$Z \rightarrow l^+ l^-$ and $W \rightarrow \nu_l l^+$ decays in the noncommutative standard model

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We study $Z \rightarrow l^+ l^-$ and $W \rightarrow \nu_l l^+$ decays in the standard model including noncommutative effects. We observe that these effects appear in the flavor-dependent part of the decay widths of the processes under consideration and, therefore, they are more effective for the heavy lepton decays.

I. INTRODUCTION

Leptonic Z decays are among the most interesting lepton flavor conserving (LFC) and lepton flavor violating (LFV) interactions. The improved experimental measurements at present stimulate the studies of these interactions. With the Giga-Z option of the DESY TeV Energy Superconducting Linear Accelerator (TESLA), there is a possibility to increase Z bosons at resonance [1]. The processes $Z \rightarrow l^- l^+$ with $l = e, \mu, \tau$ are among the lepton flavor changing (LFC) decays and they exist in the standard model (SM), even in the tree level. The experimental predictions for the branching ratios (BRs) of these decays are [2]

$$\text{BR}(Z \rightarrow e^+ e^-) = 3.366 \pm 0.0081 \%,$$

$$\text{BR}(Z \rightarrow \mu^+ \mu^-) = 3.367 \pm 0.013 \%,$$

$$\text{BR}(Z \rightarrow \tau^+ \tau^-) = 3.360 \pm 0.015 \%, \tag{1}$$

and the tree level SM predictions are

$$\text{BR}(Z \rightarrow e^+ e^-) = 3.331 \%,$$

$$\text{BR}(Z \rightarrow \mu^+ \mu^-) = 3.331 \%,$$

$$\text{BR}(Z \rightarrow \tau^+ \tau^-) = 3.328 \%. \tag{2}$$

This shows that the tree level contribution of the SM plays the main role within the experimental uncertainties. In the literature, there are various experimental and theoretical studies [3–12]. In [5], a method to determine the weak electric dipole moment was developed. The vector and axial coupling constants, v_f and a_f, in Z decays have been measured at the CERN $e^+ e^-$ collider (LEP) [7]. In [9], various additional types of interactions have been performed and a way to measure these contributions in the process $Z \rightarrow \tau^- \tau^+$ was described. Reference [12] is devoted to the possible new physics effects to the process $Z \rightarrow l^+ l^-$, in the general two Higgs doublet model.

$W \rightarrow \nu_l l^+$ ($l = e, \mu, \tau$) decays exist also in the tree level, in the SM, and the experimental predictions for the branching ratios are [2]

$$\text{BR}(W \rightarrow \nu_e e^+) = 10.9 \pm 0.4 \%,$$

$$\text{BR}(W \rightarrow \nu_\mu \mu^+) = 10.2 \pm 0.5 \%,$$

$$\text{BR}(W \rightarrow \nu_\tau \tau^+) = 11.3 \pm 0.8 \%. \tag{3}$$

The main contribution to this decay comes from the SM in the tree level, similar to the process $Z \rightarrow l^+ l^-$. There are a large number of studies in the literature on this charged process [13].

In the present work, we study $Z \rightarrow l^+ l^-$ and $W \rightarrow \nu_l l^+$ decays, with $l = e, \mu, \tau$, in the SM, including the noncommutative (NC) effects. The noncommutativity in the space-time is a possible candidate to describe the physics at very short distances of the order of the Planck length, since the nature of the space-time changes at these distances. In the noncommutative geometry, the space-time coordinates are replaced by Hermitian operators $\hat{x}_\mu$ which satisfy the equation [14]

$$[\hat{x}_\mu, \hat{x}_\nu] = i \theta_{\mu\nu}, \tag{4}$$

where $\theta_{\mu\nu}$ is a real and antisymmetric tensor with the dimensions of length-squared. Here $\theta_{\mu\nu}$ can be treated as a background field and its components are assumed as constants over cosmological scales.

