Recoil order chiral corrections to baryon octet axial vector currents and large N_c QCD

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We compute the chiral corrections to octet baryon axial vector currents through $\mathcal{O}(p^3)$ in heavy baryon chiral perturbation theory, including both octet and decuplet baryon intermediate states. We include the latter in a consistent way by using the small scale expansion. We find that, in contrast to the situation at $\mathcal{O}(p^2)$, there exist no cancellations between octet and decuplet contributions at $\mathcal{O}(p^3)$. Consequently, the $\mathcal{O}(p^3)$ corrections spoil the expected scaling behavior of the chiral expansion. We discuss this result in terms of the $1/N_c$ expansion. We also consider the implications for the determination of the strange quark contribution to the nucleon spin from polarized deep inelastic scattering data.

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I. INTRODUCTION

The chiral expansion of the octet baryon axial vector current $J^A_{\mu 5}$ has been a topic of ongoing theoretical interest for some time. At $\mathcal{O}(p^0)$, this current is parametrized by the well-known SU(3) reduced matrix elements D and F. The leading chiral corrections, which arise at $\mathcal{O}(p^2)$ contain chiral logarithms, which were first computed in Refs. [1,2]. Subsequently, the wave function renormalization correction was added in the framework of heavy baryon chiral perturbation theory (HBCPT) [3–5], which provides for a consistent power counting. While these corrections are large when only octet baryon intermediate states are kept [3], inclusion of decuplet contributions produces sizeable cancellations, leading to a significantly smaller $\mathcal{O}(p^2)$ effect [4]. The origin of these cancellations may be explained by considering the large N_c expansion [6], as noted in the work of Refs. [7–11]. In terms of this counting, the $\mathcal{O}(p^0)$ contributions are of order N_c, while the $\mathcal{O}(p^2)$ loop corrections are nominally $\mathcal{O}(N_c^2)$. As shown in Refs. [8–12], however, a spin-flavor symmetry arises at this order whose effect is to render the $\mathcal{O}(p^2)$ loop effects of relative order N_c^0. Thus inclusion of decuplet contributions is crucial to maintining the correct N_c counting as well as the convergence properties of the chiral expansion through $\mathcal{O}(p^2)$.

In a recent paper [13], we have calculated the $\mathcal{O}(p^3)$ corrections to $J^A_{\mu 5}$ arising from octet baryon intermediate states. These corrections are entirely of recoil order, scaling as inverse powers of the baryon mass. In that study, we employed baryon chiral perturbation theory with infrared regularization [14], which effectively resums an infinite tower of recoil corrections. Although this resummation is necessary to maintain the analytic properties of the currents for momenta near physical thresholds, we found that for $q^2=0$ the sum is dominated by the leading $1/M$ correction which can be obtained directly in HBCPT. We also found that the $\mathcal{O}(p^3)$ corrections were large, exacerbating the poor convergence obtained through $\mathcal{O}(p^2)$ in octet-only calculations. We left open the question as to the impact of including decuplet intermediate states, speculating that large-N_c symmetries can generate cancellations at this order as well.

In the present paper we report on an explicit calculation of the $\mathcal{O}(p^3)$ corrections which includes contributions from the decuplet. We find that, even under the symmetry constraints imposed by the large-N_c expansion, these corrections are both substantial and devoid of the cancellations arising at $\mathcal{O}(p^2)$. In several channels, the $\mathcal{O}(p^3)$ corrections can be as large as the $\mathcal{O}(p^0)$ term, in contrast to the naively expected power suppression by $(m_K/\Lambda_\chi)^2\times(m_K/M)\sim 1/8$. We also show that the reduced order in N_c arising from the $\mathcal{O}(p^2)$ spin-flavor algebra is, in retrospect, what one might expect from the N_c behavior of the relevant counterterms. In contrast, the $\mathcal{O}(p^3)$ loop corrections are finite and entirely nonanalytic (in quark mass), so there exists no counterterm at this order whose N_c behavior would imply a corresponding order in N_c for the $\mathcal{O}(p^3)$ loop corrections. While this observation does not by itself explain the apparent breakdown of the chiral expansion for $J^A_{\mu 5}$ at $\mathcal{O}(p^3)$, it does suggest that inclusion of decuplet intermediate states is not generally sufficient to maintain the proper scaling behavior of the expansion. As a practical corollary, we also note that the use of SU(3) chiral perturbation theory to extract Δs—the strange quark contribution to the nucleon spin—from polarized deep inelastic scattering data is subject to uncontrolled approximations and, therefore, untrustworthy.

