

Lepton flavor violation in tau decays

G. Cvetič,^{1,*} C. Dib,^{1,†} C. S. Kim,^{2,‡} and J. D. Kim^{2,§}¹*Department of Physics, Universidad Técnica Federico Santa María, Valparaíso, Chile*²*Department of Physics and IPAP, Yonsei University, Seoul 120-749, Korea*

(Received 25 February 2002; published 13 August 2002)

We study lepton flavor violation (LFV) in tau decays induced by heavy Majorana neutrinos within two models: (I) the standard model with additional right-handed heavy Majorana neutrinos, i.e., a typical seesaw-type model; (II) the standard model with left-handed and right-handed neutral singlets, which are inspired by certain scenarios of $SO(10)$ models and heterotic superstring models with E_6 symmetry. We calculate various LFV branching ratios and a T -odd asymmetry. The seesaw model I predicts very small branching ratios for LFV processes in most of the parameter space, although in a very restricted parameter region it can reach maximal branching ratios $Br(\tau \rightarrow \mu \gamma) \sim 10^{-9}$ and $Br(\tau \rightarrow 3\mu) \sim 10^{-10}$. In contrast, model II may show branching ratios $Br(\tau \rightarrow e \gamma) \sim 10^{-8}$ and $Br(\tau \rightarrow 3e) \leq 10^{-9}$ over a sizable region of the parameter space, large enough to be tested by experiments in the near future.

DOI: 10.1103/PhysRevD.66.034008

PACS number(s): 13.35.Dx, 14.60.Fg

I. INTRODUCTION

One of the many puzzles remaining in the current phenomenology of particle physics is to understand the smallness of the masses (≤ 1 eV) of standard neutrinos ν_e , ν_μ and ν_τ , compared to those of charged leptons. If neutrinos are of a Dirac nature, nonzero masses could be obtained in the standard model (SM) by introduction of (sterile) right-handed neutrinos. On the other hand, if neutrinos are of a Majorana nature, more appealing solutions to the small neutrino mass problem exist. In order to avoid the explicit breaking of the SM gauge symmetry and still obtain nonzero Majorana mass terms (via spontaneous symmetry breaking), an additional Higgs triplet is needed in the SM. The latter would result in physical Goldstone bosons, but these have been excluded by experiments at the CERN e^+e^- collider LEP. On the other hand, various models in the context of extended gauge structures result in Majorana mass terms and could give possible solutions to the neutrino mass problem. An appealing solution is the seesaw mechanism [1] within the framework of $SO(10)$ or left-right symmetric models. In the conventional seesaw models, the effective light neutrino masses are within the scales of eV to MeV via a relation involving the hierarchy between very large Majorana masses and Dirac masses comparable to those of charged leptons. Another possible solution was investigated in the framework of heterotic superstring models [2] with E_6 symmetry or certain scenarios of $SO(10)$ models [3], where the low-energy effective theories include new left-handed and right-handed neutral isosinglets and assume conservation of total lepton number in the Yukawa sector.

One possibility to test the neutrino sector lies in the study and measurement of lepton-flavor-violating (LFV) processes, e.g., $\mu \rightarrow e \gamma$ or $3e$; $\tau \rightarrow \mu \gamma$ or 3μ ; $\tau \rightarrow e \gamma$ or $3e$. Such pro-

cesses are practically suppressed to zero in the SM, due to the unitarity of the leptonic analogue of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix and the near masslessness of the three neutrinos. Motivated by the aforementioned models with an extended neutrino sector, the authors in Refs. [4–6] derived analytic expressions for LFV decay rates of charged leptons in such models with heavy Majorana neutrinos. The authors of Ref. [7] gave a model-independent framework for analyzing $\mu \rightarrow e \gamma$ and $\mu \rightarrow 3e$ processes and investigated specific features of several supersymmetric grand unified theories (GUTs). They focused on parity- and T -violating asymmetries involving muon polarization in the initial state.

Some generic properties of LFV processes and the corresponding constraints on the neutrino mass matrix have been studied in Ref. [8]. Phenomenological studies of various LFV and lepton-number-violating processes have appeared in the literature, including direct production of heavy Majorana neutrinos at various colliders [9], heavy Majorana mediated processes [10], and LFV processes (including $\mu \rightarrow e \gamma$ and $\tau \rightarrow \mu \gamma$) in supersymmetric frameworks [11].

In this paper we will consider LFV decays of tau leptons in the framework of the two aforementioned models with extended neutrino sectors. We will concentrate on the calculation of the corresponding LFV branching ratios and the corresponding expected numbers of events. In addition, we will calculate a T -odd asymmetry induced by these processes.¹ In Sec. II we review the two models in question. In Sec. III we present the formulas for the branching ratios of charged lepton decays, $l \rightarrow l' \gamma$ and $l \rightarrow 3l'$, and the T -odd asymmetry for $l \rightarrow 3l'$ within these two models. In Sec. IV we present approximate maximal values for LFV tau decay rates, exploring the possibility of obtaining sizable rates that

*Email address: cvetic@fis.utfsm.cl

†Email address: cdib@fis.utfsm.cl

‡Email address: cskim@yonsei.ac.kr

§Email address: jdkim@phy.yonsei.ac.kr

¹Under the assumption of CPT symmetry, CP violation is equivalent to T violation. While standard CP violation appears in the quark sector, it could also arise, for example, in processes which involve only elementary (SM) bosons [12] or (heavy) Majorana neutrinos [13,14].

can be measured in the foreseeable future, yet keeping consistency with present experimental constraints. We give a summary and state our conclusions in Sec. V.

II. TWO NEUTRINO-MIXING MODELS

To set up our notation, we briefly review the two models in question: We call model I the SM with the addition of right-handed neutrinos (singlets under the gauge group) and with the seesaw mechanism involved, and model II the SM with the addition of both left-handed and right-handed neutral singlets.

Model I. It is the SM with its N_L standard left-handed neutrinos ν_{Li} and an additional set of N_R right-handed neutrinos ν_{Ri} , where the neutrino mass terms (after gauge symmetry breaking), which can be written as

$$-\mathcal{L}_Y^\nu = \frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R^c)\mathcal{M}\begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix} + \text{H.c.}, \quad (1)$$

contain a $(N_L+N_R)\times(N_L+N_R)$ -dimensional matrix \mathcal{M} with a seesaw block form [1]. This matrix can always be diagonalized by means of a congruent transformation involving a unitary matrix U : namely,

$$\mathcal{M} = \begin{pmatrix} 0 & m_D \\ m_D^T & m_M \end{pmatrix}, \quad U\mathcal{M}U^T = \hat{\mathcal{M}}_d. \quad (2)$$

The resulting N_L+N_R mass eigenstates n_i are Majorana neutrinos, related to the interaction eigenstates ν_a by the matrix U :

$$\begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}_a = \sum_{i=1}^{N_L+N_R} U_{ia}^* n_{Li}, \quad \begin{pmatrix} \nu_L^c \\ \nu_R \end{pmatrix}_a = \sum_{i=1}^{N_L+N_R} U_{ia} n_{Ri}. \quad (3)$$

The first N_L mass eigenstates are the light standard partners of the charged leptons, while the other N_R eigenstates are heavy. It is convenient, as done in Ref. [4], to introduce a $N_L\times(N_L+N_R)$ -dimensional matrix B for charged current interactions, and a $(N_L+N_R)\times(N_L+N_R)$ -dimensional matrix C for neutral current interactions

$$B_{li} = U_{il}^*, \quad C_{ij} = \sum_{a=1}^{N_L} U_{ia} U_{ja}^*, \quad (4)$$

where the charged leptons are taken in their mass basis. The ratio between the Dirac mass (m_D) and the Majorana mass (m_M) characterizes the strength of the heavy-to-light neutrino mixings $(s_L^\nu)^2 \equiv \sum_h |U_{hl}|^2$ ($\sim |m_D|^2/|m_M|^2$), as well as the size of the physical light neutrino masses: $m_{\nu_{light}} \sim m_D^2/m_M$. In this model the very low experimental bounds on $m_{\nu_{light}}$ ($\lesssim 1$ eV) impose severe constraints on the $|m_D| \ll |m_M|$ hierarchy required, and consequently also on the heavy-to-light neutrino mixings.

