

$$m = m_Q + m_{Q'} + \epsilon_{QQ'}$$

and the baryon mass is equal to

$$M = m + m_s + \delta\mu_s + \epsilon_s .$$

Thus, we completely determine the method for the calculation of spin-independent levels in the systems with the heavy, light, and strange quarks.

C. Spin-dependent corrections

Following [16,36], we introduce a specified form of spin-dependent corrections causing the splitting of nL levels, so that in the system of a heavy diquark containing identical quarks we have

$$\begin{aligned} V_{SD}^{(d)}(\boldsymbol{r}) = {} & \frac{1}{2}\,\frac{\boldsymbol{L}_d\cdot\boldsymbol{S}_d}{2m_Q^2}\left(-\frac{dV(r)}{rdr} + \frac{8}{3}\,\frac{\alpha_s}{r^3}\right) + \frac{2}{3}\,\frac{\alpha_s}{m_Q^2}\,\frac{\boldsymbol{L}_d\cdot\boldsymbol{S}_d}{r^3} \\ & + \frac{4}{3}\,\frac{\alpha_s}{3m_Q^2}\boldsymbol{S}_{Q_1}\cdot\boldsymbol{S}_{Q_2}\,4\pi\delta(\boldsymbol{r}) - \frac{1}{3}\,\frac{\alpha_s}{m_Q^2}\,\frac{1}{4\boldsymbol{L}_d^2-3} \\ & \times[6(\boldsymbol{L}_d\cdot\boldsymbol{S}_d)^2 + 3(\boldsymbol{L}_d\cdot\boldsymbol{S}_d) - 2\boldsymbol{L}_d^2\boldsymbol{S}_d^2]\frac{1}{r^3}, \end{aligned} \tag{18}$$

where the first string corresponds to the relativistic correction to the effective *scalar* exchange, while the second string represents the terms due to the single-gluon *vector* exchange with an effective coupling constant α_s depending on flavors of quarks composing the system. In Eq. (18), we take into account the color factor corresponding to the antitriplet state of a diquark, i.e., we add 1/2 in front of the usual expression for the quark-antiquark colorless state, and substitute the static potential $V(r)$ for the color singlet sources. The last term in Eq. (18) represents the tensor forces expressed in terms of the orbital and summed spin of quarks, as was shown in [12].

Taking into account the interaction with the strange constituent quark can be done in an analogous way. So, we have to explore the following evident kinematics for the motion of two heavy quarks posed in the same point with the distance $\boldsymbol{r}$ to the strange quark:

$$\boldsymbol{S}_Q\cdot[\boldsymbol{r}\times\boldsymbol{p}_Q] = -\frac{1}{2}\boldsymbol{S}_Q\cdot\boldsymbol{L}, \quad \boldsymbol{S}_Q\cdot[\boldsymbol{r}\times\boldsymbol{p}_s] = \boldsymbol{S}_Q\cdot\boldsymbol{L},$$

$$\boldsymbol{S}_s\cdot[\boldsymbol{r}\times\boldsymbol{p}_Q] = -\frac{1}{2}\boldsymbol{S}_s\cdot\boldsymbol{L}, \quad \boldsymbol{S}_s\cdot[\boldsymbol{r}\times\boldsymbol{p}_s] = \boldsymbol{S}_s\cdot\boldsymbol{L}.$$

In this kinematics, the first term appears in the interaction with the effective scalar exchange, the second stands in the exchange by the effective gluon field, and similar expressions appear in the terms with the spin of a strange quark.

Then we adopt the current-current form of interaction between the strange and heavy quarks with the appropriate antitriplet-state factor of 1/2 in front of the static potential and sum up the terms related with the heavy-strange subsystems, so that we can explore a usual technique for the derivation of spin-dependent perturbations similar to the Breit potential in QED, and for the interaction of an S-wave diquark with the strange quark we get