II. AXIAL VECTOR CURRENTS

In writing down the octet axial vector currents, it is convenient to start with the relativistic meson-baryon Lagrangian. At the lowest order, one has

$$
\begin{aligned}
\mathcal{L}_0 = {} & i\,\mathrm{Tr}(\overline{B}(\gamma^\mu D_\mu - m_N)B) \\
& + D\,\mathrm{Tr}(\overline{B}\gamma^\mu\gamma_5\{A_\mu,B\}) + F\,\mathrm{Tr}(\overline{B}\gamma^\mu\gamma_5[A_\mu,B]) \\
& + i\overline{T}^\mu\gamma^\nu D_\nu T_\mu - m_T\overline{T}^\mu T_\mu + \mathcal{C}[\overline{T}^\mu A_\mu B + \overline{B}A_\mu T^\mu] \\
& + \mathcal{H}\overline{T}^\mu\gamma^\nu\gamma_5 A^\nu T_\mu + \frac{F_\pi^2}{4}\mathrm{Tr}((D^\mu\Sigma)^\dagger D_\mu\Sigma) \\
& + a\,\mathrm{Tr}\,M(\Sigma+\Sigma^\dagger),
\end{aligned}
\tag{1}
$$

where

$$D_\mu B=\partial_\mu B+[V_\mu,B],$$

$$D_\mu T^\nu_{abc}=\partial_\mu T^\nu_{abc}+(V_\mu)^d_a T^\nu_{dbc}+(V_\mu)^d_b T^\nu_{adc}+(V_\mu)^d_c T^\nu_{abd},$$

$$V_\mu=\frac{1}{2}(\xi\partial_\mu\xi^\dagger+\xi^\dagger\partial_\mu\xi),$$

$$A_\mu=\frac{i}{2}(\xi\partial_\mu\xi^\dagger-\xi^\dagger\partial_\mu\xi),$$

$$\xi=e^{i(\pi/F_\pi)},\quad \Sigma=\xi^2=e^{2i(\pi/F_\pi)},$$

$$\pi=\frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}}+\frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$

$$B=\begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}}+\frac{\Lambda}{\sqrt{6}} & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}}\Lambda \end{pmatrix},$$

$$M=\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix}.$$

One may obtain vector and axial vector current operators from $\mathcal{L}_0$ by including vector and axial vector sources in the covariant derivatives. The leading $[\mathcal{O}(p^0)]$ operator contains only baryon fields and the SU(3) reduced matrix elements D and F. Axial vector currents involving both baryons and mesons first appear at $\mathcal{O}(p)$. Additional purely baryonic axial currents appear at $\mathcal{O}(p^2)$ [13]. They arise from the SU(3) symmetry breaking (SB) Lagrangian

$$\mathcal{L}_1=\frac{m_K^2}{\Lambda_\chi^2}\{d_1\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5\{A_\mu,\chi_+\}B)+d_2\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5 A_\mu B\chi_+)+d_3\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5\chi_+ BA_\mu)+d_4\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5 B\{A_\mu,\chi_+\})\},\tag{2}$$

where

$$\chi_+=\frac{1}{2}(\xi^+\chi\xi^+ +\xi\chi^+\xi),$$

$$\chi=\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Using $\mathcal{L}_{0,1}$ one obtains the axial vector current:

$$\begin{aligned}J^A_\mu={}&\frac{1}{2}D\,\mathrm{Tr}(\bar{B}\gamma_\mu\gamma_5\{\xi T^A\xi^\dagger+\xi^\dagger T^A\xi,B\})+\frac{1}{2}F\,\mathrm{Tr}(\bar{B}\gamma_\mu\gamma_5[\xi T^A\xi^\dagger+\xi^\dagger T^A\xi,B])+\frac{1}{2}d_1\frac{m_K^2}{\Lambda_\chi^2}\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5\{\xi T^A\xi^\dagger+\xi^\dagger T^A\xi,\chi_+\}B)\\&+\frac{1}{2}d_2\frac{m_K^2}{\Lambda_\chi^2}\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5(\xi T^A\xi^\dagger+\xi^\dagger T^A\xi)B\chi_+)+\frac{1}{2}d_3\frac{m_K^2}{\Lambda_\chi^2}\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5\chi_+B(\xi T^A\xi^\dagger+\xi^\dagger T^A\xi))\\&+\frac{1}{2}d_4\frac{m_K^2}{\Lambda_\chi^2}\mathrm{Tr}(\bar{B}\gamma^\mu\gamma_5 B\{\xi T^A\xi^\dagger+\xi^\dagger T^A\xi,\chi_+\})+\frac{1}{2}\bar{T}^\nu\gamma_\mu(\xi T^A\xi^\dagger-\xi^\dagger T^A\xi)T_\nu+\frac{1}{2}\mathcal{H}\,\bar{T}^\nu\gamma_\mu\gamma_5(\xi T^A\xi^\dagger+\xi^\dagger T^A\xi)T_\nu\\&+\frac{1}{2}\mathcal{C}\,\bar{T}_\mu(\xi T^A\xi^\dagger+\xi^\dagger T^A\xi)B+\frac{1}{2}\mathcal{C}\,\bar{B}(\xi T^A\xi^\dagger+\xi^\dagger T^A\xi)T_\mu+\frac{1}{2}\mathrm{Tr}(\bar{B}\gamma_\mu[\xi T^A\xi^\dagger-\xi^\dagger T^A\xi,B])\\&+\frac{i}{2}F_\pi^2\mathrm{Tr}\,T^A((\partial_\mu\Sigma)^\dagger\Sigma-\partial_\mu\Sigma\Sigma^+).\end{aligned}\tag{3}$$

