

Polyakov loops versus hadronic states

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The order parameter for the pure Yang-Mills phase transition is the Polyakov loop which encodes the symmetries of the Z_N center of the $SU(N)$ gauge group. On the other side the physical degrees of freedom of any asymptotically free gauge theory are hadronic states. Using the Yang-Mills trace anomaly and the exact Z_N symmetry we construct a model able to communicate to the hadrons the information carried by the order parameter.

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I. INTRODUCTION

Investigating the $SU(N)$ deconfinement phase transition is, in general, a complex problem. At zero quark density important sampling lattice simulations are able to provide vital information about the nature of the temperature-driven phase transition for two- and three-color Yang-Mills theories with and without matter fields (see [1,2] for three colors). Different approaches [3–21] are used in the literature to tackle or study the features of this phase transition.

At zero temperature $SU(N)$ the Yang-Mills theory is asymptotically free and the physical spectrum of the theory consists of a tower of hadronic states referred to as glueballs and pseudo-scalar glueballs. The theory develops a mass gap and the lightest glueball has a mass of the order of few times the confining scale. The classical theory is conformal while quantum corrections lead to a nonvanishing trace of the energy momentum tensor.

At a nonzero temperature the Z_N center of $SU(N)$ is a relevant global symmetry [22] and it is possible to construct a number of gauge-invariant operators charged under Z_N , among which the most notable one is the Polyakov loop,

$$\begin{aligned} \ell(x) &= \frac{1}{N} \text{Tr}[\mathbf{L}] \\ &\equiv \frac{1}{N} \text{Tr} \left\{ \mathcal{P} \exp \left[i g \int_0^{1/T} A_0(x, \tau) d\tau \right] \right\}. \end{aligned} \quad (1)$$

\mathcal{P} denotes path ordering, g is the $SU(N)$ coupling constant, and x is the coordinate for the three spatial dimensions while τ is Euclidean time. The ℓ field is real for $N=2$ while otherwise complex. This object is charged with respect to the center Z_N of the $SU(N)$ gauge group [22] under which it transforms as $\ell \rightarrow z\ell$ with $z \in Z_N$. A relevant feature of the Polyakov loop is that its expectation value vanishes in the low-temperature regime while it is nonzero in the high-temperature phase. The Polyakov loop is a suitable order parameter for the Yang-Mills temperature-driven phase transition [22].

This behavior recently led Pisarski [11] to model the Yang-Mills phase transition as a mean-field theory of Polyakov loops. This model is often referred to as the Polyakov loop model (PLM). Using this model one can infer that the $SU(2)$ phase transition is second order while a phase transi-

tion (as function of the temperature) is a weak first order for $SU(3)$. The predictions are in reasonable agreement with lattice results. Moreover, the PLM is used to model the Yang-Mills free energy. Recently some interesting phenomenological PLM-inspired models aimed to understand BNL Relativistic Heavy Ion Collider (RHIC) physics were constructed [18,19].

Here we will consider pure gluon dynamics. This allows us to have a well-defined framework where the Z_N symmetry is exact. The hadronic states are the glueball fields (H) and their effective theory at the tree level is constrained by the Yang-Mills trace anomaly.

A real puzzle to me is how the information about the Yang-Mills phase transition encoded, for example, in the Z_N global symmetry can be communicated to the hadronic states of the theory. Here we propose a concrete model which can help resolve this puzzle.

This model is constructed using trace anomaly and the Z_N symmetry. We will demonstrate that the information carried by ℓ is efficiently transferred to the glueballs. More generally the glueball field is a function of ℓ :

$$H \equiv H[\ell]. \quad (2)$$

Our results can be tested via first-principle lattice simulations [23] and support the recent phenomenological investigations [18,19].

In Sec. II we present the model. In Sec. III we consider the two-color Yang-Mills theory while in Sec. IV the three-color theory is considered. We finally conclude in Sec. V.

