Hard production in multiple parton scattering

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In some previous treatments of multiple scattering in hadron-hadron collisions a sharp distinction was introduced between a soft part of the interaction, which generates the parton population and the hard part which produces the scattering between forward and backward partons, this last being followed by other soft processes that give rise to the hadronization. So at the elementary partonic level the interaction is elastic. An attempt to complete this description is now presented; it introduces into the dynamics the possibility of hard production. The topic is developed at the level in which in an elementary partonic collision at most one secondary particle is produced but this production can happen any number of times and may be followed by the reabsorption of the produced partons and by their elastic scattering. Some possible consequences of these basically inelastic processes are outlined.

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I. INTRODUCTION

The phenomena of multiple productions in hadron collisions may be approached by two procedures at least. One of them starts focusing from the beginning on collective features, so that a relevant problem is to find which are the correct variables for the description and how they are to be used in getting predictions; another, more conservative, point of view is to assume that the fundamental variables are given, in terms of the parton model. Speaking of parton model means, according to the common theoretical point of view, speaking of QCD, we do not yet have a way of extracting prevision from QCD valid for all the kinematical situations. What we have at our disposal is a sort of compromise in which we use fundamental variables whenever it is possible, but we cannot avoid resorting to phenomenological prescriptions where a more deductive procedure does not work. In this paper we continue to follow this second line, trying to extend an already given treatment of the hadronic collisions. In that treatment [1,2,3] a sharp distinction between ''hard'' and ''soft'' dynamics was constantly used; the hard dynamics was thought of as a sort of probe of the soft part, in the same way as the electroweak interactions are currently seen as a probe of the hadronic structure. There are two relevant differences in comparison to an electroweak probe. First: the distinction between the two components of the process is somehow artificial because the fundamental dynamics is, presumably, the same, QCD; second, contrary to the electroweak case, multiple interactions are important, also in the hard dynamics.

The multiple scattering is important when there are many partons; since the elementary process must be hard it is clear that the incoming hadronic system must be very energetic, so that even a parton with low Feynman-x, where the population is high, may suffer collisions with high momentum transfer. In this condition α_s is small enough to allow the perturbative description; another way of realizing high parton densities, not in contrast with the previous one, is to have a heavy nucleus; in the following, unless differently stated, the expression ''hadron'' is used both for a single nucleon and for a nucleus. Even in the collisions with a high momentum transfer most of the partons shall not have enough energy to allow the production of a secondary particle to be a hard process, but going on with the incoming energy also processes of elementary hard production may become relevant. It can be noted, also, that from the theoretical side, the relevance of the hard inelastic processes in QCD has been repeatedly stressed [4,5]. For these reasons an attempt has been made to include in the description of the hadronic collisions some aspects originating from the presence of elementary inelastic collisions. The limit is always that only the 2 into 3 elementary process is included, this is justified by the requirement that all the subenergies appearing in the elementary processes must be large. When the produced partons are rescattered by the primary partons, no further hard production is allowed, in fact if we would consider energies for which the produced particle could in turn produce further secondaries, then also the process 2 into 4 should be included.

The aim of the paper is mainly formal and systematic, i.e., to include multiple inelastic interactions in a way which, accepting some sharp limitations, should be consistent. In the next section the essential of the formalism for the multiple production that is originated from elastic scattering of the parton is reviewed in the aim of fixing notations and definitions. In the subsequent section the main purpose of the paper is worked out. As a first step the process of inelastic production without reabsorption or rescattering of the produced particles is treated. This part of the problem is considerably simpler than the more complete treatment, in fact for this limited dynamics the formalism used in order to deal with the pure elastic parton collision can be enlarged, while keeping its main characteristic, to include the elementary production process, and even the inclusion of more complicated steps like 2 into 4 seems possible. However, considering only production without subsequent interactions seems to be reasonable only if the production itself is not very important; so a more complete dynamical treatment is required.

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This dynamics is expressed in the form of a transport equation for the parton which is produced in the hard interaction and then scatters against other primary partons or is reabsorbed. The kinematical limits are always those that are imposed for a perturbative interpretation of the elementary process, in particular all the particles must be well separated in rapidity. The perturbative inputs needed are two, the elastic scattering cross section and the 2 into 3 probability, in terms of these elements the solution of the transport equation is given. Some general comments, together with the mention of the open related problems, are presented in last section.

Some previous work and results regarding the same problem were presented in two conferences in Torino (2000) and in Da Tong (2001) [6].