The heavy baryon expansion of $\mathcal{L}_{0,1}$ and J^A_μ is obtained by defining the heavy baryon field $H(x)=\exp(im_N v\cdot x)(1+\hat{v}/2)B(x)$ (v_μ is the baryon velocity) and projecting out the postive energy states as in [3]. In this case, all baryon mass terms are removed from the Lagrangian at leading order, leaving only a dependence on the octet-decuplet mass splitting, $\delta=m_\Delta-m_N$. At subleading orders, there are recoil corrections in the form of $1/m_N$. In order to consistently include the decuplet we follow the small scale expansion proposed in [16]. In this approach the energy and momenta and the decuplet and octet mass difference δ are both treated as small expansion parameters in chiral counting. Note also that in the

TABLE I. The coefficients $\overline{\lambda}_{ij}^X$ for the wave function renormalization due to the decuplet intermediate states.

	π loop	kaon loop	η loop
$\overline{\lambda}_{pn}$	1	$\frac{1}{4}$	0
$\overline{\lambda}_{\Lambda\Sigma^-}$	$\frac{11}{24}$	$\frac{2}{3}$	$\frac{1}{8}$
$\overline{\lambda}_{\Xi^0\Xi^-}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$
$\overline{\lambda}_{p\Lambda}$	$\frac{7}{8}$	$\frac{3}{8}$	0
$\overline{\lambda}_{\Lambda\Xi^-}$	$\frac{1}{2}$	$\frac{5}{8}$	$\frac{1}{8}$
$\overline{\lambda}_{n\Sigma^-}$	$\frac{7}{12}$	$\frac{13}{24}$	$\frac{1}{8}$
$\overline{\lambda}_{\Sigma^0\Xi^-}$	$\frac{5}{24}$	$\frac{19}{24}$	$\frac{1}{4}$
$\overline{\lambda}_{pp}$	1	$\frac{1}{4}$	0
$\overline{\lambda}_{\Lambda\Lambda}$	$\frac{3}{4}$	$\frac{1}{2}$	0
$\overline{\lambda}_{\Sigma\Sigma}$	$\frac{1}{6}$	$\frac{5}{6}$	$\frac{1}{4}$
$\overline{\lambda}_{\Xi\Xi}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$

heavy baryon expansion, one makes the replacement $\gamma_\mu \to v_\mu$, $\gamma_\mu\gamma_5 \to 2S_\mu$, etc., where S_μ is the spin operator.

Renormalized matrix elements of $J_{\mu 5}^A$ between octet baryon states up to $\mathcal{O}(p^3)$ may be written as

$$\langle B_i|J_\mu^A|B_j\rangle = \Big\{ \alpha_{ij} + \overline{\alpha}_{ij}\frac{m_K^2}{\Lambda_\chi^2} + \sum_{X=\pi,K,\eta} [(\lambda_{ij}^X I_d^X + \overline{\lambda}_{ij}^X I_e^X)\alpha_{ij} + (\beta_{ij}^X I_a^X + \overline{\beta}_{ij}^X I_f^X + \widetilde{\beta}_{ij}^X I_g^X) + \gamma_{ij}^X I_b^X \alpha_{ij} + \theta_{ij}^X I_c^X \alpha_{ij}] \Big\} \overline{u}_{B_i}\gamma_\mu\gamma_5 u_{B_j}, \tag{4}$$

where the first term on the right-hand side is the lowest order one. The second term arises from the SB terms in Eq. (2). The third term in Eq. (4) arises from the wave function renormalization. The fourth term comes from the vertex correction diagram. The fifth term is the vertex correction from the tadpole diagram. The last term in Eq. (4) arises from the $\mathcal{O}(p)$ one-meson operators in J_μ^A. Details of the last three terms can be found in [13].

Terms with α_{ij}^X, $\overline{\alpha}_{ij}^X$, λ_{ij}^X, β_{ij}^X, γ_{ij}^X, and θ_{ij}^X arise from the contribution of octet states only and their expressions are given in [13]. The remaining terms come from the insertion of decuplet states in the loop. The expressions of $\overline{\lambda}_{ij}^X$, $\overline{\beta}_{ij}^X$, and $\widetilde{\beta}_{ij}^X$ are presented in Tables I, II, and III respectively.

TABLE II. The coefficients $\overline{\beta}_{ij}^X$ for the vertex corrections.