II. THE MODEL

The hadronic states of the Yang-Mills theory are the glueballs. At zero temperature the Yang-Mills trace anomaly has been used to constrain the potential of the lightest glueball state H [24]:

$$V[H] = \frac{H}{2} \ln \left[\frac{H}{\Lambda^4} \right]. \quad (3)$$

Λ is chosen to be the confining scale of the theory and H is a mass dimension-four field. This potential correctly saturates the trace anomaly when H is assumed to be proportional to $\text{Tr}[G_{\mu\nu}G^{\mu\nu}]$ and $G_{\mu\nu}$ is the standard Yang-Mills

field strength. The potential nicely encodes the properties of the Yang-Mills vacuum at zero temperature and it has been used to deduce a number of phenomenological results [24].

At high temperature Pisarski conjectured that the Yang-Mills pressure can be written in terms of the field ℓ . This free energy must be invariant under Z_N and it takes the general form:

$$V[\ell] = T^4 \mathcal{F}[\ell]. \quad (4)$$

$\mathcal{F}[\ell]$ is a polynomial in ℓ invariant under Z_N and the coefficients depend on the temperature itself allowing for a mean-field description of the Yang-Mills phase transitions.

We now marry the two models by requiring both fields to be present simultaneously at nonzero temperature. The theory must reproduce the ordinary glueball Lagrangian at zero and low temperatures while the PLM must reproduce at high temperatures. We propose the following effective potential:

$$V[H, \ell] = \frac{H}{2} \ln \left[\frac{H}{\Lambda^4} \right] + V_T[H] + H\mathcal{P}[\ell] + T^4 \mathcal{V}[\ell], \quad (5)$$

where $\mathcal{V}[\ell]$ and $\mathcal{P}[\ell]$ are general (but real) polynomials in ℓ invariant under Z_N whose coefficients depend on the temperature. The explicit dependence is not known and should be fit to lattice data. Dimensional analysis and analyticity in H when coupling it with ℓ severely restricts the effective potential terms. We stress that $H\mathcal{P}[\ell]$ is the most general interaction term which can be constructed without spoiling the zero-temperature trace anomaly.

Further nonanalytic interaction terms can arise when considering thermal and quantum corrections and are partially contained in $V_T[H]$ which schematically represents the temperature of a gas of glueballs. In the following we will not investigate in detail such a term. Our theory cannot be the full story since we neglected (as customary) all of the tower of glueballs and pseudoscalar glueballs as well as the infinite series of dimensionless gauge-invariant operators with different charges with respect to Z_N . Nevertheless the potential is sufficiently general to hope to capture the essential features of the Yang-Mills phase transition.

When the temperature T is much less than the confining scale Λ the last term in Eq. (5) can be safely neglected. Since the glueballs are relatively heavy compared to the Λ scale their temperature contribution $V_T[H]$ can also be disregarded. At low temperatures the theory reduces to the standard glueball potential augmented by the third term which does not affect the trace anomaly.

At very high temperature (compared to Λ) the last term dominates (H itself is very small), recovering the picture in which ℓ dominates the free energy. In this regime we have $\mathcal{F}[\ell] = \mathcal{V}[\ell]$.

We can, in principle, compute all of the relevant thermodynamical quantities in our approach, i.e., entropy, pressure, etc.

A relevant object is the trace of the energy-momentum tensor Θ_μ^μ . At zero temperature and when the potential is a

general function of a set of bosonic fields $\{\Phi_n\}$ with mass-dimensions d_n one can construct the associated trace of the energy-momentum tensor via

$$\Theta_\mu^\mu = 4V[\Phi_n] - \sum_n \frac{\delta V[\Phi_n]}{\delta \Phi_n} \Phi_n d_n. \quad (6)$$

At finite temperature we still define our temperature-dependent energy-momentum tensor as in Eq. (6). Here H possesses engineering mass dimensions 4 while ℓ is dimensionless, yielding the following temperature-dependent stress energy tensor:

$$\Theta_\mu^\mu(T) = -2H + 4T^4 \mathcal{V}[\ell] + 4 \left[1 - H \frac{\delta}{\delta H} \right] V_T[H]. \quad (7)$$

Θ_μ^μ is normalized such that $\langle 0 | \Theta_\mu^\mu | 0 \rangle = \epsilon - 3p$ with ϵ the vacuum energy density and p the pressure. At zero temperature only the first term survives, yielding magnetic-type condensation typical of a confining phase, while at extremely high temperature the second term dominates, displaying an energy density and pressure of the deconfined phase.