$$V_{SD}^{(s)}(\boldsymbol{r})=\frac{1}{4}\left(\frac{\boldsymbol{L}\cdot\boldsymbol{S}_d}{2m_Q^2}+\frac{4\boldsymbol{L}\cdot\boldsymbol{S}_s}{2\mu_s^{\star 2}}\right)\left(-\frac{dV(r)}{rdr}+\frac{8}{3}\frac{\alpha_s}{r^3}\right)+\frac{2}{3}\frac{\alpha_s}{m_Q\mu_s^\star}\frac{\boldsymbol{L}\cdot\boldsymbol{S}}{r^3}+\frac{4}{3}\frac{\alpha_s}{3m_Q\mu_s^\star}\boldsymbol{S}_d\cdot\boldsymbol{S}_s 4\pi\delta(\boldsymbol{r})-\frac{1}{3}\frac{\alpha_s}{m_Q\mu_s^\star}\frac{1}{4\boldsymbol{L}^2-3}$$
$$\times[6(\boldsymbol{L}\cdot\boldsymbol{S})^2+3(\boldsymbol{L}\cdot\boldsymbol{S})-2\boldsymbol{L}^2\boldsymbol{S}^2-6(\boldsymbol{L}\cdot\boldsymbol{S}_d)^2-3(\boldsymbol{L}\cdot\boldsymbol{S}_d)+2\boldsymbol{L}^2\boldsymbol{S}_d^2]\frac{1}{r^3}, \tag{19}$$

where $\boldsymbol{S}=\boldsymbol{S}_d+\boldsymbol{S}_s$. In Eq. (19), we see that this form of spin-dependent forces coincides with the expression that could be derived under the assumption of a local doubly heavy diquark with the spin $\boldsymbol{S}_d$ interacting with the strange quark, so that the perturbation in the quark-antiquark system with the diquark mass $m_{QQ}=2m_Q$ is exactly reproduced.

The value of the effective parameter α_s can be determined by

$$\alpha_s=\frac{4\pi}{\beta_0\cdot\ln(2\langle T\rangle m_{\rm red}/\Lambda_{\rm QCD}^2)}, \tag{20}$$

where $\beta_0=11-2n_f/3$ and $n_f=3$, $m_{\rm red}$ is the reduced mass of quarks composing the two-particle system, and T is the kinetic energy in the quark system, so that numerically we get $\Lambda_{\rm QCD}\approx 113$ MeV from a comparison of the theoretical expression

$$\Delta M(ns)=\frac{8}{9}\frac{\alpha_s}{m_1m_2}|R_{nS}(0)|^2, \tag{21}$$

with the experimental data on the system of $c\bar{c}$,

$$\Delta M(1S,c\bar{c})=117\pm 2\ \text{MeV}, \tag{22}$$

where $R_{nS}(r)$ is the radial wave function of quarkonium, and it is calculated in the potential model under study.

In the above estimates, we explore the fact that the average kinetic energy of quarks in the bound state practically does not depend on the flavors of quarks, and it is given by the following values:

$$\langle T_d\rangle\approx 0.19\ \text{GeV}, \tag{23}$$

and

$$\langle T_s\rangle\approx 0.38\ \text{GeV}, \tag{24}$$

for the antitriplet and singlet color states, correspondingly.

For the identical quarks inside the diquark, the scheme of LS coupling well known for the corrections in the heavy quarkonium is applicable. Otherwise, for the interaction with the strange quark we use the scheme of jj coupling. Then, $\boldsymbol{L}\cdot\boldsymbol{S}_s$ is diagonal at the given $\boldsymbol{J}_s$ ($\boldsymbol{J}_s=\boldsymbol{L}+\boldsymbol{S}_s$, $\boldsymbol{J}=\boldsymbol{J}_s+\bar{\boldsymbol{J}}$), where $\boldsymbol{J}$ denotes the total spin of a baryon and $\bar{\boldsymbol{J}}$ is the total spin of a diquark, $\bar{\boldsymbol{J}}=\boldsymbol{S}_d+\boldsymbol{L}_d$.

In order to estimate various terms and mixings of states, we use the transformations of bases,

$$|J;J_s\rangle=\sum_S |J;S\rangle(-1)^{(\bar{J}+S_s+L+J)}\times\sqrt{(2S+1)(2J_s+1)}\begin{Bmatrix}\bar{J} & S_s & S\\ L & J & J_s\end{Bmatrix} \tag{25}$$

and

$$|J;J_s\rangle=\sum_{J_d} |J;J_d\rangle(-1)^{(\bar{J}+S_s+L+J)}\times\sqrt{(2J_d+1)(2J_s+1)}\begin{Bmatrix}\bar{J} & L & J_d\\ S_s & J & J_s\end{Bmatrix}, \tag{26}$$

where $\boldsymbol{S}=\boldsymbol{S}_s+\bar{\boldsymbol{J}}$ and $\boldsymbol{J}_d=\boldsymbol{L}+\bar{\boldsymbol{J}}$.