It is possible to pass to the noncommutative field theory by introducing $*$ product of functions, instead of the ordinary one,

$$(f * g)(x) = e^{(i/2)\theta_{\mu\nu}\partial_\mu^y \partial_\nu^z} f(y) g(z)|_{y=z=x}. \tag{5}$$

The commutation of the Hermitian operators $\hat{x}_\mu$ [see Eq. (4)] holds with this new product, namely,

$$[\hat{x}_\mu, \hat{x}_\nu]_* = i \theta_{\mu\nu}. \tag{6}$$

With the remotivation due to the string theory arguments [15,16], various studies on the noncommutative field theory (NCFT) have been done in the literature. However, NCFT have a nonlocal structure and the Lorentz symmetry is explicitly violated. The violation of the Lorentz symmetry has been handled in [17,18], and bounding noncommutative QCD due to the Lorentz violation has been studied in [18]. In this work, it was emphasized that the collider limits were not competitive with low energy tests of Lorentz violation for bounding the scale of space-time noncommutativity. Furthermore, the renormalizability and the unitarity of NC theo-

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ries have been studied in the series of works [19–22]. The noncommutative quantum electrodynamics (NCQED) has been examined in [23,24] and the noncommutativity among extra dimensions for QED has been studied in [25]. Furthermore, the noncommutativity in the non-Abelian case has been formulated in [26] and this formulation has been applied to the SM in [27]. Recently, a unique model for strong and electroweak interactions with their unification has been constructed in [28]. In the work [29], the SM forbidden processes $Z \to \gamma\gamma$ and $Z \to gg$ has been studied by including the NC effects. In [30], the form factors, appearing in the inclusive $b \to sg$ decay, have been calculated in the NCSM, using the approximate phenomenology, and the new operators existing in $b \to sg$ decay due to the NC effects have been obtained in [31]. In the recent work, the possible effects of NC geometry on weak CP violation and the untarity triangles have been examined [32].

The paper is organized as follows. In Sec. II, we present the explicit expressions for the branching ratios of $Z \to l^+ l^-$ and $W \to \nu_l l^+$ in the framework of the NCSM. Section III is devoted to a discussion and our conclusions.

II. THE NONCOMMUTATIVE EFFECTS ON THE $Z \to l^+ l^-$ AND $W \to \nu_l l^+$ DECAYS IN THE SM

The flavor-conserving $Z \to l^+ l^-$, $l = e, \mu, \tau$ decays appear in the tree level in the SM. When the noncommutative effects are switched on, there exists a new contribution which is proportional to the function of the noncommutative parameter θ. Our starting point is the effective action [27]

$$
S_{\text{matter, leptons}} = \int d^4 x \left(\sum_i (\bar{L}_L^{(i)} + \bar{L}_L^{(i)1} + \bar{L}_L^{(i)2}) * i(\slashed{D}^{SM} + \slashed{\Gamma}) * (L_L^{(i)} + L_L^{(i)1} + L_L^{(i)2}) \right.
$$

$$
\left. + \sum_i (\bar{e}_R^{(i)} + \bar{e}_R^{(i)1} + \bar{e}_R^{(i)2}) * i(\slashed{D}^{SM} + \slashed{\Gamma}) * (e_R^{(i)} + e_R^{(i)1} + e_R^{(i)2}) \right) + O(\theta^3), \tag{7}
$$

with

$$
D_\mu^{SM} L_L = (\partial_\mu - ig' Y_L A_\mu - ig B_{\mu a} T_L^a) L_L,
$$

$$
D_\mu^{SM} e_R = (\partial_\mu - ig' Y_R A_\mu) e_R, \tag{8}
$$

and

$$
L_L^{(i)1} = -\frac{1}{2} \theta_{\mu\nu} (g' Y_L A^\mu + g B_a^\mu T_L^a) \partial^\nu L_L^{(i)} + O(A^2, B^2, AB),
$$

$$
L_L^{(i)2} = -\frac{i}{8} \theta_{\mu\nu} \theta_{\alpha\beta} (g' Y_L \partial^\mu A^\alpha + g \partial^\mu B_a^\alpha T_L^a) \partial^\nu \partial^\beta L_L^{(i)}
$$

$$
+ O(A^2, B^2, AB),
$$

$$
e_R^{(i)1} = -\frac{1}{2} \theta_{\mu\nu} (g' Y_R A^\mu) \partial^\nu e_R^{(i)} + O(A^2),
$$

$$
e_R^{(i)2} = -\frac{i}{8} \theta_{\mu\nu} \theta_{\alpha\beta} (g' Y_R \partial^\mu A^\alpha) \partial^\nu \partial^\beta e_R^{(i)} + O(A^2), \tag{9}
$$

where $*$ in Eq. (7) denotes the Moyal-Weyl star product [see Eq. (5)], $L_L^{(i)}$ ($e_R^{(i)}$) is the left- (right-) handed lepton doublet of the ith family, $Y_L = -\frac{1}{2}$, $Y_R = -1$, and $O(A^2, B^2, AB)[O(A^2)]$ is the part of $L_L^{(i)1,2}$ ($e_R^{(i)1,2}$) which includes the interactions of more than one gauge field. Here the function Γ has no interest since it contains two gauge field interactions, which do not give any contribution to our processes $Z(W) \to l^+ l^- (\nu_l l^+)$. Furthermore, we do not present the

parts of $L_L^{(i)1,2}$ and $e_R^{(i)1,2}$, $O(A^2, B^2, AB)$, and $O(A^2)$, which include the interactions of more than one gauge field (see [26] and [27] for details).