The theory containing just ℓ can be obtained integrating out H via the equation of motion

$$\frac{\delta V[H, \ell]}{\delta H} = 0. \quad (8)$$

Formally this is justifiable if the glueballs degrees of freedom are very heavy. For simplicity we neglect the contribution of $V_T[H]$ as well as the mean-field theory corrections for ℓ . However, in the future a more careful treatment, which also includes the kinetic terms, should be considered. Within these approximations the equation of motion yields

$$H[\ell] = \frac{\Lambda^4}{e} \exp(-2\mathcal{P}[\ell]). \quad (9)$$

The previous expression shows the intimate relation between ℓ and the physical states of strongly interacting theories.

After substituting Eq. (9) back into potential (5) and having neglected $V_T[H]$ we have

$$V[\ell] = T^4 \mathcal{V}[\ell] - \frac{\Lambda^4}{2e} \exp(-2\mathcal{P}[\ell]). \quad (10)$$

This formula shows that for large temperatures the only relevant energy scale is T and we recover the PLM model. However at low temperatures the scale Λ allows for new terms in the Lagrangian. In addition to the T^4 and the Λ^4 terms we also expect terms with coefficients of the type $T\Lambda^3$ and $T^2\Lambda^2$ and $T^3\Lambda$. However, in our simple model these terms do not seem to emerge.

Expanding the exponential we have

$$V[\ell] = T^4 \mathcal{V}[\ell] + \frac{\Lambda^4}{e} \mathcal{P}[\ell] - \frac{\Lambda^4}{2e} + \dots \quad (11)$$

Since $\mathcal{V}[\ell]$ and $\mathcal{P}[\ell]$ are real polynomials in ℓ invariant under Z_N we immediately recover a general potential in ℓ .

III. THE TWO-COLOR THEORY

To illustrate how our formalism works we first consider in detail the case $N=2$ and neglect for simplicity the term $V_T[H]$. This theory has been extensively studied via lattice simulations [25,26] and it constitutes the natural playground to test our model. Here ℓ is a real field and the Z_2 invariant $\mathcal{V}[\ell]$ and $\mathcal{P}[\ell]$ are taken to be

$$\begin{aligned}\mathcal{V}[\ell] &= a_1 \ell^2 + a_2 \ell^4 + \mathcal{O}(\ell^6), \\ \mathcal{P}[\ell] &= b_1 \ell^2 + \mathcal{O}(\ell^4),\end{aligned}\quad (12)$$

with a_1, a_2 , and b_1 unknown temperature-dependent functions which should be derived directly from the underlying theory. Lattice simulations can, in principle, fix all of the coefficients. In order for us to investigate in some more detail the features of our potential and inspired by the PLM model mean-field type of approximation we first assume a_2 and b_1 to be positive and temperature-independent constants while we model $a_1 = \alpha(T_* - T)/T$, with T_* a constant and α another positive constant. We will soon see that due to the interplay between the hadronic states and ℓ , T_* need not be the critical Yang-Mills temperature while a_1 displays the typical behavior of the mass square term related to a second-order type of phase transition.