For the hyperfine spin-spin splitting in the system of a quark-diquark, we use the presentation with the local diquark for both the interaction of an S-wave diquark with the strange quark and that of a P-wave diquark with the s-wave strange quark. Then we introduce the perturbation analogous to the spin-spin term in Eq. (19) with the substitution of $\boldsymbol{S}_d\to\bar{\boldsymbol{J}}$.

Thus, we have defined the procedure of calculations for the mass spectra of doubly heavy baryons presented in the next section.

III. NUMERICAL RESULTS

In this section, we calculate the mass spectra, taking into account the spin-dependent splitting of levels. As we have clarified in the Introduction, the doubly heavy baryons with

TABLE I. The spectrum of diquark levels without the spin-dependent splittings: masses and mean-squared radii.

Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)	Diquark level	Mass (GeV)	$\langle r^2\rangle^{1/2}$ (fm)
		Diquark bb			
$1S$	9.72	0.33	$2P$	9.93	0.54
$2S$	10.01	0.69	$3P$	10.13	0.87
$3S$	10.19	0.99	$4P$	10.30	1.14
$4S$	10.35	1.26	$5P$	10.44	1.39
$5S$	10.49	1.50	$6P$	10.56	1.62
$3D$	10.07	0.72	$4D$	10.24	1.02
$5D$	10.38	1.28	$6D$	10.51	1.52
$4F$	10.18	0.87	$5F$	10.33	1.14
$6F$	10.46	1.40	$5G$	10.27	1.01
$6G$	10.41	1.28	$6M$	10.36	1.15
		Diquark bc			
$1S$	6.45	0.48	$3P$	6.91	1.17
$2S$	6.77	0.95	$4P$	7.11	1.52
$3S$	6.99	1.34	$3D$	6.82	0.97
$2P$	6.67	0.75	$4D$	7.03	1.35
$4F$	6.95	1.17	$5F$	7.14	1.53
$5G$	7.07	1.35	$6H$	7.17	1.51
		Diquark cc			
$1S$	3.13	0.58	$3P$	3.62	1.37
$2S$	3.47	1.12	$4P$	3.85	1.78
$3S$	3.72	1.57	$3D$	3.52	1.14
$2P$	3.35	0.88	$4D$	3.76	1.58

the identical heavy quarks allow quite a reliable interpretation in terms of diquark quantum numbers (the summed spin and the orbital momentum). Dealing with the excitations of a bc diquark, we show the results on the spin-dependent splitting of the ground $1S$ state, since the emission of a soft gluon breaks the simple classification of levels for the higher excitations of such a diquark.

The quark-diquark model of bound states is the most reliable for the system with the more heavy quark. Therefore, the calculations for Ξ_{bb} and Ω_{bb} are the most accurate, while the corrections due to the finite size of a diquark can be valuable in Ξ_{cc} and Ω_{cc}.

In what follows, we consequently refer to the diquark and strange quark quantum numbers: a principal quantum number n and an orbital momentum $n_d L_d n_s l_s$. The results on the characteristics of diquarks—the masses, sizes, and wave functions, calculated with the potential K^2O [1]—are very close to the values estimated in the Buchmüller-Tye potential. The estimates are presented in Tables I and II. The wave functions and binding energies of a strange quark depend slightly on the diquark mass.

A. Ω_{bb} baryons

Denote the shift of level by $\Delta^{(J)}$ marked by the total spin of baryon J. So, for $1S2p$ we have

$$\Delta^{(5/2)} = -10.5 \ \ \text{MeV}. \tag{27}$$

The states with the total spin $J=\frac{3}{2}$ (or $\frac{1}{2}$) can have different values of J_s, and, hence, they have a nonzero mixing when we perform the calculations in the perturbation theory built over the states with the definite total momentum J_s of the strange constituent quark. For $J=\frac{3}{2}$, the mixing matrix equals

TABLE II. The characteristics of the radial wave function for the diquarks: $R_{d(ns)}(0)$ (GeV$^{3/2}$), $R'_{d(np)}(0)$ (GeV$^{5/2}$).