II. PURELY ELASTIC PARTON SCATTERING

In this section the main features of some previous treatments [1,2] are summarized. The collision of two hadronic systems is described in term of multiple collision of elementary partons: these collisions are intended to be hard processes and therefore it is possible to give them dynamical properties extracted from perturbative QCD, more particularly the elementary collisions are considered elastic, the rest of the processes, i.e., the incoming parton population and the hadronization of the scattered off partons, are soft processes that cannot be described by perturbative QCD. For this reason the cross sections and the other features refer necessarily to processes where at least one hard scattering took place, experimentally where at least one jet was identified. A very general constraint, which will be continuously used, is that there is a good distinction between longitudinal and transverse directions, this property must hold even in collisions with large momentum transfer, these collisions are referred to as semihard ones.

The simplest form of probability density of having n partons in a hadronic system A with fractional momenta $x_1,\dots,x_n$ and transverse coordinates $\mathbf{b}_1,\dots,\mathbf{b}_n$ is then given by a Poissonian distribution $G_{1\cdots n}/n!$

$$G^A_{1\cdots n}=\Gamma_A(x_1,\mathbf{b}_1)\cdots\Gamma_A(x_n,\mathbf{b}_n)\exp\left[-\int \Gamma_A(x,\mathbf{b})dx d^2b\right], \tag{2.1}$$

where $\Gamma_A(x,\mathbf{b})$ is the average number of partons in the A system with momentum fraction x (with respect to the nucleon momentum), and transverse coordinate $\mathbf{b}$; discrete indices referring to the quantum numbers as spin, flavor, and color are suppressed.

The normalization of $\Gamma_A(x,\mathbf{b})$ is $\mathcal{N}_A$ times that of the nucleon parton distributions when it refers to a nucleus with $\mathcal{N}_A$ nucleons.

In terms of these distributions the semihard cross section, i.e., the cross section for a process where at least one semihard interaction happens, is expressed as

$$\begin{aligned}\sigma_H^{AB}=&\int d^2\beta\sum_{n=1}^{\infty}\frac{1}{n!}\int G^A_{1\cdots n}(x_1,\mathbf{b}_1;\dots;x_n,\mathbf{b}_n)\\&\times\sum_{l=1}^{\infty}\frac{1}{l!}\int G^B_{1\cdots l}(x'_1,\mathbf{b}'_1-\beta;\dots;x'_l,\mathbf{b}'_l-\beta)\\&\times\left[1-\prod_{i=1}^{n}\prod_{j=1}^{l}(1-\hat{\sigma}_{ij})\right]dx_1d^2b_1\cdots dx_nd^2b_n\\&\times dx'_1d^2b'_1\cdots dx'_ld^2b'_l\,.\end{aligned}\tag{2.2}$$

It is not possible to bring to a close form the general expression given by Eq. (2.2); two examples of approximate treatment are then given. A first wholly explicit expression is obtained in this way: let us assume that while the multiple interaction, i.e., the participation of many partons from both sides to the hadronic collision, is relevant the rescattering of the partons that already suffered an interaction is less relevant, then one gets

$$1-\prod_{i=1}^{n}\prod_{j=1}^{l}(1-\hat{\sigma}_{ij})\approx\sum_{ij}\hat{\sigma}_{ij}-\frac{1}{2}\sum_{ij}\sum_{k\neq i,l\neq j}\hat{\sigma}_{ij}\hat{\sigma}_{kl}+\cdots. \tag{2.3}$$

It is straightforward although lengthy to find in this way a close expression for the cross section

$$\sigma_H^{AB}=\int d^2\beta[1-e^{-\phi(\beta)}]$$

with

$$\phi(\beta)=\int \Gamma_A(\mathbf{b},x)\Gamma_B(\mathbf{b}-\beta,x')\sigma(xx')d^2b\,dx\,dx'. \tag{2.4}$$

This form corresponds to the expression obtained in eikonal models for high energy hadronic interactions [7,8].

A second example could be appropriate in order to study the collision of a nucleon on a heavy nucleus; in this case the rescattering of the single-nucleon partons are important because they impinge on a thick target, whereas the rescattering of the nuclear partons is less relevant because they find a thin target. Assuming A to indicate the nucleon and B to indicate the heavy nucleus, it results, for one of the products appearing in Eq. (2.3),

$$\prod_{i=0}^{n}(1-\hat{\sigma}_{ij})\approx 1-\sum_i\hat{\sigma}_{ij}$$

because the partons of B see at most one parton of A. The sum over n can be performed and the cross section still acquires an eikonal form, but with a more complicated phase

$$\sigma_H^{AB}=\int d^2\beta[1-e^{-\Phi(\beta)}],$$

$$\Phi(\beta)=\int \Gamma_A(\mathbf{b},x)[1-e^{-\Gamma_B(\mathbf{b}-\beta,x)\sigma(xx')dx'}]d^2b\,dx. \tag{2.5}$$

This form may be compared with the scattering amplitude from a composite object as given by Glauber [9]. If Γ_B becomes small, then $\Phi\to\phi$; Eq. (2.4).