The extrema are obtained by differentiating the potential with respect to H and ℓ :

$$\frac{\partial V}{\partial H} = \frac{\ln}{2} \left[\frac{eH}{\Lambda^4} \right] + \mathcal{P}[\ell] = \frac{\ln}{2} \left[\frac{eH}{\Lambda^4} \right] + b_1 \ell^2 = 0, \quad (13)$$

$$\frac{\partial V}{\partial \ell} = 2\ell T^4 \left(a_1 + \frac{H}{T^4} b_1 + 2a_2 \ell^2 \right) = 0. \quad (14)$$

A. Small and intermediate temperatures

At small temperatures the second term in Eq. (14) dominates and the only solution is $\ell = 0$. A vanishing ℓ leads to a null $\mathcal{P}[\ell]$ yielding the expected minimum for H :

$$\langle H \rangle = \frac{\Lambda^4}{e}. \quad (15)$$

Here ℓ and H decouple.

We now study the solution near the critical temperature for the deconfinement transition. For all the temperatures for which

$$T^4 a_1 + H b_1 = T^3 \alpha (T_* - T) + H b_1 > 0, \quad (16)$$

the solution for ℓ is still $\ell = 0$ yielding Eq. (15). The critical temperature is reached for

$$T_c = T_* + \frac{b_1 \Lambda^4}{e \alpha T_c^3}. \quad (17)$$

The critical temperature can be determined via lattice simulations. We see that within our framework the latter is related

to the glueball (gluon-condensate) coupling to two Polyakov loops and it would be relevant to measure it on the lattice. At $T = T_c$, $\ell = 0$, and $H = \Lambda^4/e$.

Let us now consider the case $T = T_c + \Delta T$ with

$$\frac{\Delta T}{T_c} \ll 1. \quad (18)$$

Expanding $\langle \ell \rangle^2$ at the leading order in $\Delta T/T_c$ yields

$$\langle \ell \rangle^2 = \frac{\alpha}{2a_2} \frac{1 + 3 \frac{b_1 \Lambda^4}{e \alpha T_c^4} \frac{\Delta T}{T_c}}{1 - \frac{b_1^2 \Lambda^4}{e a_2 T_c^4}}. \quad (19)$$

We used Eq. (17) and Eq. (13) which relate the temperature dependence of H to the one of ℓ . At high temperatures (see Sec. III B) $\langle \ell \rangle$ can be normalized to one by imposing $\alpha/2a_2 = 1$ and the previous expression reads

$$\langle \ell \rangle^2 = \frac{1 + 3 \frac{b_1 \Lambda^4}{e \alpha T_c^4} \frac{\Delta T}{T_c}}{1 - \frac{2b_1^2 \Lambda^4}{e \alpha T_c^4}} \equiv \frac{4T_c - 3T_*}{(1 - 2b_1)T_c + 2b_1 T_*} \frac{\Delta T}{T_c}. \quad (20)$$

For a given critical temperature consistency requires b_1 and T_* to be such that

$$\frac{4T_c - 3T_*}{(1 - 2b_1)T_c + 2b_1 T_*} \geq 0. \quad (21)$$

The temperature dependence in this regime of the gluon condensate is

$$\langle H \rangle = \frac{\Lambda^4}{e} \exp[-2b_1 \langle \ell \rangle^2]. \quad (22)$$

We find the mean-field exponent for ℓ , i.e., ℓ^2 increases linearly with the temperature near the phase transition [27]. Interestingly the gluon condensate drops exponentially. The drop in the gluon condensate is triggered by the rise of ℓ and it happens in our simple model exactly at the deconfining critical temperature. Although the drop might be sharp it is continuous in temperature and this is related to the fact that the phase transition is second order. Our findings strongly support the common picture according to which the drop of the gluon condensate signals, in the absence of quarks, deconfinement.

