Low Q^2 wave functions of pions and kaons and their parton distribution functions

C. Avila* and J. C. Sanabria†
Departamento de Física, Universidad de los Andes, AA 4976, Santafé de Bogotá, Colombia

J. Magnin‡
Centro Brasileiro de Pesquisas Físicas, Rua Dr. Xavier Sigaud 150, Urca, 22290-180, Rio de Janeiro, Brazil
(Received 27 May 2002; published 15 August 2002)

The low Q^2 wave functions of pions and kaons are studied in terms of an expansion in hadron-like Fock state fluctuations. In this formalism, the pion and kaon wave functions are related to one another, allowing us, in principle, to obtain the full parton structure of both pions and kaons from the measurement of light valence quark distributions. We show that the model also predicts the intrinsic sea of pions and kaons needed at the low Q^2 scale where perturbative evolution starts. Finally, we analyze the feasibility of a method to extract the kaon's parton distribution functions within this approach and compare it with available experimental data.

DOI: 10.1103/PhysRevD.66.034016 PACS number(s): 14.40.Aq, 12.39.−x, 14.65.−q

I. MOTIVATION

At present the only information available on the structure of unstable mesons is on parton distributions functions (PDFs) in pions. These PDFs are extracted from Drell-Yan (DY) dilepton and prompt photon production in pion-nucleon interactions.

In principle, the same type of experiments can be used to extract information on parton distributions in kaons by just replacing the pion beam by a kaon one. However, this requires high intensity kaon beams, which are not easily attainable in present day experiments. Furthermore, there is an additional difficulty which is inherent in the kaon structure: strange valence quarks in the kaon must annihilate with strange quarks in the target particles to produce a DY dilepton pair. As targets are made of nucleons, and strange quarks are in the sea of them, these s-$\bar{s}$ annihilation processes are expected to have a small contribution to the total DY dilepton cross section. Consequently, it is very difficult to have a precise measurement of the strange valence quark distribution in kaons. Other procedures involving the detection of a DY dilepton pair accompanied by a fast pion in the final state have been proposed to measure the strange quark distribution in kaons [1], but they rely on several assumptions coming from recombination models.

The situation, however, is radically different with the light valence quark distribution. In fact, the $\bar{u}_K$ distribution can be measured rather well in DY experiments since this is, by far, the major contribution to $q\bar{q}\to l^+l^-$ processes in K^--nucleon interactions. The first, and, to our knowledge, the only attempt in this direction was made by the NA3 Collaboration [2], who measured the ratio of the u-quark distribution in kaons to the u-quark distribution in pions, $\bar{u}_K/\bar{u}_\pi$. In this experiment, the ratio $\bar{u}_K/\bar{u}_\pi$ was extracted from DY dimuon production in 150 GeV/c K^-,π^--nucleus interactions. The measurement is, however, subject to large uncertainties due to the limited statistics of the experiment and several assumptions made in the extraction of the $\bar{u}_K/\bar{u}_\pi$ ratio.

These facts indicate that a different approach must be followed in order to obtain information on the parton structure of unstable hadrons, in particular for kaons.

In Refs. [3–5], a model of nucleons in terms of meson-baryon bound state fluctuations was developed. This model describes well the $\bar{d}$-$\bar{u}$ and $\bar{d}/\bar{u}$ asymmetries in the nucleon sea as measured by the E866 Collaboration [6]. The model also describes qualitatively the s-$\bar{s}$ asymmetry in nucleons observed in a recent global analysis of deep inelastic scattering (DIS) data in nucleons [7]. Moreover, it is interesting to note that this model provides a consistent scheme to generate nonperturbative sea quark and gluon distributions at the low Q^2 scale. Let us remark that, as already noted by several authors [8], these nonperturbative, valencelike, sea quark and gluon distributions at the low Q^2 input scale are needed to fit DIS data.

The facts above indicate that this could be a sensible approach to the problem of having a low Q^2 model of hadrons. In view of this, it is worth extending the model to pions and kaons. As we shall show in the following, within this model the parton structures of pions and kaons are related to one another. This allows us, in principle, to extract information about the kaon structure from precise measurements of the pion one by means of a rather direct procedure. It is also interesting to note that this procedure only needs the measurement of the pion's PDF and the ratio $\bar{u}_K/\bar{u}_\pi$, avoiding any measurement of the strange quark distribution in kaons.