The assumed Poissonian distribution is not essential. A version where the distributions $G_{1\cdots n}/n!$ are much more general has already been elaborated [3] by using the generating functional formalism [3,10], the Poissonian model is very simple and very useful to explore the inelastic case where also, anyhow, more general incoming distributions may be considered.

III. HARD PRODUCTION

A. Kinematics

When we have only elastic elementary collisions the fractional momentum is a convenient variable because, even in a very asymmetrical configuration, there is a clear distinction between forward and backward particles, in the presence of elementary inelastic processes the fractional momentum is not convenient, it is better to use the rapidity.

If P^+,Y,p^+,y are, respectively, the light-cone momenta and the rapidities of the hadron and of the parton, then $x=p^+/P^+$ and $P^+=m_\perp e^Y$, $p^+=m_\perp e^y$ then $x=e^{y-Y}$. (The formalism must contain a transverse cutoff $p_t^{\min}$, for the intrinsic transverse momentum there is the limitation $m_\perp \ll p_t^{\min}$, with $m_\perp=\sqrt{m^2+p_\perp^2}$.) The hadron structure may be expressed in term of the rapidities just redefining the basic ingredients of Eq. (2.1) as

$$W_{1\cdots n}=C(\mathbf{b}_1,y_1)\cdots C(\mathbf{b}_n,y_n)\times\exp\left[-\int C(\mathbf{b},y)d^2b\,dy\right]. \tag{3.1}$$

The normalization requires

$$C(\mathbf{b},y)dy=\Gamma(\mathbf{b},x)dx,\quad \text{i.e.,}\quad C(\mathbf{b},y)=x\Gamma(\mathbf{b},x). \tag{3.2}$$

This relation holds for the forward hadronic system $y>0$, in a symmetric reference the backward hadronic system has $y<0$. From now on the short-hand $u\equiv(\mathbf{b},y)$ will be frequently used with the convention $u-\beta\equiv(\mathbf{b}-\beta,y)$.

The subenergy of a colliding pair made up by the i forward parton and the j backward parton is $s_{ij}\approx 1/2m_\perp^2 e^{y_i-y_j}$.

The subenergies involving a produced parton of rapidity η and one of the primary partons are $s_i\approx 1/2m_\perp^2 e^{y_i-\eta}$, $s_j\approx 1/2m_\perp^2 e^{\eta-y_j}$, so $s_is_j\approx 1/2m_\perp^2 s_{ij}$.

B. Dynamics: pure emission

When one is interested in hard production all the subenergies must be large. As already said the process in which one only new parton is produced is considered, it can be said that only one rung, not the whole Balitskiĭ-Fadin-Kuraev-Lipatov (BFKL) ladder [4] is taken into account. It is not intended that kinematics strictly forbids processes with more rungs; they happen very seldom because they correspond to regions where the structure functions are small. So the elementary collision probability $\hat\sigma$ is the sum of two addenda

$$\hat\sigma=\hat\sigma_o+\int dv\,\rho(v), \tag{3.3}$$

where $\hat\sigma_o$ comes from the elastic cross section and $\rho(v)$ from the cross section for the production of a particle with quantum number v whose actual meaning may be $v\equiv(\mathbf{b},\eta)$ or else $v\equiv(\mathbf{q},\eta)$.

The rapidity η can be either positive or negative, but in any case $|\eta|\ll Y$. The formulation presented above is particularly suited for describing processes where the produced particles are never reabsorbed; this condition is not too unrealistic for what concerns some global variables, in fact one could well allow the elastic rescattering of the secondary particles with the primary ones, provided no further particles are in this collision. In fact, if the secondary particle could in turn produce a further hard parton, then the primary kinematics would be consistent with a hard 2 into 4 process, which has been, for the moment, excluded; what is less justified is the exclusion of the reverse 3 into 2 process, this possibility will be further considered.

Now the case with pure production is described in detail. One may rewrite the expression for the cross section, see Eq. (2.2), at fixed β as

$$\sigma_H(\beta)=\int\sum_n\frac{1}{n!}W^A_{1\cdots n}(u_1,\ldots,u_n)\times\sum_l\frac{1}{l!}W^B_{1\cdots l}(u'_1-\beta,\ldots,u'_l-\beta)\times\left\{1-\prod_{i=1}^n\prod_{j=1}^m[1-\hat\sigma_{i,j}(u,u')]\right\}\prod du\,du'. \tag{3.4}$$