Finally, the additional vertex to the $Z \to l^+ l^-$ decay to the second order in θ can be obtained as

$$
V_{\mu, \text{NC}}^Z = \left((\theta_{\mu\nu} \gamma_\alpha + \theta_{\nu\alpha} \gamma_\mu + \theta_{\alpha\mu} \gamma_\nu) p_Z^\nu p_1^\alpha \right.
$$

$$
- \frac{i}{4} (\theta_{\mu\nu} \gamma_\alpha + \theta_{\nu\alpha} \gamma_\mu + \theta_{\alpha\mu} \gamma_\nu) \theta_{\gamma\sigma} p_Z^\gamma p_Z^\alpha p_1^\sigma p_1^\nu \right)
$$

$$
\times (c_1 L + c_2 R), \tag{10}
$$

where $c_1 = -e[(2 \sin^2 \theta_W - 1)/(4 \sin \theta_W \cos \theta_W)]$, $c_2 = -e(\tan \theta_W / 2)$, $L(R) = [(1 - \gamma_5)/2][(1 + \gamma_5)/2]$, and p_Z $(-p_1)$ is the incoming (out going) four-momentum of the Z boson with polarization vector ϵ^μ (antilepton). Notice that the part of the vertex proportional with $\theta_{\nu\alpha}$ would be the whole contribution in the case in which the NC effects enter into the expressions as an exponential factor $e^{(i/2) \theta_{\mu\nu} p_Z^\mu p_1^\nu}$, which is consistent with in approximate phenomenology (see [33] and references therein).

Now we present the BR of the process $Z \to l^+ l^-$ including the noncommutative effects at the least order in θ, in the Z boson rest frame:

$$
\text{BR} = \frac{\alpha_{\text{em}} m_Z}{6 \Gamma_Z \sin^2 2\theta_W} \left[(1 - 4 \sin^2 \theta_W + \sin^4 \theta_W) \right.
$$

$$
\left. - \frac{m_l^2}{m_Z^2} \left(1 + 8 \sin^2 \theta_W - 16 \sin^4 \theta_W + \frac{m_Z^4}{16} f(\theta) \right) \right], \tag{11}
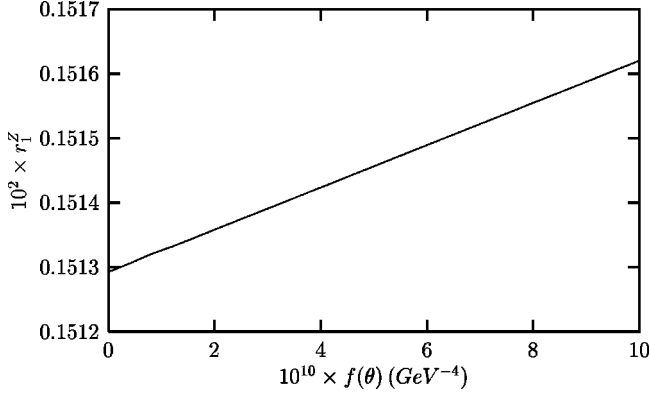$$

FIG. 1. $f(\theta)$ dependence of ratio $r_1^Z = \mathrm{BR}_{\mathrm{flavor}}/\mathrm{BR}_{\mathrm{tot}}$, where $\mathrm{BR}_{\mathrm{flavor}}$ is the flavor-dependent part of the BR, and $\mathrm{BR}_{\mathrm{tot}}$ is the total BR, for the process $Z \to \tau^+ \tau^-$.

where Γ_Z is the total decay width of the Z boson, $\Gamma_Z = 2.490$ GeV, and $\alpha_{\mathrm{em}} = e^2/4\pi$. As shown in this equation, the NC effects appear as the function of θ,

$$f(\theta) = (\vec{\theta}_T . \hat{p}_1)^2 + (\vec{\theta}_S . \hat{p}_1)^2 - (|\vec{\theta}_T|^2 + |\vec{\theta}_S|^2)$$
$$+ 2\hat{p}_1 . (\vec{\theta}_T \times \vec{\theta}_S). \tag{12}$$

