Identifying the quark content of the isoscalar scalar mesons $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ from weak and electromagnetic processes

Frieder Kleefeld,[1,*] Eef van Beveren,[2,†] George Rupp,[1,‡] and Michael D. Scadron[3,§]
[1]*Centro de Física das Interacções Fundamentais, Instituto Superior Técnico, P-1049-001 Lisboa, Portugal*
[2]*Departamento de Física, Universidade de Coimbra, P-3004-516 Coimbra, Portugal*
[3]*Physics Department, University of Arizona, Tucson, Arizona 85721*
(Received 19 September 2001; published 12 August 2002)

The assignments of the isoscalar scalar mesons $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ in terms of their flavor substructure is still a matter of heated dispute. Here we employ the weak and electromagnetic decays $D_s^+ \to f_0\pi^+$ and $f_0 \to \gamma\gamma$, respectively, to identify the $f_0(980)$ and $f_0(1500)$ as mostly $\bar{s}s$, and the $f_0(1370)$ as dominantly $\bar{n}n$, in agreement with previous work. The two-photon decays can be satisfactorily described with quark as well as with meson loops, though the latter ones provide a less model-dependent and more quantitative description.

DOI: 10.1103/PhysRevD.66.034007 PACS number(s): 14.40.Cs, 13.25.Ft, 13.40.Hq, 13.75.Lb

I. INTRODUCTION

A proper classification of the scalar mesons is still being clouded by two major problems, which mutually hamper the resolution of either. The first difficulty is the apparent excess of experimentally confirmed scalar resonances with respect to the number of theoretically expected $\bar{q}q$ states. The second problem is to unambiguously identify the $\bar{q}q$ configuration of the isoscalar scalar mesons, i.e., the $f_0(400\text{–}1200)$ (or σ), $f_0(980)$, $f_0(1370)$, $f_0(1500)$, and $f_0(1710)$. In previous work, especially the former issue has been addressed, showing that the light (below 1 GeV) scalars can be described as a complete nonet of $\bar{q}q$ states, resulting from either the dynamical breaking of chiral symmetry [1] or the coupling of bare P-wave $\bar{q}q$ systems to the meson-meson continuum in a unitarized approach [2,3]. We believe that these two mechanisms are intimately related to one another, though in a not yet completely understood fashion. In any case, in both pictures the scalar mesons between 1.3 and 1.5 GeV form another nonet, and so forth. So we conclude there is no excess of observed resonances, thus dispensing with the introduction of new degrees of freedom.

Here, we want to focus on the second issue, namely the identification of the isoscalars, especially the vehemently disputed $f_0(980)$, $f_0(1370)$, and $f_0(1500)$, in an as model-independent way as one may achieve. In Refs. [4,5] qualitative arguments from observed hadronic decays have already been presented that favor, in our view, a mainly $\bar{s}s$ configuration for the $f_0(980)$ and $f_0(1500)$, and a dominantly nonstrange $\bar{q}q$ content for the $f_0(1370)$. Furthermore, we are engaged in substantiating these arguments by analyzing also the four-pion decays of these scalars via intermediate $\rho\rho$ and $\sigma\sigma$ two-resonance states, in a similar way as done for the $\omega \to \rho\pi \to \pi\pi\pi$ cascade process in Ref. [6]. In the present work, we shall employ the weak and electromagnetic decays (as opposed to the more complicated strong-interaction dynamics) $D_s^+ \to f_0\pi^+$ and $f_0 \to \gamma\gamma$, respectively, which will give quantitative support for our $\bar{q}q$ assignments. These processes will be analyzed in a simple $\bar{q}q$ picture for the corresponding f_0 resonances, with a minimum of model-dependent input.

This paper is organized as follows. In Sec. II we compute the weak decays $D_s^+ \to \pi^+ f_0(980)$, $\pi^+ f_0(1500)$, $\pi^+ f_0(1710)$ using W^+ emission. In Sec. III we calculate the $f_0(980)$, $f_0(1370) \to 2\gamma$ electromagnetic decays, employing quark as well as meson loops. Conclusions are drawn in Sec. IV.

II. WEAK DECAYS $D_s^+ \to \pi^+ f_0$

First we compute the parity-conserving weak decays $D_s^+ \to \pi^+ f_0(980)$ and $\pi^+ f_0(1500)$, supposing for the moment that both of these final-state scalar mesons are purely $\bar{s}s$. Given the Fermi Hamiltonian density $H_W = (G_F/2\sqrt{2})(J J^+ + J^+ J)$ with [7] $G_F = 1.16639(1)\times 10^{-5}$ GeV^{-2} and $F_\pi = f_{\pi^+}/\sqrt{2} \simeq (92.42 \pm 0.27)$ MeV, the magnitudes of the corresponding weak decay amplitudes of W^+ emission are [8] (also see Ref. [9])

$$|M(D_s^+ \to \pi^+ f_0(980))| = \frac{G_F |V_{ud}||V_{cs}|}{2} F_\pi (m^2_{D_s^+} - m^2_{f_0(980)}) = (159 \pm 24)\times 10^{-8}\ \text{GeV}, \tag{1}$$

$$|M(D_s^+ \to \pi^+ f_0(1500))| = \frac{G_F |V_{ud}||V_{cs}|}{2} F_\pi (m^2_{D_s^+} - m^2_{f_0(1500)}) = (89 \pm 13)\times 10^{-8}\ \text{GeV}, \tag{2}$$

*Author to whom correspondence should be addressed. Email address: kleefeld@cfif.ist.utl.pt
†Email address: eef@teor.fis.uc.pt
‡Email address: george@ajax.ist.utl.pt
§Email address: scadron@physics.arizona.edu

being both close to the data [7] $(178\pm 40)\times 10^{-8}$ GeV and $(96\pm 28)\times 10^{-8}$ GeV, respectively. The latter amplitudes are extracted from the observed decay rates Γ according to $|M| = m_{D_s^+}\sqrt{8\pi\Gamma/q_{cm}}$. The agreement of Eqs. (1) and (2) with the data, which has already been noted in Refs. [5] and [10], respectively, shows that first-order perturbative weak graphs have impressive predictive power.

The formulas of Eqs. (1) and (2) are based on the standard description of weak interactions in terms of Fermi theory, which is a low-energy tree-level approximation of the standard model Lagrangian, or in other words, a lowest-order description in the spirit of Wilson's operator product expansion (OPE). In the language of Ref. [11], we only consider the current-current operator Q_2 multiplied by the Wilson coefficient C_2. Higher orders could be included by taking into account further operators, Q_1, Q_3, Q_4, Q_5, Q_6, multiplied by the corresponding Wilson coefficients C_1, C_3, C_4, C_5, C_6. From Ref. [11] we learn that the corresponding contributions are suppressed and often negative for K and D decays. Throughout this work we assume $C_2 = 1$, thereby absorbing the anticipated negative marginal contributions of the further operators as a correction to the value $C_2 \approx 1.25$ quoted in Ref. [11]. Furthermore, we may observe that, since decay rates of $q\bar{q}$ systems are to a good approximation proportional to $q\bar{q}$ probability distributions at the $q\bar{q}$ center of mass [12], the higher-order OPE terms seem to cancel corrections from the $q\bar{q}$ wave function, such that we meet the experimental data.