The paper is organized as follows. In the next section we will study the low Q^2 wave functions of pions and kaons. In Sec. III, we shall try to extract information on the kaon's PDF in terms of the model and the available experimental data, and Sec. IV is devoted to conclusions and further discussion.

II. LOW Q^2 WAVE FUNCTIONS OF PIONS AND KAONS

To start with, let us consider the π^- and K^-. Following Ref. [5], their wave functions can be expanded as

*Electronic address: cavila@uniandes.edu.co
†Electronic address: jsanabri@uniandes.edu.co
‡Electronic address: jmagnin@cbpf.br

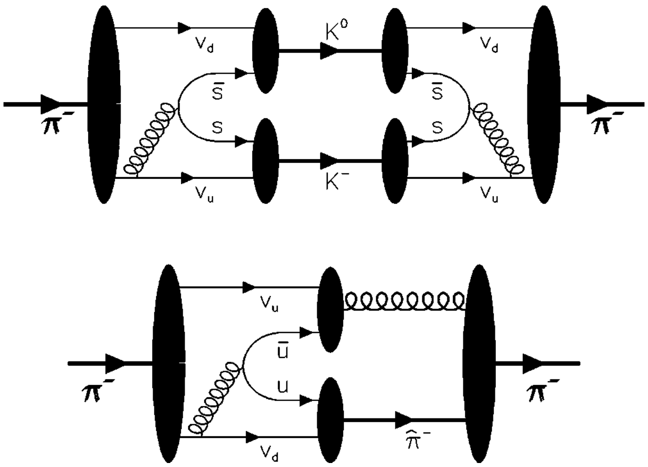

FIG. 1. Schematic representation of a $|\pi^-\rangle \to |K^0 K^-\rangle$ fluctuation (upper) and a process giving rise to the $|\pi^-\rangle \to |\pi^- g\rangle$ fluctuation (lower).

$$|\pi^-\rangle = a_0^\pi |\pi^-\rangle + a_1^\pi |\pi^- g\rangle + a_2^\pi |K^0 K^-\rangle + \cdots,$$

$$|K^-\rangle = a_0^K |K^-\rangle + a_1^K |K^- g\rangle + a_2^K |\overline{K}^0 \pi^-\rangle + \cdots \tag{1}$$

at some low Q_v^2 scale. The first terms in the right-hand side (RHS) of Eqs. (1) are the bare meson states, which are formed by dressed valence quarks or *valons* [9]. Fluctuations in the above expansions have a two-step origin: first a valon emits a gluon which subsequently splits into a $q\bar{q}$ pair, and secondly, the $q\bar{q}$ pair interacts with valons so as to form the Fock state fluctuation [3]. As discussed in Ref. [5], we assume that a quark or antiquark and a valon of the same flavor do not interact to form a neutral, unflavored, virtual meson structure but annihilate nonperturbatively to a gluon. This $\bar{q}v_q$ or $qv_{\bar{q}}$ anihilation gives rise to the second terms in the expansions of Eqs. (1) (see Fig. 1).

Hadrons in the Fock state fluctuations on the RHS of Eqs. (1) are assumed to be formed only by *valons*. These fluctuations are responsible for the nonperturbative—*intrinsic*—$q\bar{q}$ sea, which should provide the necessary binding among constituent quarks to form hadrons [5,10]. In addition, the second term in the RHS of Eqs. (1) provides the intrinsic gluon distribution in the pion and kaon, respectively.

It is worth noting that, on a very general basis, individual hadrons in the $|MM'\rangle$ Fock states must be colored. The same is also true for the $|Mg\rangle$ fluctuations in the second term of Eqs. (1) as long as the gluon is in a color octet state. However, the fluctuation itself is colorless. Notice that, if the two components of a generic fluctuation are in the **8** representation of color SU(3), and since $\mathbf{8}\otimes\mathbf{8} = \mathbf{1}\oplus\mathbf{8}\oplus\mathbf{8}\oplus\cdots$, then there is a singlet (colorless) representation where the fluctuation can be accommodated.