Here we use the definitions $(\theta_T)_i = \theta_{0i}$ and $(\theta_S)_i = \frac{1}{2}\epsilon_{ijk}\theta^{jk}$, $i,j,k = 1,2,3$ and $\vec{p}_1 = (m_Z/2)\hat{p}_1$. $(\theta_T)_i$ and $(\theta_S)_i$ are responsible for time-space and space-space noncommutativity, respectively. The noncommutative effects enter into the expression with lepton mass and they are much more suppressed in the case of light leptons. Notice that the terms of the vertex Eq. (10), which is second order in θ, do not give any contribution to the BR of the decay $Z \to l^+ l^-$ in the Z boson rest frame.

The charged $W \to \nu_l l^+$ decays exist with the charged current and they also appear at the tree level in the SM. Similar to the $Z \to l^+ l^-$ decay, the noncommutative effects are controlled by the additional vertex

$$V_{\mu,\mathrm{NC}}^W = -\frac{e}{2\sqrt{2}\sin\theta_W}\Bigg((\theta_{\mu\nu}\gamma_\alpha + \theta_{\nu\alpha}\gamma_\mu + \theta_{\alpha\mu}\gamma_\nu)p_W^\nu p_1^\alpha$$
$$-\frac{i}{4}(\theta_{\mu\nu}\gamma_\alpha + \theta_{\nu\alpha}\gamma_\mu + \theta_{\alpha\mu}\gamma_\nu)\theta_{\gamma\sigma}p_W^\gamma p_W^\alpha p_1^\sigma p_1^\nu\Bigg)L, \tag{13}$$

where p_W $(-p_1)$ is the incoming (ougoing) four-momentum of the W boson with polarization vector ϵ^μ (antilepton). The BR of the process $W \to \nu_l l^-$ including the noncommutative effects, at the least order in θ, in the W boson rest frame reads

$$\mathrm{BR} = \frac{\alpha_{\mathrm{em}} m_W}{384\Gamma_W \sin^2\theta_W}\Bigg(32 + \frac{m_l^2}{m_W^2}[16 - m_W^4 f(\theta)]\Bigg), \tag{14}$$

where Γ_W is the total decay width of the W boson, $\Gamma_W = 2.060$ GeV. Here the function $f(\theta)$ [see Eq. (12)] represents the noncommutative effects. The terms of the vertex
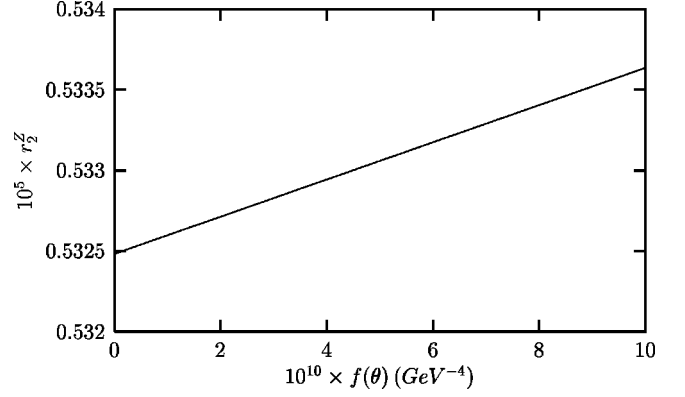
Eq. (13), which is second order in θ, give a nonzero contribution to the BR of the decay $W \to \nu_l l^+$, in the W boson rest frame. This contribution is proportional to $m_l^2 m_W^2 (\vec{\theta}_T . \hat{p}_1)^2$. However, it is cancelled by the part coming from the vertex linear in θ.

At this stage we will try to parametrize the vectors $(\theta_T)_i$ and $(\theta_S)_i$ which are responsible for time-space and space-space noncommutativity, respectively. With the assumption that the matrix $\theta_{\mu\nu}$ is real and constant, we take

$$\vec{\theta}_T = A_1\hat{p}_1 + A_2\hat{p}_{1\perp}^T,$$
$$\vec{\theta}_S = B_1\hat{p}_1 + B_2\hat{p}_{1\perp}^S, \tag{15}$$

where $\hat{p}_1$ $(\hat{p}_{1\perp}^T, \hat{p}_{1\perp}^S)$ is the unit vector in the direction of (the perpendicular direction to) the incoming lepton three-momentum $\vec{p}_1$ (for $\vec{\theta}_T, \vec{\theta}_S$), and A_i, B_i are the corresponding real coefficients. Using this parametrization, $f(\theta)$ can be written as