The multiple interaction term

$$f=1-\prod[1-\hat\sigma_{i,j}(u,u')]$$

may be decomposed into the sum of the term with a fixed number of interactions [1], with the definitions $Q=mn$, and $\mathcal{S}$ as a symbol indicating the symmetrization over the indices $R\equiv(i,j)$, the formal result can be written:

$$f=Q\mathcal{S}\hat\sigma_1\prod_{R>1}^{Q}[1-\hat\sigma_R]+\binom{Q}{2}\mathcal{S}\hat\sigma_1\hat\sigma_2\prod_{R>2}^{Q}[1-\hat\sigma_R]+\cdots+\binom{Q}{K}\mathcal{S}\hat\sigma_1\cdots\hat\sigma_K\prod_{R>K}^{Q}[1-\hat\sigma_R]+\cdots.$$

It is convenient to define an auxiliary functional $\overline{\sigma}[J]=\hat\sigma_o+\int dv\,J(v)\rho(v)$ so that $\overline{\sigma}[1]=\hat\sigma$ and $\overline{\sigma}[0]=\hat\sigma_o$.

Then one inserts the expression $\overline{\sigma}[J]$ into the sum in the place of the interaction probability $\hat\sigma_o$, leaving untouched the noninteraction probability $1-\hat{\sigma}$; this newly produced expression is called $f[J]$. It is evident that $J=0$ selects out the elastic process and $\delta/\delta J(v)$ selects out the production of a secondary process with quantum numbers v, so every kind of production parton can be extracted from $f[J]$. The expression may also be resummed back and it gives:

$$f[J]=\prod_{R=1}^{Q}[1-\hat{\sigma}_R+\overline{\sigma}_R[J]]-\prod_{R=1}^{Q}[1-\hat{\sigma}_R].$$

By inserting the expression of $f[J]$ into the general form Eq. (3.4) and taking the appropriate functional derivatives one can then get the expression for the cross sections for the hard production of secondaries with assigned quantum numbers v_i. It has been already said that, even in the absence of production at the partonic level, if a close general expression for the cross section is not available the situation is even worse when the production arises, so in order to produce more transparent expressions the approximations already introduced are used. With no rescattering at all, Eq. (6), there is a simple enough form for the generating functional:

$$\begin{aligned}\mathcal{G}[J]=\exp\Big[\int du du'(C^A(u)\overline{\sigma}[u,u';J]C^B(u'-\beta)\\ -C^A(u)\hat{\sigma}(u,u')C^B(u'-\beta))\Big]\\ -\exp\Big[-\int du du' C^A(u)\hat{\sigma}(u,u')C^B(u'-\beta)\Big].\end{aligned} \tag{3.5}$$

The distribution of the secondaries at fixed β is given by

$$\begin{aligned}P_r(v_1,\ldots,v_r;\beta)&=T(v_1;\beta)\cdots T(v_r;\beta)\\ &\quad\times\exp\Big[\int dv\, T(v;\beta)\Big]\\ &\quad\times\{1-\exp[-T_o(\beta)]\},\end{aligned}$$

$$T(v;\beta)=\int du du' C^A(u)\rho(u,u',v)C^B(u'-\beta), \tag{3.6}$$

$$T_o(\beta)=\int du du' C^A(u)\hat{\sigma}_o(u,u')C^B(u'-\beta).$$

It is possible to investigate how much the distribution of the secondary particles remembers the distribution of the primary ones. In fact one could give a much more general form for the functional of Eq. (3.5) if we introduce the functional generators for the primary partons: $Z[I]$. In this case, in fact, it results in:

$$\begin{aligned}\mathcal{G}[J]=\Big\{\exp\Big[\int du du'\Big(\frac{\delta}{\delta I(u)}\{\overline{\sigma}[u,u';J]\\ -\hat{\sigma}(u,u')\}\frac{\delta}{\delta I'(u'-\beta)}\Big)\Big]\\ -\exp\Big[-\int du du'\frac{\delta}{\delta I(u)}\hat{\sigma}(u,u')\frac{\delta}{\delta I'(u'-\beta)}\Big]\Big\}\\ \times Z_A[I]Z_B[I']|_{I=I'=0}.\end{aligned} \tag{3.7}$$

A general calculation is very complicated and perhaps not very interesting, but it is possible to perform it for some particular distributions, e.g., if one starts from a negative binomial for the primaries[1] then the distribution of the secondaries is more complicated than the binomial one started with, it may be expressed in terms of hypergeometric functions, we find therefore that, with the dynamics here considered only the Poissonian distribution reproduces itself in the secondary particles.

This treatment seems consistent when all the rescatterings are not very important, but is not very satisfactory when rescattering is important in this case in fact, if there is production there should be also absorption, moreover the kinematics shows that also hard scattering between a produced parton and some preexisting parton may happen, so in order to deal with a situation of this kind a different procedure is needed.