Model II. It is similar to model I, except that it contains an equal number N_R of left-handed (S_{Li}) and right-handed (ν_{Ri}) neutral singlets [2,3], and the form of the mass matrix \mathcal{M} is such that total lepton number is conserved (although

lepton flavor mixing is still possible). After electroweak symmetry breaking, the neutrino mass terms are

$$-\mathcal{L}_Y^\nu = \frac{1}{2}(\bar{\nu}_L, \bar{\nu}_R^c, \bar{S}_L)\mathcal{M}\begin{pmatrix} \nu_L^c \\ \nu_R \\ S_L^c \end{pmatrix} + \text{H.c.},$$

$$\mathcal{M} = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & m_M^T \\ 0 & m_M & 0 \end{pmatrix}. \quad (5)$$

The mass matrix \mathcal{M} is $(N_L+\tilde{N}_R)\times(N_L+\tilde{N}_R)$ -dimensional, where $\tilde{N}_R=2N_R$ (the Dirac block m_D is $N_R\times N_L$ -dimensional). When $N_R=N_L$, this model predicts, for each of the N_L generations, a massless Weyl neutrino and two degenerate neutral Majorana neutrinos. Consequently, the seesaw-type restriction $m_{\nu_{light}} \sim m_D^2/m_M$ of model I does not apply here [15,16]. Here it is not the smallness of light neutrino masses but the present experimental bounds on the heavy-to-light mixing parameters $(s_L^\nu)^2 \sim |m_D|^2/|m_M|^2$ ($\lesssim 10^{-2}$, see below) which impose a certain level of hierarchy $|m_D| < |m_M|$ between the Dirac and Majorana mass sector. This hierarchy is in general much weaker than in seesaw models. Although model II features (N_L) massless neutrinos in the light sector, nonzero masses for the light neutrinos can be generated by introducing small perturbations in the lower right block of \mathcal{M} , i.e., small Majorana mass terms for the neutral singlets S_{Li} , without much effect on the mixings of heavy-to-light fields.

III. FLAVOR-VIOLATING TAU DECAYS WITHIN THE TWO MODELS

Recently LFV processes have been investigated extensively because SUSY GUTs predict that the branching ratios for $\mu \rightarrow e\gamma$ and $\mu \rightarrow 3e$ and the $\mu-e$ conversion rate in a nucleus can reach just below present experimental bounds [7,17]. Here we address the predictions for LFV decays of the form $l \rightarrow l'\gamma$ and $l \rightarrow 3l'$ within the models of Sec. II.

The amplitudes for $l \rightarrow l'\gamma$ and $l \rightarrow 3l'$ in terms of the model parameters were obtained in Ref. [4]. These processes occur only at one loop level or higher in the (extended) electroweak theory. The amplitude for $l \rightarrow l'\gamma$ arises from a γ -penguin with the photon on the mass shell and is given by an expression of the form

$$\mathcal{M}(l \rightarrow l'\gamma) = i \frac{e\alpha_w}{8\pi M_W^2} \epsilon^\mu G_\gamma^{ll'} \bar{u}_{l'} i\sigma_{\mu\nu} q^\nu (m_{l'} P_L + m_l P_R) u_l, \quad (6)$$

while the amplitude for $l \rightarrow 3l'$ receives contributions from γ penguin diagrams, Z penguin diagrams and box diagrams, $\mathcal{M}(l \rightarrow 3l') = \mathcal{M}_\gamma + \mathcal{M}_Z + \mathcal{M}_{Box}$:

$$\begin{aligned} \mathcal{M}_\gamma(l \rightarrow 3l') &= -i \frac{\alpha_w^2 s_w^2}{2M_W^2} \bar{u}_{l'} \gamma^\mu v_l \bar{u}_l \\ &\times \left[F_\gamma^{ll'} \left(\gamma_\mu - \frac{q_\mu \not{q}}{q^2} \right) P_L - G_\gamma^{ll'} i \sigma_{\mu\nu} \right. \\ &\left. \times \frac{q^\nu}{q^2} (m_{l'} P_L + m_l P_R) \right] u_l, \end{aligned} \quad (7)$$

$$\begin{aligned} \mathcal{M}_Z(l \rightarrow 3l') &= -i \frac{\alpha_w^2}{8M_W^2} \bar{u}_{l'} \gamma_\mu P_L u_l \bar{u}_l \gamma^\mu \\ &\times [(2 - 4s_w^2) P_L - 4s_w^2 P_R] v_l F_Z^{ll'}, \end{aligned} \quad (8)$$

$$\begin{aligned} \mathcal{M}_{Box}(l \rightarrow 3l') &= -i \frac{\alpha_w^2}{4M_W^2} \bar{u}_{l'} \gamma_\mu P_L u_l \bar{u}_l \gamma^\mu \\ &\times P_L v_l F_{Box}^{ll'}. \end{aligned} \quad (9)$$

In the above expressions, ϵ^μ [Eq. (6)] is the photon polarization, $P_{R/L} = (1 \pm \gamma_5)/2$, and the factors $G_\gamma^{ll'}$, \dots , $F_{Box}^{ll'}$ are combinations of mixing matrix elements and some special functions that appear in the loop diagrams of the corresponding processes:

$$G_\gamma^{ll'} = \sum_{i=1}^{N_L + \tilde{N}_R} B_{li}^* B_{l'i} G_\gamma(\lambda_i), \quad (10)$$

$$F_\gamma^{ll'} = \sum_{i=1}^{N_L + \tilde{N}_R} B_{li}^* B_{l'i} F_\gamma(\lambda_i), \quad (11)$$

$$\begin{aligned} F_Z^{ll'} &= \sum_{i,j=1}^{N_L + \tilde{N}_R} B_{li}^* B_{l'j} [\delta_{ij} F_Z(\lambda_i) + C_{ij} H_Z(\lambda_i, \lambda_j) \\ &+ C_{ij}^* G_Z(\lambda_i, \lambda_j)], \end{aligned} \quad (12)$$

$$\begin{aligned} F_{Box}^{ll'} &= \sum_{i,j=1}^{N_L + \tilde{N}_R} [2B_{l'i} B_{l'j} B_{li}^* B_{lj}^* F_{Box}(\lambda_i, \lambda_j) \\ &+ B_{l'i} B_{l'i} B_{l'j}^* B_{lj}^* G_{Box}(\lambda_i, \lambda_j)]. \end{aligned} \quad (13)$$

The explicit expressions for the loop functions $G_\gamma(x)$, \dots , $F_{Box}(x, y)$ are given in Ref. [4], and their arguments are $\lambda_i = m_i^2/M_W^2$, i.e. the masses (squared) of the Majorana neutrinos inside the loop, in units of M_W . Equations (10)–(13) involve a summation over all Majorana neutrinos, \tilde{N}_R being the number of heavy ones ($= N_R, 2N_R$ in models I, II, respectively).

From the amplitudes (6)–(9), the decay rates are obtained. Using the notation of Ref. [7], they take the form

$$\begin{aligned} \Gamma(l \rightarrow l' \gamma) &= \Gamma(l \rightarrow e \bar{\nu}_e \nu_l) \\ &\times 384 \pi^2 (1 + m_{l'}/m_l) |A_R|^2, \end{aligned} \quad (14)$$

$$\begin{aligned} \Gamma(l \rightarrow 3l') &= \Gamma(l \rightarrow e \bar{\nu}_e \nu_l) \cdot \{2|g_4|^2 + |g_6|^2 \\ &+ 8 \operatorname{Re}(eA_R(2g_4^* + g_6^*)) + (1 \\ &+ m_{l'}/m_l)(32 \log(m_l^2/3m_{l'}^2) \\ &- 104/3) |eA_R|^2\} \\ &\times \{1 + \mathcal{O}(m_{l'}/m_l)\}, \end{aligned} \quad (15)$$

where g_4 , g_6 and A_R are the coefficients of the operators in the effective Lagrangian relevant to these processes, and are given by

$$g_4 = \frac{\alpha_w}{8\pi} \{2s_w^2 F_\gamma^{ll'} + (1 - 2s_w^2) F_Z^{ll'} + F_{Box}^{ll'}\}, \quad (16)$$

$$g_6 = \frac{\alpha_w}{8\pi} \{2s_w^2 F_\gamma^{ll'} + (-2s_w^2) F_Z^{ll'}\}, \quad (17)$$

$$eA_R = \frac{\alpha_{em}}{8\pi} G_\gamma^{ll'}, \quad (18)$$

where $\alpha_w \equiv g_2^2/(4\pi) = \sqrt{2} G_F M_W^2/\pi$ and $\alpha_{em} \equiv e^2/(4\pi)$ are the weak and electromagnetic fine structure constants, and $s_w^2 \equiv \sin^2 \theta_w$.