Diquark bb				Diquark bb				Diquark bc				Diquark cc			
nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$	nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$	nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$	nL	$R_{d(ns)}(0)$	nL	$R'_{d(np)}(0)$
$1S$	1.345	$2P$	0.479	$3S$	0.782	$4P$	0.585	$1S$	0.722	$2P$	0.200	$1S$	0.523	$2P$	0.102
$2S$	1.028	$3P$	0.539	$4S$	0.681	$5P$	0.343	$2S$	0.597	$3P$	0.330	$2S$	0.424	$3P$	0.155

TABLE III. The mass spectrum of Ω_{bb} baryons in comparison with the Ξ_{bb} one.

$(n_dL_dn_lL_l)$ J^P	$M[\Omega_{bb}]$ (GeV)	$M[\Xi_{bb}]$ (GeV) [16]	$(n_dL_dn_lL_l)$ J^P	$M[\Omega_{bb}]$ (GeV)	$M[\Xi_{bb}]$ (GeV) [16]
$(1S1s)1/2^+$	10.210	10.093	$(3P1s)1/2^-$	10.617	10.493
$(1S1s)3/2^+$	10.257	10.113	$(3D1s)5/2'^+$		10.497
$(2P1s)1/2^-$	10.416	10.310	$(3D1s)7/2^+$	10.627	10.510
$(2P1s)3/2^-$	10.462	10.343	$(3P1s)3/2^-$	10.663	10.533
$(2S1s)1/2^+$	10.493	10.373	$(1S2p)1/2^-$	10.651	10.541
$(2S1s)3/2^+$	10.540	10.413	$(1S2p)3/2^-$	10.661	10.567
$(3D1s)5/2^+$		10.416	$(1S2p)1/2'^-$	10.700	10.578
$(3D1s)3/2'^+$		10.430	$(1S2p)5/2^-$	10.670	10.580
$(3D1s)1/2^+$	10.617	10.463	$(1S2p)3/2'^-$	10.720	10.581
$(3D1s)3/2^+$		10.483	$(3S1s)1/2^+$	10.682	10.563

$$\begin{pmatrix} -18.8 & -4.7 \\ -4.7 & 39.4 \end{pmatrix} \text{ MeV}, \tag{28}$$

so that the mixing practically can be neglected, and the level shifts are determined by the values

$$\Delta'^{(3/2)} = \lambda_1' = -19.2 \text{ MeV}, \tag{29}$$

$$\Delta^{(3/2)} = \lambda_1 = 39.8 \text{ MeV}$$

with

$$|1S2p(\tfrac{3}{2}')\rangle = -0.997|J_s = \tfrac{3}{2}\rangle - 0.080|J_s = \tfrac{1}{2}\rangle, \tag{30}$$

$$|1S2p(\tfrac{3}{2})\rangle = 0.080|J_s = \tfrac{3}{2}\rangle - 0.997|J_s = \tfrac{1}{2}\rangle.$$

For $J = \frac{1}{2}$, the mixing matrix has the form

$$\begin{pmatrix} -23.8 & -15.6 \\ -15.6 & 14.2 \end{pmatrix} \text{ MeV}, \tag{31}$$

with the eigenvectors given by

$$|1S2p(\tfrac{1}{2}')\rangle = -0.941|J_s = \tfrac{3}{2}\rangle - 0.338|J_s = \tfrac{1}{2}\rangle, \tag{32}$$

$$|1S2p(\tfrac{1}{2})\rangle = 0.338|J_s = \tfrac{3}{2}\rangle - 0.941|J_s = \tfrac{1}{2}\rangle,$$

and the eigenvalues equal to

$$\Delta'^{(1/2)} = \lambda_2' = -29.5 \text{ MeV}, \tag{33}$$

$$\Delta^{(1/2)} = \lambda_2 = 19.8 \text{ MeV}.$$

We straightforwardly check that the difference between the wave functions as caused by the different masses of a diquark subsystem is unessential, so that for the $2S2p$ level the splittings are very close to the values calculated above.