C. Dynamics: emission, reabsorption, and scattering

In this case one concentrates the attention to the secondary particles leaving a more passive role to the primary ones; this is possible because in the mean the energy taken away by the secondaries is small with respect to the available energy of the colliding pair, as previously discussed.

The physical model consists of a nucleon colliding with a heavy nucleus and the guiding idea is to follow the secondary population in its development, the basic quantity is the probability distribution for a definite configuration of the secondary partons $P_r(v_1,\ldots,v_r)$. In order to follow this evolution one needs a parameter τ which could be interpreted as the depth at which the nucleon is penetrated into the nucleus, or the mean number of partons of the nucleus that have been hit. In fact the asymmetry of the physical system is used to assume that the partons of the incoming projectile can interact subsequently with the partons of the thick nuclear target while the latter seldom interact more than once with the components of the thin nucleon. Within this frame a transport equation[2] for the probability distribution of precisely r partons with given quantum numbers can be written:

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[1]This distribution is produced by the choice $Z[I]=(1-y[1])^k(1-y[I])^{-k}$ with $y[I]=\int du C(u)I(u)$.

[2]Forms of transport equations in the hadronic system have been used in particular in connection with the possibility of producing the QCD plasma, see [11] and references therein.

$$P_r(v_1,\ldots,v_r;\tau+\Delta\tau)=P_r(v_1,\ldots,v_r;\tau)+\sum_{s=1}^{r}P_{r-1}(v_1,\ldots,v_{s-1},v_{s+1},\ldots,v_r;\tau)E(v_s;\tau)\Delta\tau$$
$$+\int dwP_{r+1}(v_1,\ldots,v_r,w;\tau)A(w;\tau)\Delta\tau$$
$$+\int dw\sum_{s=1}^{r}P_r(v_1,\ldots,v_{s-1},w,v_{s+1},\ldots,v_r;\tau)T(v_s,w;\tau)\Delta\tau$$
$$-\sum_{s=1}^{r}P_r(v_1,\ldots,v_r;\tau)A(v_s;\tau)\Delta\tau-\int dwP_r(v_1,\ldots,v_r;\tau)E(w;\tau)\Delta\tau$$
$$-\sum_{s=1}^{r}P_r(v_1,\ldots,v_r;\tau)\int dwT(w,v_s;\tau)\Delta\tau. \tag{3.8}$$

It is understood that the element $\Delta\tau$ is so small that only one secondary particle is involved in the emission or absorption or scattering. So six basic steps are foreseen: two emission steps, two absorption steps, and two scattering steps. The coefficients E, A are emission and absorption probabilities, and the coefficients T are the elastic scattering probability, all of them may depend on τ. (The overall impact parameter β is fixed and it will no longer be written.) The system of equation is solved by defining a generating functional

$$\mathcal{F}[I;\tau]=\sum_r\frac{1}{r!}\int dv_1,\ldots,dv_rI(v_1)\cdots I(v_r)P_r(v_1,\ldots,v_r;\tau). \tag{3.9}$$

Performing a continuum limit $\Delta\tau\to0$ a differential equation is produced,

$$\frac{\partial}{\partial\tau}\mathcal{F}[I;\tau]=\mathcal{F}[I;\tau]\int dwI(w)E(w;\tau)+\int dwA(w;\tau)\frac{\delta}{\delta I(w)}\mathcal{F}[I;\tau]+\int dwdw'T(w,w',\tau)I(w)\frac{\delta}{\delta I(w')}\mathcal{F}[I;\tau]$$
$$-\mathcal{F}[I;\tau]\int dwE(w;\tau)-\int dwA(w;\tau)I(w)\frac{\delta}{\delta I(w)}\mathcal{F}[I;\tau]-\int dwdw'T(w',w;\tau)I(w)\frac{\delta}{\delta I(w)}\mathcal{F}[I;\tau]. \tag{3.10}$$

A useful simplification is gained by setting

$$\mathcal{F}[I;\tau]=\exp\mathcal{L}[I;\tau] \tag{3.9$'$}$$

so that the new form is produced:

$$\frac{\partial}{\partial\tau}\mathcal{L}[I;\tau]=\int dwI(w)E(w;\tau)+\int dwA(w;\tau)\frac{\delta}{\delta I(w)}\mathcal{L}[I;\tau]+\int dwdw'T(w,w';\tau)I(w)\frac{\delta}{\delta I(w')}\mathcal{L}[I;\tau]-\int dwE(w;\tau)$$
$$-\int dwA(w;\tau)I(w)\frac{\delta}{\delta I(w)}\mathcal{L}[I;\tau]-\int dwdw'T(w',w;\tau)I(w)\frac{\delta}{\delta I(w)}\mathcal{L}[I;\tau]. \tag{3.11}$$