	π loop	kaon loop	η loop
$\overline{\beta}_{pn}$	$\frac{5}{6}$	$\frac{1}{6}$	0
$\overline{\beta}_{\Lambda\Sigma^-}$	$\frac{1}{2\sqrt{6}}$	$\frac{1}{4\sqrt{6}}$	0
$\overline{\beta}_{\Xi^0\Xi^-}$	$\frac{1}{24}$	$-\frac{1}{6}$	$-\frac{1}{8}$
$\overline{\beta}_{p\Lambda}$	$-\frac{\sqrt{6}}{4}$	$-\frac{\sqrt{6}}{8}$	0
$\overline{\beta}_{\Lambda\Xi^-}$	$\frac{\sqrt{6}}{8}$	$\frac{\sqrt{6}}{8}$	0
$\overline{\beta}_{n\Sigma^-}$	$-\frac{1}{6}$	$-\frac{1}{12}$	0
$\overline{\beta}_{\Sigma^0\Xi^-}$	$\frac{1}{6\sqrt{2}}$	$\frac{7}{12\sqrt{2}}$	$\frac{1}{4\sqrt{2}}$

The functions I_a^X etc., are defined as[1]

$$I_a^X = \left(\frac{m_X}{\Lambda_\chi}\right)^2 \ln\left(\frac{\mu}{m_X}\right)^2 + \pi\frac{m_X^3}{m_N\Lambda_\chi^2},$$

$$I_b^X = -\left(\frac{m_X}{\Lambda_\chi}\right)^2 \ln\left(\frac{\mu}{m_X}\right)^2,$$

$$I_c^X = \frac{\pi}{2}\frac{m_X^3}{m_N\Lambda_\chi^2},$$

$$I_d^X = \frac{3}{4} I_a^X. \tag{5}$$

For $X=K,\ \eta$ we have

$$I_e^X = 2\frac{\mathcal{C}^2}{\Lambda_\chi^2}\Big\{ -\Big[(2\delta^2 - m_X^2)\ln\left(\frac{\mu}{m_X}\right)^2 + 4\delta\sqrt{m_X^2-\delta^2}\arccos\frac{\delta}{m_X}\Big] + \frac{1}{m_N}\Big[\frac{2}{3}(m_X^2 - 4\delta^2)\sqrt{m_X^2-\delta^2}\arccos\frac{\delta}{m_X} + \delta\left(m_X^2 - \frac{4}{3}\delta^2\right)\ln\left(\frac{\mu}{m_X}\right)^2\Big]\Big\}, \tag{6}$$

[1]Here, $\mu \sim 1$ GeV denotes the renormalization scale. In our previous analysis [13], this scale was effectively set equal to m_N. Moreover, in that work, the variable μ denoted the ratio m_π/m_N and not the renormalization scale.

TABLE III. The coefficients $\tilde{\beta}^X_{ij}$ for the vertex corrections.

	π loop	kaon loop	η loop
$\tilde{\beta}_{pn}$	$\frac{8}{3}(D+F)$	$\frac{F+3D}{3}$	0
$\tilde{\beta}_{\Lambda\Sigma^-}$	$\frac{2}{\sqrt{6}}F$	$\frac{4}{\sqrt{6}}\left(F+\frac{2}{3}D\right)$	$\frac{1}{\sqrt{6}}D$
$\tilde{\beta}_{\Xi^0\Xi^-}$	$-\frac{D-F}{3}$	$\frac{5F+D}{3}$	$\frac{3F+D}{3}$
$\tilde{\beta}_{p\Lambda}$	$-\frac{1}{2\sqrt{6}}(3F+11D)$	$-\frac{3}{2\sqrt{6}}(F+D)$	0
$\tilde{\beta}_{\Lambda\Xi^-}$	$-\frac{1}{2\sqrt{6}}(3F-D)$	$\frac{3}{2\sqrt{6}}(D-F)$	$\frac{1}{\sqrt{6}}D$
$\tilde{\beta}_{n\Sigma^-}$	$\frac{1}{3}(D+5F)$	$\frac{1}{6}(5F+D)$	$\frac{1}{6}(3F-D)$
$\tilde{\beta}_{\Sigma^0\Xi^-}$	$\frac{\sqrt{2}}{6}(2D+F)$	$\frac{\sqrt{2}}{12}(15D+13F)$	$\frac{\sqrt{2}}{4}(D+F)$

$$I_f^X=\frac{20}{27}\frac{\mathcal{H}\mathcal{C}^2}{\Lambda_\chi^2}\left\{-\left[(2\delta^2-m_X^2)\ln\left(\frac{\mu}{m_X}\right)^2+4\delta\sqrt{m_X^2-\delta^2}\arccos\frac{\delta}{m_X}\right]+\frac{1}{m_N}\left[\frac{2}{3}(m_X^2-4\delta^2)\sqrt{m_X^2-\delta^2}\times\arccos\frac{\delta}{m_X}+\delta\left(m_X^2-\frac{4}{3}\delta^2\right)\ln\left(\frac{\mu}{m_X}\right)^2\right]\right\}, \tag{7}$$

$$I_g^X=\frac{2}{3}\frac{\mathcal{C}^2}{\Lambda_\chi^2}\left\{-\left[\left(m_X^2-\frac{2}{3}\delta^2\right)\ln\left(\frac{\mu}{m_X}\right)^2+\frac{4}{3}\frac{(m_X^2-\delta^2)^{3/2}}{\delta}\times\arccos\frac{\delta}{m_X}-\frac{2}{3}\pi\frac{m_X^3}{\delta}\right]-\frac{1}{m_N}\left[\frac{4}{3}(m_X^2-\delta^2)^{3/2}\times\arccos\frac{\delta}{m_X}+\delta\left(m_X^2-\frac{2}{3}\delta^2\right)\ln\left(\frac{\mu}{m_X}\right)^2\right]\right\}. \tag{8}$$