The splitting of a diquark, $\Delta^{(J_d)}$, is numerically small: $|\Delta^{(J_d)}| < 10$ MeV. Such corrections are inessential up to the current accuracy of the method ($\delta M \approx 30-40$ MeV). They can be neglected for the excitations whose sizes are less than the distance to the strange quark, i.e., for diquarks with small values of principal quantum number.

The mass spectra of Ω_{bb} and Ξ_{bb} baryons are compared in Table III, wherein we restrict ourselves by the presentation of *S*-, *P*-, and *D*-wave levels.

The most reliable predictions are the masses of baryons $1S1s$ ($J^P = 3/2^+, 1/2^+$), $2P1s$ ($J^P = 3/2^-, 1/2^-$), and $3D1s$ ($J^P = 7/2^+, \ldots, 1/2^+$). The $2P1s$ level is quasistable. In the Ξ_{bb} family, the transition into the ground state requires the instantaneous change of both the orbital momentum and the summed spin of quarks inside the diquark. Therefore, when the splitting between $2P1s$ and $1S2p$, $\Delta E \sim \Lambda_{\rm QCD}$, is not small, their mixing is suppressed as $\delta V/\Delta E \sim (1/m_Q m_q)(r_d/r_s^4)(1/\Delta E) \ll 1$. Since the admixture of $1S2p$ in the $2P1s$ state is low, the $2P1s$ levels are quasistable, i.e., their hadronic transitions into the ground state with the emission of π mesons are suppressed as we have derived, though an additional suppression is given by a small value of phase space. In contract to the Ξ_{bb} family, in the Ω_{bb} system the transition of $2P1s$ to the ground level with the emission of a pion is forbidden because of the conservation of flavor in the strong interactions, while the emission of a kaon with the transition into the ground state of Ξ_{bb} is forbidden by insufficient phase space. An alternative possibility is the transition under the emission of two pions in the singlet of the isospin group, which is the open channel for the $2S1s$ levels.

As for the higher excitations, the $3P1s$ states are close to the $1S2p$ levels with $J^P = 3/2^-, 1/2^-$, so that the operators changing both the orbital momentum of the diquark and its spin can lead to the essential mixing with an amplitude $\delta V_{nn'}/\Delta E_{nn'} \sim 1$, despite the suppression by the inverse heavy quark mass and the small size of the diquark. The mixing slightly shifts the masses of states. The most important effect is a large admixture of $1S2p$ in $3P1s$. It causes the state to be unstable because of the transition into the ground $1S1s$ state with the emission of a gluon (the E1 transition). This transition leads to decays with the emission of kaons.[2]

The level $1S2pJ^P = 5/2^-$ has definite quantum numbers of diquark and light quark motion, because there are no lev-

[2]Remember that the $\Xi_{QQ'}$ baryons are the isodoublets, while the $\Omega_{QQ'}$ ones are the isosinglets.

TABLE IV. The mass spectrum of Ω_{cc} baryons in comparison with the Ξ_{cc} one.

$(n_d L_d n_l L_l)$ J^P	$M[\Omega_{cc}]$ (GeV)	$M[\Xi_{cc}]$ (GeV) [16]	$(n_d L_d n_l L_l)$ J^P	$M[\Omega_{cc}]$ (GeV)	$M[\Xi_{cc}]$ (GeV) [16]
$(1S1s)1/2^+$	3.594	3.478	$(3P1s)1/2^-$	4.073	3.972
$(1S1s)3/2^+$	3.730	3.61	$(3D1s)3/2'^+$		4.007
$(2P1s)1/2^-$	3.812	3.702	$(1S2p)3/2'^-$	4.102	4.034
$(3D1s)5/2^+$		3.781	$(1S2p)3/2^-$	4.176	4.039
$(2S1s)1/2^+$	3.925	3.812	$(1S2p)5/2^-$	4.134	4.047
$(3D1s)3/2^+$		3.83	$(3D1s)5/2'^+$		4.05
$(2P1s)3/2^-$	3.949	3.834	$(1S2p)1/2'^-$	4.145	4.052
$(3D1s)1/2^+$	3.973	3.875	$(3S1s)1/2^+$	4.172	4.072
$(1S2p)1/2^-$	4.050	3.927	$(3D1s)7/2^+$	4.204	4.089
$(2S1s)3/2^+$	4.064	3.944	$(3P1s)3/2^-$	4.213	4.104

els with the same values of J^P in its vicinity. However, its width of transition into the ground state of Ξ_{bb} and a kaon is not suppressed and a seems to be large, $\Gamma\sim 100$ MeV.