A T -odd asymmetry can be defined in the decays $l \rightarrow 3l'$ which is sensitive to the CP phases of the neutrino mixing matrices, but has the experimental drawback that it requires independent knowledge of the initial lepton polarization (in this case, the tau lepton polarization). Defining, in the c.m. frame, the decay plane as the plane that contains the three final momenta, A_T is the asymmetry between the cases when the polarization of the initial lepton points to one or to the other side of the decay plane. Geometrically, A_T is a ϕ -angle asymmetry, where ϕ is the angle between the decay plane and the plane that contains the polarization vector of the initial lepton l and the momentum of the final lepton with charge opposite to l (see Ref. [7] for details). The explicit expression for A_T is then

$$A_T = \left(\int_0^\pi \frac{\pi d\Gamma}{d\phi} d\phi - \int_\pi^{2\pi} \frac{\pi d\Gamma}{d\phi} d\phi \right) / \Gamma \quad (19)$$

and in terms of parameters (16)–(18) it is

$$\begin{aligned} A_T &= \frac{\Gamma(l \rightarrow e \bar{\nu}_e \nu_l)}{\Gamma(l \rightarrow 3l')} \left\{ \frac{192}{35} \operatorname{Im}(eA_R g_4^*) - \frac{128}{35} \operatorname{Im}(eA_R g_6^*) \right\} \\ &\times \{1 + \mathcal{O}(m_{l'}/m_l)\}. \end{aligned} \quad (20)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

Several experiments provide constraints for the masses and mixings of light and heavy neutrinos: Tritium beta decay provides the present bound on the electron neutrino mass $m_{\nu_e} < 3$ eV [18]. The solar neutrino deficit [19] can be interpreted either by matter enhanced neutrino oscillations if $\Delta m_{sol}^2 \sim 1 \times 10^{-5}$ eV² with small or large mixing, or by

vacuum oscillations if $\Delta m_{sol}^2 \sim 10^{-10} \text{ eV}^2$ with maximal mixing [20]. Atmospheric neutrino experiments show evidence for $\Delta m_{atm}^2 \sim 2.2 \times 10^{-3} \text{ eV}^2$ with maximal mixing [21,22]. We will assume that $\Delta m_{sol}^2 = |m_{\nu_\mu}^2 - m_{\nu_e}^2|$ and $\Delta m_{atm}^2 = |m_{\nu_\tau}^2 - m_{\nu_\mu}^2|$. Since $\Delta m_{sol}^2 \ll \Delta m_{atm}^2$, then $|m_{\nu_\tau}^2 - m_{\nu_e}^2| = \Delta m_{atm}^2$ as well. Since $\Delta m_{atm}^2 \ll 3^2 \text{ eV}^2$, the 3 eV upper bound applies to all three light neutrino masses: $m_{\nu_e}, m_{\nu_\mu}, m_{\nu_\tau} < 3 \text{ eV}$. Experimental evidence indicates that $\nu_\tau - \nu_e$ mixing is (nearly) zero [23]. Further, Refs. [23] investigated possible patterns of the Majorana neutrino mass matrix which are compatible with these results and the non-observation of neutrinoless double beta decay [24,25]. In the models we are considering, a number of low-energy experiments set upper bounds on possible non-SM couplings, which are characterized in Refs. [4,26] as $(s_L^{\nu_l})^2 \equiv \sum_h |B_{lh}|^2$ (where h indicates heavy neutrinos). Recent analyses [27], for models where the additional neutrinos are $SU(2)_L$ singlets, give

$$(s_L^{\nu_e})^2 < 0.005, \quad (s_L^{\nu_\mu})^2 < 0.002, \quad (s_L^{\nu_\tau})^2 < 0.010. \quad (21)$$

There is also a theoretical constraint, a perturbative unitarity condition (PUB) [28], which states that perturbation theory to one loop is applicable only if the decay width Γ_{n_h} of a heavy Majorana neutrino is small compared to its mass, say, $\Gamma_{n_h} < M_{n_h}/2$. In the limit of large masses $M_{n_h} \gg M_W, M_Z, M_H$, the PUB constitutes an upper bound for heavy neutrino masses [4,29,30]:

$$M_{n_h}^2 \sum_{l=1}^{N_L} |B_{lh}|^2 < \frac{2}{\alpha_w} M_W^2, \quad h = 1, \dots, \tilde{N}_R. \quad (22)$$

In addition, there is a lower bound, $M_{n_h} > 100 \text{ GeV}$, arising from the non-observation of heavy neutrinos in experiments to date.

As the bound on $(s_L^{\nu_\mu})^2$ in Eq. (21) is tighter than those on $(s_L^{\nu_e})^2$ and $(s_L^{\nu_\tau})^2$, LFV muon decays are more suppressed than tau decays. We will therefore study LFV tau decays in the two models, trying to see if the present experimental upper bounds on $\tau \rightarrow l \gamma$ and $\tau \rightarrow 3l$ [18]

$$\begin{aligned} Br(\tau^- \rightarrow e^- \gamma) &< 2.7 \times 10^{-6}, \\ Br(\tau^- \rightarrow \mu^- \gamma) &< 1.1 \times 10^{-6}, \end{aligned} \quad (23)$$

$$\begin{aligned} Br(\tau^- \rightarrow e^- e^- e^+) &< 2.9 \times 10^{-6}, \\ Br(\tau^- \rightarrow \mu^- \mu^- \mu^+) &< 1.9 \times 10^{-6}, \end{aligned} \quad (24)$$

can be reached theoretically once we account for all the aforementioned constraints.

A numerical analysis of such reactions in model I, for $N_L=3$ and $N_R=2$, has been performed in Ref. [4]. A comprehensive numerical analysis in model II has been performed in Ref. [31]. In these two references, the analyses were performed by starting with the neutrino eigenmasses and specific combinations of the mixing matrix coefficients

B_{ij} on which restrictions were imposed. Our approach will be somewhat different, starting with explicit mass matrices, Eqs. (2) and (5), and from there deriving the masses and mixings. This approach is cumbersome if we include all three light generations, so we will take $N_L=2$ and $N_R=2$. In this way, $Br(\tau \rightarrow \mu \mu e, e e \mu)$ will not be considered. However, since we are particularly interested in the largest possible branching ratios, and since either $Br(\tau \rightarrow 3e)$ or $Br(\tau \rightarrow 3\mu)$ is very suppressed (as argued below), then $Br(\tau \rightarrow \mu \mu e, e e \mu)$ is also expected to be suppressed in comparison with the largest of the LFV branching ratios.

A. Model I

In the considered case ($N_L=2$ and $N_R=2$) for LFV τ decays, we have two light lepton generations (ν_l, l^-) and (ν_τ, τ^-) , where l is either e or μ . The structure of the CP -violating phases in these types of models was studied in Ref. [13]. In the considered case there are two independent physical phases (δ_i , $i=1,2$), so that the Dirac and Majorana submatrices in \mathcal{M} [see Eq. (2)] can be taken to be of the form

$$m_D = \begin{pmatrix} a & b e^{i\delta_1} \\ c e^{i\delta_2} & d \end{pmatrix}, \quad m_M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}, \quad (25)$$

where a, b, c, d are real. We take the convention $M_2 \geq M_1$. The matrix \mathcal{M} can be diagonalized via the congruent transformation of Eq. (2)—in numerical calculations we use the diagonalization approach as described in Ref. [32].