In the expansions of Eqs. (1) we have neglected higher order contributions involving heavier mesons and fluctuations to Fock states containing more than two mesons. These fluctuations should be far off shell and they can be safely ignored at this point.

The coefficients a_i^M, $M=\pi,K$, $i=1,2,3,\ldots$, in Eqs. (1) are constrained by probability conservation: $\Sigma_i |a_i^M|^2 = 1$.

Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution to higher Q^2 generates the perturbative—*extrinsic*—sea of $q\bar{q}$ pairs and gluons.

Since valon distributions in pions and kaons are related by

$$v_\pi(x) \equiv v_{\bar{u}/\pi^-}(x) = v_{d/\pi^-}(x) = v_{u/\pi^+}(x) = v_{\bar{d}/\pi^+}(x),$$

$$v_K(x) \equiv v_{\bar{u}/K^-}(x) = v_{d/K^\circ}(x) = v_{\bar{d}/\bar{K}^\circ}(x) = v_{u/K^+}(x),$$

$$v_{s/K} \equiv v_{s/K^-}(x) = v_{\bar{s}/K^+}(x) = v_{s/\bar{K}^\circ}(x) = v_{\bar{s}/K^\circ}(x), \tag{2}$$

due to isospin invariance, then at the Q_v^2 scale parton distribution functions are given by

$$\bar{u}_\pi(x) = d_\pi(x)$$
$$= |a_0^\pi|^2 v_\pi(x) + |a_1^\pi|^2 P_{\pi g}\otimes v_\pi + |a_2^\pi|^2 P_{KK}\otimes v_K,$$

$$s_\pi(x) = \bar{s}_\pi(x) = |a_2^\pi|^2 P_{KK}\otimes v_{s/K},$$

$$g_\pi(x) = |a_1^\pi|^2 P_{g\pi}(x) \tag{3}$$

for pions, and

$$\bar{u}_K(x) = |a_0^K|^2 v_K(x) + |a_1^K|^2 P_{Kg}\otimes v_K + |a_2^K|^2 P_{\pi K}\otimes v_\pi,$$

$$s_K(x) = |a_0^K|^2 v_{s/K^-}(x) + |a_1^K|^2 P_{Kg}\otimes v_{s/K^-} + |a_2^K|^2 P_{K\pi} \otimes v_{s/K},$$

$$d_K(x) = |a_2^K|^2 P_{\pi K}\otimes v_\pi,$$

$$\bar{d}_K(x) = |a_2^K|^2 P_{K\pi}\otimes v_K,$$

$$g_K(x) = |a_1^K|^2 P_{gK}(x) \tag{4}$$

for kaons. In Eqs. (3) and (4)

$$P_{MM'}\otimes v_{q/M} \equiv \int_x^1 \frac{dy}{y} P_{MM'}(y)\, v_{q/M}\left(\frac{x}{y}\right) \tag{5}$$

is the probability density of the nonperturbative contribution to the parton distribution coming from the $|MM'\rangle$ fluctuation [4,5]. A brief description about how to obtain the $P_{MM'}$ probability densities is given in the Appendix.

Looking at Eqs. (3) and (4) it follows that, since the $P_{MM'}$ probability densities are calculable within the model, the knowledge of the valon distributions will determine the entire structure of pions and kaons up to the coefficients giving the probability of each individual fluctuation. Furthermore, only three valon distributions must be determined, namely, v_π, v_K, and $v_{s/K}$.

In order to determine the valon distributions, one must note that the second of Eqs. (3) gives the strange quark distributions in pions only in terms of $v_{s/K}$. Then, by using the convolution theorem applied to Mellin transforms we can formally invert that expression to obtain

$$v_{s/K}(n)=\frac{s_\pi(n)}{|a_2^\pi|^2 P_{KK}(n)}, \tag{6}$$

where $f(n)$ is the Mellin transform of $f(x)$. A deevolution from the experimental Q^2 to the Q_v^2 scale is understood in s_π. Notice that the above equation not only determines the shape of $v_{s/K}$, but also fixes the coefficient $|a_2^\pi|^2$, since the strange valon distribution must be normalized to 1.