$$f(\theta) = 2A_2B_2\hat{p}_1 . (\hat{p}_{1\perp}^T \times \hat{p}_{1\perp}^S) - (A_2^2 + B_2^2), \tag{16}$$

and this shows that the transverse components of the vectors $\hat{p}_{1\perp}^T$ and $\hat{p}_{1\perp}^S$ to the incoming lepton three-momentum $\vec{p}_1$ play the main role for the NC effects. In the case of
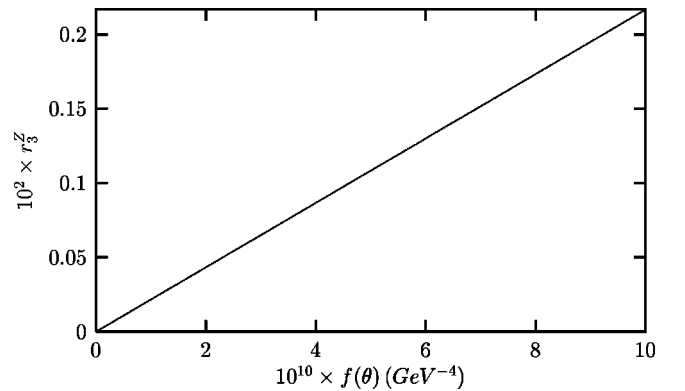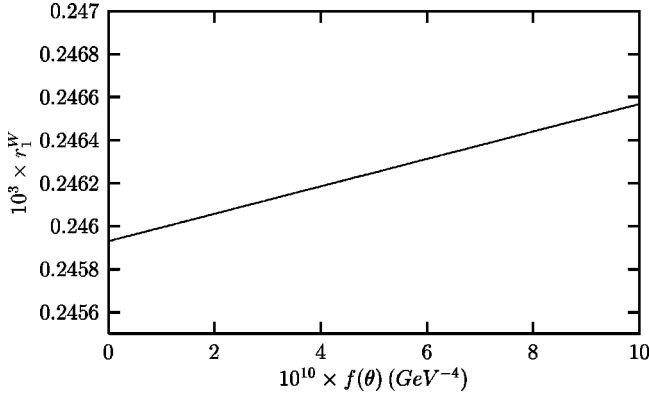

FIG. 2. The same as Fig. 1 but for $Z \to \mu^+ \mu^-$ decay.

FIG. 3. $f(\theta)$ dependence of ratio $r_3^Z = \mathrm{BR}_{\mathrm{flavor}_\theta}/\mathrm{BR}_{\mathrm{flavor}}$, where $\mathrm{BR}_{\mathrm{flavor}_\theta}$ is the noncummutative flavor-dependent part of the BR, for the process $Z \to \tau^+ \tau^-$.

FIG. 4. The same as Fig. 1 but for $W \to \nu_\tau \tau^+$ decay.

FIG. 5. The same as Fig. 3 but for $W \to \nu_\tau \tau^+$ decay.

$\hat{p}_{1\perp}^T \perp \hat{p}_{1\perp}^S \perp \hat{p}_1$ with $A_2 = B_2$, the noncommutative effects are switched off. Furthermore, for $\vec{\theta}_T \| \hat{p}_1$ ($\vec{\theta}_S \| \hat{p}_1$), the coefficient $A_2 = 0$ ($B_2 = 0$) and, therefore, only the space-space (space-time) noncommutativity is responsible for the noncommutative effects. This is interesting in the determination of the noncommutative directions with the help of the future sensitive experimental results.

III. DISCUSSION

In this section, we analyze the NC effects on the BR of the flavor conserving $Z \to l^+ l^-$ and charged $W \to \nu l^+$ decays, in the framework of the SM. The processes under consideration exist in the tree level in the SM and the theoretical calculation of the BRs obeys the experimental results within the measurement errors.
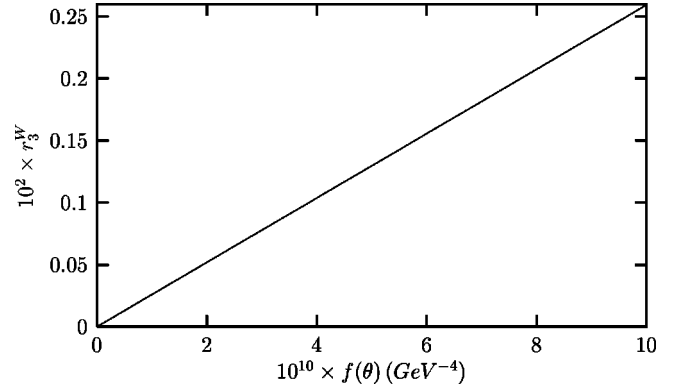
The flavor $l = e, \mu, \tau$ dependence of the part of the BR($Z \to l^+ l^-$) is extremely weak,