The coincidence that both effects—one perturbative and one nonperturbative—compensate each other may have some physical roots. It is also important to notice that, although we do not rely on wave functions in this paper, we bear in mind the nonet assignment given in Refs. [2,3], which classifies both the light nonet of scalar resonances and the nonet between 1.3 and 1.5 GeV as ground states, each from a different origin. As a consequence, we do not foresee the usual suppression factors for radial excitations in the case of the $f_0(1500)$ [and also the $f_0(1370)$], as for instance used in Ref. [13].

Another way to study Eqs. (1) and (2) above is to take the ratio

$$\left|\frac{M(D_s^+ \to \pi^+ f_0(980))}{M(D_s^+ \to \pi^+ f_0(1500))}\right|_{|f_0\rangle = |s\bar{s}\rangle} = \frac{m^2_{D_s^+} - m^2_{f_0(980)}}{m^2_{D_s^+} - m^2_{f_0(1500)}} = 1.79 \pm 0.04, \tag{3}$$

[using $m_{f_0(980)} = (980\pm 10)$ MeV, $m_{f_0(1500)} = (1500\pm 10)$ MeV, and $m_{D_s^+} = (1968.6\pm 0.6)$ MeV], which is independent of the weak scale G_F, the Cabibbo-Kobayashi-Maskawa (CKM) parameters $|V_{ud}|$, $|V_{cs}|$, and the pion decay constant F_π. As such, Eq. (3) is the kinematic (model-independent) infinite-momentum-frame (IMF) (see, e.g., Ref. [14]) version. The data [7] depend on the branching ratio and center-of-mass (c.m.) momenta as

$$\left|\frac{M(D_s^+ \to \pi^+ f_0(980))}{M(D_s^+ \to \pi^+ f_0(1500))}\right|_{\rm PDG} = \sqrt{\frac{\Gamma(D_s^+ \to \pi^+ f_0(980))\, q_{cm}(D_s^+ \to \pi^+ f_0(1500))}{\Gamma(D_s^+ \to \pi^+ f_0(1500))\, q_{cm}(D_s^+ \to \pi^+ f_0(980))}} = 1.86 \pm 0.68, \tag{4}$$

showing again a very good agreement. Here, we have used the measured branching ratios [7] $\Gamma(D_s^+ \to \pi^+ f_0(980))/\Gamma(D_s^+) = (1.8\pm 0.8)\%$ and $\Gamma(D_s^+ \to \pi^+ f_0(1500))/\Gamma(D_s^+) = (0.28\pm 0.16)\%$, and the corresponding extracted c.m. momenta $q_{cm}(D_s^+ \to \pi^+ f_0(980)) = (732.1\pm 5.1)$ MeV/c and $q_{cm}(D_s^+ \to \pi^+ f_0(1500)) = (393.8\pm 8.1)$ MeV/c. The large error ± 0.68 in Eq. (4) stems from the uncertainties in the measured branching ratios rather than from the quite accurately known c.m. momenta. These uncertainties leave quite some room to allow for significant $n\bar{n}$ admixtures in the $f_0(980)$ as well as the $f_0(1500)$, without calling into question their $s\bar{s}$ dominance. On the other hand, from the failure to observe the decay $D_s^+ \to \pi^+ f_0(1370)$ [7] (see, however, Ref. [15]) it seems safe to conclude that the $f_0(1370)$ does not have a large $s\bar{s}$ component.

To conclude the weak processes, let us look at the situation for the $f_0(1710)$. Although the weak decay $D_s^+ \to \pi^+ f_0(1710)$ has been observed, the quoted rate $(1.5\pm 1.9)\times 10^{-3}$ [7], corresponding to an amplitude of $(97\pm 123)\times 10^{-8}$ GeV, only accounts for K^+K^- decays of this resonance. The theoretical W^+-emission amplitude has a magnitude of 52×10^{-8} GeV, if we again ignore possible corrections from the internal $q\bar{q}$ wave function of the $f_0(1710)$, which may be questionable for this probably excited state. Also in view of the huge experimental error, no definite conclusions on the $q\bar{q}$ (or any other) substructure of the $f_0(1710)$ are possible for the time being. Nevertheless, the sheer observation of the weak decay process seems to preclude a dominantly $n\bar{n}$ configuration. Indeed, the meson particle listings conclude that the $f_0(1710)$ "*is consistent with a large $s\bar{s}$ component*" (Ref. [7], page 470).

III. ELECTROMAGNETIC SCALAR DECAYS $S \to 2\gamma$

An alternative process to analyze the flavor content of the f_0 mesons is the two-photon decay, since the corresponding amplitude is very sensitive to the masses and especially the charges of the particles involved. Moreover, this process may also provide a tool to determine whether some of these isoscalar scalar mesons are in fact glueballs [16]. In our analysis, we shall restrict ourselves to those f_0 states for which two-photon decays have been observed.

A. The decay $f_0(980) \to 2\gamma$

The Particle Data Group (PDG) tables [7] now report the scalar $f_0(980) \to 2\gamma$ decay rate as (0.39 ± 0.12)

keV. Given the scalar amplitude structure [17–20] $M\varepsilon_\mu(k')\varepsilon_\nu(k)(g^{\mu\nu}k'\cdot k-k'^{\mu}k^{\nu})$, the two-photon decay rate is

$$\Gamma(f_0\to 2\gamma)=\frac{m_{f^0}^3|M|^2}{64\pi}\quad\text{or}$$

$$|M(f_0(980)\to 2\gamma)|=(0.91\pm 0.14)\times 10^{-2}\ \ \text{GeV}^{-1}. \quad (5)$$