B. High temperature

At very high temperatures the second term in Eq. (14) can be neglected and the minimum for ℓ is

$$\langle \ell \rangle = \sqrt{\frac{\alpha}{2a_2}}. \quad (23)$$

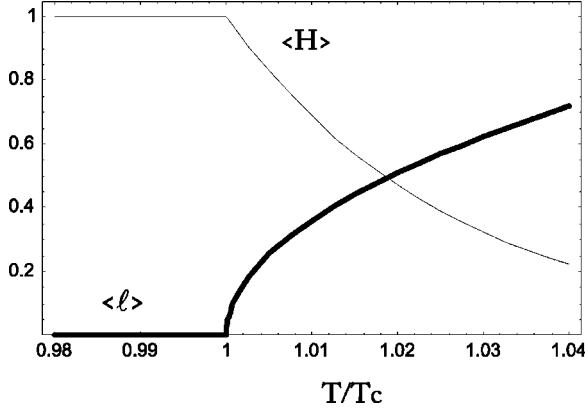


FIG. 1. The thin line is the gluon condensate $\langle H \rangle$ normalized to Λ^4/e as a function of the temperature. The thick line represents the normalized to unity $\langle \ell \rangle$ as a function of the temperature. We have chosen for illustration $\alpha=1$, $b_1=1.45$, and $T_c \approx 1.16\Lambda$.

For H we have now

$$\langle H \rangle = \frac{\Lambda^4}{e} \exp\left[-2b_1 \frac{\alpha}{2a_2}\right] = \frac{\Lambda^4}{e} \exp[-2b_1]. \quad (24)$$

In the last step we normalized $\langle \ell \rangle$ to unity at high temperature. In order for the previous solutions to be valid we need to operate in the following temperature regime:

$$T \gg^4 \sqrt{\frac{b_1}{\alpha}} \langle H \rangle \approx T_c. \quad (25)$$

We find that at sufficiently high temperature $\langle H \rangle$ is exponentially suppressed and the suppression rate is determined solely by the glueball— ℓ^2 mixing term encoded in $\mathcal{P}[\ell]$. The coefficient b_1 should be large (or increase with the temperature) since we expect a vanishing gluon condensate at asymptotically high temperatures. Clearly it is crucial to determine all of these coefficients via first-principle lattice simulations. The qualitative picture which emerges in our analysis is summarized in Fig. 1.

IV. THE THREE-COLOR THEORY

Z_3 is the global symmetry group for the three-color case while ℓ is a complex field. The functions $\mathcal{V}[\ell]$ and $\mathcal{P}[\ell]$ are

$$\begin{aligned} \mathcal{V}[\ell] &= a_1 |\ell|^2 + a_2 |\ell|^4 - a_3 (\ell^3 + \ell^{*3}) + \mathcal{O}(\ell^5), \\ \mathcal{P}[\ell] &= b_1 |\ell|^2 + \mathcal{O}(\ell^3), \end{aligned} \quad (26)$$

with a_1 , a_2 , a_3 , and b_1 unknown temperature-dependent coefficients which can be determined using lattice data. In this paper we want to investigate the general relation between glueballs and ℓ so we will not try to find the best parametrization to fit the lattice data. In the spirit of the mean-field theory we take a_2 , a_3 , and b_1 to be positive

constants while $a_1 = \alpha(T_* - T)/T$. With $\ell = |\ell|e^{i\varphi}$ the extrema are now obtained by differentiating the potential with respect to H , $|\ell|$, and φ ,

$$\frac{\partial V}{\partial H} = \frac{\ln\left[\frac{eH}{\Lambda^4}\right]}{2} + \mathcal{P}[\ell] = \frac{\ln\left[\frac{eH}{\Lambda^4}\right]}{2} + b_1 |\ell|^2 = 0,$$

$$\frac{\partial V}{\partial |\ell|} = 2|\ell|T^4 \left(a_1 + \frac{H}{T^4} b_1 - 3a_3 |\ell| \cos(3\varphi) + 2a_2 |\ell|^2 \right) = 0,$$

$$\frac{\partial V}{\partial \varphi} = 6|\ell|^3 \sin(3\varphi) = 0. \quad (27)$$