$$R_{\mu e} = \frac{\text{BR}(Z \to \mu^+ \mu^-)}{\text{BR}(Z \to e^+ e^-)} = 1.0008 \pm 0.005,$$

$$R_{\tau e} = \frac{\text{BR}(Z \to \tau^+ \tau^-)}{\text{BR}(Z \to e^+ e^-)} = 0.998 \pm 0.005. \quad (17)$$

This part, which controls the flavor effects, is proportional to the factor m_l^2 / m_Z^2 and it includes the noncommutative effects. Therefore, it is more informative to study the heavy lepton decays to determine the noncommutativity of the geometry. Notice that we choose the noncommutative parameter $\theta = |\theta_{\mu\nu}|$ as at the order of magnitude of $\sim 10^{-6} - 10^{-5}$ GeV^{-2}.

In Fig. 1, we present the noncommutative parameter $f(\theta)$ dependence of the ratio $r_1^Z = \text{BR}_{\text{flavor}} / \text{BR}_{\text{tot}}$, where $\text{BR}_{\text{flavor}}$ is the flavor-dependent part of the BR, and BR_{tot} is the total BR, for the process $Z \to \tau^+ \tau^-$. This figure shows that the noncommutative effects are at most at the order of magnitude of 0.001%, even for the heavy lepton τ decay. This dependence becomes extremely small, 10^{-6}%, for Z $\to \mu^+ \mu^-$ decay (see Fig. 2), since the mass of the lepton μ is small and there is a strong suppression factor m_μ^2 / m_Z^2 for $\text{BR}_{\text{flavor}}$.

Figure 3 is devoted to the $f(\theta)$ dependence of ratio $r_3^Z = \text{BR}_{\text{flavor}_\theta} / \text{BR}_{\text{flavor}}$, where $\text{BR}_{\text{flavor}_\theta}$ is the noncommutative flavor-dependent part of the BR, for the process $Z \to \tau^+ \tau^-$. It is observed that the noncommutative effects on the flavor-dependent part can reach 0.1%.

Now we would like to study the charged $W \to \nu_l l^+$ decay and the noncommutative effects on this process. The BR for this process is

$$\text{BR}(W \to \nu_l l^+) = 10.74 \pm 0.33\% \quad (18)$$

and the flavor $l = e, \mu, \tau$ dependence of this value is weak. Similar to the $Z \to l^+ l^-$ decay, the part of the BR($W \to \nu l^+$) which controls the flavor effects is proportional to the factor m_l^2 / m_W^2 and the noncommutative effects appear in this part.

In Fig. 4, we present the noncommutative parameter $f(\theta)$ dependence of ratio $r_1^W = \text{BR}_{\text{flavor}} / \text{BR}_{\text{tot}}$, where $\text{BR}_{\text{flavor}}$ is the flavor-dependent part of the BR, and BR_{tot} is the total BR for the process $W \to \nu_\tau \tau^+$. It is observed that the noncommutative effects are at most at the order of magnitude of 0.001%, for the heavy lepton τ decay.

Figure 5 represents the $f(\theta)$ dependence of ratio $r_3^W = \text{BR}_{\text{flavor}_\theta} / \text{BR}_{\text{flavor}}$, where $\text{BR}_{\text{flavor}_\theta}$ is the noncommutative flavor-dependent part of the BR for the process $W \to \nu_\tau \tau^+$. Here, the noncommutative effects on the flavor-dependent part can reach 0.1%, similar to the process $Z \to \tau^- \tau^+$.

In conclusion, the NC effects in the decays under consideration are effective in the flavor-dependent part of their BRs. With the possible future experiments, which will be sensitive to the flavor-dependent part of these processes, those effects can be extracted and the noncommutative direction can be determined.

$Z \rightarrow l^+ l^-$ AND $W \rightarrow \nu_l l^+$ DECAYS IN THE . . .

PHYSICAL REVIEW D **66**, 034011 (2002)

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