One could expect the transitions of

$$\Omega_{bb}\left(\frac{3}{2}^-\right)\to\Xi_{bb}\left(\frac{3}{2}^+\right)K \text{ in the } S \text{ wave},$$

$$\Omega_{bb}\left(\frac{3}{2}^-\right)\to\Xi_{bb}\left(\frac{1}{2}^+\right)K \text{ in the } D \text{ wave},$$

$$\Omega_{bb}\left(\frac{1}{2}^-\right)\to\Xi_{bb}\left(\frac{3}{2}^+\right)K \text{ in the } D \text{ wave},$$

$$\Omega_{bb}\left(\frac{1}{2}^-\right)\to\Xi_{bb}\left(\frac{1}{2}^+\right)K \text{ in the } S \text{ wave}.$$

The D-wave transitions are suppressed by the ratio of low recoil momentum to the mass of the baryon.

The width of the $1S1s$ state with $J^P=3/2^+$ is completely determined by the radiative electromagnetic M1 transition into the basic $J^P=1/2^+$ state.

B. Ω_{cc} baryons

The calculation procedure described above leads to the results for the doubly charmed baryons as presented below.

For $1S2p$, the splitting is equal to

$$\Delta^{(5/2)}=5.1 \text{ MeV}. \tag{34}$$

For $J=\frac{3}{2}$, the mixing is determined by the matrix

$$\begin{pmatrix} -18.5 & -14.8 \\ -14.8 & 42.9 \end{pmatrix} \text{ MeV}, \tag{35}$$

so that the eigenvectors

$$|1S2p(\tfrac{3}{2}')\rangle=-0.975|J_s=\tfrac{3}{2}\rangle-0.223|J_s=\tfrac{1}{2}\rangle, \tag{36}$$

$$|1S2p(\tfrac{3}{2})\rangle=0.223|J_s=\tfrac{3}{2}\rangle-0.975|J_s=\tfrac{1}{2}\rangle,$$

have the eigenvalues

$$\Delta'^{(3/2)}=\lambda_1'=-21.9 \text{ MeV}, \tag{37}$$

$$\Delta^{(3/2)}=\lambda_1=46.3 \text{ MeV}.$$

For $J=\frac{1}{2}$, the mixing matrix equals

$$\begin{pmatrix} -32.6 & -47.6 \\ -47.6 & -31.4 \end{pmatrix} \text{ MeV}, \tag{38}$$

where the vectors

$$|1S2p(\tfrac{1}{2}')\rangle=-0.712|J_s=\tfrac{3}{2}\rangle-0.703|J_s=\tfrac{1}{2}\rangle, \tag{39}$$

$$|1S2p(\tfrac{1}{2})\rangle=0.703|J_s=\tfrac{3}{2}\rangle-0.712|J_s=\tfrac{1}{2}\rangle,$$

have the eigenvalues

$$\Delta'^{(1/2)}=\lambda_2'=-79.6 \text{ MeV}, \tag{40}$$

$$\Delta^{(1/2)}=\lambda_2=15.6 \text{ MeV}.$$

For the $1S$-, $2S$-, and $3S$-wave levels of a diquark, the shifts of vector states are equal to

$$\Delta(1S)=6.4 \text{ MeV},$$

$$\Delta(2S)=4.7 \text{ MeV},$$

$$\Delta(3S)=4.2 \text{ MeV}.$$

The mass spectra of the Ω_{cc} and Ξ_{cc} baryons are presented in Table IV.

C. Ω_{bc} baryons

As we have already mentioned in the Introduction, the heavy diquark composed of the quarks of different flavors turns out to be unstable under the emission of soft gluons. We suppose that the calculations of masses for the excited Ω_{bc} baryons are not justified in the given scheme. Therefore, we present only the result for the lowest states with $J^P=1/2^+$,

$$M_{\Omega_{bc}'}=6.97 \text{ GeV}, \quad M_{\Omega_{bc}}=6.93 \text{ GeV},$$

and with $J^P=3/2^+$,

$$M_{\Omega^*_{bc}}=7.00\ \text{GeV},$$

whereas for the vector diquark we have assumed that the spin-dependent splitting due to the interaction with the strange quark is determined by the standard contact coupling of magnetic moments for the pointlike systems.