At small temperature the H/T^4 term in the second equation dominates and the solution is $\langle |\ell| \rangle = 0$, $\langle H \rangle = \Lambda^4/e$, and the last equation is verified for any $\langle \varphi \rangle$, so we choose $\langle \varphi \rangle = 0$. The second equation can have two more solutions:

$$\frac{3}{4} \frac{a_3}{a_2} \pm \sqrt{\frac{9}{16} \frac{a_3^2}{a_2^2} + \frac{\alpha(T-T_*)}{2Ta_2} - \frac{b_1 H}{2a_2 T^4}}, \quad (28)$$

whenever the square root is well defined (i.e., at sufficiently high temperatures). The negative sign corresponds to a relative maximum while the positive one corresponds to a relative minimum. We have then to evaluate the free energy value (i.e., the effective thermal potential) at the relative minimum and compare it with the one at $\ell=0$. The temperature value for which the two minima have the same free energy is defined as the critical temperature and is

$$T_c = \left[T_* + \frac{b_1 \Lambda^4}{e \alpha T_c^3} \right] \frac{\alpha a_2}{\alpha a_2 + a_3^2}. \quad (29)$$

When a_3 vanishes we recover the second-order-type critical temperature T_c . To derive the previous expression we held fix the value of H to Λ^4/e at the transition point. In a more refined treatment one should not make such an assumption. Below this temperature the minimum is still for $\langle \ell \rangle = 0$ and $\langle H \rangle = \Lambda^4/e$.

Just above the critical temperature the fields jump to the new values:

$$\langle |\ell| \rangle = \frac{a_3}{a_2}, \quad \langle H \rangle = \frac{\Lambda^4}{e} \exp[-2b_1 \langle |\ell| \rangle^2]. \quad (30)$$

Close but above T_c (i.e., $T = T_c + \Delta T$) we have

$$\langle |\ell| \rangle \approx \frac{a_3}{a_2} + \rho \frac{\Delta T}{T_c}, \quad (31)$$

with

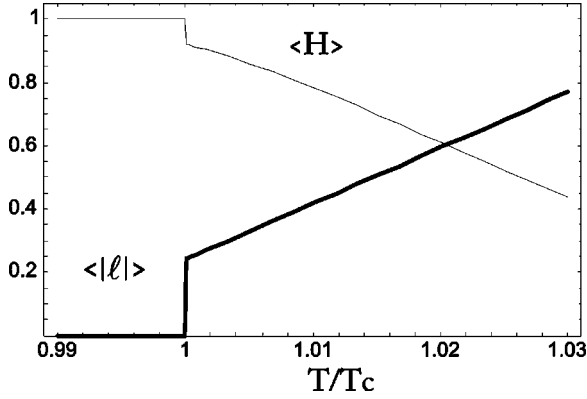


FIG. 2. A cartoon sketching the gluon condensate $\langle H \rangle$ normalized to Λ^4/e and the $\langle |\ell| \rangle$ (thick line) behaviors as function of the temperature. We have chosen for illustration $a_3=0.3$, $a_2=1$, $\alpha=2$, $b_1=0.7$, and $T_c=1.2\Lambda$.

$$\rho \simeq \frac{\alpha a_2}{a_3} \frac{4\kappa T_c - 3T_*}{a_2 T_c - 4b_1 \alpha (\kappa T_c - T_*)},$$

$$\kappa = \frac{\alpha a_2 + a_3^2}{\alpha a_2}, \quad (32)$$

a positive function of the coefficients of the effective potential. In this regime

$$\langle H \rangle = \frac{\Lambda^4}{e} \exp \left[-2b_1 \left(\frac{a_3}{a_2} + \rho \frac{\Delta T}{T_c} \right)^2 \right]. \quad (33)$$

At high temperature we expect a behavior similar to the one presented for the two-color theory case. A cartoon representing the behavior of the Polyakov loop and the gluon condensate is presented in Fig. 2.

Since we are in the presence of a first-order phase transition higher-order terms in Eq. (26) may be important. However, lattice simulations have shown that the behavior of the Polyakov loop, for three colors, resemble a weak first-order transition (i.e., small a_3) partially justifying our simple approach.