This is an inhomogeneous equation of first order; a particular solution is looked for in the form

$$\mathcal{L}[I;\tau]=\int dw[I(w)p(w;\tau)-q(w,\tau)]. \tag{3.12}$$

The auxiliary functions satisfy the set of equations (the *dot* means the derivative with respect to τ; sometimes this variable is not written):

$$\dot{q}(w)=E(w)-A(w)p(w), \tag{3.13}$$

$$\dot{p}(w)=E(w)-\int dw'R(w,w')p(w'),$$

where

$$R(w,w')=\delta(w-w')\left[A(w)+\int dw'',T(w'',w)\right]$$
$$-T(w,w'). \tag{3.14}$$

It is useful to define an operator $\hat{R}$ such that

$$(w|\hat{R}(\tau)|w')=R(w,w';\tau)$$

and a kernel

$$K(w,w';\tau,t)=(w|e^{-\int_t^\tau \hat{R}(\theta)d\theta}|w'), \tag{3.15}$$

the integral at the exponent must be interpreted as t-ordered. The formal solution for $p(\tau)$ is

$$p(w;\tau)=\int_o^\tau dt\int dw' K(w,w';\tau,t)E(w';t), \tag{3.16}$$

and the corresponding solution for q is

$$q(w;\tau)=\int_o^\tau dt[E(w;t)-A(w,t)p(w,t)]. \tag{3.16$'$}$$

With these solutions for the auxiliary functions one gets $\mathcal{L}[I;0]=0$, $\forall I$, this is required by the physical initial conditions, in fact for $\tau=0$ there must be no secondary partons at all, so we must have

$$P_0=1,\ \ P_r=0,\ \ \forall r>0,\ \ \mathcal{F}=1,\ \ \forall I.$$

So the particular solution is precisely the one required by the initial conditions.

The structure of the functional, independently of the actual form of the functions p and q, says that the particles are produced according to a Poisson distribution if we consider the unrealistic case of a definite and fixed distribution of the primary partons. The problem of the interplay between these two distributions is discussed in the next section. For the moment we face the problem of making more explicit the content of Eqs. (3.16), (3.16$'$). As a first step it has to be noted that from Eq. (3.14) it results

$$\int dwdw' R(w,w')p(w')=\int A(w)p(w),$$

then

$$\int \dot{p}(w)dw=\int \dot{q}(w)dw, \tag{3.17}$$

since the auxiliary function q appears always integrated in $\mathcal{L}$ it is possible to substitute it with p. Then $\mathcal{L}[1;\tau]=0$, so $\mathcal{F}[1;\tau]=1$ for every τ. By construction, see Eqs. (3.9), (3.9$'$), this expression is the sum of the probabilities so the evolution equation conserves correctly the overall probability.

The price of making the content of Eqs. (3.16), (3.16$'$) more transparent is to have them in a less compact form. It is convenient to separate the absorption from the scattering by writing

$$R(w,w';\tau)=A(w,w';\tau)-S(w,w';\tau);$$

it is then possible to write the evolution equation for K_t as a function of τ:

$$K(w,w';\tau,t)=J(w,w';\tau,t)\exp\left[-\int_t^\tau d\theta A(w;\theta)\right],$$

$$\frac{d}{d\tau}J(w,w';\tau,t)=\int du\,\exp\left[\int_t^\tau d\theta A(w;\theta)\right]S(w,w';\tau)\times\exp\left[-\int_t^\tau d\theta A(u;\theta)\right]J(u,w';\tau,t).$$

This equation gives an iterative solution for J and so for the kernel K

$$K_t(w,w';\tau,t)=\exp\left[-\int_t^\tau dt' A(w;t')\right]\delta(w-w')+\int_t^\tau d\theta\,\exp\left[-\int_\theta^\tau dt' A(w;t')\right]S(w,w';\theta)\exp\left[-\int_t^\theta dt' A(w';t')\right]$$
$$+\int du\int_t^\tau d\theta\int_t^\theta d\theta'\,\exp\left[-\int_\theta^\tau dt' A(w;t')\right]S(w,u;\theta)\exp\left[-\int_{\theta'}^\theta dt' A(u,t')\right]S(u,w';\theta')$$
$$\times\exp\left[-\int_t^{\theta'} dt' A(w';t')\right]+\cdots. \tag{3.18}$$

In this way the sequence of absorptions and scatterings along the line of flight of the secondary partons appears evident, the term E present in Eq. (3.16) completes the description saying that the secondary partons are continuously produced.

If we stop the iterative solution at the first term we get the solution in absence of scattering that has been already presented [6].

In every case the actual form of the functional $\mathcal{F}$ and the relation Eq. (3.17) yields a Poissonian distribution of the secondary partons.