We then find the values of m_D (i.e., $a, b, c, d; \delta_1, \delta_2$) which give, for given heavy Majorana masses M_1 and M_2 , the largest possible LFV branching ratios $Br(\tau \rightarrow \gamma l)$ and $Br(\tau \rightarrow 3l)$.

In model I, the transformation matrix U of Eq. (2) can be presented as a product of a seesaw transformation block matrix U_s and a light-sector mixing matrix V^\dagger : $U = V^\dagger U_s$. The seesaw transformation U_s produces an effective light neutrino mass matrix $m_{\nu_{light}} \approx m_D m_M^{-1} m_D^T$ in the case $m_D \ll m_M$, namely

$$\begin{aligned} m_{\nu_{light}} &= \begin{bmatrix} \left(\frac{a^2}{M_1} + \frac{b^2}{M_2} e^{2i\delta_1} \right) & \left(\frac{ac}{M_1} e^{i\delta_2} + \frac{bd}{M_2} e^{i\delta_1} \right) \\ \left(\frac{ac}{M_1} e^{i\delta_2} + \frac{bd}{M_2} e^{i\delta_1} \right) & \left(\frac{c^2}{M_1} e^{2i\delta_2} + \frac{d^2}{M_2} \right) \end{bmatrix} \\ &\times (1 + \mathcal{O}(m_D^2 m_M^{-2})), \end{aligned} \quad (26)$$

and LFV mixings of order $\sim m_D m_M^{-1}$. The light sector mixing matrix V^\dagger , which is the upper left part of U , is approximately unitary and of the form

$$V^\dagger \approx \begin{pmatrix} \cos \theta & \sin \theta \exp(-i\varepsilon) \\ -\sin \theta \exp(i\varepsilon) & \cos \theta \end{pmatrix}, \quad (27)$$

where $\theta=0$ and $\pi/4$ correspond to zero and maximal mixing, respectively, and where ε is a CP phase, in general a complicated function of δ_1 and δ_2 .

In the case $\tau \rightarrow \mu$, we indeed have maximal mixing $\theta \rightarrow \pi/4$, according to atmospheric neutrino experiments. We will consider first this case. If we demand that this maximal mixing is obtained independently of the values M_1 and M_2 of the heavy Majorana sector [see Eq. (25)], then the following simple relations in the light Dirac sector are implied: $a^2 = c^2$, $b^2 = d^2$, and $\delta_1 = \delta_2 \equiv \delta$. The value of δ can be restricted to lie in the range $-\pi/2 < \delta \leq \pi/2$. The eigenmasses of the two light neutrinos are then

$$m_{\nu_1, \nu_2} = \left[\left(\frac{a^2}{M_1} \right)^2 + \left(\frac{b^2}{M_2} \right)^2 + 2 \frac{a^2}{M_1} \frac{b^2}{M_2} \cos(2\delta) \right]^{1/2} \pm \left(\frac{ac}{M_1} + \frac{bd}{M_2} \right), \quad (28)$$

while the heavy-to-light mixing parameters (21) $(s_L^{\nu_i})^2 \equiv \Sigma_{h=3}^4 |B_{lh}|^2$ are

$$(s_L^{\nu_\mu})^2 = (s_L^{\nu_\tau})^2 = \frac{a^2}{M_1^2} + \frac{b^2}{M_2^2} (\equiv s_L^2), \quad (29)$$

and the CP -violating parameter ε of Eq. (27) is

$$\tan \varepsilon = \tan \delta \times \frac{(a^2/M_1) - (b^2/M_2)}{(a^2/M_1) + (b^2/M_2)}. \quad (30)$$

Now, conditions $a^2 = c^2$, $b^2 = d^2$ mean two possible cases: (1) $a = \pm c$ and $b = \pm d$; or (2) $a = \pm c$ and $b = \mp d$.

(1) Case $a = \pm c$ and $b = \pm d$: then $m_{\nu_\tau} \geq (a^2/M_1 + b^2/M_2)$, so $s_L^2 \leq m_{\nu_\tau}/M_1 < 3 \text{ eV}/M_1 \leq 3 \times 10^{-11}$. Since $Br(\tau \rightarrow \mu \gamma)$ and $Br(\tau \rightarrow 3\mu)$ are approximately proportional to $(s_L^{\nu_\mu})^2 (s_L^{\nu_\tau})^2 (\equiv s_L^4)$, it follows that $Br(\tau \rightarrow \mu \gamma)$ and $Br(\tau \rightarrow 3\mu)$ are below 10^{-24} . For muon decays, $Br(\mu \rightarrow e \gamma)$ and $Br(\mu \rightarrow 3e)$ are obtained by dividing the previous values by $Br(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) \approx 0.174$, thus obtaining $Br(\mu \rightarrow e \gamma)$ and $Br(\mu \rightarrow 3e)$ below 10^{-23} , values which are well below their respective present experimental bounds (10^{-11} and 10^{-12}).

(2) Case $a = \pm c$ and $b = \mp d$: then $m_{\nu_\tau} \geq 2|a^2/M_1 - b^2/M_2|$, with the equality being reached only when $\delta = \pi/2$. In the latter case, $m_{\nu_\mu} = 0$, and $m_{\nu_\tau} = 2|a^2/M_1 - b^2/M_2| = (\Delta m_{atm}^2)^{1/2} \approx 0.047 \text{ eV}$. This case ($\delta = \pi/2$) thus avoids the suppression of $s_L^2 = (a^2/M_1^2 + b^2/M_2^2)$ while keeping a^2/M_1 extremely close to b^2/M_2 (a fine tuning situation). The value of s_L^2 can then be saturated to $(s_L^2)_{\max} = 0.002$ [Eq. (21)] with the following parameters in the Dirac matrix m_D :

$$a = c = M_1 (s_L)_{\max} / \sqrt{1 + M_1/M_2},$$

$$b = -d = a(1 \pm \eta) \sqrt{M_2/M_1}, \quad (31)$$

$$\eta = \sqrt{\Delta m_{atm}^2} \left(1 + \frac{M_1}{M_2} \right) \frac{1}{4M_1 (s_L^2)_{\max}}$$

$$\approx 1.17 \times 10^{-13} \times \frac{1}{(s_L^2)_{\max}} \left(1 + \frac{M_1}{M_2} \right), \quad (32)$$

and where, as mentioned, $\delta = \pi/2$. The rates $\tau \rightarrow \mu \gamma$ and $\tau \rightarrow 3\mu$ are again practically proportional to (s_L^4) and approach their maximum (for fixed chosen values of M_1 and M_2) in the case given by Eqs. (31),(32), as shown in the Appendix. The conditions (31),(32), which are only reached by fine tuning, give the largest possible branching ratios in model I, $Br(\tau \rightarrow \mu \gamma) \sim 10^{-9}$ and $Br(\tau \rightarrow \mu \mu \mu) \leq 10^{-10}$. In Fig. 1 we show the two branching ratios as functions of M_2 , for two different ratios $M_1/M_2 = 0.1$ and 0.5 , accounting also for the PUB conditions (22). The CP -violating asymmetry parameter A_T (20) is in this case, unfortunately, equal to zero, since $\delta = \pi/2$ implies $\varepsilon = 0$ [Eq. (30)] and thus no CP violation. If we move δ away from $\pi/2$, the allowed branching ratios drop sharply, mainly due to the upper bounds $m_{\nu_\mu}, m_{\nu_\tau} < 3 \text{ eV}$, i.e., a situation similar to case (1) sets in. Accordingly, we do not consider other situations of CP violation in model I, as the branching ratios fall dramatically to unobservable values outside the fine-tuning condition.