Replacing the combination

$$\frac{\arccos\frac{\delta}{m_X}}{\sqrt{m_X^2-\delta^2}}$$

in Eqs. (6)–(8) by

$$\frac{1}{\sqrt{\delta^2-m_X^2}}\ln\left(\frac{\delta+\sqrt{\delta^2-m_X^2}}{m_X}\right)$$

we obtain expressions for $I^\pi_{e,f,g}$. In this work we explicitly keep the pion loop contribution. If we truncate at order $\mathcal{O}(p^2)$ and ignore the pion loops and take $\delta=0$ and $m_\eta^2=\frac{4}{3}m_K^2$, we reproduce the expressions in [3,4] exactly. Note that we retain only loop corrections having nonanalytic dependence on quark masses. Analytic terms (e.g., $\propto m_K^2$) have been absorbed into the counterterms d_{1-4}.

III. N_c COUNTING

As discussed in a beautiful series of papers [8–12], the baryon axial vector currents have an expansion in $1/N_c$ involving SU(6) spin-flavor operators:

$$G^{ia}=q^\dagger\frac{\sigma^i}{2}\frac{\lambda^a}{2}q, \tag{9}$$

$$T^a=q^\dagger\frac{\lambda^a}{2}q, \tag{10}$$

$$J^i=q^\dagger\frac{\sigma^i}{2}q, \tag{11}$$

where q and $q^\dagger$ are SU(6) quark creation and annihilation operators and λ^a and σ^i are the Gell-Mann and Pauli matrices, respectively. At leading order in $1/N_c$, one has

$$J^a_{i5}\equiv A^{ia}\propto G^{ia}, \tag{12}$$

where the coefficient of proportionality is of order unity and where terms of relative order $1/N_c$ have been dropped. The N_c counting rules give $G^{ia}\sim N_c$. Thus the $\mathcal{O}(p^0)$ current is $\mathcal{O}(N_c)$, while loop corrections, which contain three insertions of A^{ia} divided by $F_\pi^2\sim N_c$ are nominally of order N_c^2. However, the SU(6) commutator algebra

$$[G^{ia},G^{jb}]=\frac{i}{4}\delta^{ij}f^{abc}T^c+\frac{i}{6}\delta^{ab}\epsilon^{ijk}J^k+\frac{i}{2}d^{abc}\epsilon^{ijk}G^{kc} \tag{13}$$

implies that the $\mathcal{O}(p^2)$ loop corrections, which depend on double commutators of A^{ia}, are actually of order N_c^0, since each commutator reduces the naive counting by one power of N_c.

Because the $\mathcal{O}(p^2)$ loops are divergent, there must exist counterterms of the same order which absorb the infinities. The most general $\mathcal{O}(p^2)$ operators arising at this order include those proportional to $d_{1\ldots4}$ in Eq. (2).[2] These operators involve one insertion of A^{ia} times m_P^2/Λ_χ^2, where m_P is the Goldstone boson mass. The latter is $\mathcal{O}(N_c^0)$ whereas $\Lambda_\chi^2=(4\pi F_\pi)^2$ is $\mathcal{O}(N_c)$. Thus, the $\mathcal{O}(p^2)$ counterterms are $\mathcal{O}(N_c^0)$. Self-consistency of the theory implies that the $\mathcal{O}(p^2)$ loop corrections must also be of $\mathcal{O}(N_c^0)$. Otherwise, there would exist a mismatch between the divergent loops and the counterterms which render them finite in the large N_c limit. In retrospect, then, one might have anticipated the existence

[2]There exist additional operators proportional to m_π^2 and q^2 as well. The finite parts of the former are numerically insignificant while the latter do not contribute to the $q^2=0$ currents. Thus we do not show them explicitly, though their presence is required to remove the divergences.

TABLE IV. The separation of fit results into pure $\mathcal{O}(p^0)$ and $\mathcal{O}(p^2)$ pieces where we have used $\delta=0.3$ GeV, $m_N\to\infty$, $\mathcal{C}=-1.5$, and $\mathcal{H}=-2.25$ as inputs. The fit yields $D=0.63$, $F=-0.45$, $d_1=0.79$, $d_2=1.87$, $d_3=1.43$, and $d_4=-1.13$ with $\chi^2=0.15$.

	Full fit results	Tree level only	$\mathcal{O}(p^2)$ only
g^A_{pn}	1.28	0.18	1.10
$g^A_{\Lambda\Sigma^-}$	0.59	0.51	0.08
$g^A_{p\Lambda}$	-0.83	0.29	-1.12
$g^A_{\Lambda\Xi^-}$	0.29	-0.81	1.10
$g^A_{n\Sigma^-}$	0.32	1.08	-0.76
$g^A_{\Sigma^0\Xi^-}$	0.97	0.13	0.84
$g^A_{\Xi^0\Xi^-}{}^\dagger$	-0.02	1.08	-1.10
$g^{A\dagger}_8$	0.32	-0.57	0.89

of a large-N_c spin-flavor algebra whose affect is to reduce the nominal N_c order of the loops to match that of the counterterms.