D. Perturbative inputs

The elementary components of the previously given expressions are the scattering amplitudes T, the production amplitudes A, and the absorption amplitudes E.

For the scattering term we have as the starting point the usual expression: cross section=flux factor $\times|$matrix element$|^2\times$phase space,

$$\sigma=\frac{1}{2\hat{s}}\int |M|^2\prod_i \left(\frac{d^3k}{2k_o}\right)_i \delta^4\left(\sum_i k_i-p_a-p_b\right),$$

$$\sigma=\frac{1}{4\hat{s}^2}\int |M|^2 d^2q.$$

Performing the Fourier transform of amplitude with respect to the momentum transfer we have

$$\tilde{M}(\mathbf{b})=\frac{1}{2\pi}\int M(\mathbf{q})e^{i\mathbf{q}\cdot\mathbf{b}}d^2q. \tag{3.19}$$

We consider only hard scattering[3] so the size of the interaction region is $\mathcal{O}(1/p_{t\,\mathrm{min}}^2)$, much smaller than the size of the hadron, we use therefore a "$\delta(\mathbf{b})$" approximation.

$$\frac{1}{4\hat{s}^2}|\tilde{M}(\mathbf{b})|^2\approx\delta(\mathbf{b})\sigma(x_ix_j). \tag{3.20}$$

In perturbative estimate σ is a Rutherford cross section, aside the constants, integrating it from $p_{t\,\mathrm{min}}$ to ∞ in the transverse momenta it results:

$$\sigma(x_ix_j)\approx\mathrm{const}\frac{1}{p_{t\,\mathrm{min}}^2}.$$

For the production amplitude the starting point is the explicit form [4] of the perturbative production probability "Lipatov vertex:"[4]

$$|M_{gg\to ggg}|^2=54g^6\frac{\hat{s}^2}{k_{0\perp}^2k_{1\perp}^2k_{2\perp}^2},\ \sum_i \mathbf{k}_{i\perp}=\mathbf{p}_a+\mathbf{p}_b=0$$

to which the corresponding cross section is

$$\sigma=\frac{1}{2\hat{s}}\int |M|^2\prod_i\left(\frac{d^3k}{2k_o}\right)_i\delta^4\left(\sum_i k_i-p_a-p_b\right).$$

Within the kinematical region of interest all the subenergies are large and this gives a cut in the range of the rapidity of the produced particle y_1 so that the energy taken away by the intermediate parton is small.

$$\sigma=\int\frac{1}{4\hat{s}^2}|M|^2\prod_i d^2k_i\delta^2\left(\sum_i k_\perp\right)dy_1$$

or with the definition $q=(k_2-k_0)/2=(q_1+q_2)/2$

$$\sigma=\int\frac{1}{4\hat{s}^2}|M|^2d^2qd^2k_1dy_1.$$

We try now to follow again the steps leading from Eq. (3.19) to Eq. (3.20). Standard Fourier transform gives

$$\sigma=\int\frac{1}{4\hat{s}^2}|M(\mathbf{b},\mathbf{b_1};y_1)|^2d^2bd^2b_1dy_1,$$

we use again the δ approximation with respect to $\mathbf{b}$ and write

$$\sigma=\int\rho(\mathbf{b_1};y_1)d^2b_1dy_1. \tag{3.21}$$

Finally we have to express the absorption term A. Were we considering transition between normalized states, both primaries and secondaries, the terms A and E would be trivially related by time reversal, the fact is that the partonic states are normalized in the continuum and this makes the two-body incoming flux, entering in E, different from the three-body flux, entering in A. In fact the dimensions of A are different from the dimensions of E. Within the approximations we have used the flux of the primaries is the same in the emission and in the absorption because the loss of energy of the incoming particles is neglected, in the definition of A the new factor is the secondary flux, the speed of the secondary is of the order of c, i.e., 1, the density is referred to the (transverse) region where the secondary particles must be found and may reinteract, but we have seen, just in discussing the δ approximation, that the transverse dimensions associated to the perturbative processes are $\mathcal{O}(1/p_{t\,\mathrm{min}}^2)$. This is then the factor that relates the absorption and the emission term.

The above considerations also show that the rescattering term $T(w,w')$, once we perform the identification $w\equiv(\mathbf{b},y)$, becomes diagonal in the impact parameter, but there is no reason to expect that it is diagonal in the rapidities.

E. Interplay between primary and secondary partons

Until now the kinematical variables of the primary partons have been ignored, in reality E and A depend on these variables, and we can write a new transport equation with u coordinates for the primary partons and v coordinates for the produced partons. The basic distribution will then be

$$P_{r,n}(u_1,\ldots,u_n;v_1,\ldots,v_r;\tau).$$

The set of the primaries' variables will be frozen in the functional equation, so apparently nothing relevant happens; the new result will come when we look at the distribution of the secondaries after summing over the distributions of the primaries.