The results for $Br(\mu \rightarrow e \gamma)$ and $Br(\mu \rightarrow 3e)$ are again obtained by dividing the above results by 0.174. However, we will then obtain values above the present experimental bounds 1.2×10^{-11} and 1.0×10^{-12} , respectively. We are thus led to conclude that the assumed fine-tuning condition is not really met in this case.

If we now consider the case $\tau \rightarrow e$, i.e. the processes $\tau \rightarrow e \gamma$ and $\tau \rightarrow eee$, the neutrino oscillation experiments indicate that the mixing is almost zero [23]: $\theta \approx 0$ in Eq. (27). If we assume that the zero mixing condition is satisfied independently of the heavy Majorana sector, we obtain the relations $ac = 0$ and $bd = 0$. The cases where $a = b = 0$ or $c = d = 0$ give us $(s_L^{\nu_e})^2 = 0$ and $(s_L^{\nu_\tau})^2 = 0$, respectively, and thus extremely suppressed branching ratios. The cases where $a = d = 0$ or $b = c = 0$ give $(s_L^{\nu_e})^2 (s_L^{\nu_\tau})^2 < m_{\nu_\tau}/M_1 < 3 \text{ eV}/100 \text{ GeV} = 3 \times 10^{-11}$, i.e., as in case (1) discussed previously we obtain extremely suppressed branching ratios.

B. Model II

The neutrino mass matrix has the form of Eq. (5). In the considered two-generation case ($N_L = 2$, $\tilde{N}_R = 4$) for LFV τ decays, the submatrices m_D and m_M can be taken in the form

$$m_D = \begin{pmatrix} a & be^{i\xi} \\ ce^{i\xi} & d \end{pmatrix}, \quad m_M = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix}. \quad (33)$$

In the two-generation scheme of model II there is only one CP -violating phase ξ [15].

Since the (N_L) light neutrinos in the model are massless, the LFV τ decay rates will neither be affected by the experimental light neutrino mass bounds, nor by the solar and atmospheric neutrino experiments and their requirements of

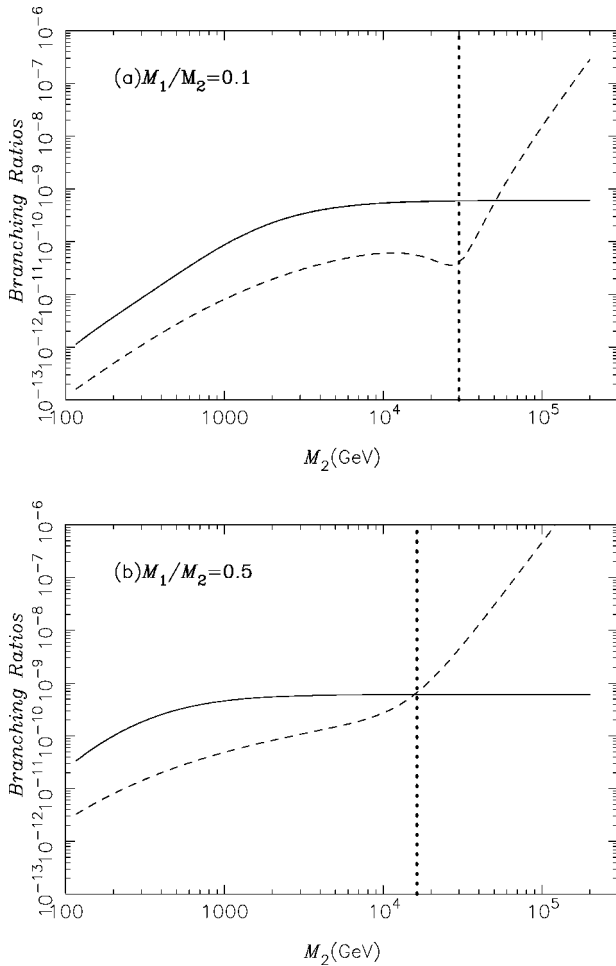


FIG. 1. Maximal branching ratios for $\tau \rightarrow \mu \gamma$ (solid lines) and $\tau \rightarrow 3\mu$ (dashed lines) as functions of M_2 in model I, for a fixed ratio $M_1/M_2=0.1$ (a) and $M_1/M_2=0.5$ (b). M_1 and M_2 are restricted to be above 100 GeV and below the perturbative unitarity bounds (22), indicated by the vertical line. The Dirac mass parameters are taken in the form (31),(32) (and $\delta_1 = \delta_2 = \pi/2$) which give maximal branching ratios at any given M_1 and M_2 .

the maximal $(\nu_\mu - \nu_\tau, \nu_e - \nu_\mu)$ or minimal $(\nu_e - \nu_\tau)$ mixing.² However, the rates will be affected by the PUB restrictions (22), as well as by the mixing parameter bounds (21) as the rates are proportional to $(s_L^{\nu\tau})^2(s_L^{\nu\mu})^2$ or $(s_L^{\nu\tau})^2(s_L^{\nu e})^2$. The $\tau \rightarrow \mu$ rates are suppressed in comparison to $\tau \rightarrow e$ rates, because the upper bound for $(s_L^{\nu\mu})^2$ is smaller than that for $(s_L^{\nu e})^2$ [cf. Eq. (21)]. Therefore, we will consider $\tau \rightarrow e \gamma$ and $\tau \rightarrow 3e$ LFV rates.

Similarly as in model I, we first find the Dirac parameters in model II such that the LFV branching ratios, at fixed Majorana masses M_1 and M_2 , are maximal. This occurs when

²Nonetheless, it is possible to obtain nonzero light neutrino masses in model II to accommodate neutrino oscillation experiments. Introduction of small mass terms for the neutral singlets S_{Li} gives non-zero and non-degenerate masses of the light neutrinos, thus the possibility to accommodate Δm_{atm}^2 without significantly affecting the presented LFV rates.

inequality (A3) in the Appendix becomes equality, and the values of the mixing parameters $(s_L)^2$ are maximized, i.e., saturated according to Eq. (21). In the case of no CP violation ($\xi=0$), the requirement (A4) for the inequality (A3) to become an equality gives the relation $ad=bc$, while the saturation of the values of $(s_L^{\nu\tau})^2$ and $(s_L^{\nu e})^2$ gives two other conditions, for the four Dirac parameters a, b, c, d . This still allows us the freedom of fixing one of the four Dirac parameters without affecting the rates. We can, for example, require the symmetry of the (real) m_D matrix: $b=c$. All of the above results in the following approximately ‘‘optimized’’ choice of m_D parameters (when $\xi=0$):

$$a = \frac{M_2}{\sqrt{(M_2/M_1)^2 + (s_{2m}/s_{1m})^2}} \frac{s_{1m}}{\sqrt{1 - s_{1m}^2 - s_{2m}^2}}, \quad (34)$$

$$b = c = a \times (s_{2m}/s_{1m}), \quad d = a \times (s_{2m}/s_{1m})^2, \quad (35)$$

where $s_{1m}^2 = (s_L^{\nu e})_{\max}^2 = 0.005$ and $s_{2m}^2 = (s_L^{\nu\tau})_{\max}^2 = 0.010$, according to the bounds of Eq. (21).³ In Fig. 2 we present the two branching ratios $Br(\tau \rightarrow e \gamma)$ and $Br(\tau \rightarrow 3e)$ as functions of M_2 , for two fixed ratios $M_1/M_2=0.1$ and 0.5 , and for the CP phase $\xi=0$. We see from Fig. 2 that the LFV branching ratios in model II are $Br(\tau \rightarrow e \gamma) \lesssim 10^{-8}$ and $Br(\tau \rightarrow 3e) \lesssim 10^{-9}$. These values decrease relatively slowly when the parameters of the Dirac sector $(a, b, c, d; \xi)$ are moved away from the ‘‘optimal’’ values. This contrasts with model I, where the maximal rates are reached only in a finely tuned region of parameter space. Our maximal values of $Br(\tau \rightarrow e \gamma)$ agree with those of Ilakovac [31]—ours are by about factor three lower only because we took a different upper bound $(s_L^{\nu\tau})^2 < 0.010$ (21).