Once $v_{s/K}(x)$ is known, the $v_K(x)$ distribution becomes fixed by momentum conservation; thus

$$v_K(x)=v_{s/K}(1-x). \tag{7}$$

Then, by measuring only the strange quark distribution in pions, the valon probability densities in kaons are determined. Now, by measuring $\overline{u}_\pi$ and g_π we can determine the remaining v_π through

$$v_\pi(n)=\frac{\overline{u}_\pi(n)-|a_2^\pi|^2 P_{KK}(n) v_K(x)}{|a_0^\pi|^2+|a_1^\pi|^2 P_{\pi g}(n)}, \tag{8}$$

where we have used again the convolution theorem applied to Mellin transforms. Once again, a deevolution from the experimental Q^2 to the Q_v^2 scale is understood in both $\overline{u}_\pi$ and g_π. Since the coefficients $|a_1^\pi|^2$ and $|a_2^\pi|^2$ are fixed once the strange quark and gluon distributions in pions are measured [see the second and third of Eqs. (3)], there are no free parameters in Eq. (8) due to probability conservation.

To extract the shape of the required valon distributions and the values of the coefficients $|a_i^\pi|^2$ $(i=1,2,3)$, it is worth noting that $\pi p \to \gamma X$ cross sections are dominated by qg scattering; thus they are sensitive to the gluon distribution. On the other hand, the difference of cross sections $\sigma(\pi^- p) - \sigma(\pi^+ p)$ is dominated by $q\overline{q}$ anihilation, allowing in this way the determination of quark distributions in pions. Furthermore, in Ref. [11] it was argued that adequate linear combinations of $\pi^\pm$-nucleon DY cross sections should allow the determination of valence densities in pions, independently of sea quark distributions. Having the gluon and valence quark distributions, the sea quark densities can be determined.

Thus, the determination of the valon densities v_π, v_K, and $v_{s/K}$ and the parameters $|a_i^\pi|^2$ can be achieved by measuring only the pion parton structure. In practice, however, some sort of iterative procedure is needed to determine v_π, v_K, and $v_{s/K}$ from experimental data since v_π enters in the calculation of the $P_{\pi g}$ and P_{KK} probability densities.

Once the valon distributions are known, we can proceed to determine the kaon PDF. To this end, however, some experimental input on kaon parton distributions is needed. The $\overline{u}_K$ distribution can be measured in K^--nucleon DY experiments; thus we can use the relationship in the first of Eqs. (4) to fix the coefficients a_i^K $(i=1,2,3)$ in the kaon wave function. Since v_K and $v_{s/K}$ have been determined previously, the kaon PDFs become completely fixed. This will be done in the next section.

III. KAON PARTON DISTRIBUTIONS

Most of the existing parametrizations of pion PDFs have been obtained assuming an $\mathrm{SU}(3)_{flavor}$ symmetric sea. Furthermore, in most cases the $q\overline{q}$ sea has been generated perturbatively through DGLAP evolution [12–14]. Thus it is not possible to determine the valon distributions v_π, v_K, and $v_{s/K}$ from present parametrizations. This will require us first to determine separately the valence and sea distributions in pions, and secondly to reanalyze the existing data on $\pi^\pm$-nucleon dilepton and prompt photon production in terms of the model presented here. Consequently, meanwhile, we refrain from any attempt at determining the valon distributions from experimental data. Instead, we will use the valon distributions in pions and kaons proposed in Ref. [9],

$$\begin{aligned} v_\pi(x)&=1,\\ v_K(x)&=\alpha_K x^{a-1}(1-x)^{b-1},\\ v_{s/K}(x)&=\alpha_K x^{b-1}(1-x)^{a-1}, \end{aligned} \tag{9}$$

where a and b are related by $a/b=m_l/m_h \sim 2/3$. m_l and m_h are the light and strange valon masses in kaons and α_K is a normalization constant.

These simple valon parametrizations would allow us to have both an idea of the feasibility of this method to extract the kaon's PDF, as well as some insight into the kaon low Q_v^2 structure.