In contrast, there exist no counterterm operators at $\mathcal{O}(p^3)$, and the loop contributions of this order are entirely finite and nonanalytic in m_q. Thus one has no self-consistency requirement at $\mathcal{O}(p^3)$ involving counterterms and loops to force a reduction in the nominal N_c order of the latter. In particular, the $\mathcal{O}(p^3)$ wave function renormalization and vertex corrections involve three insertions of A^{ia} divided by $F^2_\pi\times m_N$. Since m_N is $\mathcal{O}(N_c)$, these loop effects are nominally order N_c. In the absence of any algebra which reduces this nominal order, one might expect them to be numerically significant. As a practical matter, we find that inclusion of decuplet intermediate states produces no cancellations indicative of an algebraic reduction in the nominal N_c order of these graphs. Similarly, the $\mathcal{O}(p^3)$ seagull graphs involving the chiral connection, which have nominal chiral order $\mathcal{O}(N_c^0)$, receive only octet contributions, so no cancellations are possible in this case. We also find that these contributions can be significant. Indeed, as we show below, the $\mathcal{O}(p^3)$ contributions are generally as large or larger than the $\mathcal{O}(p^2)$ terms, in accordance with naive scaling arguments.

TABLE V. The separation of fit results into pure $\mathcal{O}(p^0)$ and $\mathcal{O}(p^2)$ pieces where we have used $\delta=0.3$ GeV, $m_N\to\infty$, $\mathcal{C}=-2D$, and $\mathcal{H}=-3D$ as inputs. The fit yields $D=0.46$, $F=0.31$, $d_1=-0.80$, $d_2=0.93$, $d_3=-0.63$, and $d_4=0.78$ with $\chi^2=0.002$.

	Full fit results	Tree level only	$\mathcal{O}(p^2)$ only
g^A_{pn}	1.26	0.77	0.49
$g^A_{\Lambda\Sigma^-}$	0.62	0.38	0.24
$g^A_{p\Lambda}$	-0.89	-0.57	-0.32
$g^A_{\Lambda\Xi^-}$	0.32	0.19	0.13
$g^A_{n\Sigma^-}$	0.34	0.15	0.19
$g^A_{\Sigma^0\Xi^-}$	0.92	0.54	0.38
$g^A_{\Xi^0\Xi^-}{}^\dagger$	0.15	0.15	0
$g^{A\dagger}_8$	0.17	0.13	0.04

TABLE VI. The separation of fit results into pure $\mathcal{O}(p^0)$ and $\mathcal{O}(p^2)$ pieces where we have used $\delta=0.3$ GeV, $m_N\to\infty$, $d_{1-4}=0$, $\mathcal{C}=-2D$, and $\mathcal{H}=-3D$ as inputs. The fit yields $D=0.51$ and $F=0.25$ with $\chi^2=1.1$.

	Full fit results	Tree level only	$\mathcal{O}(p^2)$ only
g^A_{pn}	1.10	0.76	0.34
$g^A_{\Lambda\Sigma^-}$	0.66	0.42	0.24
$g^A_{p\Lambda}$	-0.88	-0.51	-0.37
$g^A_{\Lambda\Xi^-}$	0.31	0.10	0.21
$g^A_{n\Sigma^-}$	0.29	0.26	0.03
$g^A_{\Sigma^0\Xi^-}$	1.05	0.54	0.51
$g^A_{\Xi^0\Xi^-}{}^\dagger$	0.35	0.26	0.09
$g^{A\dagger}_8$	0.26	0.07	0.19

IV. NUMERICAL ANALYSIS

In Tables IV–VII we present various fits to the octet axial vector currents, showing the contributions arising at various orders in p. For notational simplicity we define the axial couplings g^A_{ij} as

$$\langle B_i|J^A_\mu|B_j\rangle = g^A_{ij}\overline{u_{B_i}}\gamma_\mu\gamma_5 u_{B_j}, \tag{14}$$

where we have omitted the induced pseudoscalar terms. In general, we have eight low-energy constants (LEC's) to be determined: D, F, d_{1-4}, $\mathcal{H}$, and $\mathcal{C}$. However, there exist experimental data for only six octet matrix elements [15]. Consequently, we must invoke additional assumptions in order to complete the analysis.