Figure 3 shows the T -odd asymmetry A_T of Eq. (20) for the same choices of mass matrix parameters as in Fig. 2(b), but for $\xi = \pi/4$ (solid line) and $\xi = 3\pi/4$ (dashed line) (maximal CP violation). When all four heavy Majorana neutrinos are degenerate, there is no CP violation [15] and $A_T=0$. Also $A_T=0$ if $\xi=0$ or $\pi/2$. Notice that the maximal Br 's are reached for $\xi=0$, thus no CP violation; in the cases of Fig. 3 ($\xi = \pi/4, 3\pi/4$) the values of Br 's are about one half of the corresponding ones in Fig. 2(b) ($\xi=0$). The searches for maximal rates and for CP violation are in this sense complementary in their most optimistic cases.

In the $\tau \rightarrow \mu$ processes, the maximal branching ratios in model II are suppressed by an additional factor of $(s_L^{\nu\mu})_{\max}^2 / (s_L^{\nu e})_{\max}^2 = 0.002/0.005 \approx 0.4$ [cf. Eq. (21)]. The approximate maximal branching ratios for the $\mu \rightarrow e$ LFV processes in model II are obtained from the corresponding $\tau \rightarrow e$ branching ratios by multiplying them with $(s_L^{\nu\mu})_{\max}^2 / (s_L^{\nu\tau})_{\max}^2 = 0.2$ and dividing by $Br(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau) \approx 0.174$. Thus, $Br(\mu \rightarrow e \gamma) \lesssim 10^{-8}$ and $Br(\mu \rightarrow 3e) \lesssim 10^{-9}$,

³In contrast to model I, we do not have the requirement of $(s_L^{\nu\tau})^2 = (s_L^{\nu e})^2$ which followed there from the maximal $(\nu_l - \nu_\tau)$ mixing condition ($l = \mu$ there), so that in model II we can saturate each of the two upper bounds (21) separately.

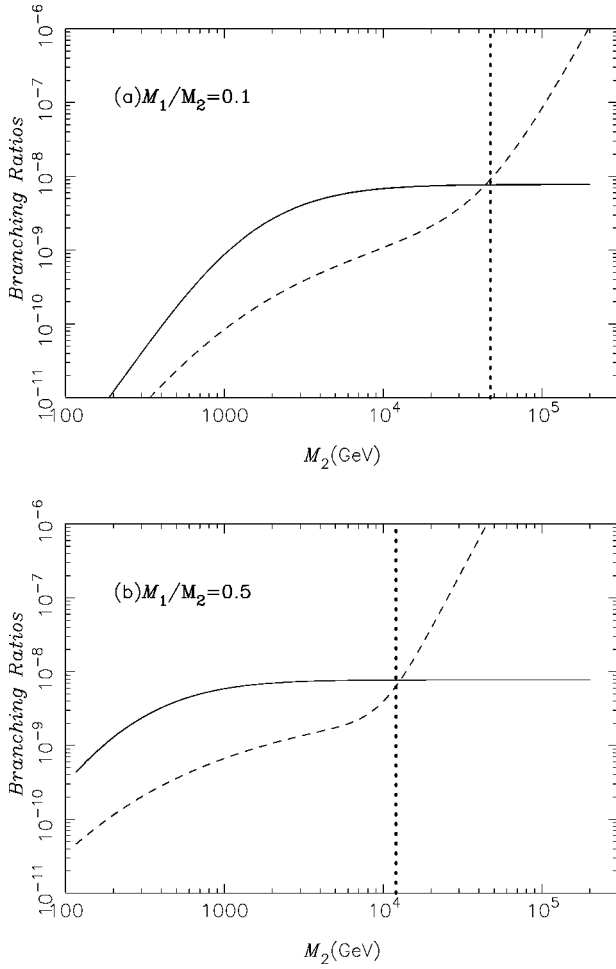


FIG. 2. Maximal branching ratios for $\tau \rightarrow e\gamma$ (solid line) and $\tau \rightarrow 3e$ (dashed line) as functions of M_2 in model II, for a fixed ratio $M_1/M_2=0.1$ (a) and $M_1/M_2=0.5$ (b). M_1 and M_2 are restricted to be above 100 GeV and below the perturbative unitarity bounds (22), indicated by the vertical line. The Dirac mass parameters a, b, c, d are taken in the form (34),(35) which give approximately maximal branching ratios. Here we use the CP phase $\xi=0$.

which is above the present experimental bounds 1.2×10^{-11} and 1.0×10^{-12} . Therefore, the maximizing conditions (34), (35) cannot be met in this case.

C. Expected numbers of events

The explicit numbers of expected events in the considered processes depend on the way the τ leptons are produced and on the luminosities involved. For example, the τ pairs could be produced via $e^+e^- \rightarrow \tau^+\tau^-$ close to the production threshold, or by sitting on a specific vector meson resonance V : $e^+e^- \rightarrow V \rightarrow \tau^+\tau^-$. In this case, $\sigma(e^+e^- \rightarrow V)$ as a function of the CMS energy \sqrt{s} can be approximated as a Breit-Wigner function

$$\sigma(s; e^+e^- \rightarrow V) = K \frac{1}{[(\sqrt{s} - M_V)^2 + (\Gamma_V/2)^2]}, \quad (36)$$

where M_V and Γ_V are the mass and the total decay width of

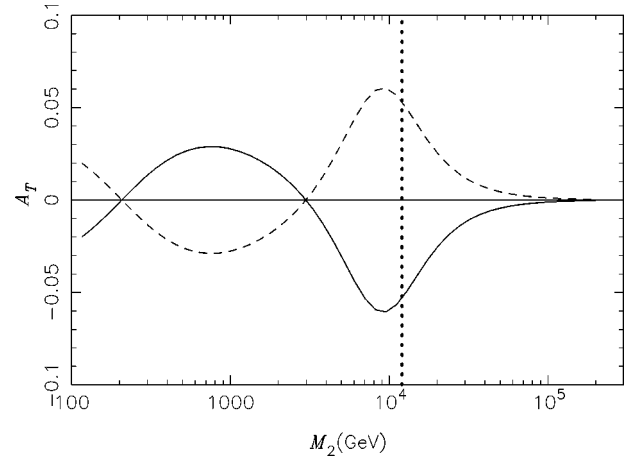


FIG. 3. The T asymmetry A_T in model II, for the decay $\tau \rightarrow 3e$ as a function of M_2 , keeping $M_1/M_2=0.5$ and adjusting the mass parameters in order to obtain maximal branching ratios [i.e., Fig. 2(b)]. The CP phase is taken to be $\xi = \pi/4$ (solid line) and $\xi = 3\pi/4$ (dashed line). $Br(\tau \rightarrow 3e)$ for $\xi = \pi/4$ or $3\pi/4$ is lower by about factor two in comparison to the case $\xi=0$ of Fig. 2(b).

the resonance. Constant K in Eq. (36) can be fixed by invoking the narrow width approximation (nwa) formula

$$\sigma_{\text{nwa}}(s; e^+e^- \rightarrow V) = \frac{12\pi^2 \Gamma_{ee}(V)}{M_V} \delta(s - M_V^2), \quad (37)$$

where $\Gamma_{ee}(V)$ is the partial decay width for $V \rightarrow e^+e^-$. Integration of Eq. (37) over the variable s gives $12\pi^2 \Gamma_{ee}(V)/M_V$, fixing the constant K in Eq. (36): $K = 3\pi \Gamma_{ee}(V) \Gamma_V / M_V^2$. The production cross section is maximal on the top of the resonance $\sqrt{s} = M_V$:

$$\begin{aligned} \sigma(e^+e^- \rightarrow V \rightarrow \tau^+\tau^-)^{\text{max}} &= \sigma(e^+e^- \rightarrow V)^{\text{max}} \frac{\Gamma_{\tau\tau}(V)}{\Gamma_V} \\ &\approx 12\pi \frac{\Gamma_{ee}(V)}{\Gamma_V} \frac{\Gamma_{\tau\tau}(V)}{\Gamma_V} \frac{1}{M_V^2}. \end{aligned} \quad (38)$$

Multiplying this cross section by twice the branching ratio $Br(\tau \rightarrow e\gamma(eee))$ we obtain the cross section for the process $e^+e^- \rightarrow V \rightarrow \tau^+\tau^- \rightarrow e\gamma(eee) + \tau$. These branching ratios are $\leq 10^{-8}(10^{-9})$ in model II, as shown in Fig. 2. For example, if the resonance is taken to be $V = Y(1S)$ [$M_V = 9.46$ GeV; $\Gamma_{ee}(V)/\Gamma_V \approx \Gamma_{\tau\tau}(V)/\Gamma_V \approx 0.025$], then $\sigma(e^+e^- \rightarrow V \rightarrow \tau^+\tau^- \rightarrow e\gamma(eee) + \tau)$ would be about 2 (0.2) fb. For a luminosity of $10 \text{ fb}^{-1}/\text{yr}$, this corresponds to 20 (2) events per year. Increased luminosities would give correspondingly larger numbers of events.