To obtain the values of the $|a_i^K|^2$ coefficients in the kaon wave function, data for $\overline{u}_k/\overline{u}_\pi$ by the NA3 Collaboration [2] were fitted using

$$\frac{\overline{u}_K(x)}{\overline{u}_\pi(x)}=\frac{|a_0^K|^2 v_K(x)+|a_1^K|^2 P_{Kg}\otimes v_K(x)+|a_2^K|^2 P_{\pi K}\otimes v_\pi(x)}{\overline{u}_\pi(x)}, \tag{10}$$

together with the valon distributions of Eqs. (9). Notice that the fitting function has only two free parameters due to probability conservation. For the $\overline{u}_\pi$ distribution we have used two different parametrizations; the Gluck-Reya-Vogt set P (GRV-P) [14] distribution at the input scale $Q^2=0.4$ GeV^2, and the $\overline{u}_\pi$ distribution at $Q_0^2 \simeq 0.5$ GeV^2 obtained in Ref. [15] by means of a Monte Carlo model of hadrons. This enables us to test the sensitivity of the model to the shape of the $\overline{u}_\pi$ distribution.

In order to do the fits we assumed that the $\overline{u}_K/\overline{u}_\pi$ ratio is independent of Q^2. We recognize that this assumption could be questionable, but given the large error bars of the NA3 data, and the uncertainty in the valon distributions, the fits are hardly sensitive to QCD evolution.

The results of the fits are displayed in Fig. 2 and in Table I. All the fits were done using $a=1.5$ and $b=2.25$ in the v_K distribution of Eqs. (9). As can be seen in the figure, the main effect of using a different $\overline{u}_\pi$ distribution is in the low x (<0.2) region. But in this region, the pion valence distribution is not well known.

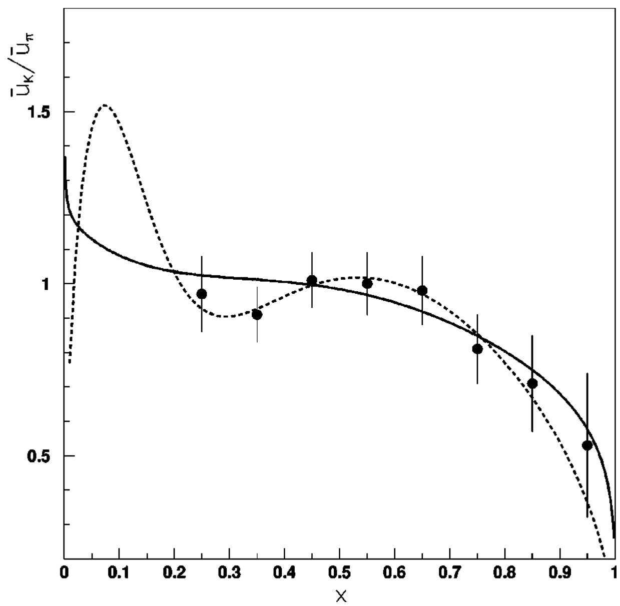

FIG. 2. $\overline{u}_K/\overline{u}_\pi$ as a function of x. Data are from Ref. [2]. Dashed line is the fit using the $\overline{u}_\pi$ distribution of Ref. [14]; the solid line is the fit using the $\overline{u}_\pi$ distribution given in Ref. [15].

A noticeable effect is also evident in the value of the $|a_i^K|^2$ coefficients (see Table I). In fact, using the GRV-P parametrization, the probability of the $|\overline{K}^0\pi^-\rangle$ fluctuation of the kaon is bigger than the probability of the $|K_0\rangle$ state, opposite to intuition. On the other hand, using the $\overline{u}_\pi$ distribution of Ref. [15], one gets $|a_1^K|^2 \sim |a_2^K|^2$ and $|a_0^K|^2 > |a_1^K|^2$, as expected. Given the uncertainties coming mainly from the shape of the valon distributions, these facts have no special meaning regarding valence distributions in pions. However, the general scheme proposed seems to be significant in order to extract the kaon's PDF.

In Fig. 3 we display the $\overline{u}_K$ distribution obtained from fits using the $\overline{u}_\pi$ parametrization of Ref. [15] in comparison to the $\overline{u}_\pi$ distribution itself. As expected, the $\overline{u}_K$ probability density is peaked at lower x than the one of the pion, indicating that light valence quarks in kaons carry on average less momentum than valence quarks in pions.

In Fig. 4 we show the full set of valence and intrinsic $q\overline{q}$ and gluon distributions in K^- at the low Q_v^2 scale. These distributions were calculated using the coefficients in the second row of Table I together with Eqs. (4), (5), and (9).