The modifications of the formalism are complicated to write out but easy to understand, the functional $\mathcal{F}$ depends now also on the variables of the primary partons $\{u\}\equiv(u_1,u_2,\ldots,u_n)$ and the same happens for the expressions derived from it. If we perform an integration over the u variables this reflects on the distributions of the secondaries: a simple but clear example is the following. We have seen, in Sec. III C, that the distribution at fixed $\{u\}$ for r-produced partons is Poissonian:

$$\frac{1}{n!}p(\{u\},v_1)\cdots p(\{u\},v_r)\exp\left[-\int p(\{u\},v)dv\right]\Pi_n(\{u\}).$$

[3] The kinematical constraints are $x\leqslant1$, $xx's>4p_{t\,\mathrm{min}}$.

[4] This name of "Lipatov vertex" is perhaps a bit emphatic, we simply mean that it originates from the 2 into 3 amplitude obtained by summing a set of graphs that are required to guarantee gauge invariance, bounds on the kinematical variables are used so that they never approach the regions where infrared singularities come out.

Π_n is the probability distribution of the primary partons. Then the inclusive one-parton and two-parton distributions at fixed primaries are

$$p(\{u\},w)\Pi_n(\{u\}), \tag{3.22}$$

$$p(\{u\},w)p(\{u\},w')\Pi_n(\{u\}). \tag{3.22$'$}$$

We perform then the integration over the variables $\{u\}$, and also the sum over their multiplicity n; clearly a variety of results may be produced depending on the shape of the distribution Π_n. Even if the simplest, Poissonian form is assumed,

$$\Pi_n(\{u\})=\frac{1}{n!}g(u_1)\cdots g(u_n)\exp\left[-\int g(u)du\right],$$

care must be taken that the same parton u_j may be involved both in the emission of v and in the emission of v' and therefore the two body distribution, obtained integrating Eq. (3.22′), cannot be factorized any more.

IV. OPEN PROBLEMS AND CONCLUSIONS

There are many open problems; some of them are purely technical, which does not mean that they are trivial, others seem to belong to a more fundamental level.

To the first family we can ascribe a better clarification of the kinematical relation between emission and absorption, an estimate of the cumulative effect of the elastic scattering, beyond the iterative representation. The subsequent rescattering processes have been studied in the case of the elastic fundamental process [2] and the result was found to be interpretable as a random walk in the transverse plane. A much harder question is the rescattering on both sides, which is evidently relevant for nucleus-nucleus collision; perhaps the treatment presented here is not able to cope with this problem. It has been seen that in the case of no hard production the general formalism also covering this case exists and arises in a natural way, but also there the extraction of more explicit information requires in fact various approximations.

What may be called a deeper level has to do with the fact that all the formalism deals with cross sections, production probabilities, i.e., with real quantities, where the quantum-mechanical phases have been washed out and together with them the color structure has also been lost.

We think that these limitations are acceptable until one wishes to look for a description of the purely hard processes, when another part of the dynamics is relevant, the scheme as it stands, cannot work because of coherence effects and the related compensations between absolute squares and nondiagonal interference terms; this kind of compensation is known to be particularly effective in dealing with infrared singularities.

There is no formal limit to the number of the inelastic collisions, in other words the formalism is not perturbative in the hard production process. As it happens for the elastic partonic case the problem is easier for multiple interactions without rescattering, what one could call "disconnected graphs," at the perturbative level; when the rescattering, which now also means absorption, is relevant the treatment is more involved and, one should say, less elegant, but some definite results are also available in this case.

The typical collective observable affected by the processes described here is the transverse energy; in fact the transverse flux of energy seen in the produced hadrons can be related to the transverse flux of energy carried out by partons with the standard assumption that hadronization is a "soft" process and therefore does not alter very much this observable. The presence of hard production, however, has necessarily the result of making the total amount of transverse energy larger. The particles that are produced according to the process described here are gluons, the rapidity regions where the produced parton lie and the rapidity region of the original partons partially superimpose, so the effect is a variation of the overall gluon population; if something similar happens also for quarks, this redefinition of the partonic population could be relevant in processes of the Drell-Yan kind; however, the extension to the hard production of quarks is not completely trivial, in particular some inputs defined in Sec. III D must be modified.

The principal aim of the present research has been precisely to investigate how far, even in these complicated phenomena, one could go on starting from elementary dynamics; it is found that a class of emission and reabsorption processes can be included into the formalism originally designed to deal with elastic partonic scattering. In so doing the formalism becomes more complicated; still it yields explicit answers. The kinematical limits of validity remain the same; a step towards a more complete dynamical description has been made.

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