V. CONCLUSIONS AND SUMMARY

We investigated and compared heavy Majorana neutrino effects on lepton flavor violating (LFV) decay rates of tau leptons, in two popular models: (I) the interfamily seesaw-type model realized in the SM with right-handed neutrinos,

and (II) the SM with left-handed and right-handed neutral singlets. Further, we calculated a T -odd asymmetry A_T for $\tau \rightarrow 3l'$. Model I is severely constrained in most of its parameter space by the actual eV-scale experimental upper bound on the light neutrino masses. It can give maximal LFV branching ratios $Br(\tau \rightarrow \mu \gamma) \sim 10^{-9}$ and $Br(\tau \rightarrow 3\mu) \sim 10^{-10}$ in a very restricted region of parameter space (where, incidentally, the CP -violating asymmetry A_T is zero), but otherwise it gives branching ratios many orders of magnitude smaller. On the other hand, in model II the LFV branching ratios can be larger over a wide range of parameter values, $Br(\tau \rightarrow e \gamma) \sim 10^{-8}$ and $Br(\tau \rightarrow 3e) \sim 10^{-9}$, and A_T can reach values up to 5%. The results in model II are insignificantly affected by the experimental bounds on the light neutrino masses. Model II can predict large enough LFV branching ratios (Br 's) to be tested with near future experiments. The maximal LFV Br 's in model II are obtained when $A_T = 0$, and are reduced by about a factor of two when $A_T \approx 5\%$. Model II thus indicates a certain complementarity in the searches of the LFV decay rates and of the associated T violation.

ACKNOWLEDGMENTS

The work of G.C. was supported by FONDECYT (Chile), Grants No. 1010094 and No. 7010094. C.D. received support in part from FONDECYT (Chile), Grant No. 8000017 and Fundacion Andes (Chile), Grant C-13680/4. G.C. and C.D. also acknowledge partial support from the program MECE-SUP FSM9901 of the Chilean Ministry of Education. The work of C.S.K. was supported in part by CHEP-SRC Program and Grant No. 20015-111-02-2, R03-2001-00010 of the KOSEF, in part by BK21 Program and Grant No. 2001-042-D00022 of the KRF. The work of J.D.K. was supported by the Korea Research Foundation Grant (2001-042-D00022). We wish to thank S. Kovalenko and A. Pilaftsis for helpful discussions.

APPENDIX: APPROXIMATE MAXIMIZATION OF THE BRANCHING RATIOS

The amplitude squared for the $l \rightarrow l' \gamma$ LFV process is, according to Eq. (6), approximately proportional to

$$|A|^2 \propto \left| \sum_{l=1}^{N_L} B_{il}^* B_{l'l} \times f(0) + \sum_{h=N_L+1}^{N_L+\tilde{N}_R} B_{lh}^* B_{l'h} \times f(m_h^2) \right|^2, \quad (\text{A1})$$

where the first sum runs over the light, practically massless, intermediate neutrinos, and the second sum over the heavy neutrinos with masses $m_h (\sim M_1 \sim M_2)$. The function f depends on the mass of the exchanged neutrino; in the specific case, it is the loop function G_γ appearing in Eq. (10). We now approximate the second sum by assuming that all the heavy neutrinos eigenmasses m_h are the same: $m_h = M$. Then this, together with the unitarity of the matrix U (note: $B_{ii} = U_{ii}^*$), implies

$$|A|^2 \propto \left| \sum_{h=N_L+1}^{N_L+\tilde{N}_R} U_{h2} U_{h1}^* \right|^2 \times |f(M^2) - f(0)| \\ \propto \left| \sum_{h=N_L+1}^{N_L+\tilde{N}_R} U_{h2} U_{h1}^* \right|^2. \quad (\text{A2})$$

Here we denoted the flavor index l of the heavier charged lepton as 2 and the index l' of the lighter one as 1.

The amplitude squared for the $l \rightarrow 3l'$ LFV process is, according to Eqs. (7)–(9) and (10)–(13), more complicated. However, in the leading order in $m_D m_M^{-1}$, and when there is no CP violation (when matrix U is real), it is straightforward to show that $|A|^2$ is proportional to the same kind of combination (A1). Thus, the proportionality (A2) is approximately satisfied also for the $l \rightarrow 3l'$ LFV process.

The above proportionality can be supplemented by the Schwarz inequality

$$|A|^2 \propto \left| \sum_h U_{h2} U_{h1}^* \right|^2 \\ \leq \sum_h |U_{h2}|^2 \times \sum_{h'} |U_{h'1}|^2 \\ \equiv (s_L^{v_2})^2 (s_L^{v_1})^2. \quad (\text{A3})$$

The equality is achieved only when there is a proportionality:

$$\frac{U_{h2}}{U_{h1}} \Big|_{h=N_L+1} = \frac{U_{h2}}{U_{h1}} \Big|_{h=N_L+2} = \dots = \frac{U_{h2}}{U_{h1}} \Big|_{h=N_L+\tilde{N}_R}. \quad (\text{A4})$$

Thus, the approximate maximal value of $|A|^2$, and thus of the LFV branching ratios, is achieved when the values of the heavy-to-light mixing parameters $(s_L^{v_2})^2$ and $(s_L^{v_1})^2$ are saturated according to the upper bounds (21) and, at the same time, the mixing matrix U elements in the heavy-to-light sector satisfy the equalities (A4).

In the seesaw model I (with $N_L = \tilde{N}_R = 2$), the mixing matrix U elements in the heavy-to-light sector are $U_{h2} = (m_M^{-1} m_D^\dagger)_{h'2}$ and $U_{h1} = (m_M^{-1} m_D^\dagger)_{h'1}$, where $h' \equiv h - 2$. In our specific case of maximal mixing ($a = c$, $b = -d$, $\delta_1 = \delta_2 = \pi/2$) we have $U_{31} = a/M_1$, $U_{41} = -ib/M_2$, $U_{32} = -ia/M_1$, $U_{42} = -b/M_2$; the equality (A4) is satisfied; and $(s_L^{v_1})^2 = (s_L^{v_2})^2 = (a^2/M_1^2 + b^2/M_2^2)$.

In model II (with $N_L = 2$ and $\tilde{N}_R = 4$), with $\xi = 0$, it can be shown, e.g. by using MATHEMATICA, that the equality (A4) is satisfied when $ad = bc$. If in this case also the values of $(s_L^{v_2})^2 = \sum_h |U_{h2}|^2$ and $(s_L^{v_1})^2 = \sum_h |U_{h1}|^2$ are saturated by the corresponding upper bounds of Eq. (21), then the approximate maximal branching LFV ratios are achieved.