If the $f_0(980)$ were $\bar{n}n$, the isoscalar u, d quark-loop analogue of the isovector $\pi^0\to 2\gamma$ amplitude, given by [18] $\sqrt{2}\alpha N_c\ \text{Tr}[Q^2Q_{\bar{n}n}]/(\pi F_\pi)=5\alpha N_c/(9\pi F_\pi)\simeq 0.042\ \text{GeV}^{-1}$ with $N_c=3$, would generate an $f_0(980)\to 2\gamma$ decay rate a factor of *21 times too large*.[1] If, instead, the $f_0(980)$ is a pure $\bar{s}s$ state, the $f_0\to 2\gamma$ amplitude magnitude becomes [18] $\alpha N_c g_{f_0 ss}/(9\pi m_s)\simeq 0.81\times 10^{-2}\ \ \text{GeV}^{-1}$, using $g_{f_0\,SS}=\sqrt{2}\ \ 2\pi/\sqrt{3}$ and constituent strange quark mass [21,1] $m_s=490\ \ \text{MeV}\simeq 1.44\hat{m}$ [from Ref. [21], $F_K/F_\pi=(\hat{m}+m_s)/(2\hat{m})\simeq 1.22$] with the constituent nonstrange mass $\hat{m}\simeq 340\ \ \text{MeV}$. This value lies reasonably close to the observed amplitude in Eq. (5).[2] However, at this point we should note that the quark-loop result for the two-photon decay rate is very sensitive to a possible $\bar{n}n$ admixture in the $f_0(980)$, due to an enhancement factor of 25 of the $\bar{n}n$ component with respect to the $\bar{s}s$ component. This factor comes from the electric charge of the quarks, yielding $[(\frac{2}{3})^2+(\frac{1}{3})^2]^2$ for the nonstrange isoscalar $(1/\sqrt{2})(\bar{u}u+\bar{d}d)$, and $(\frac{1}{3})^4$ for the strange isoscalar.

Therefore, rather than involving the model-dependent quark coupling and constituent quark masses as above, we instead consider a combination of the decay chains $f_0\to K^+K^-\to 2\ \gamma$ and $f_0\to\pi^+\pi^-\to 2\ \gamma$ [17–20]. According to Refs. [17,20], the kaon loop is suppressed by 10% due to a, so far experimentally unconfirmed, scalar $\kappa(900)$. [However, very recent results from the E791 Collaboration present preliminary evidence for a light κ (see the e-print in Ref. [15]), which would confirm the prediction [1,2] of such a state.] In order to proceed, we have to remind the reader of the standard mixing scheme between the ''physical'' states ($|\sigma(600)\rangle$ and $|f_0(980)\rangle$), and the nonstrange and strange basis states $|\bar{n}n\rangle$ and $|\bar{s}s\rangle$, i.e.,

$$|\sigma(600)\rangle=\cos\phi_s|\bar{n}n\rangle-\sin\phi_s|\bar{s}s\rangle,$$
$$|f_0(980)\rangle=\sin\phi_s|\bar{n}n\rangle+\cos\phi_s|\bar{s}s\rangle. \quad (6)$$

With quadratic mass mixing, one can define for the states $|\bar{n}n\rangle$ and $|\bar{s}s\rangle$ the nonstrange and strange mass parameters $m_{\bar{n}n}$ and $m_{\bar{s}s}$ by [18] as

$$\begin{aligned} m_{\bar{n}n}^2&=\cos^2\phi_s m_\sigma^2+\sin^2\phi_s m_{f_0}^2\\ &=[(646\pm 10)\ \ \text{MeV}]^2,\\ m_{\bar{s}s}^2&=\sin^2\phi_s m_\sigma^2+\cos^2\phi_s m_{f_0}^2\\ &=[(950\pm 11)\ \ \text{MeV}]^2. \end{aligned} \quad (7)$$

Throughout this paper we choose a mixing angle of[3] $\phi_s\simeq 18°\pm 2°$ [1,22,21,18] or $\phi_s\simeq -(18°\pm 2°)$ [20], and assume the scalar-meson masses to be $m_{f_0(980)}=(980\pm 10)$ MeV [7] and $m_{\sigma(600)}=600$ MeV. Since the interaction Lagrangians between the f_0 and the pseudoscalars $\pi^\pm$ and $K^\pm$ are proportional to f_0, the Lagrangians can, within the same mixing scheme, be simultaneously reexpressed in terms of nonstrange and strange fields, i.e.,

$$\mathcal{L}(f_0\pi\pi)+\mathcal{L}(f_0KK)=\sin\phi_s[\mathcal{L}(\bar{n}n\pi\pi)+\mathcal{L}(\bar{n}nKK)]+\cos\phi_s[\mathcal{L}(\bar{s}s\pi\pi)+\mathcal{L}(\bar{s}sKK)]. \quad (8)$$

Within the usual nonet, that is, the U(3) picture, the scalar (S) and pseudoscalar (P) fields are proportional to linear combinations of the Gell-Mann matrices $\lambda_0,\lambda_1,\ldots,\lambda_8$ (λ_0 denotes here $\sqrt{2/3}\ 1_3$ with 1_3 being the 3-dimensional unit matrix), denoted by Q_S and Q_P, respectively. From the quark content of the corresponding mesonic systems, it is easy to derive

$$\bar{n}n=\frac{1}{\sqrt{2}}(\bar{u}u+\bar{d}d)\Rightarrow Q_{\bar{n}n}=\frac{1}{\sqrt{3}}\left(\lambda_0+\frac{1}{\sqrt{2}}\lambda_8\right),$$
$$\bar{s}s\Rightarrow Q_{\bar{s}s}=\frac{1}{\sqrt{3}}\left(\frac{1}{\sqrt{2}}\lambda_0-\lambda_8\right), \quad (9)$$
$$\pi^+=\bar{d}u,\ \ \pi^-=\bar{u}d\Rightarrow Q_{\pi^\pm}=\frac{1}{2}(\lambda_1\pm i\lambda_2),$$
$$K^+=\bar{s}u,\ \ K^-=\bar{u}s\Rightarrow Q_{K^\pm}=\frac{1}{2}(\lambda_4\pm i\lambda_5).$$

In the linear σ model (LSM), the interaction Lagrangian $\mathcal{L}(SP_1P_2)$ is proportional to the flavor trace Tr $(Q_S\{Q_{P_1},Q_{P_2}\})$, and so are the corresponding coupling con-

[1]We introduced the SU(3) charge matrix $Q=T_3+Y/2=\text{diag}[2/3,-1/3,-1/3]=(\lambda_3+\lambda_8/\sqrt{3})/2$ and the $\bar{n}n=(\bar{u}u+\bar{d}d)/\sqrt{2}$ analogue $Q_{\bar{n}n}=\text{diag}[1/\sqrt{2},1/\sqrt{2},0]=(\lambda_0+\lambda_8/\sqrt{2})/\sqrt{3}$.

[2]Without changes, we could of course also use the identity $\sqrt{2}\alpha N_c\ \text{Tr}[Q^2Q_{\bar{s}s}]/(\pi F_{\bar{s}s})=\sqrt{2}\alpha N_c/(9\pi F_{\bar{s}s})\simeq 0.81\times 10^{-2}\text{GeV}^{-1}$, with $F_{\bar{s}s}=\sqrt{3}m_s/(2\pi)=135.1\ \ \text{MeV}\simeq 1.2F_K\simeq 2F_K-F_\pi\simeq\sqrt{2}F_\pi$ and $Q_{\bar{s}s}=\text{diag}[0,0,1]=(\lambda_0/\sqrt{2}-\lambda_8)/\sqrt{3}$. The use of [7] $F_K=f_{K^+}/\sqrt{2}=(113.00\pm 1.04)$ MeV instead of $F_{\bar{s}s}$ would bring us even closer to the data, as $\sqrt{2}\alpha N_c/(9\pi F_K)\simeq 0.972\times 10^{-2}\ \ \text{GeV}^{-1}$.