IV. CONCLUSIONS

In this work we addressed the problem of the low Q^2 structure of the π^- and K^-. We showed, by using a hadron-like Fock state expansion of the pion and kaon wavefunctions, that the π^- and K^- structures are related one another. The same is also true concerning the structure of the π^+ and K^+, where similar relationships to those displayed in Eqs. (1)–(4) can be found.

TABLE I. Parameters in the $\overline{u}_K/\overline{u}_\pi$ fitting function.

$x\overline{u}_\pi(x)$	$\|a_0^K\|^2$	$\|a_1^K\|^2$	$\|a_2^K\|^2$
GRV-P [14]	0.392 ± 0.050	0.101 ± 0.059	0.507 ± 0.077
$2.3x^{1.1}(1-x)$ [15]	0.646 ± 0.083	0.177 ± 0.097	0.177 ± 0.127

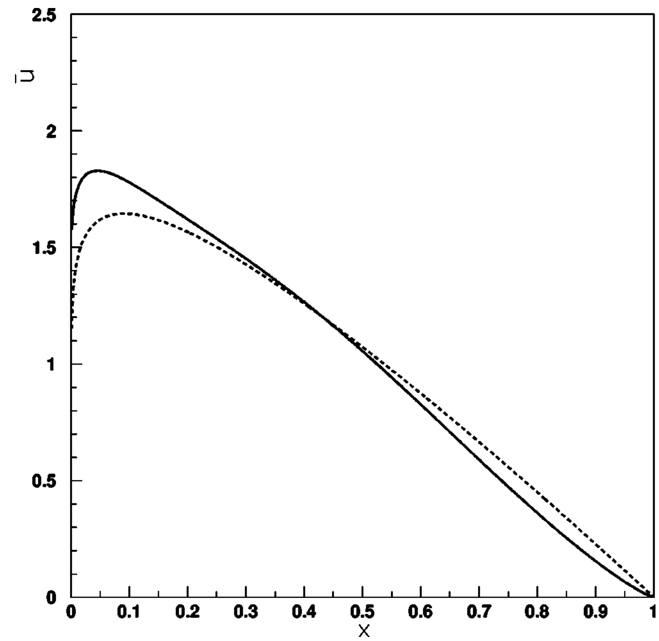

FIG. 3. $\overline{u}_K$ (solid line) compared to the $\overline{u}_\pi$ (dashed line) distribution as a function of x at the low Q_v^2 scale. The $\overline{u}_K$ distribution comes from the coefficients in the second row of Table I. The valence distribution in pions uses the the parametrization of Ref. [15].

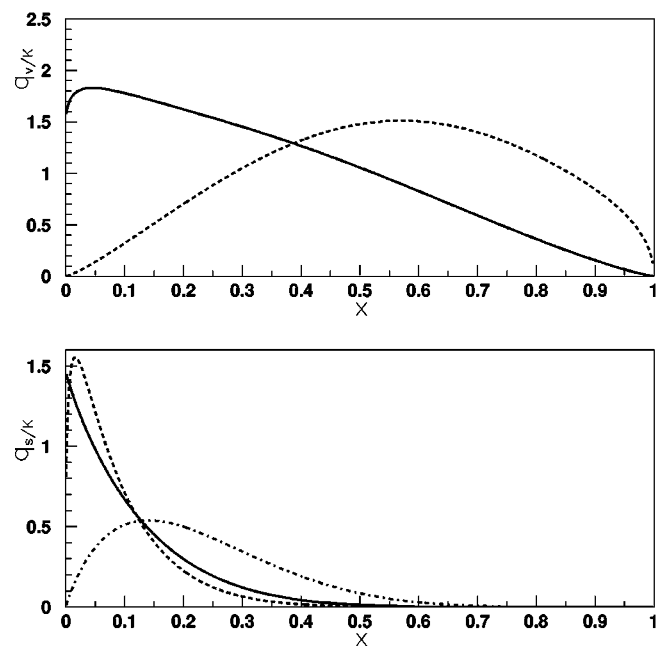

FIG. 4. K^- parton distributions at the Q_v^2 scale as a function of x obtained from fits using the $\overline{u}_\pi$ parameterization given in Ref. [15]. Upper: $\overline{u}_K$ (solid line) and s_K (dashed line) distributions. Lower: d_K (solid line), $\overline{d}_K$ (dashed line), and g_K (dot-dashed) distributions.