The constants $\mathcal{C}$ and $\mathcal{H}$ can be treated using one of several approaches. Drawing entirely on experimental data, the magnitude of $\mathcal{C}$ can be determined from the decay width of the Δ. At leading order, one has $|\mathcal{C}|=1.5$ [16], which is consistent with the large N_c prediction [10,11]. Loop corrections to this result arise at $\mathcal{O}(p^2)$. Since $\mathcal{C}$ enters the axial vector currents at $\mathcal{O}(p^2)$, chiral corrections to the value of $\mathcal{C}$ as determined from the Δ decay width affect our analysis at $\mathcal{O}(p^4)$. Unfortunately, the phase of $\mathcal{C}$ cannot be determined in this manner, and so one must rely on auxiliary considerations. For ex-

TABLE VII. The separation of fit results into pure $\mathcal{O}(p^0)$, $\mathcal{O}(p^2)$, and $\mathcal{O}(p^3)$ pieces where we have used $\delta=0.3$ GeV, $m_N=0.94$ GeV, $\mathcal{C}=-2D$, and $\mathcal{H}=-3F$ as inputs. The fit yields $D=0.39$, $F=0.22$, $d_1=-1.97$, $d_2=1.14$, $d_3=-0.45$, and $d_4=-0.06$ with $\chi^2=0.12$.

	Full fit results	Tree level only	$\mathcal{O}(p^2)$ only	$\mathcal{O}(p^3)$ only
g^A_{pn}	1.26	0.61	0.41	0.24
$g^A_{\Lambda\Sigma^-}$	0.58	0.32	0.14	0.12
$g^A_{p\Lambda}$	-0.92	-0.43	-0.11	-0.38
$g^A_{\Lambda\Xi^-}$	0.26	0.11	0.05	0.10
$g^A_{n\Sigma^-}$	0.33	0.17	0.03	0.13
$g^A_{\Sigma^0\Xi^-}$	0.87	0.43	0.05	0.39
$g^A_{\Xi^0\Xi^-}{}^\dagger$	0.22	0.17	-0.02	0.07
$g^{A\dagger}_8$	0.32	0.08	0.17	0.07

ample, SU(6) symmetry implies $\mathcal{C}$ and D have the opposite phase. In what follows, we make this choice for the phase.

The situation regarding $\mathcal{H}$ is more problematic. This LEC does not appear at leading order in any physical decay amplitude. It does, however, give the strong $\pi\Delta\Delta$ coupling at leading order [16]. A determination of this constant is, therefore, highly dependent on model assumptions. In the large N_c limit, for example, $\mathcal{H}=-\frac{9}{5}(D+F)$. Various quark models yield the same result [18–20]. On the other hand, a light cone QCD sum rule analysis [17] yields $|\mathcal{H}|=1.35$, which is only half of a large N_c or quark model prediction and is approximately the same value as extracted from from the isobar production experiments in $\pi^- p\to\pi^+\pi^- n$ near threshold [21]. This constant has also been extracted from decay widths using HBCPT to $\mathcal{O}(p^2)$ [22]. Recently, $\mathcal{H}$ was determined from a fit to phase shift data in the fourth order chiral perturbation theory analysis [23]. The results imply $0.94\leqslant\mathcal{H}\leqslant 2.65$. While the magnitude of $\mathcal{H}$ for this range is consistent with both the large N_c and QCD sum rule analyses, the phase differs from all other approaches. It was emphasized in Ref. [23], however, that $\mathcal{H}$ enters pion nucleon scattering at third order loop so it cannot be pinned down precisely. Fortunately, in the case of the axial vector currents, $\mathcal{H}$ arises at $\mathcal{O}(p^3)$, so the impact of uncertainty in this constant is not as pronounced as in the case of $\mathcal{C}$.

A final possibility for treating $\mathcal{C}$ and $\mathcal{H}$ is to follow the analysis of Refs. [8–12] and invoke the SU(6) relations: $\mathcal{C}=-2D$, $\mathcal{H}=-3D$.[3] Doing so reduces the number of fit parameters to six.[4] The authors of Refs. [8–12] found that use of SU(6) relations among the LECs minimizes the size of the $\mathcal{O}(p^2)$ loop corrections, in accordance with the cancellations expected from large N_c arguments. It is not possible to apply similar relations to d_{1-4}, however, since they parametrize explicit symmetry-breaking terms in the Lagrangian.

In Table IV we give a fit through $\mathcal{O}(p^2)$ using the experimentally determined magnitude for $\mathcal{C}$, a phase opposite to that of D, and the quark model value for $\mathcal{H}$. The remaining six LECs are determined from the nucleon and hyperon semileptonic decay data. Under these conditions, the $\mathcal{O}(p^2)$ corrections (including both loop effects and symmetry breaking terms) are generally as large as the $\mathcal{O}(p^0)$ contributions. However, invoking the SU(6) relations among D, $\mathcal{C}$, and $\mathcal{H}$ changes this situation considerably, as illustrated in Table V. In this case, the relative importance of the $\mathcal{O}(p^2)$ terms is considerably reduced and the χ^2 improved. In Table VI we show the corresponding fit using the SU(6) relations but setting $d_{1-4}=0$. The latter fit corresponds roughly to the analysis of Refs. [8–12], which illustrated the impact of $\mathcal{O}(p^2)$ loop cancellations in the symmetry limit. Generally speaking, inclusion of d_{1-4} improves the quality of the fit as well as the scaling behavior of the chiral expansion through $\mathcal{O}(p^2)$.