[3]The sign of the mixing angle, which cannot be identified from a quadratic mass mixing scheme, has still to be determined from theoretical consistency arguments, as it has a strong influence on the interference terms in the present work.

stants. It should be mentioned that the charge of a mesonic system ϕ is determined by $\mathrm{Tr}(Q\,[Q_\phi,Q_\phi^T])$. Thus, we derive for the relevant channels under consideration, i.e., $\overline{n}n\to\pi\pi$, $\overline{n}n\to KK$, $\overline{s}s\to\pi\pi$, and $\overline{s}s\to KK$:

$$\begin{aligned}
d_{\overline{n}n\pi^+\pi^-}&=\frac{1}{\sqrt{2}}\mathrm{Tr}(Q_{\overline{n}n}\{Q_{\pi^+},Q_{\pi^-}\})=1,\\
d_{\overline{n}nK^+K^-}&=\frac{1}{\sqrt{2}}\mathrm{Tr}(Q_{\overline{n}n}\{Q_{K^+},Q_{K^-}\})=\frac{1}{2},\\
d_{\overline{s}s\pi^+\pi^-}&=\frac{1}{\sqrt{2}}\mathrm{Tr}(Q_{\overline{s}s}\{Q_{\pi^+},Q_{\pi^-}\})=0,\\
d_{\overline{s}sK^+K^-}&=\frac{1}{\sqrt{2}}\mathrm{Tr}(Q_{\overline{s}s}\{Q_{K^+},Q_{K^-}\})=\frac{1}{\sqrt{2}}.
\end{aligned}\tag{10}$$

The corresponding equivalent symmetric structure constants $d_{\overline{n}n\,33}$, $d_{\overline{n}n\,K^0K^0}$, $d_{\overline{s}s\,33}$, $d_{\overline{s}s\,K^0K^0}$, with $d_{abc}=\mathrm{Tr}(\lambda_a\{\lambda_b,\lambda_c\})/4$, for two neutral pseudoscalars in the final state have already been derived in Ref. [21]. In accordance with the σ-model results, we determine the corresponding SU(3) couplings for $\phi_s\simeq+(18°\pm2°)$ and $\phi_s\simeq-(18°\pm2°)$ as

$$\begin{aligned}
g'_{\overline{n}n\pi\pi}&=d_{\overline{n}n\pi^+\pi^-}\frac{m^2_{\overline{n}n}-m^2_{\pi^\pm}}{2F_\pi}\\
&=\frac{\cos^2\phi_s m_\sigma^2+\sin^2\phi_s m_{f_0}^2-m^2_{\pi^\pm}}{2\,F_\pi}\\
&=(2.152\pm0.068)\ \ \mathrm{GeV},\\
g'_{\overline{n}nKK}&=d_{\overline{n}nK^+K^-}\frac{m^2_{\overline{n}n}-m^2_{K^\pm}}{F_K}\\
&=\frac{\cos^2\phi_s m_\sigma^2+\sin^2\phi_s m_{f_0}^2-m^2_{K^\pm}}{2F_K}\\
&=(0.768\pm0.056)\ \ \mathrm{GeV},\\
g'_{\overline{s}s\pi\pi}&=d_{\overline{s}s\pi^+\pi^-}\frac{m^2_{\overline{s}s}-m^2_{\pi^\pm}}{2F_\pi}=0,\\
g'_{\overline{s}sKK}&=d_{\overline{s}sK^+K^-}\frac{m^2_{\overline{s}s}-m^2_{K^\pm}}{F_K}\\
&=\frac{\sin^2\phi_s m_\sigma^2+\cos^2\phi_s m_{f_0}^2-m^2_{K^\pm}}{\sqrt{2}F_K}\\
&=(4.126\pm0.141)\ \ \mathrm{GeV},
\end{aligned}\tag{11}$$

yielding for $\phi_s\simeq+(18°\pm2°)$

$$\begin{aligned}
(\sin\phi_s g'_{\overline{n}n\pi\pi}+\cos\phi_s g'_{\overline{s}s\pi\pi})&=(0.665\pm0.093)\ \ \mathrm{GeV},\\
(\sin\phi_s g'_{\overline{n}nKK}+\cos\phi_s g'_{\overline{s}sKK})&=(4.162\pm0.138)\ \ \mathrm{GeV},
\end{aligned}\tag{12}$$

and for $\phi_s\simeq-(18°\pm2°)$

$$\begin{aligned}
(\sin\phi_s g'_{\overline{n}n\pi\pi}+\cos\phi_s g'_{\overline{s}s\pi\pi})&=(-0.665\pm0.093)\ \ \mathrm{GeV},\\
(\sin\phi_s g'_{\overline{n}nKK}+\cos\phi_s g'_{\overline{s}sKK})&=(3.687\pm0.194)\ \ \mathrm{GeV}.
\end{aligned}\tag{13}$$

In order to compute these numbers, we used $F_\pi\simeq(92.42\pm0.27)$ MeV, $F_K\simeq(113.00\pm1.04)$ MeV, i.e. $F_K/F_\pi\simeq1.22$. Putting all this together, we obtain for the pion- and kaon-loop amplitudes [17]

$$\begin{aligned}
M_{\pi\ \mathrm{loop}}&=\frac{2\alpha(\sin\phi_s g'_{\overline{n}n\pi\pi}+\cos\phi_s g'_{\overline{s}s\pi\pi})}{\pi m_{f_0}^2}\left[-\frac{1}{2}+\xi_\pi I(\xi_\pi)\right]\\
&=[-0.177\pm0.025+i(+0.079\pm0.012)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq+(18°\pm2°)\\
&=[+0.177\pm0.025+i(-0.079\pm0.012)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq-(18°\pm2°),\\
M_{K\ \mathrm{loop}}&=\frac{2\alpha(\sin\phi_s g'_{\overline{n}nKK}+\cos\phi_s g'_{\overline{s}sKK})}{\pi m_{f_0}^2}\left[-\frac{1}{2}+\xi_K I(\xi_K)\right]\\
&=(1.138\pm0.254)\times10^{-2}\mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq+(18°\pm2°)\\
&=(1.008\pm0.229)\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq-(18°\pm2°),\\
M_{\pi\ \mathrm{loop}}+M_{K\ \mathrm{loop}}&=[0.960\pm0.255+i(+0.079\pm0.012)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq+(18°\pm2°)\\
&=[1.185\pm0.230+i(-0.079\pm0.012)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq-(18°\pm2°),\\
|M_{\pi\ \mathrm{loop}}+M_{K\ \mathrm{loop}}|&=(0.964\pm0.255)\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq+(18°\pm2°)\\
&=(1.188\pm0.230)\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s\simeq-(18°\pm2°).
\end{aligned}\tag{14}$$