The approximation for our coefficients is too crude and it would certainly be relevant to fit them to lattice simulations.

What we learned is that the gluon condensate, although not a real order parameter, encodes the information of the underlying Z_3 symmetry. More generally we have shown that once the map between hadronic states and the true order parameter is known we can use directly hadronic states to determine when the phase transition takes place and the order of the phase transition. For example by comparing Fig. 1 and Fig. 2 we immediately notice the distinct difference in the gluon condensate temperature dependence near the phase transition.

V. CONCLUSIONS AND SELF-CRITICISM

Our simple model is able to account for many features inherent to the Yang-Mills deconfining phase transition. We related two very distinct and relevant sectors of the theory:

the hadronic sector (the glueballs), and some dimensionless fields (ℓ) charged under the discrete group Z_N understood as the center of the underlying $SU(N)$ Yang-Mills theory [11].

The gluon-condensate is, strictly speaking, not an order parameter for the deconfining Yang-Mills phase transition. However, we have shown that the information encoded in the true order parameter ℓ is efficiently communicated to the gluon condensate. Since the exponential drop of the condensate just above the Yang-Mills critical temperature is a direct consequence of the behavior of the true order parameter at the transition we can consider this drop as a strong signal of deconfinement. This drop has already been used in various models for the Yang-Mills phase transition. We have also seen that the reduction in the gluon condensate is associated with the increase of the Polyakov loop condensate ℓ . The information about the order of the phase transition is also transferred to the behavior of the gluon condensate. Indeed from Fig. 1 and Fig. 2 we deduce that the drop is continuous for the gluon condensate in the two-color case while it is discontinuous for the three-color theory. Physically the glueballs start decaying into gluons and this information is encoded in the $HP[\ell]$ term of the present theory. We now better understand the mechanism for transferring information from the Yang-Mills order parameter to the physical states.

It is important to stress that our model is very limited since we neglect the temperature corrections associated with the glueball gas as well as other dimensionless operators with different charges under Z_N . Finally we did not include any of the excited glueball and pseudoscalar glueball states present in the theory. Besides the temperature dependence of the coefficients in $\mathcal{P}[\ell]$ and $\mathcal{V}[\ell]$ is not known and we have just adopted the simplest guess consistent with mean-field theory. We also know that mean field cannot be the whole story and corrections need to be included.

It is worth mentioning that the Polyakov loop need not be the only acceptable order parameter. For example, using an Abelian projection one can define a new (nonlocal in the chromomagnetic variables) order parameter [20]. Our model can be, in principle, modified to be able to couple the hadronic state to any reasonable Yang-Mills order parameter.

The same holds true when considering the introduction of quarks. Once we identify a true order parameter for QCD with quarks we can first construct a model Lagrangian which satisfies the ordinary symmetries at zero temperature for the hadronic states and then extend it to describe at the same time the order parameter and the hadronic states. Although ℓ is not a good order parameter when quarks are added to the theory, since the Z_N symmetry is explicitly broken, we can still construct a theory containing ℓ and the hadronic states (mesons and glueballs) provided we introduce explicit Z_N breaking terms. This approach might be relevant for understanding RHIC physics [18,19].

Although the model is at a very early stage of development at the tree level some of the results are already fairly robust. For example the exponential drop of $\langle H \rangle$ as function of $\langle \ell \rangle$ is a prediction not expected to be very sensitive to different sources of corrections. We also note that the first-order behavior of the deconfining three-color Yang-Mills theory is directly inherited by the $\langle H \rangle$. Indeed this quantity is discontinuous at the phase transition for three colors while it

displays a smooth behavior in the two-color case. We expect these results to be quite general. We also stress that they are connected to the saturation of trace anomaly and the Z_N symmetry at the effective Lagrangian level when considering simultaneously ℓ and H .

By computing the temperature dependence of the coefficients in the present effective theory lattice simulations can test the validity of the present model while improving the present results.

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