IV. CONCLUSION

In this paper, we have evaluated the spectroscopic characteristics of baryons containing two heavy quarks and the single strange quark, in the framework of the potential model. The calculations have been based on the assumption of a stringlike structure of the doubly heavy baryon when the small diquark interacts with the strange quark in the limit of quark-diquark factorization in the wave functions. We have explored the QCD-motivated model of the static potential [1], which takes into account two known asymptotic regimes at small and large distances. The first limit is the asymptotic freedom up to three-loop running of the coupling constant consistent with the measurements of α_s at large virtualities. The second regime is the linearly rising confining potential. The spin-dependent corrections have been taken into account. The region of factorization applicability as well as the uncertainties have been discussed.

Below the threshold of decay into the heavy baryon and heavy strange meson, we have found the system of excited bound states, which are quasistable under the hadronic transitions into the ground state. In the baryonic systems with two heavy quarks and the strange quark, the quasistability of diquark excitations is provided by the absence of transitions with the emission of both a single kaon and a single pion. These transitions are forbidden because of the small splitting between the levels and the conservation of the isospin and strangeness. Further studies on the electromagnetic and hadronic transitions between the states of doubly heavy baryons are of interest.

In conclusion, we compare the results obtained in the present paper with the estimates in potential models and in lattice simulations as shown in Table V.

TABLE V. The masses of ground states M (in GeV) for the baryons with two heavy quarks calculated in various approaches (* denotes the results of authors in this work). The accuracy of predictions under the variation of model parameters is about 30–50 MeV. The systematic uncertainties are discussed in the text.

baryon	*	[37]	[39]	[40]	[41]	[42]	[43]	[45]	[46]
Ξ_{cc}	3.48	3.66	3.74	3.66	3.61	3.65	3.71	3.62	3.57
Ξ^*_{cc}	3.61	3.81	3.86	3.74	3.68	3.73	3.79	3.73	3.63
Ω_{cc}	3.59	3.76	3.76	3.74	3.71	3.75	3.89	3.78	3.69
Ω^*_{cc}	3.73	3.89	3.90	3.82	3.76	3.83	3.91	3.87	3.75
Ξ_{bb}	10.09	10.23	10.30	10.34			10.43	10.20	
Ξ^*_{bb}	10.11	10.28	10.34	10.37			10.48	10.24	
Ω_{bb}	10.21	10.32	10.34	10.37			10.59	10.36	
Ω^*_{bb}	10.26	10.36	10.38	10.40			10.62	10.39	
Ξ_{cb}	6.82	6.95	7.01	7.04			7.08	6.93	6.83
Ξ'_{cb}	6.85	7.00	7.07	6.99			7.10	6.96	6.84
Ξ^*_{cb}	6.90	7.02	7.10	7.06			7.13	6.98	6.88
Ω_{cb}	6.93	7.05	7.05	7.09			7.23	7.09	6.94
Ω'_{cb}	6.97	7.09	7.11	7.06			7.24	7.12	6.95
Ω^*_{cb}	7.00	7.11	7.13	7.12			7.27	7.13	6.98

The quark-diquark factorization in calculating the masses of ground states for the baryon systems with two heavy quarks was also considered in Ref. [37], where the quasipotential approach [38] was explored. There is a numerical difference in the choice of heavy quark masses that leads to the mass of a doubly charmed diquark in [37] being about 100 MeV greater than the mass used in the above calculations. This difference determines the discrepancy of estimates for the masses of ground states presented in this paper and in [37]. We believe that this deviation between the quark masses is caused by the use of the Cornell potential with the constant value of the effective Coulomb exchange coupling in contrast to the above consideration with the running coupling constant, which eliminates the uncertainty in the arbitrary additive shift of energy. Furthermore, in the potential approach the masses of heavy quarks depend on the mentioned additive shift, which is adjusted in the phenomemological models by comparing, say, the leptonic constants of heavy quarkonium calculated in the model with the values known from experiments. In the QCD-motivated potential, such ambiguity of the potential because of the additive shift is absent, so that the estimates of heavy quark masses have fewer uncertainties. Let us stress that in the Cornell model the leptonic constants were calculated by taking into account the one-loop corrections caused by the hard gluons. This correction is quite essential, in part, for the charmed quarks. The two-loop corrections are also important for the consideration of leptonic constants in the potential approach [1]. Moreover, in [37] the constituent mass of a light quark is posed with no correlation with the normalization of the potential, while we put the constituent mass to be a part of the nonperturbative energy in the potential. This can lead to an additional deviation between the estimates of baryon masses of about 50 MeV. Taking into account the above remarks on the systematic differences, we can claim that the estimates of ground-state masses for the baryons with two heavy quarks in [37] agree with the values obtained in the presented approach (see Table V).