As $\xi_\pi = m^2_{\pi^+}/m^2_{f_0(980)} = 0.02028 \pm 0.00042 < 1/4$, the value of the pion-loop integral is obtained from (see also pages 230 and 422 in Ref. [23])

$$\begin{aligned} I(\xi_\pi) &= \int_0^1 dy \int_0^1 dx \frac{y}{\xi_\pi - xy(1-y)} \\ &= 2\left[\frac{\pi}{2} + i \ln\left(\sqrt{\frac{1}{4\xi_\pi}} + \sqrt{\frac{1}{4\xi_\pi} - 1}\right)\right]^2 \\ &= \frac{\pi^2}{2} - 2\ln^2\left[\sqrt{\frac{1}{4\xi_\pi}} + \sqrt{\frac{1}{4\xi_\pi} - 1}\right] \\ &\quad + 2\pi i \ln\left[\sqrt{\frac{1}{4\xi_\pi}} + \sqrt{\frac{1}{4\xi_\pi} - 1}\right] \\ &= -2.500 \pm 0.083 + i(12.114 \pm 0.067), \end{aligned}$$

while, as $\xi_K = m^2_{K^+}/m^2_{f_0(980)} = 0.2538 \pm 0.0052 > 1/4$, the kaon loop follows from

$$\begin{aligned} I(\xi_K) &= \int_0^1 dy \int_0^1 dx \frac{y}{\xi_K - xy(1-y)} \\ &= 2\left[\arcsin\sqrt{\frac{1}{4\xi_K}}\right]^2 = 4.197 \pm 0.482, \end{aligned} \tag{15}$$

yielding, respectively,

$$\begin{aligned} -\frac{1}{2} + \xi_\pi I(\xi_\pi) &= -0.5507 \pm 0.0020 \\ &\quad + i(0.2457 \pm 0.0037), \end{aligned} \tag{16}$$

$$-\frac{1}{2} + \xi_K I(\xi_K) = 0.5651 \pm 0.1242.$$

Reducing the kaon-loop amplitude in Eq. (14) by 10% [owing to the scalar $\kappa(900)$ loop], but leaving the value of its error unaltered, predicts $(0.85 \pm 0.26) \times 10^{-2}$ GeV^{-1} [$\phi_s \simeq +(18° \pm 2°)$] or $(1.09 \pm 0.23) \times 10^{-2}$ GeV^{-1} [$\phi_s \simeq -(18° \pm 2°)$] for the modulus of the $f_0(980) \to 2\gamma$ amplitude, reasonably near the data [7] in Eq. (5). Therefore, whether we employ quark loops or instead π and K loops as in Eq. (14), it is clear that the $f_0(980) \to 2\gamma$ amplitude can only be understood, if the $f_0(980)$ is mostly $s\bar{s}$.[4] This is the same conclusion as obtained, more easily, from the weak decay $D_s^+ \to \pi^+ f_0(980)$ in Eq. (1).

Similar conclusions for the flavor content of the $f_0(980)$ can be found in Refs. [24,25]. Furthermore, in Ref. [26] two possibilities are indicated, either dominantly $s\bar{s}$, or flavor octet, which is dominantly $s\bar{s}$ as well.

[4]Surely, the error bars of the presented analysis rely strongly on the assumption that we choose a sharp σ-meson mass $m_\sigma = 600$ MeV, without any uncertainty.

B. The decay $f_0(1370) \to 2\gamma$

Now we study the process $f_0(1370) \to 2\gamma$, using the same techniques as above. In the meson listings of the Particle Data Group [7], two values are given for the two-photon partial width of the $f_0(1370)$, i.e., (3.8 ± 1.5) keV and (5.4 ± 2.3) keV, from Refs. [27] and [28], respectively. In these analyses, the 2γ coupling is determined from the S-wave $\gamma\gamma \to \pi\pi$ cross section in the energy region under the $f_2(1270)$. However, the peaking of this cross section above 1 GeV is explained by the authors as a consequence of a low-mass-scalar suppression due to gauge invariance (see also Ref. [16]), pushing the corresponding distribution towards the high-mass end of the $f_0(400–1200)$, rather than as a signal of the $f_0(1370)$. For the purpose of our present study, we abide by the current PDG interpretation favoring the $f_0(1370)$, but keeping in mind that the experimental situation is anything but settled. Furthermore, we average the two data on the two-photon partial width, providing us with a, albeit preliminary, theoretical value of (4.6 ± 2.8) keV, with the amplitude given by [using $m_{f_0(1370)} = (1370 \pm 170)$ MeV]

$$\Gamma(f_0 \to 2\gamma) = \frac{m^3_{f^0}|M|^2}{64\pi} \quad \text{or} \tag{17}$$

$$|M(f_0(1370) \to 2\gamma)| = (1.90 \pm 0.68) \times 10^{-2} \text{ GeV}^{-1}.$$

In order to apply again a meson-loop approach, we develop once more a meson-mixing scheme, namely,

$$\begin{aligned} |f_0(1370)\rangle &= \cos\phi'_s|n\bar{n}\rangle - \sin\phi'_s|s\bar{s}\rangle \\ |f_0(1500)\rangle &= \sin\phi'_s|n\bar{n}\rangle + \cos\phi'_s|s\bar{s}\rangle. \end{aligned} \tag{18}$$

Again we define, using quadratic mass mixing with respect to the states $|n\bar{n}\rangle$ and $|s\bar{s}\rangle$, the nonstrange and strange mass parameters m'_{nn} and m'_{ss} by

$$\begin{aligned} m'^2_{nn} &= \cos^2\phi'_s m^2_{f_0(1370)} + \sin^2\phi'_s m^2_{f_0(1500)}, \\ m'^2_{ss} &= \sin^2\phi'_s m^2_{f_0(1370)} + \cos^2\phi'_s m^2_{f_0(1500)}. \end{aligned} \tag{19}$$