The π^0 structure, however, deserves separate consideration. In fact, the π^0 wave function at the low Q_v^2 scale can be writen as [5]

$$|\pi^0\rangle = b_0|\pi^0\rangle + b_1|\pi^0,g\rangle + b_2|\pi^-\pi^+\rangle + \frac{b_3}{\sqrt{2}}[|K^-K^+\rangle - |K^0\overline{K^0}\rangle] + \cdots. \tag{11}$$

Thus, unlike charged pions, where the intrinsic sea is formed only by strange quarks and gluons, in the π^0 the intrinsic sea is formed also by $u\overline{u}$ and $d\overline{d}$ quarks due to the $|\pi^+\pi^-\rangle$ fluctuation. It should be noted also that, as the $|\pi^0 g\rangle$ and the $|\pi^+\pi^-\rangle$ fluctuations have the same origin, namely, the splitting of a gluon to a $u\overline{u}$ or $d\overline{d}$ pair, it can be assumed that $|b_1|^2 \sim |b_2|^2$, thus possibly reducing the intrinsic $s\overline{s}$ sea due to probability conservation. This indicates a remarkable difference between the structures of charged and neutral pions.

The K^0 and $\overline{K}^0$ parton structures do not suffer from the above complications and can be determined, just by using isospin symmetry, from the structure of the charged ones.

We want to stress that one can determine the complete structure of pions and kaons from a minimal set of measurements of the π and K^- PDFs just by extracting the three valon distributions v_π, v_K, and $v_{s/K}$. In this sense the model has an interesting predictive power. Note that, as a matter of fact, the experimental information one can get on the kaon structure is only on the light valence quark distribution. Measurements of the strange and even sea quark distributions in kaons are not possible for practical reasons. Actually, strange and sea quarks contribute to the total DY dilepton cross section only through valence-sea and sea-sea $q\overline{q}$ annihilation. Thus their contributions are small and cannot be easily separated, and one must rely on other methods to obtain information on the kaon structure.

Moreover, the model predicts the structure of pions and kaons at the low Q_v^2 scale, where perturbative QCD (PQCD) evolution starts. This gives a plausible solution to the long standing problem of the valencelike sea quark and gluon distributions needed, at the low Q^2 input scale for evolution, to describe experimental data on hadron structure. Furthermore, the model predicts, for each hadron, what initial (intrinsic) sea quark flavors and gluon distributions must be considered. This is important not only for nucleons, but also for pions and kaons. Note that the pion PDFs usually given in the literature have been determined by using a SU(3) symmetric sea, and in most cases this sea has been generated by means of PQCD evolution alone.

Finally, we would like to remark that a confident determination of pion and kaon PDFs requires a reanalysis of the pion data in terms of the model presented here.

ACKNOWLEDGMENTS

J.M. is grateful for the warm hospitality at the Physics Deptartment of Universidad de los Andes, where part of this work was done.

APPENDIX: IN-MESON HADRON DISTRIBUTIONS

The meson probability density $P_{MM'}(x)$ in the $|MM'\rangle$ fluctuation has been calculated in Refs. [4,5]. It is given by

$$P_{MM'}(x) = \int_0^1 \frac{dy}{y} \int_0^1 \frac{dz}{z} F(y,z) R(x,y,z) \tag{A1}$$

with

$$F(y,z) = \beta y v_q(y) z q'(z)(1-y-z)^a,$$
$$R(x,y,z) = \alpha \frac{yz}{x^2} \delta\left(1 - \frac{y+z}{x}\right). \tag{A2}$$