In Table VII we give the best fit through $\mathcal{O}(p^3)$. Here, we have used the SU(6) relations for D, $\mathcal{C}$, and $\mathcal{H}$ in order to produce the cancellations at $\mathcal{O}(p^2)$. We observe that the $\mathcal{O}(p^3)$ contributions are generally as large or larger than the $\mathcal{O}(p^2)$ terms and, in several channels, as large as the $\mathcal{O}(p^0)$ terms. This pattern becomes even more pronounced away from the SU(6) limit for the LECs, in which case neither the $\mathcal{O}(p^2)$ nor the $\mathcal{O}(p^3)$ terms scale as expected.

The breakdown of the chiral expansion which we observe at $\mathcal{O}(p^3)$ reflects a number of factors: the large magnitude of the kaon mass, which appears in the numerator of the expressions in Eqs. (5); the apparent absence of cancellations (and an underlying large N_c spin-flavor algebra) among the recoil order corrections; and the appearance of factors of π in integrals I_a^X and I_c^X arising at this order.

V. DISCUSSION

It has been known for many years that tree-level SU(3) relations are remarkably successful in describing a number of the low-lying properties of hadrons, such as pseudoscalar masses and baryon axial vector currents. Ideally, chiral perturbation theory—together with the large N_c expansion—should suffice to explain why these relations work so well. With such an understanding in hand, one would have had considerable confidence in exploiting these relations to determine quantities for which one has no direct measurement, such as the strange quark contribution to the nucleon spin, Δs. In the present study, however, we observe that the chiral expansion for baryon octet axial vector currents does not appear to be under control. While large N_c considerations imply that the expansion works reasonably well through $\mathcal{O}(p^2)$, it breaks down completely at $\mathcal{O}(p^3)$.[5] While a theoretical justification for applying SU(3) symmetry to the octet axial vector currents may exist,[6] we are unable to provide one at this time.

As a practical consequence of this situation, we consider the determination of Δs from polarized deep inelastic scattering (DIS) data. As shown in Ref. [25], one may express Δs in terms of the polarized structure function integrals

$$\Gamma_{p,n}=\int_0^1 dx\, g_1^{p,n}(x) \tag{15}$$

as

$$\Delta s=\frac{3}{2}[\Gamma_p+\Gamma_n]-\frac{5\sqrt{3}}{6}g_8^A, \tag{16}$$

[3]We have also used the relations arising from the inclusion of $1/N_c$ corrections to Eq. (12): $\mathcal{C}=-2D$, $\mathcal{H}=3D-9F$ in our fit. The fit results turn out to be the same. We thank E. Jenkins for suggesting this point.

[4]A further reduction in the number of parameters may occur when the double expansion in m_q and $1/N_c$ of Ref. [24] is used.

[5]However, we observe that at $\mathcal{O}(p^2)$ there exist some channels for which the large N_c cancellations are not strong (see, e.g., Table V).

[6]See, e.g., the regulator scheme proposed in Ref. [26].

where g_8^A is the axial vector coupling associated with the matrix element $\langle p|J_\mu^8|p\rangle$. The combinations of LECs required for this matrix element are

$$\alpha_{pp}^8 = \frac{1}{2\sqrt{3}}(3F-D),$$

$$\beta_{pp}^{8,\pi} = \frac{\sqrt{3}}{8}(3F-D)(D+F)^2,$$

$$\beta_{pp}^{8,K} = \frac{1}{\sqrt{3}}\left(\frac{2}{3}D^3 - 2D^2F\right),$$

$$\beta_{pp}^{8,\eta} = \frac{1}{24\sqrt{3}}(3F-D)^3,$$

$$\overline{\beta}_{pp}^{8,\pi} = \frac{\sqrt{3}}{2},$$

$$\overline{\beta}_{pp}^{8,K,\eta} = 0,$$

$$\tilde{\beta}_{pp}^{8,\pi,\eta} = 0,$$

$$\tilde{\beta}_{pp}^{8,K} = \frac{\sqrt{3}}{2}(D-F),$$

$$\overline{\alpha}_{pp}^8 = \frac{1}{\sqrt{3}}\left(\frac{1}{2}d_2 - 2d_4\right),$$

$$\gamma_{pp}^{8,K} = -\frac{3}{2}, \quad \gamma_{pp}^{8,\pi,\eta} = 0,$$

$$\theta_{pp}^{8,\pi,K,\eta} = -4\gamma_{pp}^{8,\pi,K,\eta}.$$

The numerical separation of g_8^A through $\mathcal{O}(p^3)$ is given in Table VII and yields

$$\Delta s = 0.14 - [0.12 + 0.25 + 0.10], \tag{17}$$

where the numbers in square brackets correspond, respectively, to the order p^0, p^2, and p^3 contributions to g_8^A. Since the chiral expansion is not converging for Δs, we do not quote a total for this quantity nor can we estimate a theoretical uncertainty. In contrast, extractions of Δs from semi-inclusive measurements performed by the Hermes collaboration [27] or from elastic neutrino-nucleon scattering [28,29] are not plagued by large SU(3)-breaking uncertainties, making them in principle more reliable probes of the flavor content of the nucleon spin.

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