Consequently, we use the couplings

$$
\begin{aligned}
g'_{nn\pi\pi} &= d^{-}_{nn\pi^+\pi^-}\frac{m'^2_{nn}-m^2_{\pi^\pm}}{2F_\pi} = \frac{\cos^2\phi'_s m^2_{f_0(1370)}+\sin^2\phi'_s m^2_{f_0(1500)}-m^2_{\pi^\pm}}{2F_\pi} \\
&= (10.05\pm 2.53)\ \text{GeV for } \phi'_s = 0° \\
&= (10.24\pm 2.29)\ \text{GeV for } \phi'_s \simeq \pm(18°\pm 2°), \\
g'_{nnKK} &= d^{-}_{nnK^+K^-}\frac{m'^2_{nn}-m^2_{K^\pm}}{F_K} = \frac{\cos^2\phi'_s m^2_{f_0(1370)}+\sin^2\phi'_s m^2_{f_0(1500)}-m^2_{K^\pm}}{2F_K} \\
&= (7.23\pm 2.07)\ \text{GeV for } \phi'_s = 0° \\
&= (7.38\pm 1.87)\ \text{GeV for } \phi'_s \simeq \pm(18°\pm 2°), \qquad (20)\\
g'_{ss\pi\pi} &= d^{-}_{ss\pi^+\pi^-}\frac{m'^2_{ss}-m^2_{\pi^\pm}}{2F_\pi} = 0, \\
g'_{ssKK} &= d^{-}_{ssK^+K^-}\frac{m'^2_{ss}-m^2_{K^\pm}}{F_K} = \frac{\sin^2\phi'_s m^2_{f_0(1370)}+\cos^2\phi'_s m^2_{f_0(1500)}-m^2_{K^\pm}}{\sqrt{2}F_K} \\
&= (12.56\pm 0.23)\ \text{GeV for } \phi'_s = 0° \\
&= (12.33\pm 0.35)\ \text{GeV for } \phi'_s \simeq \pm(18°\pm 2°),
\end{aligned}
$$

yielding, respectively,

$$
\begin{aligned}
(\cos\phi'_s g'_{nn\pi\pi}-\sin\phi'_s g'_{ss\pi\pi}) &= (10.05\pm 2.53)\ \text{GeV for } \phi'_s = 0° \\
&= (9.74\pm 2.17)\ \text{GeV for } \phi'_s \simeq +(18°\pm 2°) \\
&= (9.74\pm 2.17)\ \text{GeV for } \phi'_s \simeq -(18°\pm 2°), \qquad (21)\\
(\cos\phi'_s g'_{nnKK}-\sin\phi'_s g'_{ssKK}) &= (7.23\pm 2.07)\ \text{GeV for } \phi'_s = 0° \\
&= (3.21\pm 1.75)\ \text{GeV for } \phi'_s \simeq +(18°\pm 2°) \\
&= (10.83\pm 1.90)\ \text{GeV for } \phi'_s \simeq -(18°\pm 2°)
\end{aligned}
$$

to determine the pion- and kaon-loop amplitudes

$$
\begin{aligned}
M_{\pi\ \text{loop}} &= \frac{2\alpha(\cos\phi'_s g'_{nn\pi\pi}-\sin\phi'_s g'_{ss\pi\pi})}{\pi m^2_{f_0(1370)}}\left[-\frac{1}{2}+\xi_\pi I(\xi_\pi)\right], \qquad (22)\\
M_{K\ \text{loop}} &= \frac{2\alpha(\cos\phi'_s g'_{nnKK}-\sin\phi'_s g'_{ssKK})}{\pi m^2_{f_0(1370)}}\left[-\frac{1}{2}+\xi_K I(\xi_K)\right].
\end{aligned}
$$

Using $\xi_\pi = m^2_{\pi^+}/m^2_{f_0(1370)} = 0.0104\pm 0.0026 < 1/4$ and $\xi_K = m^2_{K^+}/m^2_{f_0(1370)} = 0.1299\pm 0.0323 < 1/4$, we obtain [17] (see also pages 230 and 422 in Ref. [23])

$$
\begin{aligned}
I(\xi_\pi) &= \frac{\pi^2}{2} - 2\ln^2\left[\sqrt{\frac{1}{4\xi_\pi}}+\sqrt{\frac{1}{4\xi_\pi}-1}\right] + 2\pi i \ln\left[\sqrt{\frac{1}{4\xi_\pi}}+\sqrt{\frac{1}{4\xi_\pi}-1}\right] = -5.40\pm 1.16 + i(14.28\pm 0.80), \qquad (23)\\
I(\xi_K) &= \frac{\pi^2}{2} - 2\ln^2\left[\sqrt{\frac{1}{4\xi_K}}+\sqrt{\frac{1}{4\xi_K}-1}\right] + 2\pi i \ln\left[\sqrt{\frac{1}{4\xi_K}}+\sqrt{\frac{1}{4\xi_K}-1}\right] = 3.48\pm 0.62 + i(5.37\pm 1.13),
\end{aligned}
$$

yielding, respectively,

$$-\tfrac{1}{2}+\xi_\pi I(\xi_\pi)=-0.556\pm0.002+i(0.148\pm0.029), \tag{24}$$

$$-\tfrac{1}{2}+\xi_K I(\xi_K)=-0.049\pm0.192+i(0.697\pm0.027).$$

Combining all the previous results, we arrive at

$$\begin{aligned}
M_{\pi\ \mathrm{loop}}&=[-1.383\pm0.008+i(0.369\pm0.071)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'=0^\circ\\
&=[-1.341\pm0.037+i(0.357\pm0.070)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq+(18^\circ\pm2^\circ)\\
&=[-1.341\pm0.037+i(0.357\pm0.070)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq-(18^\circ\pm2^\circ),\\
M_{K\ \mathrm{loop}}&=[-0.087\pm0.343+i(1.247\pm0.068)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'=0^\circ\\
&=[-0.039\pm0.153+i(0.554\pm0.173)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq+(18^\circ\pm2^\circ)\\
&=[-0.131\pm0.514+i(1.869\pm0.173)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq-(18^\circ\pm2^\circ),\\
M_{\pi\ \mathrm{loop}}+M_{K\ \mathrm{loop}}&=[-1.470\pm0.343+i(1.615\pm0.099)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'=0^\circ\\
&=[-1.379\pm0.157+i(0.912\pm0.187)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq+(18^\circ\pm2^\circ)\\
&=[-1.471\pm0.515+i(2.226\pm0.186)]\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq-(18^\circ\pm2^\circ),\\
|M_{\pi\ \mathrm{loop}}+M_{K\ \mathrm{loop}}|&=(2.184\pm0.242)\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'=0^\circ\\
&=(1.653\pm0.167)\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq+(18^\circ\pm2^\circ)\\
&=(2.668\pm0.324)\times10^{-2}\ \ \mathrm{GeV}^{-1}\ \ \text{for}\ \ \phi_s'\simeq-(18^\circ\pm2^\circ).
\end{aligned} \tag{25}$$

If we again reduce the kaon-loop amplitude by 10% owing to the $\kappa(900)$, and assume for the moment that the $f_0(1370)$ is purely $\bar{n}n$, we get for the modulus of the decay amplitude the value $(2.09\pm0.25)\times10^{-2}$ GeV^{-1}, in good agreement with the experimental result in Eq. (17). Taking instead a mixing angle of $\phi_s'=18^\circ\pm2^\circ$ produces an amplitude value of $(1.62\pm0.17)\times10^{-2}$ GeV^{-1}, also well within the experimental error bars. On the other hand, choosing a negative mixing angle of $\phi_s'=-18^\circ\pm2^\circ$ gives rise to a somewhat too large amplitude, albeit still compatible with the experimentally allowed range of values, namely $(2.51\pm0.34)\times10^{-2}$ GeV^{-1}. So a positive mixing angle seems to be clearly favored. Further increasing a positive ϕ_s' from $+18^\circ$ will yield smaller and smaller amplitudes, until at about 60° a minimum is reached of $\approx0.94\times10^{-2}$ GeV^{-1}, after which the amplitude increases again. For $\phi_s'>80^\circ$, there would be agreement again with experiment. However, such a large mixing angle, which would imply an almost pure $\bar{s}s$ substructure for the $f_0(1370)$, seems to be excluded by the weak processes discussed in Sec. II, as well as by hadronic decays [5].