In Ref. [39], following [37] in the framework of the quasipotential approach, the analysis of spin-dependent relativistic corrections was performed so that the overestimated (in our opinion) value of heavy diquark mass from [37] was used. Unfortunately, there is an evident mistake in the description of calculations in [39], because both the parameter giving the relative contribution of the scalar and vector parts in the potential and the anomalous chromomagnetic moment of the heavy quark are denoted by the same symbol, which leads to numerical errors, since in [37] it was shown that these quantities have different values. This mistake enlarges the uncertainty by about 100 MeV into the estimates of [39], so that we can consider that the results of [39] do not contradict the presented description (see Table V).

The estimates based on the hypothesis of pair interactions were presented in Ref. [40], so that in light of the discussion given in the Introduction, the difference of about 200–300 MeV, which follows from values in Table V, is not amazing. This deviation is, in general, related with the different character of interquark forces in the doubly heavy baryon, though the uncertainty in the heavy quark masses is also important.

In Ref. [41], simple speculations based on the HQET with the heavy diquark were explored, so that the estimates depend on the supposed mass of a diquark composed of two heavy quarks. In this way, if we neglect the binding energy in the diquark, which is evidently related with the choice of heavy quark masses, then we get the estimates of ground-state masses shown in Table V.

Next, in [43] the analysis given in [44] was modified on the basis of interpolation formulas for the mass of a ground state, taking into account the dependence of spin forces on both the wave functions and the effective coupling constant, which were changed with the quark contents of the hadrons. In this way, the energy shift parameter enters the fitting function, so that this parameter essentially changes under the transition from the description of mesons to baryons: $\delta_M \approx 80$ MeV$\to \delta_B \approx 210$ MeV. This energy shift provides a good agreement of fitting with the mass values for the mesons and baryons observed experimentally. However, if we suggest that the doubly heavy baryon is similar to the meson containing the local heavy source in the picture of strong interactions, then we should use the energy shift prescribed to the heavy mesons but not to the heavy baryons, wherein the presence of a system with two light quarks leads to the essential difference in the calculation of bound-state masses, hence to the energy shift that is different from the mesonic one. Such substitution of parameters would lead to a better agreement between the results of [43] (see Table V) and the values obtained in this work.

Recently, the analysis presented in [37] was modified in [45] in order to take into account the running of QCD coupling in the static potential in the framework of the quasipotential approach. The authors of [45] claim that the difference between the estimates of masses for doubly heavy baryons in [16] and [45] is due to three sources. The first is the choice of heavy quark masses, as we have mentioned above, which gives a shift of about 50 MeV by the estimates in [45]. The second source is the relativistic dispersion law of free light or strange quarks. The third is the spin-independent relativistic corrections for the orbitally excited states. Nevertheless, we emphasize that the estimates obtained in these two approaches are in good agreement within the limits of systematic uncertainties of about 70 MeV.

Finally, the lattice simulations based on the Lagrangian of NRQCD are also presented in Table V [46]. One can see that the lattice results show approximately twice the reduction of spin-dependent splitting of the ground level, which agrees with the estimates obtained in the potential models.

Summarizing, we can claim that, first of all, in the framework of the potential approach in the calculations of masses for the doubly heavy baryons, the dominant uncertainty is caused by the choice of heavy quark masses, so that due to the adjustment on the systems with heavy quarks, the analysis presented in the QCD-motivated model of the potential with the running coupling constant at short distances and the linear nonperturbative term confining quarks at large distances gives the most reliable predictions. Further, the calculations in the framework of NRQCD sum rules [17] gave the results for the ground-state masses with no account of spin-dependent forces, so that the sum-rule estimates are in good agreement with the values obtained in the potential models.

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