In Eqs. (A2), v_q and q' are the valon and the quark or antiquark distributions which will form the meson M in the $|MM'\rangle$ fluctuation. The q' distribution is generated through gluon emission from a valon followed by $q'\overline{q'}$ pair creation, which are basic processes in QCD. Thus its distribution is given by [3]

$$q'(x) = \overline{q'}(x) = N \frac{\alpha_{st}^2(Q_v^2)}{(2\pi)^2} \int_x^1 \frac{dy}{y} P_{qg}\left(\frac{x}{y}\right) \times \int_y^1 \frac{dz}{z} P_{gq}\left(\frac{y}{z}\right) v_q(z), \tag{A3}$$

where $P_{qg}(z)$ and $P_{gq}(z)$ are the Altarelli-Parisi splitting functions [16] given by

$$P_{gq}(z) = \frac{4}{3} \frac{1+(1-z)^2}{z},$$
$$P_{qg}(z) = \frac{1}{2}[z^2 + (1-z)^2]. \tag{A4}$$

It is worth noting that the only scale dependence appearing in Eq. (A3) arises through $\alpha_{st}(Q^2)$. Since the valon scale is typically of the order of $Q_v^2 \sim 0.64$ GeV^2 [9], then $q'\overline{q'}$ pair creation can be safely evaluated perturbatively because $\alpha_{st}^2/(2\pi)^2$ is still sufficiently small. The normalization constants α, β, and N in Eqs. (A2) and (A3) contribute to the global normalization coefficient of the corresponding Fock state fluctuation in the expansion of Eqs. (1).

Momentum conservation also requires

$$P_{MM'}(x) = P_{M'M}(1-x), \tag{A5}$$

a condition which relates the in-meson M and M' probability densities. Additionally, hadrons in the $|MM'\rangle$ fluctuation must be correlated in velocity in order to form a bound state. This implies that

$$\frac{\langle x P_{MM'}(x)\rangle}{m_M} = \frac{\langle x P_{M'M}(x)\rangle}{m'_M}, \tag{A6}$$

fixing in this way the exponent a in Eqs. (A2). Notice also that P_{gM} is calculated from the "recombination" of an antiquark with a valon of the same flavor [5]. Then, formally, the P_{gM} corresponds to the π^0 distribution in a hypothetical $|M\pi^0\rangle$ fluctuation.

[1] R.G. Badalyan and G.R. Gulkanyan, Sov. J. Nucl. Phys. **50**, 115 (1989); N.N. Badalyan, R.G. badalyan, and H.R. Gulkanian, Z. Phys. C **42**, 69 (1989).
[2] J. Badier *et al.*, Phys. Lett. **93B**, 354 (1980).
[3] H.R. Christiansen and J. Magnin, Phys. Lett. B **445**, 8 (1998).
[4] J. Magnin and H.R. Christiansen, Phys. Rev. D **61**, 054006 (2000).
[5] H.R. Christiansen and J. Magnin, Phys. Rev. D **63**, 014001 (2001).
[6] E.A. Hawker *et al.*, Phys. Rev. Lett. **80**, 3715 (1998); J.C. Peng *et al.*, Phys. Rev. D **58**, 092004 (1998).
[7] V. Barone, C. Pascaud, and F. Zomer, Eur. Phys. J. C **12**, 243 (2000).
[8] M. Glück, E. Reya, and A. Vogt, Z. Phys. C **53**, 127 (1992); **67**, 433 (1995); J. Botts *et al.*, Phys. Lett. B **304**, 159 (1993); H.L. Lai *et al.*, Phys. Rev. D **51**, 4763 (1995); A.D. Martin, W.J. Stirling, and R.G. Roberts, Phys. Lett. B **354**, 155 (1995).
[9] R.C. Hwa, Phys. Rev. D **22**, 1593 (1980).
[10] S.J. Brodsky, C. Peterson, and N. Sakai, Phys. Rev. D **23**, 2745 (1981).
[11] J.T. Londergan, G.Q. Liu, E.N. Rodionov, and A.W. Thomas, Phys. Lett. B **361**, 110 (1995).
[12] P. Aurenche *et al.*, Phys. Lett. B **233**, 517 (1989); P.J. Sutton, A.D. Martin, R.G. Roberts, and W.J. Stirling, Phys. Rev. D **45**, 2349 (1992).
[13] M. Glück, E. Reya, and I. Schienbein, Eur. Phys. J. C **10**, 313 (1999).
[14] M. Glück, E. Reya, and A. Vogt, Z. Phys. C **53**, 651 (1992).
[15] A. Edin and G. Ingelman, Phys. Lett. B **423**, 402 (1998).
[16] G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977).