Alternatively, if instead we try the $\bar{n}n$ u,d quark loops, the $f_0(1370)\to2\gamma$ amplitude would be [17], for $\xi\simeq m_u^2/m_{f_0(1370)}^2\simeq m_d^2/m_{f_0(1370)}^2\leqslant1/4$,

$$\begin{aligned}
M(f_0(1370)\to2\gamma)&=\sqrt{2}\mathrm{Tr}[Q^2Q_{\bar{n}n}]\frac{\alpha N_c}{\pi F_\pi}\\
&\quad\times2\xi[2+(1-4\xi)I(\xi)]\\
&=\frac{5\alpha N_c}{9\pi F_\pi}2\xi[2+(1-4\xi)I(\xi)].
\end{aligned} \tag{26}$$

For $\xi<1/4$, the values $0.053<\xi\simeq m_u^2/m_{f_0(1370)}^2\simeq m_d^2/m_{f_0(1370)}^2<0.086$ are compatible with the experimental estimate in Eq. (17), i.e., $|M(f_0(1370)\to2\gamma)|=(1.90\pm0.68)\times10^{-2}\mathrm{GeV}^{-1}$. For $m_{f_0(1370)}\simeq1370$ MeV, the allowed ranges for $\xi<1/4$ yield $315\ \mathrm{MeV}<m_u\simeq m_d<402$ MeV (see Fig. 1). Using $I(\xi)$ given in Eq. (15), we observe that for all $\xi>1/4$, which would anyhow imply unrealistically large quark masses, the quark-loop rate is not consonant with the experimental estimate. The allowed range for the constituent u, d mass is quite consistent with the $f_0(1370)$ being purely $\bar{n}n$, or with a small $\bar{s}s$ admixture, of course. On the other hand, taking the $f_0(1370)$ to be mostly $\bar{s}s$, it is almost impossible to find any reasonable quark masses and mixing angles to get agreement with experiment.

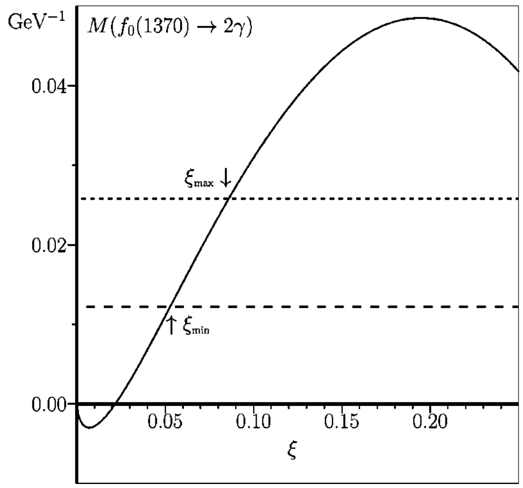

FIG. 1. Two-photon-decay amplitude of the $f_0(1370)$ determined by u,d quark loops. Here, $\xi = m_n^2/m_{f0(1370)}^2$, with m_n representing the constituent nonstrange quark mass $m_u = m_d$, and ξ_{min}, ξ_{max} stand for the one-standard-deviation boundaries of the experimental estimate given in Eq. (17) for the two-photon decay rate of the $f_0(1370)$ meson. The corresponding nonstrange quark masses range from 315 to 402 MeV.

IV. CONCLUSIONS

In this paper we have studied weak and electromagnetic decay processes with isoscalar scalar mesons in the final and initial state, respectively, in order to identify the quark substructure of especially the $f_0(980)$, $f_0(1370)$, and $f_0(1500)$ resonances. Calculating the weak process $D_s^+ \to f_0 \pi^+$, which has been observed for the $f_0(980)$, $f_0(1500)$, and $f_0(1710)$, via the standard W^+-emission graph, leads to good agreement with experiment for the $f_0(980)$ and $f_0(1500)$, if these states are assumed to be mostly $\bar{s}s$. For the $f_0(1710)$, the large experimental error does not allow a definite conclusion about a possible dominant $\bar{s}s$ configuration, but a mostly $\bar{n}n$ substructure of this resonance is unlikely. As to the $f_0(1370)$, the PDG tables do not report the process $D_s^+ \to f_0(1370)\pi^+$ at all, which would exclude a mostly $\bar{s}s$ nature of this resonance. Not even the observation of the process by the E791 Collaboration seems to affect this conclusion, since $D_s^+ \to f_0(1370)\pi^+ \to K^+K^-\pi^+$ is *not* observed [15].

Regarding the electromagnetic processes, calculation of the experimentally observed two-photon decays $f_0(980) \to \gamma\gamma$ and $f_0(1370) \to \gamma\gamma$, using either quark or meson loops, leads to good agreement with the experimentally measured rates, provided that the $f_0(980)$ is assumed to be mostly $\bar{s}s$ and the $f_0(1370)$ mainly $\bar{n}n$, and taking, moreover, the controversial PDG data on the $f_0(1370)$ at face value (see the discussion in Sec. III B). While the quark-loop results depend rather sensitively on the (model-dependent) quark masses and mixing angles, especially in the case of the $f_0(980)$, the meson-loop results only depend on the $\bar{n}n$ vs $\bar{s}s$ mixing and, therefore, are more stable and reliable.

At this point we should remark that, in a strict SU(3) extension of the quark-level LSM (qlLSM) [21], which to some extent underlied our approach here, both quark *and* meson loops should be included in the two-photon decay amplitude of the $f_0(980)$, being a ground-state scalar meson. As a matter of fact, the contributions of both kinds of loops are needed for the $\sigma(600)$—in the SU(2) case—so as to get near the not-so-well known experimental two-photon width of the $f_0(400\text{–}1200)$ ([29]). However, as mentioned in the text, the quark-loop result for the $f_0(980)$ is very sensitive to the quark masses and the mixing angle due to a rate-enhancement factor of 25 for the nonstrange $\bar{q}q$ component. By a judicious but not unreasonable choice of these parameters, one can easily make the quark-loop contribution vanish, which would occur (using $g_{f_0SS} = \sqrt{2}\,2\pi/\sqrt{3}$) for, e.g., $m_{u,d} = 340$ MeV, $m_s = 490$ MeV, $\phi_s = 12.4°$, or $m_{u,d} = 300$ MeV, $m_s = 432$ MeV, $\phi_s = 18.3°$, or all kinds of intermediate values. Therefore, our conclusion on the dominantly $\bar{s}s$ nature of the $f_0(980)$ is upheld no matter which framework is used, i.e., either the rigorous SU(3) qlLSM or the more phenomenological meson-loops-only approach.