- [1] T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe*, edited by O. Swada and A. Sugamoto (KEK, Tsukuba, Japan, 1979), p. 95; M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by P. van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979), p. 315; R.N. Mohapatra and G. Senjanović, *Phys. Rev. Lett.* **44**, 912 (1980).
- [2] E. Witten, *Nucl. Phys.* **B268**, 79 (1986); R.N. Mohapatra and J.W. Valle, *Phys. Rev. D* **34**, 1642 (1986); J.L. Hewett and T.G. Rizzo, *Phys. Rep.* **183**, 193 (1989).
- [3] D. Wyler and L. Wolfenstein, *Nucl. Phys.* **B218**, 205 (1983).
- [4] A. Ilakovac and A. Pilaftsis, *Nucl. Phys.* **B437**, 491 (1995).
- [5] J.G. Körner, A. Pilaftsis, and K. Schilcher, *Phys. Rev. D* **47**, 1080 (1993).
- [6] M.C. Gonzalez-Garcia and J.W. Valle, *Mod. Phys. Lett. A* **7**, 477 (1992).
- [7] Y. Okada, K.i. Okumura, and Y. Shimizu, *Phys. Rev. D* **58**, 051901 (1998); **61**, 094001 (2000).
- [8] C. Dib, V. Gribov, S. Kovalenko, and I. Schmidt, talk given at International Workshop on Neutrinos and Physics Beyond the Standard Model NANPino (Non-Accelerator New Physics in Neutrino Observations), Dubna, Moscow region, Russia, 2000, hep-ph/0011213.
- [9] F. del Aguila, E. Laermann, and P. Zerwas, *Nucl. Phys.* **B297**, 1 (1988); E. Ma and J. Pantaleone, *Phys. Rev. D* **40**, 2172 (1989); J. Kogo and S.Y. Tsai, *Prog. Theor. Phys.* **86**, 183 (1991); W. Buchmüller and C. Greub, *Nucl. Phys.* **B363**, 345 (1991); **B381**, 109 (1992); J. Maalampi, K. Mursula, and R. Vuopionperä, *ibid.* **B372**, 23 (1992); A. Datta and A. Pilaftsis, *Phys. Lett. B* **278**, 162 (1992). A. Datta, M. Guchait, and A. Pilaftsis, *Phys. Rev. D* **50**, 3195 (1994); J. Gluza and M. Zralek, *ibid.* **48**, 5093 (1993); **51**, 4695 (1995); **51**, 4707 (1995); A. Hofer and L.M. Sehgal, *ibid.* **54**, 1944 (1996); G. Cvetič, C.S. Kim, and C.W. Kim, *Phys. Rev. Lett.* **82**, 4761 (1999); G. Cvetič and C.S. Kim, *Phys. Lett. B* **461**, 248 (1999); **471**, 471(E) (2000); F.M. Almeida, Y.A. Coutinho, J.A. Martins Simoes, and M.A. do Vale, *ibid.* **494**, 273 (2000); *Eur. Phys. J. C* **22**, 277 (2001).
- [10] M. Flanz, W. Rodejohann, and K. Zuber, *Eur. Phys. J. C* **16**, 453 (2000); K. Zuber, *Phys. Lett. B* **479**, 33 (2000); W. Rodejohann and K. Zuber, *Phys. Rev. D* **62**, 094017 (2000); C. Dib, V. Gribov, S. Kovalenko, and I. Schmidt, *Phys. Lett. B* **493**, 82 (2000).
- [11] A. Kageyama, S. Kaneko, N. Shimoyama, and M. Tanimoto, *Phys. Lett. B* **527**, 206 (2002); A. Kageyama, S. Kaneko, N. Shimoyama, and M. Tanimoto, *Phys. Rev. D* **65**, 096010 (2002), and references therein.
- [12] G. Cvetič, M. Nowakowski, and A. Pilaftsis, *Phys. Lett. B* **301**, 77 (1993); G. Cvetič, *Phys. Rev. D* **48**, 5280 (1993).
- [13] T. Endoh, T. Morozumi, T. Onogi, and A. Purwanto, *Phys. Rev. D* **64**, 013006 (2001).
- [14] J.F. Nieves and P.B. Pal, *Phys. Rev. D* **64**, 076005 (2001).
- [15] G.C. Branco, M.N. Rebelo, and J.W.F. Valle, *Phys. Lett. B* **225**, 385 (1989).
- [16] M.C. Gonzalez-Garcia and J.W. Valle, *Phys. Lett. B* **216**, 360 (1989).
- [17] J. Hisano and K. Tobe, *Phys. Lett. B* **510**, 197 (2001).
- [18] Particle Data Group, D.E. Groom *et al.*, *Eur. Phys. J. C* **15**, 1 (2000).
- [19] B.T. Cleveland *et al.*, *Astrophys. J.* **496**, 505 (1998); GALLEX Collaboration, W. Hampel *et al.*, *Phys. Lett. B* **447**, 127 (1999); SAGE Collaboration, J.N. Abdurashitov *et al.*, *Phys. Rev. C* **60**, 055801 (1999); GNO Collaboration, M. Altmann *et al.*, *Phys. Lett. B* **490**, 16 (2000); Super-Kamiokande Collaboration, Y. Fukuda *et al.*, *Phys. Rev. Lett.* **82**, 2430 (1999); **86**, 5651 (2001); SNO Collaboration, Q.R. Ahmad *et al.*, *ibid.* **87**, 071301 (2001).
- [20] J.N. Bahcall, P.I. Krastev, and A.Yu. Smirnov, *Phys. Rev. D* **58**, 096016 (1998); M.C. Gonzalez-García *et al.*, *Nucl. Phys.* **B573**, 3 (2000); G.L. Fogli *et al.*, *Phys. Rev. D* **62**, 013002 (2000); A.S. Joshipura and M. Nowakowski, *ibid.* **51**, 2421 (1995).
- [21] T. Kajita and Y. Totsuka, *Rev. Mod. Phys.* **73**, 85 (2001); C.K. Jung, C. McGrew, T. Kajita, and T. Mann, *Annu. Rev. Nucl. Part. Sci.* **51**, 451 (2001).
- [22] V.D. Barger, S. Pakvasa, T.J. Weiler, and K. Whisnant, *Phys. Lett. B* **437**, 107 (1998); H. Georgi and S.L. Glashow, *Phys. Rev. D* **61**, 097301 (2000); M. Jezabek and Y. Sumino, *Phys. Lett. B* **457**, 139 (1999); C.H. Albright and S.M. Barr, *ibid.* **461**, 218 (1999); S.K. Kang and C.S. Kim, *Phys. Rev. D* **59**, 091302 (1999); C.S. Kim and J.D. Kim, *ibid.* **61**, 057302 (2000).
- [23] S.K. Kang and C.S. Kim, *Phys. Rev. D* **63**, 113010 (2001); K. Kang, S.K. Kang, C.S. Kim, and S.M. Kim, *Mod. Phys. Lett. A* **16**, 2169 (2001).
- [24] For recent reviews, see F. Simkovic *et al.*, *Phys. Rev. C* **64**, 035501 (2001).
- [25] F. Simkovic, P. Domin, and A. Faessler, hep-ph/0204278.
- [26] P. Langacker and D. London, *Phys. Rev. D* **38**, 886 (1988).
- [27] E. Nardi, E. Roulet, and D. Tommasini, *Phys. Lett. B* **327**, 319 (1994); **344**, 225 (1995); D. Tommasini, G. Barenboim, J. Bernabeu, and C. Jarlskog, *Nucl. Phys.* **B444**, 451 (1995).
- [28] M.S. Chanowitz, M.A. Furman, and I. Hinchliffe, *Nucl. Phys.* **B153**, 402 (1979); L. Durand, J.M. Johnson, and J.L. Lopez, *Phys. Rev. Lett.* **64**, 1215 (1990); L. Durand, J.M. Johnson, and J.L. Lopez, *Phys. Rev. D* **45**, 3112 (1992).
- [29] A. Pilaftsis, *Z. Phys. C* **55**, 275 (1992).
- [30] S. Fajfer and A. Ilakovac, *Phys. Rev. D* **57**, 4219 (1998).
- [31] A. Ilakovac, *Phys. Rev. D* **62**, 036010 (2000).
- [32] J. Bernabeu and P. Pascual, *Nucl. Phys.* **B228**, 21 (1983); J.A. Aguilar-Saavedra, *Int. J. Mod. Phys. C* **8**, 147 (1997).