Summarizing, weak and electromagnetic processes lend quantitative evidence to a dominantly $\bar{s}s$ interpretation of the $f_0(980)$ and $f_0(1500)$, and a mostly $\bar{n}n$ assignment for the $f_0(1370)$.

ACKNOWLEDGMENTS

We wish to thank D. V. Bugg and M. R. Pennington for valuable exchanges of ideas. This work was partly supported by the Fundação para a Ciência e a Tecnologia (FCT) of the Ministério da Ciência e da Tecnologia of Portugal, under Grant No. PRAXIS XXI/BPD/20186/99 and under contract numbers POCTI/35304/FIS/2000 and CERN/P/FIS/40119/2000.

[1] M.D. Scadron, Phys. Rev. D **26**, 239 (1982).
[2] E. van Beveren, T.A. Rijken, K. Metzger, C. Dullemond, G. Rupp, and J.E. Ribeiro, Z. Phys. C **30**, 615 (1986).
[3] Eef van Beveren and George Rupp, Eur. Phys. J. C **22**, 493 (2001).
[4] Eef van Beveren and George Rupp, Eur. Phys. J. C **10**, 469 (1999).
[5] George Rupp, Eef van Beveren, and Michael D. Scadron, Phys. Rev. D **65**, 078501 (2002).
[6] Frieder Kleefeld, Eef van Beveren, and George Rupp, Nucl. Phys. **A694**, 470 (2001). Also see J.L. Lucio-Martinez, M. Napsuciale, M.D. Scadron, and V.M. Villanueva, Phys. Rev. D **61**, 034013 (2000).
[7] Particle Data Group, D.E. Groom *et al.* Eur. Phys. J. C **15**, 1 (2000).
[8] To test the parity-conserving decay formulas Eqs. (1) and (2),

we give here also the analogous parity-violating amplitudes:

$$|M(K^+\to\pi^+\pi^0)|=\frac{G_F|V_{ud}||V_{us}|}{2\sqrt{2}}F_\pi(m_{K^+}^2-m_{\pi^0}^2)$$
$$=(1.837\pm0.020)\times10^{-8}\ \text{GeV}$$
$$(\text{data}\,[7]:(1.832\pm0.007)\times10^{-8}\ \text{GeV}),$$

$$|M(D^+\to\pi^+\pi^0)|=\frac{G_F|V_{ud}||V_{cd}|}{2\sqrt{2}}F_\pi(m_{D^+}^2-m_{\pi^0}^2)$$
$$=(28.9\pm2.1)\times10^{-8}\ \text{GeV}$$
$$(\text{data}\,[7]:(38.6\pm5.4)\times10^{-8}\ \text{GeV}),$$

$$|M(D^+\to\pi^+\overline{K}^0)|=\frac{G_F|V_{ud}||V_{cs}|}{2}F_\pi(m_{D^+}^2-m_{\overline{K}^0}^2)$$
$$=(177\pm27)\times10^{-8}\ \text{GeV}$$
$$(\text{data}[7]:(136\pm6)\times10^{-8}\ \text{GeV}).$$

[9] M.D. Scadron, Phys. Rev. D **29**, 1375 (1984); Proceedings of the Workshop on the Future of High Sensitivity Charm Experiments: CHARM2000, Batavia, Illinois, 1994, edited by D. M. Kaplan and S. Kwan, pp. 407–409.

[10] Eef van Beveren, George Rupp, and Michael D. Scadron, Phys. Lett. B **495**, 300 (2000); **509**, 365(E) (2000). Note that an erroneous factor $\sqrt{2}$ appeared in the denominator of Eq. (3) of the original paper, instead of the correct factor 2. So the theoretical prediction is reduced by a factor $\sqrt{2}$, being still in good agreement with experiment (see the Erratum).

[11] Gerhard Buchalla, Andrzej J. Buras, and Markus E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[12] R. Van Royen and V.F. Weisskopf, Nuovo Cimento A **50**, 617 (1967); **51**, 583(E) (1967).

[13] A.V. Anisovich, V.V. Anisovich, and V.A. Nikonov, Eur. Phys. J. A **12**, 103 (2001).

[14] M.D. Scadron, Mod. Phys. Lett. A **7**, 669 (1992).

[15] As a matter of fact, observation of the process $D_s^+\to f_0(1370)\pi^+$ has been reported in a recent e-print by the E791 Collaboration: E791 Collaboration, Carla Göbel, hep-ex/0012009. However, the author(s) interpret(s) this as an indication of W^+-annihilation amplitudes or strong rescattering in the final state, and also notes that no $f_0(1370)$ is observed in the $D_s^+\to K^+K^-\pi^+$ final state, pointing to the $f_0(1370)$ being a non-$s\bar{s}$ particle.

[16] M.R. Pennington, talk given at PHOTON 2000, Ambleside, Lake District, England, 2000, hep-ph/0009267.

[17] A.S. Deakin, V. Elias, D.G.C. McKeon, M.D. Scadron, and A. Bramon, Mod. Phys. Lett. A **9**, 2381 (1994).

[18] R. Delbourgo, Dong-sheng Liu, and M.D. Scadron, Phys. Lett. B **446**, 332 (1999).

[19] L. Babukhadia, Ya.A. Berdnikov, A.N. Ivanov, and M.D. Scadron, Phys. Rev. D **62**, 037901 (2000).

[20] J.L. Lucio Martinez and Mendivil Napsuciale, Phys. Lett. B **454**, 365 (1999).

[21] R. Delbourgo and M.D. Scadron, Int. J. Mod. Phys. A **13**, 657 (1998).

[22] M.D. Scadron, Phys. Rev. D **29**, 2076 (1984).

[23] J.F. Donoghue, E. Golowich, and Barry R. Holstein, Cambridge Monogr. Part. Phys. Nucl. Phys. Cosmol. **2**, 1 (1992).

[24] F. De Fazio and M.R. Pennington, Phys. Lett. B **521**, 15 (2001).

[25] A. Deandrea, R. Gatto, G. Nardulli, A.D. Polosa, and N.A. Törnqvist, Phys. Lett. B **502**, 79 (2001).

[26] A.V. Anisovich, V.V. Anisovich, D.V. Bugg, and V.A. Nikonov, Phys. Lett. B **456**, 80 (1999).

[27] Mariaelena Boglione and M.R. Pennington, Eur. Phys. J. C **9**, 11 (1999).

[28] D. Morgan and M.R. Pennington, Z. Phys. C **48**, 623 (1990).

[29] M.D. Scadron, Mod. Phys. Lett. A **14**, 1273 (1999).